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Roman Kvit

RELIABILITY PREDICTION OF COMPOSITE MATERIALS WITH RANDOM ELASTIC INCLUSIONS

The object of the research is the construction of an algorithm that allows finding a number of strength (failure loading) statistical characteristics of a composite material plate under the conditions of a complex stress state. The relationships that determine the most probable value, mean value, dispersion and coefficient of variation of strength for an elastic homogeneous plate in which elliptical inclusions of another elastic material are uniformly distributed are written. Inclusions do not interact with each other and their geometric parameters are statistically independent random variables whose distribution laws are written for certain physical reasons.

The combination of the known deterministic solution of the composite materials failure theory and probabilistic statistical methods that take into account the randomness of the material structure makes it possible to study the failure of composite materials taking into account the stochasticity of their structure.

The main content of this article is the construction and analysis of the strength statistical characteristics algorithm of two-component lamellar composite materials. The mechanism of composite plate's failure initiation in the inclusion is considered. The recorded relationships make it possible to calculate the most probable value, mean value, dispersion and coefficient of variation of strength and to investigate their dependence on the type of applied loading, structural heterogeneity of the composite and its dimensions (number of inclusions).

The obtained results allow effective assessment of the reliability of stochastically defective two-component composite structural materials under complex stress conditions. This is due to the fact that the combined consideration of defectiveness and randomness in the composite material structure as interconnected, inseparable phenomena open new opportunities for researching of the strength problem and failure of composite materials under various types of applied loading.

Keywords: *elastic inclusions, composite materials failure, distribution function, statistical characteristics of strength, stochasticity of the structure.*

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1. Introduction

Composite materials occupy an important place among structural materials. Descriptive parameters of the composite material structure are characterized by randomness and a certain probability distribution (stochasticity), which is defining aspects in assessing their strength. Taking this stochasticity into account is an urgent task for a more complete display of the composite materials strength properties. Complex application of probabilistic statistical approaches and deterministic solutions of brittle fracture mechanics allows for a more adequate assessment of composite materials reliability. The problem of composite structural materials strength reliability in the probabilistic aspect was investigated in the works of a number of authors.

The paper [1] presents a state of the review of ultimate strength prediction and reliability analysis for composite material structures with emphasis on laminated composite structures. In research [2] a numerical simulation and analytical probabilistic methods for the reliability evaluation of composite structures are considered. The author of [3] proposed a mechanical multi-scale model describing relationship between the crack-opening and composite bridging stress in brittle matrix composites with heterogeneous reinforcement. The publication [4] concerned with a statistical distributions of the critical fracture toughness values with due consideration given to the scale size effect. According to the experimental results, a probability analysis was conducted on the degradation of tensile strength [5]. Experimental investigations of the composite glass fiber materials tensile strength and the statistical analysis of the results obtained on the basis of the two-parameter Weibull distribution have been carried out in [6]. The proposed approach [7] explicitly represents damage as discrete cracks and leads to more accurate prediction of brittle and push-out failure patterns for sublaminate-scaled and ply-scaled laminates, respectively, including size and lay-up effects. In paper [8], an adaptive multi-fidelity (AMF) modelling approach is proposed, wherein actively damaging areas are modelled with high-fidelity three-dimensional (3D) brick elements and discrete cracks, while dormant and inactive sites are modelled with lower-fidelity shell elements and smeared cracks. The transition criteria between the two levels of modelling are studied in order to preserve as much fidelity to the physics as necessary while improving computational efficiency. In work [9], metamodels such as support vector machines, radial basis function, and logistic regression in conjunction with Latin hypercube, Sobol, and Halton sequence sampling methods were used to calculate the failure probability in the carbon fibre/epoxy-based composite material. In [10] was investigated the role of microstructural bridging on the fracture toughness of composite materials. To achieve this, a new computational framework is presented that integrates phase field fracture and cohesive zone models to simulate fibre breakage, matrix cracking and fibre-matrix debonding. The composite microstructure is represented by an embedded cell at the vicinity of the crack tip, whilst the rest of the sample is modeled as an anisotropic elastic solid.

Joint consideration of defectiveness and randomness when assessing the structural composite materials reliability is an urgent task.

Thus, *the object of the research* is the construction of an algorithm that allows finding a number of strength (failure loading) statistical characteristics of a composite material plate under the conditions of a complex stress state.

