DEVELOPMENT OF A DIFFERENTIAL BLOCK CODING METHOD FOR APPLICATION IN MOBILE RADIO COMMUNICATION SYSTEMS USING MIMO SYSTEMS

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1. Introduction

When implementing coherent reception or precoding in multi-antenna multiple communication (Multiple Input Multiple Output – MIMO) systems [1–3], it is necessary to know the system of information about the state of the communication channel in order to compensate it. For channel estimation, together with information signals, pilot signals known at the receiving side are transmitted. The transmission of pilot signals consumes the resource of

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the system and thus hinders the efficient use of the radio frequency spectrum [4, 5]. Currently, work is underway to overcome this problem, to the resolution of which a significant contribution has been made [6–8].

If it is possible to pay attention to the Differential phase shift keying (DPSK) [9–11], then when using it, useful information is contained in the phase difference of two sequentially transmitted signals. The receiver selects useful information by comparing the phases of two adjacent received signals, which eliminates the need for the receiver to have information about the state of the communication channel. The condition to which the communication channel must meet is insignificant changes in the state of the communication channel in the transmission interval of two adjacent signals or signal matrices during space-time coding (STC). Failure to comply with this condition when transmitting one signal (signal matrix) will lead to erroneous detection of no more than two transmitted signals (signal matrices) and will not cause errors to multiply [12, 13]. The given specificity of DPSK allows its application in conditions of fast fading and with high mobility of movement of stations.

Based on the above, the development and study of transmission schemes that do not require the receiver and/or transmitter to know information about the state of the communication channel, which is the aim of this work, is relevant today. Thus, the object of research is the methods and algorithms of space-time block coding, also used in MIMO systems.

2. Methods of research

A MIMO system with \( M \) transmitting and \( N \) receiving antennas can be represented in the following general form (Fig. 1).

\[
\sum_{n=1}^{N} |x_{n,t}| = 1, \text{ at } t = 1, \ldots, T. \tag{1}
\]

The signals transmitted by the antenna \( m \) can serve as signals of the constellation L-PSK:

\[
x_s = \frac{1}{\sqrt{M}} \exp \left( \frac{2\pi ki}{L} + \frac{\pi i}{4} \right), \quad k = 0, 1, 2, \ldots, L-1,
\]

where \( k \) – \( k \)-th signal of the modulation constellation; \( L \) – the number of modulation constellation signals. The signal amplitude is divided by \( \sqrt{M} \) in view of condition (1).

Let’s use the matrix form for the analysis of the MIMO system:

\[
Y = HX + W,
\]

where \( t \) is the time index for the matrices; \( Y \) – received matrix size \( N \times T \); \( H \) – matrix of channel coefficients \( N \times M \); \( X \) – the transmitted information matrix \( M \times 1 \), and \( W \) – matrix of additive white Gaussian noise.

If the condition \( T_0 \gg T \) is satisfied, where \( T_0 \) is the coherence time (duration) and \( T \) is the symbol duration, the matrix of channel coefficients \( H \) during \( T_0 \) is relatively constant. This condition was confirmed by calculation and taken as a basis.

2.1. Coding algorithm. The transmitted sequence of bits in each of the transmission periods \( p \) is divided into groups of \( ML \) bits – \( c_p = (c_{p1}, c_{p2}, \ldots, c_{pm}) \). Then, according to the state table of the encoder compiled earlier (to be presented later), based on the combinations and previous transmitted values of the signals of the constellation L-PSK \( x_{i,p} = (x_{1,i}, x_{2,i}, \ldots, x_{M,i}) \) (belong to the constellation L-PSK) are calculated according to the formula – differential encoding rule:

\[
x_p = r_p X_{i,p+1}^T \tag{2.1}
\]

where \( r_p \) – the vector of complex differential coefficients \( r_p = (r_{p1}, r_{p2}, \ldots, r_{pm}) \); \( X_{i,p} \) – the complex matrix composed of signals \( x_{i,p} = (x_{1,i}, x_{2,i}, \ldots, x_{M,i}) \) and satisfying the orthogonality condition:

\[
X^H X = \sum_{n=1}^{M} |x_n|^2 I_N
\]

where \( X^H \) the Hermitian conjugation of the matrix \( X \). In this case, the transmit diversity order will be equal to the number of transmitting antennas \( M \).

