1. Introduction

ARPA (automatic radar plotting aid) is used on modern ships to track targets and pass from them. ARPA is an automated system that assumes the presence of an operator in the loop of information processing and control [1–3]. Today, the operator’s interventions in control processes are quite significant and often lead to an increase in the number of accidents and disasters. Recently, specialists have been paying more and more attention to the automation of ship control processes. The most promising direction of automation is the use of automatic control modules in automated systems. In this case, the shipmaster only decides to use the automatic module and observes its operation. An example of an automatic module in an automated system is autosteering, which has been used on ships for over 100 years.

The paper considers the method of automatic optimal passing of ships in the field of risks. The method allows to minimize the path of passing, provided that the given collision risk is not exceeded. The obtained results are explained by the use of an on-board computer for the calculation of controls. In the on-board computer, at each step of the calculation, a field of risks is built. For the position point of the ship in the field of risks, there is a field gradient and a direction of movement of the ship perpendicular to the gradient. The direction of movement of the ship at each point is tangent to the trajectory of passing – an ellipse of equal risk. The ellipse of equal risks is used as a motion program for the formation of controls that ensure the movement of the ship along the ellipse of a given risk during the passing process.

The developed method can be used for the development of automatic modules for managing the passing of ships in the field of risks.

Keywords: risk field, optimal passing, gradient procedure, minimization of passing path, given risk ellipse, automatic module.
In papers [7, 8], the issue of applying the model of a multi-step matrix game for the synthesis of optimal maneuvering, which allows determining a safe game trajectory of one’s own ship in situations where it meets a large number of objects, is considered. The trajectory was described as a certain sequence of course and speed maneuvers.

The works [9–12] present the application of selected methods of the theory of optimal and game control to determine the safe trajectory of one’s own ship when passing other ships that meet in conditions of good and limited visibility at sea. Five algorithms for determining the safe trajectory of one’s own ship in a collision risk situation are compared: non-cooperative position game (NPG), non-cooperative matrix game (NMG), cooperative position game (CPG), dynamic optimization (DO) and kinematic optimization (KO). The considered control algorithms are, in a certain sense, formal models of the mental processes of a navigator who controls its own ship. The developed algorithms take into account the rules of COLREG, dynamic properties of the ship, and allow to estimate the deviation of the real trajectory from the given one.

The papers [13, 14] considered the application of selected game theory methods for the automation of marine object management processes. The parameters of the state vector, control and restrictions on the parameters of the state vector are defined, the target control function is defined in the form of payments - integral payment and final payment. Multi-stage positional and multi-step matrix, non-cooperative and cooperative, game and optimal control algorithms in a conflict situation are presented.

In works [15, 16], the issue of automatic passing with many targets, including maneuvering ones, is considered. The aim of research is to develop a method of automatic optimal passing of the ship in the field of risks. This will make it possible to automate the processes of passing, reduce the influence of the human factor on control processes, reduce the length of the passing trajectory and fuel consumption during passing, and reduce the exhaustion of the crew.

The set aim is achieved due to:
- use of an on-board computer with an automatic passing control module in the automated control system;
- constructions at each step of calculating the risk field;
- finding, for the point of the position of the ship in the field of risks, the gradient of the field and the direction of movement of the ship, perpendicular to the gradient.

The direction of movement of the ship at each point is tangent to the trajectory of passing – an ellipse of equal risk; the use of an ellipse of equal risks as a software trajectory for the formation of controls that ensure the movement of the ship along the ellipse of a given risk in the process of passing.

2. Materials and Methods

The object of research is the processes of automatic optimal passing of ships in the field of risks.

The research used a systematic approach, analysis and synthesis, mathematical analysis, methods of probability theory, automatic control and conducting an experiment. As well as equipment: a personal computer with the Windows 10 operating system and the MS Office 2016 application package, the MATLAB environment for mathematical modeling.

3. Results and Discussion

Let’s consider the problem of optimal control of the passing of ships in the form of:

\[
\begin{aligned}
L(\mathbf{x}) & \to \min \int \varphi(\mathbf{x}) d\mathbf{x}, \\
\varphi(\mathbf{x}) & = [S(\mathbf{x})] \begin{bmatrix} C(\mathbf{x}) \end{bmatrix}, \\
C(\mathbf{x}) & \leq C^*,
\end{aligned}
\]

where \(L(\mathbf{x})\) is the target functional to be optimized; \(\varphi(\mathbf{x})\) is a vector function of the target functional; \(S(\mathbf{x})\) is the length of the passing trajectory; \(C(\mathbf{x})\) is a function of the total risk; \(\mathbf{x}\) is a vector of system state parameters; \(C^*\) is an acceptable risk.

Formulation of the problem in the form of system (1) means finding the shortest passing trajectory \(S(\mathbf{x})\) on which the risk of collision with the target ship does not exceed the specified one.

The risk function \(C(\mathbf{x})\) used in formula (1) must take into account the uncertainties associated with: errors in measuring the parameters of the ship and the target, errors in the operation of executive devices due to the presence of backlashes, delays in the transmission of information, errors in estimating the geometric dimensions of targets, their behavior, etc. [7, 8]. Also, the risk function \(C(\mathbf{x})\) should take into account economic and technical losses (ship and cargo value), other factors [9].