The aim of this research is to extend the methodology for calculating statistical strength characteristics to composite materials with elastic elliptical inclusions that are under complex stress conditions. The basis of the study is the deterministic criterion of brittle failure of the type of the law of Coulomb friction with adhesion. An analysis of the influence of the type of applied loading, the structural material heterogeneity and the dimensions of the composite material (number of inclusions) on a number of statistical strength characteristics was carried out.

2. Research methodology

Consider the algorithm for determining the strength statistical characteristics of a composite material in the form of an elastic homogeneous plate, in which elliptical inclusions from another elastic material are evenly distributed (let's assume that their number is equal to a certain number N). The inclusions do not interact with each other. The plate is under the influence of uniformly distributed forces P and $Q=\eta P$ $-\infty<\eta\leq 1$, which can be interpreted as the principal stresses in the plane stress state (Fig. 1).

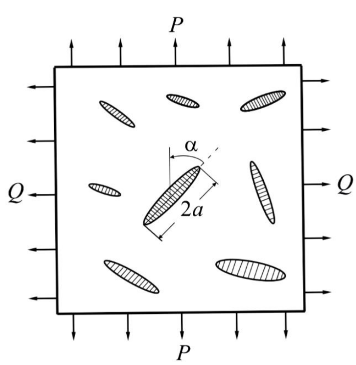

Fig. 1. Composite material research model

Geometric characteristics of inclusions are their orientation and size, which are random variables. The elastic properties of the matrix and inclusions are given. Inclusions have the form of thin flattened ellipses, are soft and significantly affect strength [11]. Let's consider a two-component material (the properties of all inclusions are the same). Such inclusions are often found in metals (graphite inclusions in cast iron, oxidized layers, etc.).

Let's introduce a random variable δ that establishes the relationship between the semi-axes of the ellipse a and b $(\delta=2b/(a+b)\approx 2b/a,\ b=a)$. Let's consider the mechanism of plate failure initiation in inclusion.

Let's set the probability distribution laws of the inclusions geometric parameters α and δ. Let's consider the plate material to be macroisotropic, for which the orientation of all inclusions is equiprobable. Then let's write the distribution density probabilities of the inclusions orientation angle α in the form of a uniform law [12]:

$$f(\alpha)=2/\pi\ (0\leq\alpha\leq\pi/2). \tag{1}$$

According to the above assumptions, let's set the range of change for a limited random variable δ: $\delta_0\leq\delta\leq\delta_1$, where $\delta_0=0$, $\delta_1=0.5$. For it, let's choose the distribution density probabilities in the form of the β-law [12]:

$$f(\delta)=2(r+1)(1-2\delta). \tag{2}$$

The parameter $r\geq 0$ characterizes the structural heterogeneity of the composite material. The probability of meeting a random variable δ decreases with its increase.

The graph of the distribution density probabilities $f(\delta)$ for different values of the parameter r is shown in Fig. 2.

The integral probability distribution function of the parameter δ will be written as follows:

$$F(\delta)=1-(1-2\delta)^{r+1}. \tag{3}$$

The graph of the integral probability distribution function $F(\delta)$ (3) for different values of the parameter δ is shown in Fig. 3.

The distribution density probabilities of the joint distribution of statistically independent random variables α and δ according to (1), (2) has the form:

$$f(\alpha,\delta)=4(r+1)(1-2\delta)/\pi. \tag{4}$$

The study of the distribution density probabilities of the joint distribution (4) was carried out in [13].