Possible values of differential coefficients \( R_{p1}, R_{p2}, \ldots, R_{pm} \) form a constellation \( R \), and their sets \( (R_1, R_2, \ldots, R_M) \), each of which corresponds to a combination of input bits \( c_{p1}, c_{p2}, \ldots, c_{pm} \), form a set \( R_d \). The number of these sets is equal to the number of encoder states \( J = L^M \), which is determined based on their possible combinations of input bits, the number of transmitting antennas \( M \), and modulation positionality PSK (Fig. 2).

Thus, differential signals \( x_p = (x_{1,p}, x_{2,p}, \ldots, x_{M,p}) \) contain transmitted bit information \( c_p = (c_{p1}, c_{p2}, \ldots, c_{pm}) \). Here is an example of the encoder state table for the case \( M=2 \), QPSK modulation, and signal values \( x_1 = 0.5 + 0.5j, x_2 = 0.5 + 0.5j \) (Table 1).
Fig. 2. The structural diagram of the encoder of differential space-time block coding (DSTBC)

![Diagram of DSTBC encoder](image)

**Table 1**

<table>
<thead>
<tr>
<th>Encoder state table</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j$</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
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<td>11</td>
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<tr>
<td>12</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>16</td>
</tr>
</tbody>
</table>

Thus, there is a one-to-one correspondence between $x_{p-1}$, combination of information bits $c_p$, differential coefficients $r_p$ and $x_p$. When compiling the state table of the encoder, it is possible to use the expression to find:

$$r_p = x_p X_{p-1}^{'},$$

where $X_{p-1}^{'}$ is the complex conjugation of the matrix $X_{p-1}$.

Let’s consider the example at $M=2$, using the complex orthogonal Alamouti form [14]. In this case, the signal transmission will be carried out according to the Table 2.

**Table 2**

<table>
<thead>
<tr>
<th>No.</th>
<th>$t$</th>
<th>$t+1$</th>
<th>$t+2$</th>
<th>$t+3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna 1</td>
<td>$x_1$</td>
<td>$-x_2$</td>
<td>$x_3$</td>
<td>$-x_4$</td>
</tr>
<tr>
<td>Antenna 2</td>
<td>$x_2$</td>
<td>$x_1$</td>
<td>$x_3$</td>
<td>$x_4$</td>
</tr>
</tbody>
</table>

The signals $x_1$ and $x_2$ can take any values $x_1$ of the constellation L-PSK and are transmitted respectively by the first and second antenna at time $t$, and the signals $-x_2$ and $x_1$ at time $t+1$. The matrix transmitted in this way:

$$X_i = \begin{bmatrix} x_i & -x_i' \\ x_i' & x_i \end{bmatrix}$$

carries information of a block $M_l$ of information bits. After transmitting the matrix $X_i$, the next block of $M_l$ bits enters the encoder and the encoding then occurs according to the specified algorithm. The coding rate is 1.

For the case $M=4$, it is possible to use the real orthogonal form [15], which involves the use of modulations of the BPSK or ASK type. Also in [15], a method for constructing real orthogonal forms for an arbitrary number of transmitting antennas with a code rate of 1 and complex orthogonal forms with a code rate of 1/2 is described.

Next, let’s consider the coding implementation, which is carried out according to the following tree-like algorithm (for the case $M=2$ and QPSK modulation) (Fig. 3).

![Diagram of tree-like algorithm](image)

Example: If the encoder received 4 bits $-0101$, then the sum of the branch weights is $0+4+0+1=5$. Adding a unit to the sum, get $j=6$ and, therefore, according to Table 1: $R_1 = -j$, $R_2 = 0$.

2.2. Decoding algorithm. As can be seen from Fig. 1, a set of signals emitted from all transmit antennas is induced on each of the receiving antennas. It is assumed that all
propagation channels are uncorrelated and, therefore, maximum diversity is achieved.

