IMPROVEMENT OF IMAGES BY USING GRADUATE TRANSFORMATIONS OF THEIR FOURIER DECIPTIONS

The object of research is low-quality digital images. The presented work is devoted to the problem of digital processing of low quality images, which is one of the most important tasks of data science in the field of extracting useful information from a large data set. It is proposed to carry out the process of image enhancement by means of tonal processing of their Fourier images. The basis for this approach is the fact that Fourier images are described by brightness values in a wide range of values, which can be significantly reduced by gradation transformations. The work carried out the Fourier transform of the image with the separation of the amplitude and phase. The important role of the phase in the process of forming the image obtained after the implementation of the inverse Fourier transform is shown. Although the information about the signal amplitude is lost during the phase analysis, nevertheless all the main details correspond accurately to the initial image. This suggests that when modifying the Fourier spectra of images, it is necessary to take into account the effect on both the amplitude and the phase of the object under study. The effectiveness of the proposed method is demonstrated by the example of space images of the Earth’s surface. It is shown that after the gradation logarithmic Fourier transform of the image and the inverse Fourier transform, an image is obtained that is more contrasting than the original one, will certainly facilitate the work with it in the process of visual analysis. To explain the results obtained, the schedule of the obtained gradation transformation into the Mercator series was carried out. It is shown that the resulting image consists of two parts. The first of them corresponds to the reproduction of the original image obtained by the inverse Fourier transform, and the second performs smoothing of its brightness, similar to the action of the combined method of spatial image enhancement. When using the proposed method, preprocessing is also necessary, which, as a rule, includes operations necessary for centering the Fourier image, as well as converting the original data into floating point format.

Keywords: digital image processing, gradation transformations, discrete Fourier transform, satellite images.

1. Introduction

The presented work is devoted to the problem of digital image processing, it is one of the most relevant branches of data science in the field of extracting useful information from a large array. Currently, there are a number of powerful computer computational methods [1, 2], which, however, do not always take into account all the features of images, and, as a rule, their use is rather laborious. The different approaches to image enhancement are divided into two ways. Namely in the spatial and frequency domain. The term «spatial domain» refers to the use of an image plane. It combines approaches that have a direct impact on image pixels [3, 4]. Frequency domain processing methods are based on signal modification by using Fourier transforms [5, 6]. At present, there is no general theory that would make it possible to improve images, since ultimately the result is assessed visually [7, 8]. The task is somewhat simplified if image processing is carried out for machine perception [9, 10]. This, in particular, concerns the problems of character and pattern recognition [11, 12]. Nevertheless, even in a situation where the problem allows to establish clear criteria for quality, it still takes a certain number of testing attempts to select a specific approach to improve images. Therefore, the presented work, devoted to the development of a new method for improving images, is to modify their Fourier images by means of gradation transformations is relevant. Thus, low-quality digital images were chosen as the object of research. And the aim of research is to improve images using the gradation transformations of their Fourier images.

2. Methods of research

Gradation transforms are traditionally used to directly enhance images. They are among the simplest of all methods, but in many cases they have shown their high efficiency [3, 4]. The pixel values before and after processing will be denoted by the symbols r and s, respectively. These values are related by the expression:
where \( T \) is the transform that reflects the \( r \) pixel value to the \( s \) pixel value. Since we are dealing with discrete (quantum) quantities, the value of the transformation function, as a rule, is stored in a one-dimensional array, and the mapping from \( r \) to \( s \) is carried out according to the table.

The most common gradation transformations include: image-to-negative transformations, power transformations, and logarithmic transformations.

The general view of the logarithmic transformation, which is used in this work, is expressed by the formula:

\[
S = c \log(1+r),
\]

where \( c \) is a constant; it is assumed that \( r \geq 0 \). This transformation reflects a narrow range of low brightness values in the original image to a wider range of final values. For large values of the input signal, the opposite is true. This type of transform is used to stretch the range of dark pixel values in an image while compressing the range of bright pixel values.

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Fig. 1. Illustration of the logarithmic transformation:

\( a \) – Fourier spectrum; \( b \) – the result of applying the logarithmic transformation according to the formula (3) with \( c = 1 \) [3]

From Fig. 1 it can be seen that in the second case the number of visible details increases significantly in comparison with the initial spectrum.

In this work, for image processing, a two-dimensional discrete Fourier transform (DFT) is used, which is described by the expression [3, 4, 10]:

\[
F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-2\pi i (ux/M+vy/N)},
\]

for \( x = 0, 1, 2, ..., M-1 \) and \( y = 0, 1, 2, ..., N-1 \). Expressions (3), (4) constitute a pair of two-dimensional discrete Fourier transforms. Since the 2D DFT is complex, it can be expressed in polar coordinates:

\[
F(u,v) = |F(u,v)| e^{i\phi(u,v)},
\]

where the amplitude:

\[
|F(u,v)| = \left[ R^2(u,v) + I^2(u,v) \right]^{1/2},
\]

is called the Fourier spectrum (or frequency spectrum), and:

\[
\phi(u,v) = \arctg \left( \frac{I(u,v)}{R(u,v)} \right),
\]

called a phase. Here \( R \) and \( I \) are the real and imaginary parts of the function \( F(u,v) \).