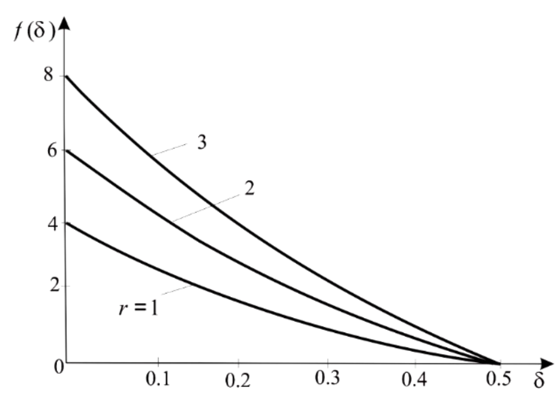

Fig. 2. The distribution density probabilities of the parameter δ

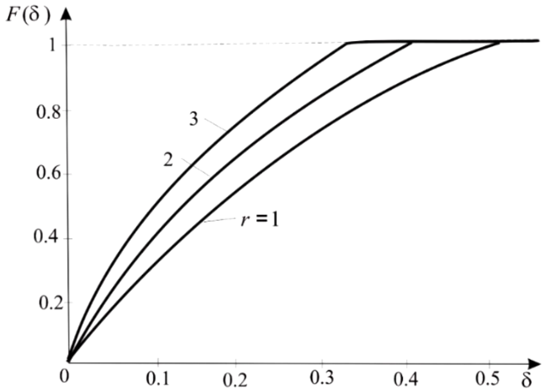

Fig. 3. The integral probability distribution function of the parameter δ

The inclusion failure criterion is taken as a type of Coulomb friction law with adhesion [14]:

$$\tau_{xy}^1 \le K^1 - \sigma_y^1 \operatorname{tg}\rho^1, \tag{5}$$

where σ_y^1, τ_{xy}^1 are the stress in inclusion; $\operatorname{tg}\rho^1$ is a coefficient of material inclusion internal friction; K^1 is a clutch coefficient.

The stress in inclusion, which inclines at an angle α to the main axis, is determined by the formulas [14]:

$$\sigma_y^1 = \frac{0.5P(\eta+1+(1-\eta)\cos 2\alpha)G_1(1+\varkappa_1)(1+\varkappa_2)}{G_1(1+\varkappa_1)(1+\varkappa_2)+2\delta G_2(\varkappa_1-1)},$$

$$\tau_{xy}^1 = \frac{0.5P(1-\eta)\sin 2\alpha\, G_1(1+\varkappa_2)}{G_1(1+\varkappa_2)+2\delta G_2}, \tag{6}$$

where $\varkappa_1$, $\varkappa_2$ are the elastic constants, which are expressed in terms of Poisson's coefficient ν ($\varkappa = \frac{3-\nu}{1+\nu}$ for a plane stressed state, $\varkappa = 3-4\nu$ for a plane deformation); G_1, G_2 are the shear modules ($G_1/G_2<1$).

In accordance with the failure criterion (5), (6), carrying out the replacement of the variable $p=P/K^1$ ($q=Q/K^1$) (introduce a dimensionless loading), let's get the following expression for loading calculating, at which a crack with the length $2a$ is formed:

$$p = \frac{L(D+\delta M)}{B\begin{pmatrix}\eta+1+\\+(1-\eta)\cos 2\alpha\end{pmatrix}+C(1-\eta)\sin 2\alpha}, \tag{7}$$

where

$$B=(1+\varkappa_1)\operatorname{tg}\rho^1,\quad C=\varkappa_1-1,$$

$$D=\varkappa_1(1+\varkappa_2),$$

$$L=\frac{4}{1+\varkappa_2},\quad M=\frac{CG_2}{G_1}.$$

The value of the parameter δ that corresponds to the given failure loading p ($p_{\min}\le p\le p_{\max}$) is determined from (7):

$$\delta(p,\eta,\alpha) = \frac{p(B(\eta+1+(1-\eta)\cos 2\alpha)+C(1-\eta)\sin 2\alpha)-LD}{LM}. \tag{8}$$

The change of the parameter $\delta(p,\eta,\alpha)$ occurs depending on the orientation of inclusion angle α and the ratio of the applied loading $\eta=q/p$.

3. Research results and discussion

3.1. The integral probability distribution function of strength. Expressions of the integral probability distribution function of strength (failure loading) under such types of applied loading have the form [13]:

1) biaxial tension ($0\le\eta\le1$):

$$F_1(p,\eta) = \begin{cases} F(p,\eta,\alpha_1,\alpha_2),\ p_{\min}\le p\le p_1\ (\eta\ne 0); \\ F(p,\eta,0,\alpha_1)+F(p,\eta,\pi/4,\alpha_3),\ p_1\le p\le p_2\ (\eta\ne 1); \\ F(p,\eta,\alpha_3,\pi/2),\ p_2\le p< p_{\max}\ (\eta\ne 0); \end{cases} \tag{9}$$