Let’s consider the case in which \( M=2 \) and \( N=1 \). The signals received by the receiving antenna at the appropriate time points can be recorded as:

\[
\begin{align*}
y_1 &= h_1 x_1 + h_2 x_2 + w_i, \\
y_{11} &= -h_1 x_1^2 + h_2 x_2^2 + w_{i1}, \\
y_{12} &= h_1 x_1 + h_2 x_2 + w_{i2}, \\
y_{13} &= -h_1 x_1^2 + h_2 x_2^2 + w_{i3}. \\
\end{align*}
\]

In matrix form:

\[
\begin{bmatrix}
y_1 \\
y_{11} \\
y_{12} \\
y_{13}
\end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_1^2 & x_2^2 \\ x_1 & x_2 \\ x_1^2 & x_2^2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} w_i \\ w_{i1} \\ w_{i2} \\ w_{i3} \end{bmatrix}.
\]

Then the restored values of the differential coefficients are defined as:

\[
\begin{bmatrix} \hat{R}_1 \\ \hat{R}_2 \end{bmatrix} = \begin{bmatrix} y_1 - y_{11} \\ y_{12} - y_{13} \end{bmatrix} \begin{bmatrix} y_1 + y_{11} \\ y_{12} + y_{13} \end{bmatrix}^{-1}.
\]

The block diagram of the DSTBC decoder is shown in Fig. 4.

![Fig. 4. The structural diagram of the differential spatial-temporal block coding (DSTBC) decoder](image)

Further, let’s agree in the notation: \( y_i \) – the upper index determines the time instant, and the lower – the number of the receiving antenna; \( Y^{(j)} \) – the upper superscript in parentheses \( (j) \) determines the column number of the matrix \( Y \).

At \( M=2 \) and \( N=2 \):

\[
\begin{align*}
y_{11}^{(2)} & = Y_1 Y_2 = X_1 H + W_{11}, \\
y_{12}^{(2)} & = \begin{bmatrix} w_{11}^{(2)} \\ w_{12}^{(2)} \end{bmatrix} X_2 H + W_{12}, \\
y_{13}^{(2)} & = X_1 H + W_{13}, \\
y_{14}^{(2)} & = X_2 H + W_{14}.
\end{align*}
\]

In this case, the restored values of the differential coefficients are defined as:

\[
\begin{bmatrix} \hat{R}_1 \\ \hat{R}_2 \end{bmatrix} = \begin{bmatrix} (y_{11}^{(2)}) - y_{12}^{(2)} & (y_{13}^{(2)}) - y_{14}^{(2)} \\ (y_{12}^{(2)}) & (y_{13}^{(2)}) \end{bmatrix}^{-1} \begin{bmatrix} y_{11}^{(2)} \\ y_{12}^{(2)} \end{bmatrix}.
\]

In a similar way, the values of differential coefficients are calculated for the case \( M=4 \).

After the reconstructed values of the differential coefficients are obtained, the receiver, by evaluating the maximum likelihood (ML), selects the closest vector \( \left( R_1, R_2, \ldots, R_M \right) \) from the set of their combinations \( R_{\text{all}} \) represented by the encoder status table. After that, the transmitted sequence of bits \( c_j \), corresponding to the state of encoder \( j \) is determined:

\[
\left( c_1, c_2, \ldots, c_M \right)_j = \arg \min_{\sum_{p=1}^M R_p} \left( R_1, R_2, \ldots, R_M \right) - \left( R_1, R_2, \ldots, R_M \right)^T.
\]

Next, let’s describe an algorithm for compensating noise components. Let’s draw attention to the fact that after receiving \( Y_1 \) and \( Y_2 \) becomes possible to estimate the values of the channel matrix (provided that the decoding \( c_j \) is valid and \( X_i \neq X_j \)):

\[
H = \left( Y_1 - X_1 \right) \left( Y_2 - X_2 \right)^T.
\]

therefore, it is possible to restore the values of the noise matrix \( W_1 \) when receiving \( Y_1 \) and \( W_2 \), when receiving \( Y_2 \):

\[
\begin{align*}
W_1 &= Y_1 - X_1 \left( Y_1 - X_1 \right)^T, \\
W_2 &= Y_2 - X_2 \left( Y_2 - X_2 \right)^T.
\end{align*}
\]