The calculations were carried out using the object-oriented language C#, since it has all the necessary applications, as well as software components in the form of stand-alone packages that implement individual functionality [13].

3. Research results and discussion

Fig. 2 shows the results of image processing (a) using the direct discrete Fourier transform with the separation of the amplitude (b) and phase (c), as well as the result of the inverse transform (d). Test photo is taken from [3].

Fig. 2. Results of processing:

\( a \) – original image; \( b \) – processed using the direct discrete Fourier transform with the separation of the amplitude; \( c \) – with the separation of the phase; \( d \) – the result of the inverse transformation

It should be noted that in Fig. 2, \( b, c, d \), there are no details that would help the visual analysis to associate them with the features of the original image.

The importance of phase in specifying the shape of the output signal becomes apparent from Fig. 3, \( a \) obtained by
calculating the inverse DFT using only phase information. Although the luminance information is lost, all the main details in this image match the original unmistakably.

![Fig. 3. The result of calculating the inverse discrete Fourier transform using exclusively: a – phase information, b – amplitude information](image)

In Fig. 3, b, the image contains only bright information in which the constant component dominates. There is no information about the form, since the phase was nullified here. Let’s note that Fig. 2, d, obtained by calculating the inverse DFT using the spectrum of the amplitude (Fig. 2, b) and phase (Fig. 2, c) of the figure.

Fig. 4, a shows the result of filtering space images in the frequency domain using a logarithmic function, followed by a transition to a spatial image. As can be seen from Fig. 4, the Fourier image, the image of which has been subjected to tonal logarithmic transformation more clear, certainly facilitates the operator’s work in the process of further analysis.

![Fig. 4. The result of filtering space images: a – the original image, b – the result of its filtering in the frequency domain using a logarithmic function with the subsequent transition to a spatial image](image)

To explain what is shown in Fig. 4 results, logarithmic tonal Fourier transform of the image 3:

\[ S = c \log(1 + F(u,v)), \]

expand in a Mercator series [14], for \( c=1 \):

\[ S = F(u,v) - \frac{F(u,v)^2}{2}. \]

Then the inverse Fourier transform will look like:

\[ g(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \left( F(u,v) - \frac{F(u,v)^2}{2} \right) e^{2\pi i (ux/M + vy/N)}. \]

Or, opening the brackets:

\[ g(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{2\pi i (ux/M + vy/N)} - \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \frac{F(u,v)^2}{2} e^{2\pi i (ux/M + vy/N)}. \]

As can be seen from expression (11), the resulting image will consist of two parts. The first of the bottom corresponds to the reproduction of the original image obtained by the inverse Fourier transform and is equal to \( f(x,y) \). The second part, which denote as \( \delta(x,y) \), will lead to an improvement in the image.

Similar changes were previously observed when combining spatial improvement methods [3]. Thus, let’s obtain the overlay on the original image of the changes caused by the gradational logarithmic Fourier transforms of the image of this image:

\[ g(x,y) = f(x,y) - \delta(x,y). \]

Based on the results obtained, a method for improving the image can be proposed, the diagram of which is shown in Fig. 5.

As seen in Fig. 5, the proposed scheme includes the stages of preliminary and final processing. In addition to multiplying by \((-1)^{x+y}\), such processing can include cropping the original image so that its dimensions take on paired values with respect to the original. This is a prerequisite for the correct centering of the Fourier image. In addition, scaling is performed, converting the input data format to floating point and converting the output data format to 8-bit integer values. Multi-stage filtration procedures and a variety of pre- and post-processing operations are also possible.

![Fig. 5. General scheme of image enhancement using tone processing Fourier transform](image)
At the next stage, the direct discrete Fourier transform of the original image \( F(u, v) \) is calculated. The function thus obtained is subjected to tonal transformation according to formulas (8), (9) and inverse transformation (10)–(12). As a result of this operation, all pixel values \( F \) are converted into pixel values \( S \). After this stage, it is possible to calculate the inverse discrete Fourier transform (10)–(12) and select the real part of the result. Post-processing includes multiplying the result by \((-1)^{x+y}\), as well as operations similar to those used in the preprocessing stage.

4. Conclusions

In the course of the study, it was revealed that the gradation transformations of the Fourier spectra effectively improve low-quality images. The results obtained are explained by expanding the Fourier spectrum in a Mercator series. It was revealed that the resulting spatial image is described by two components. The first of them corresponds to the reproduction of the original image, and the second will lead to its improvement. On the basis of the results obtained, a scheme for improving the image using Fourier tone processing is proposed. In the process of practical use, it is possible to implement various options for the presented initial scheme. It is important to remember that the proposed filtering method is based on a certain change in the modulus and phase of the Fourier image by means of gradation transformations, followed by obtaining the processed original image.

The research results will be useful to operators involved in the analysis of satellite images of the earth’s surface, which are often characterized by low quality.

References

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Ihor Polovynko, Doctor of Physical and Mathematical Sciences, Professor, Department of Optoelectronics and Information Technology, Ivan Franko National University of Lviv, Lviv, Ukraine, ORCID: https://orcid.org/0000-0003-2810-5173

Lubomyr Kniazevich, Department of Optoelectronics and Information Technology, Ivan Franko National University of Lviv, Lviv, Ukraine, ORCID: https://orcid.org/0000-0001-5039-8350