2) tension-compression ($-\infty\le\eta<0$):

$$F_1(p,\eta) = \begin{cases} F(p,\eta,\alpha_1,\alpha_2),\ p_{\min}\le p\le p_1; \\ F(p,\eta,0,\alpha_1)+F(p,\eta,\pi/4,\alpha_3),\ p_1\le p\le p_2\ (\eta\ne -1); \\ F(p,\eta,\alpha_3,\pi/2-\alpha_{**}),\ p_2\le p<\infty(\eta\ne -1). \end{cases} \tag{10}$$

A notation is introduced here:

$$F(p,\eta,\alpha_i,\alpha_j) = \frac{2}{\pi}\int_{\alpha_i}^{\alpha_j}\left(1-2\delta(p,\eta,\alpha)\right)^{r+1}d\alpha, \tag{11}$$

where the function $\delta(p,\eta,\alpha)$ is determined from (8).

The following orientation angles corresponds to the given failure loading p ($p_{\min}\le p\le p_{\max}$) and ratio $\eta=q/p$:

$$\alpha_* = 0.5\operatorname{arctg}\frac{C}{B}; \tag{12}$$

$$\alpha_{**} = \frac{\pi}{2}-0.5\arcsin\frac{B(-C(\eta+1))+\sqrt{C^2(\eta+1)^2-4(B^2+C^2)\eta}}{(B^2+C^2)(1-\eta)}; \tag{13}$$

$$\alpha_1 = 0.5\arcsin\frac{C(T-B(\eta+1))-\sqrt{C^2(T-B(\eta+1))^2-(B^2+C^2)\begin{pmatrix}T^2+4B^2\eta-\\-2TB(\eta+1)\end{pmatrix}}}{(B^2+C^2)(1-\eta)},$$

$$T=\frac{L(D+\delta M)}{p},\ (0\le\alpha_1\le\alpha_*,\ p_{\min}\le p\le p_1); \tag{14}$$

$$\alpha_2 = 0.5\arcsin\frac{C(T-B(\eta+1))+\sqrt{C^2(T-B(\eta+1))^2-(B^2+C^2)\begin{pmatrix}T^2+4B^2\eta-\\-2TB(\eta+1)\end{pmatrix}}}{(B^2+C^2)(1-\eta)},$$

$$(\alpha_*\le\alpha_2\le\pi/4,\ p_{\min}\le p\le p_2); \tag{15}$$

$$\alpha_3=\frac{\pi}{2}-\alpha_1\left(\frac{\pi}{4}\le\alpha_3\le\frac{\pi}{2},\ p_2\le p\le p_{\max}\ \text{(biaxial tension)},\ \frac{\pi}{4}\le\alpha_3\le\frac{\pi}{2}-\alpha_{**},\ p_2\le p<\infty\ \text{(tension-compression)}\right); \tag{16}$$

$$p_{\min}=p(\alpha_*,\delta_0)=\frac{LD}{B(\eta+1)+(1-\eta)\sqrt{B^2+C^2}}; \tag{17}$$

$$p_1=p(0,\delta)=\frac{L(D+\delta M)}{2B}; \tag{18}$$

$$p_2 = p(\pi/4,\delta) = \frac{L(D+\delta M)}{B(\eta+1)+C(1-\eta)}; \qquad (19)$$

$$p_{\max} = p(\pi/2,\ \delta_1) = \frac{L(D+0.5M)}{2B\eta} \text{ (for biaxial tension)}; \qquad (20)$$

$$p_{\max} = p(\alpha_{**},\ \delta_1) = \frac{L(D+0.5M)}{2B\eta}$$

$$\text{(for tension-compression).} \qquad (21)$$

In the expressions for the loading let's introduce instead of the parameter δ its mean value [13]:

$$\langle\delta\rangle = 0.5\,(r+2)^{-1}.$$

Let's consider partial cases. According to expressions (9)–(21), the integral probability distribution function of strength will be written as follows:

1) biaxial symmetric tension $(\eta=1,\ p=q>0)$:

$$F_1(p,1) = 1-\left(1-2\frac{2pB-LD}{LM}\right)^{r+1},\ \frac{LD}{2B}\le p<\frac{L(D+0.5M)}{2B}; \qquad (22)$$