But, since the decoding of the bits \( c_j \), transmitted by the matrices \( X_1 \) and \( X_2 \) has already occurred, it is possible, having determined the values of the noise matrix \( W_1 \), subtract them from \( Y_1 \) and use the obtained one (that is, the vector \( Y_2 \) without taking into account the influence of noise) and the next received vector \( Y_3 \) to restore the differential coefficients \( R_1, R_2, \ldots, R_M \) and decode the subsequent transmitted bits \( c_j \). Thus, the condition – two signals (vectors) \( Y_3 \) and \( Y_{3i} \) are involved in decoding and, upon which interference and noise of the communication channel are superimposed – changes, and, already in this situation, when decoding, only one vector of received signals will be affected by noise, and from the second it influence will be removed. The described algorithm makes it possible to approximate the DSTBC method to the methods of coherent reception in efficiency, despite the fact that these methods use pilot signals that do not contain useful user information, thereby being redundant and do not allow efficient use of the radio frequency spectrum.

### 3. Research results and discussion

The simulation was performed in the MATLAB software package for a different number of receiving and transmitting antennas. The simulation results are presented in Fig. 5 (for \( M=2 \)) as dependences of the probability of error of received symbols (BER) on the signal-to-noise ratio in the system (SNR) and are given for BPSK, QPSK, and 8-PSK modulations using Alamouti STC [14], a typical scheme of the described differential STC [16, 17] and the proposed DSTBC. The simulation was carried out using the Rayleigh fading channel, taking into account the condition for relatively constant values of channel coefficients \( h_{n, o} \) during the coherence time \( T_c \).
The use of the DSTBC method for an arbitrary number of transmitting antennas is limited by the need for the availability of appropriate complex orthogonal forms. In this work, for $M=2$, the complex orthogonal Alamouti form [14] (code rate = 1) is used, for $M=4$, the real and complex orthogonal forms [15] (code rate = 1 and 1/2).

It should be noted that an increase in the positionality of phase modulation or the number of transmitting antennas leads to an exponential increase in the computational complexity of decoding, since the number of states of the encoding/decoding table is $J = LM$. In view of this, it is advisable to use spherical decoding methods when decoding.

From the graphs in the Fig. 5 it is possible to show that in terms of efficiency with equal numbers of spatial channels of the Alamouti STC, it is almost comparable to the proposed DSTBC (when the noise compensation algorithm is turned on) and they both outperform a typical differential STC by 3 dB on average. This is a definite result. This can be said about comparing the noise immunity of coherent Alamouti STC and incoherent DSTBC. It should also be emphasized that the Alamouti PVC was modeled taking into account reliable knowledge of the state of the communication channel, which is not achievable in all cases. It is also possible to note that in the STC implementation, as already mentioned, channel estimation is necessary with the help of pilot signals that consume system resources and impede the efficient use of the radio frequency spectrum, while there is no such need for the DSTBC.

It should also be noted about the effectiveness of the DSTBC without the compensation algorithm for the noise components of the communication channel (noise immunity curves with the noise reduction algorithm disabled). This is due to the cumulative increase in the distance between the nearest states determined by the points of

Fig. 5. Interference immunity curves for the cases under consideration: $a$ – BPSK; $b$ – QPSK; $c$ – 8-PSK; $d$ – QPSK with the disabled noise reduction algorithm.
the signal constellation of differential coefficients $R$, compared with a similar distance of the L-PSK constellation.

4. Conclusions

The DSTBC method proposed in the work relates to incoherent methods. It is based on the DPSK principle, which allows there to be no need for information about the state of the communication channel at the receiving side. The method also contains the developed algorithms for tree coding and compensation of the noise components of the communication channel. This, accordingly, makes it possible to optimize the computational load of the system implementation and bring the proposed differential method closer to the coherent reception methods in terms of noise immunity.

Due to obvious advantages, the described method can find application in modern radio communication systems with rapidly changing communication channel parameters due to the high speed of movement of mobile stations.

References


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