2) uniaxial tension $(\eta=0)$:

$$F_1(p,0) = \frac{2}{\pi}\int_0^{\alpha_1}\left(1-2\frac{p\begin{pmatrix}2B\cos^2\alpha+\\+C\sin 2\alpha\end{pmatrix}-LD}{LM}\right)^{r+1}d\alpha +$$

$$+\frac{2}{\pi}\int_{\pi/4}^{\alpha_3}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}d\alpha,$$

$$\frac{LD}{B+\sqrt{B^2+C^2}}\le p\le\frac{L(D+\delta M)}{B+C}; \qquad (23)$$

3) tension-compression (net shear) $(\eta=-1)$:

$$F_1(p,-1) = \frac{2}{\pi}\int_{\alpha_1}^{\alpha_2}\left(1-2\frac{2p\begin{pmatrix}B\cos 2\alpha+\\+C\sin 2\alpha\end{pmatrix}-LD}{LM}\right)^{r+1}d\alpha,$$

$$\frac{LD}{2\sqrt{B^2+C^2}}\le p\le\frac{L(D+\delta M)}{2B}. \qquad (24)$$

3.2. Distribution density probabilities of strength. The distribution density probabilities of strength for a plate with a stochastic distribution of N inclusions are determined by the relation [15]:

$$f_N(P,\eta) = N(1-F_1(P,\eta))^{N-1}\left|\frac{dF_1(P,\eta)}{dP}\right|. \qquad (25)$$

According to (22)–(25), for the specified partial cases, let's obtain:

1) biaxial symmetric tension:

$$f_N(p,1) = \frac{4NB(r+1)}{LM}(1-F_1(p,1))^{N-1}\times$$

$$\times\left(1-2\frac{2pB-LD}{LM}\right)^r,\ \frac{LD}{2B}\le p<\frac{L(D+0.5M)}{2B}; \qquad (26)$$

2) uniaxial tension:

$$f_N(p,0)=$$

$$=\frac{4N(2B\cos^2\alpha+C\sin 2\alpha)(r+1)}{\pi LM}(1-F_1(p,0))^{N-1}\times$$

$$\times\left(\int_0^{\alpha_1}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^r d\alpha +\right.$$

$$\left.+\int_{\pi/4}^{\alpha_3}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^r d\alpha\right),$$

$$\frac{LD}{B+\sqrt{B^2+C^2}}\le p\le\frac{L(D+\delta M)}{B+C}; \qquad (27)$$

3) tension-compression:

$$f_N(p,-1)=$$

$$=\frac{4N(2B\cos 2\alpha+C\sin 2\alpha)(r+1)}{\pi LM}(1-F_1(p,0))^{N-1}\times$$

$$\times\int_{\alpha_1}^{\alpha_2}\left(1-2\frac{p(2B\cos 2\alpha+C\sin 2\alpha)-LD}{LM}\right)^r d\alpha,$$

$$\frac{LD}{2\sqrt{B^2+C^2}}\le p\le\frac{L(D+\delta M)}{2B}. \qquad (28)$$

According to (26)–(28), using the data from [12, 16], in Fig. 4–6 graphs of the distribution density probabilities of strength $f_N(p,\eta)$ for a plate with a stochastic distribution of N elastic inclusions under different types of stress state are plotted. The impact on the most probable value of strength (mode) $Mo(p)$ of the composite area at the same density of inclusions (parameter N) and type of stress state (parameter η) is considered.

The distributions of the strength random value will be unimodal. The strength threshold value (the minimum value of the failure loading) is not equal to zero and depends on the type of stress state.

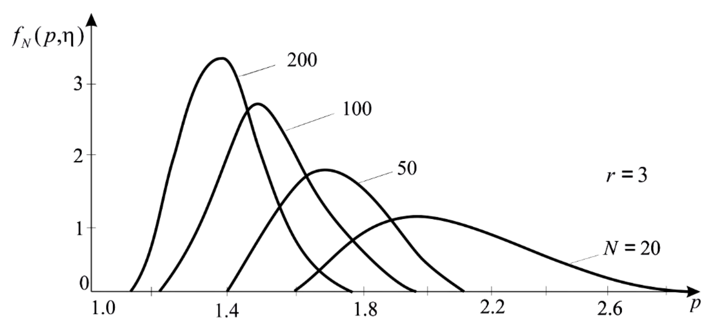

Fig. 4. Uniaxial tension

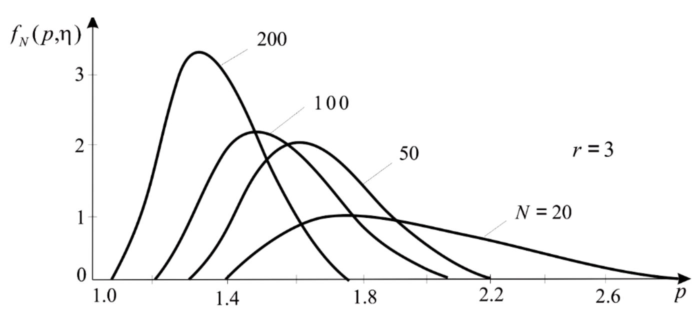

Fig. 5. Biaxial symmetric tension

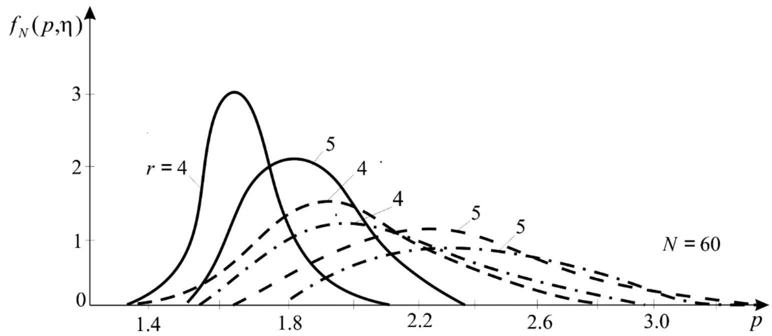

Fig. 6. Influence of the composite structure heterogeneity (solid for $\eta=1$, dashed for $\eta=-1$, dash-dotted for $\eta=0$)

3.3. Strength statistical characteristics of composite material. Let's record and conduct a study of the strength statistical characteristics of a composite material plate with a stochastic distribution of N elastic inclusions.

The most probable strength value (mode) $Mo(p)$, which corresponds to the loading level at which the distribution density probabilities of strength reaches its maximum, is determined from the equation [15]:

$$\begin{aligned}&(1-F_1(p,\eta))F_1''(p,\eta)+\\&+(1-N)(F_1'(p,\eta))^2=0.\end{aligned}\qquad(29)$$

The mean value, dispersion and coefficient variation of strength are determined by the formulas [15]:

$$\langle p\rangle=p_{\min}(\eta)+\int_{p_{\min}(\eta)}^{p_{\max}(\eta)}(1-F_1(p,\eta))^N\,\mathrm{d}p;\qquad(30)$$

$$\begin{aligned}&D(p)=p_{\min}^2(\eta)+\\&+2\int_{p_{\min}(\eta)}^{p_{\max}(\eta)}(1-F_1(p,\eta))^N p\,\mathrm{d}p-\langle p\rangle^2;\end{aligned}\qquad(31)$$

$$W(p)=\sqrt{D(p)}\big/\langle p\rangle.\qquad(32)$$

By substituting the expressions for the integral probability distribution function of strength (9), (10) into formulas (29)–(32), let's find the strength statistical characteristics of a composite material plate with a stochastic distribution of N inclusions.

In particular, for partial cases (22)–(24) there is:

1) biaxial symmetric tension:

$$\langle p\rangle=\frac{LD}{2B}+\int_{\frac{LD}{2B}}^{\frac{L(D+0.5M)}{2B}}\left(\left(1-2\frac{2pB-LD}{LM}\right)^{r+1}\right)^N\mathrm{d}p;\qquad(33)$$

$$\begin{aligned}&D(p)=\left(\frac{LD}{2B}\right)^2+\\&+2\int_{\frac{LD}{2B}}^{\frac{L(D+0.5M)}{2B}}\left(\left(1-2\frac{2pB-LD}{LM}\right)^{r+1}\right)^N p\,\mathrm{d}p-\langle p\rangle^2;\end{aligned}\qquad(34)$$

2) uniaxial tension:

$$\begin{aligned}&\langle p\rangle=\frac{LD}{B+\sqrt{B^2+C^2}}+\\&+\int_{\frac{LD}{B+\sqrt{B^2+C^2}}}^{\frac{L(D+\delta M)}{B+C}}\left(1-\frac{2}{\pi}\int_0^{\alpha_1}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}\mathrm{d}\alpha-\right.\\&\left.-\frac{2}{\pi}\int_{\pi/4}^{\alpha_3}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}d\alpha\right)^N\mathrm{d}p;\end{aligned}\qquad(35)$$

$$\begin{aligned}&D(p)=\left(\frac{LD}{B+\sqrt{B^2+C^2}}\right)^2+\\&+2\int_{\frac{LD}{B+\sqrt{B^2+C^2}}}^{\frac{L(D+\delta M)}{B+C}}\left(1-\frac{2}{\pi}\int_0^{\alpha_1}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}\mathrm{d}\alpha-\right.\\&\left.-\frac{2}{\pi}\int_{\pi/4}^{\alpha_3}\left(1-2\frac{p(2B\cos^2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}\mathrm{d}\alpha\right)^N p\mathrm{d}p-\langle p\rangle^2;\end{aligned}\qquad(36)$$

3) tension-compression:

$$\begin{aligned}&\langle p\rangle=\frac{LD}{2\sqrt{B^2+C^2}}+\\&+\int_{\frac{LD}{2\sqrt{B^2+C^2}}}^{\frac{L(D+\delta M)}{2B}}\left(1-\frac{2}{\pi}\int_{\alpha_1}^{\alpha_2}\left(1-2\frac{2p(B\cos 2\alpha+C\sin 2\alpha)-LD}{LM}\right)^{r+1}d\alpha\right)^N\mathrm{d}p;\end{aligned}\qquad(37)$$

$$\begin{aligned}&D(p)=\left(\frac{LD}{2\sqrt{B^2+C^2}}\right)^2+\\&+2\int_{\frac{LD}{2\sqrt{B^2+C^2}}}^{\frac{L(D+\delta M)}{2B}}\left(1-\frac{2}{\pi}\int_{\alpha_1}^{\alpha_2}\left(1-2\frac{2p\begin{pmatrix}B\cos 2\alpha+\\+C\sin 2\alpha\end{pmatrix}-LD}{LM}\right)^{r+1}\mathrm{d}\alpha\right)^N p\mathrm{d}p-\langle p\rangle^2.\end{aligned}\qquad(38)$$

In Fig. 7–9, in accordance with expressions (33)–(38), graphs of the specified strength probabilistic characteristics for composite materials with different numbers of inclusions under different types of stress state are plotted.

Fig. 7, shows the influence of the loading ratio and the number of inclusions on the strength mean value $\langle p\rangle$.

Fig. 8 shows the dependence of the strength dispersion on the number of inclusions and the ratio of the applied loading.

In Fig. 9, the effect of material homogeneity and the number of inclusions on the strength coefficient variation $W(p)$ under biaxial symmetric tension is investigated.

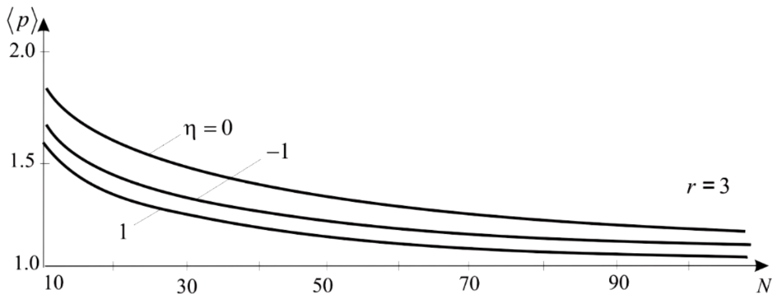

Fig. 7. Strength mean value

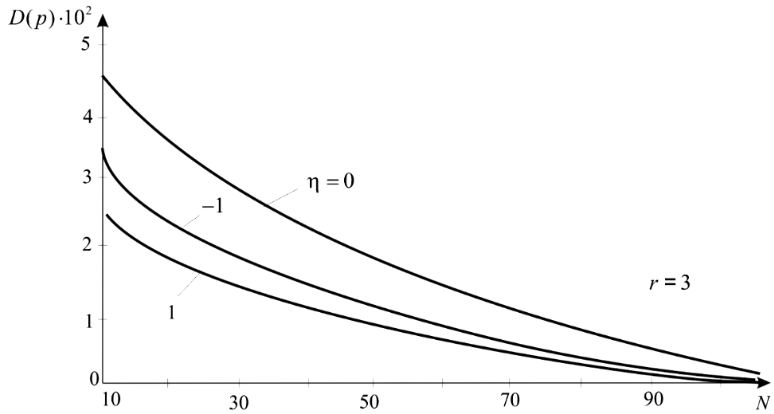

Fig. 8. Strength dispersion

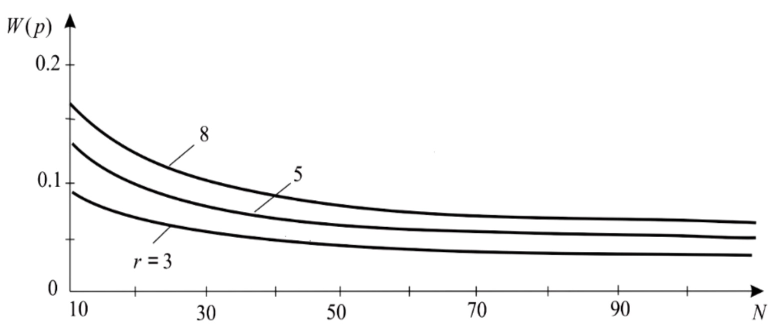

Fig. 9. Strength coefficient variation

A limitation of this study is the flat model of the composite material with elastic inclusions. Therefore, the construction of a spatial model will make it possible to make more complete use of deterministic theories of the influence of inclusions on the strength of composite materials in combination with research probabilistic statistical methods.

In this article, the algorithm for determining the strength statistical characteristics of composite material plates with stochastically distributed elastic elliptical inclusions under plane stress conditions was considered. A possible development of this study is the extended experimental approbation of the obtained theoretical results.

4. Conclusions

In the course of the study, it was shown that under uniaxial ($\eta=0$) and biaxial symmetric tension ($\eta=1$), the value of $Mo(p)$ decreases with an increase in the number of defects. The shape of the curve of the distribution density probabilities changes with a change in the parameter N and depends on the type of applied loading. The values $Mo(p)$ will be greater for uniaxial tension than for biaxial symmetric tension. The influence of the type of applied loading on $Mo(p)$ decreases with an increase in the number of defects. The maximum ordinate of the distribution curve depends on the type of stress state and is directly proportional to the parameter N.

The influence of the composite structure heterogeneity (parameter r) with the same number of defects (composite sizes) was studied. As the value r increases (the composite becomes more uniform), the value $Mo(p)$ increases and depends on the type of stress state. Its smallest value will be for biaxial symmetric tension ($\eta=1$) and the largest for uniaxial tension ($\eta=0$).

It was found that the greatest strength mean value $\langle p\rangle$ will be in the case of uniaxial tension. The intensity of the decrease in the strength mean value depends on the number of inclusions. There is a certain range of composite sizes for which the strength with asymptotic approach to its threshold value is almost independent of the number of defects. Such regularities are also observed for plates with defects-cracks [17–20].

It was established that the strength dispersion $D(p)$ is a decreasing function of the number of inclusions in the composite. At a certain interval of changes N, it is possible to see a rapid decrease in the value of $D(p)$. The nature of this decrease does not depend on the type of loading. As with the strength mean value, there is a range of composite sizes for which the strength dispersion is almost independent of the number of defects.

It is shown that the strength coefficient variation $W(p)$ decreases with an increase in the number of inclusions N and increases with an increase in the parameter r. There is a certain range of sizes of the composite, for which let's observe a significant change in value $W(p)$ and an asymptotic approach to a certain threshold value. Similar regularities can be observed for other types of stress.

Conflict of interests

The author declares that there is no conflict of interest regarding this research, including financial, personal nature, authorship or other nature that could affect the research and its results presented in this article.

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Data availability

The manuscript has no associated data.

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Roman Kvit, *PhD, Associate Professor, Department of Mathematics, Lviv Polytechnic National University, Lviv, Ukraine, ORCID: https://orcid.org/0000-0002-2232-8678, e-mail: roman.i.kvit@lpnu.ua*