For a normal distribution [10–12], the risk function will have the form:

\[
C(\mathbf{x}) = C_m f(x) = \frac{C_m}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2}\left[\frac{(x-x_0)^2}{\sigma_x^2} + \frac{(y-y_0)^2}{\sigma_y^2}\right]},
\]

where \(C_m\) is the factor taking into account the cost of the ship and cargo; \(x_0, y_0\) are the ship position; \(\sigma_x, \sigma_y\) are the root mean square values of total errors along the longitudinal and lateral axes of the ship; \(r_{xy}\) is the correlation coefficient between \(\sigma_x, \sigma_y\).

The geometric locus of equal risks is an ellipse:

\[
\frac{(E-x_0)^2}{\sigma_x^2} - \frac{r_{xy}(x-x_0)(y-y_0)}{\sigma_x^2\sigma_y^2} + \frac{(y-y_0)^2}{\sigma_y^2} = R^2,
\]

\[
R^2 = -(2-2r_{xy})\ln\left(\frac{2\pi\sigma_x\sigma_y C^*}{C_m}\right),
\]

which obtain from equation (2) for \(C(\mathbf{x}) = C^*\).

In order to simplify calculations, when constructing the risk field in the on-board computer, which is the sum of the risk fields of one’s own ship, targets and navigational hazards, it is advisable to use the reference risk field, which is the same for all ships, followed by affine transformation of the reference field into the instantaneous risk field of individual ships.

The field of risks is shown in Fig. 1.

The affine transformation of the reference field into the instantaneous risk field has the form:
where $A$ is the rotation matrix of the reference field by the angle $\varphi$ (the instantaneous course of the ship or target); $x_0, y_0$ are the coordinates of the instantaneous displacement vector of the center of the ship or target relative to the center of the reference field.

Fig. 1. Reference field of risks

Taking (4) into account, the instantaneous risk field of the ship or target is determined by transformation (5) of the reference risk field:

$$C_i(x) = C_{\alpha_i} \left( \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \right).$$

(5)

To obtain the entire instantaneous risk field, the operation of adding the risk fields of individual targets is performed:

$$C(x) = \sum_{i=1}^{n} C_i(x).$$

(6)

Fig. 2 shows the total momentary risk field of the ship and targets.

The field of risks (6) extends to the entire field of operation and at each point of the field its gradient can be determined, which indicates the direction of increasing risk:

$$x_{i+1} = x_i + \frac{V}{|\text{grad} C(x)|} \text{grad} C(x); i = 1,\ldots, m; 
 x(0) = x_0.$$  

(7)

To avoid a collision, it is necessary to move in a direction perpendicular to the direction of the gradient:

$$x_{i+1} = x_i + \frac{V}{|\text{grad} C(x)|} \text{grad} C(x) \rightarrow 0.$$  

(8)

Algorithm (8) means that at each step it is necessary to calculate the direction of movement along the line of equal gradient, on which the permissible risk $C^*$ is not exceeded, i.e., when passing, the ellipse of equal risk of the ship must «slide» along the equal-risk ellipse of the target. Deviation inside the equal-risk ellipse decreases the path but increases the risk, and deviation outward decreases the risk but increases the path.

Thus, algorithm (8) provides the optimal passing condition (1), namely, it minimizes the length of the trajectory of the ship’s passing with the target, provided that the permissible risk is not exceeded.

Fig. 2. Total instantaneous field of ship risks and targets: 1 – the risk field of a separate target; 2 – overlaying the risk fields of goals; 3 – projection of the total risk field on the horizontal plane
The workability and efficiency of the developed method, algorithmic and software of the optimal passing module in the field of risks are verified by mathematical modeling in the MATLAB environment.

Container ship was selected for mathematical modeling of the optimal passing (Dis. 32025t). Ship characteristics: engine type – low-speed diesel (1×15890) kW, propulsion type – FPP, bow thruster present, stern thruster absent, displacement \( D = 32025 \) t, maximum speed \( V_{\text{max}} = 19.4 \) kn, length \( L = 203.6 \) m, width \( B = 25.4 \) m, bow/aft draft \( d = 9.6/10 \) m.

A task has been created to simulate the passing: our ship is moving on a course of 180° at a speed of 10 m/s (20 knots); the target ship moves on a course of 90° also at a speed of 10 m/s (20 knots). The collision probability is taken as \( C(x) = 0.3 \) %. For a given probability of collision, the root mean square errors of RADAR measurement given in Resolution A.477(XII) «Operational requirements for radar equipment» and root mean square errors of ARP A measurement given in IMO Resolution A.823(19) dated 23.11.1995 «Operational requirements for automatic radar plotting aid (ARP A)», ellipses of the given risk for the own ship and the target ship are constructed. When constructing the ellipse of the given target risk, the cost of the cargo in formula (2) is additionally taken into account through the coefficient \( C_m \) of the cost of the cargo in the distribution of the total risk.

The results of mathematical modeling are shown in Fig. 3. The automatic control system, after the arrival of the own ship at point A, deflects the rudder to the starboard side and the ship begins to circulate with the calculated radius \( r \) to exit tangentially to the ellipse of the given risk 1. The ellipse of the given risk 1 moves on a course \( K_s = 900 \) together with the target at a speed targets \( V_s \). The consecutive positions of the ellipse of the given risk of the target after 10 s and the optimal passing trajectory (marked in red) are shown. When the ship approaches the home course line by the calculated distance on the optimum passing trajectory 3, the control system deflects the rudder to the right to initiate a circulation with the given radius and come to the home course line. The movement along the ellipse of the given risk of the target is the longest stage of passing, during which the ship itself «slides» along the ellipse of the given risk along the optimal trajectory. Such a movement involves a constant change in the course of one’s own ship, however, the passing distance in this case is smaller, compared to the passing distance using the traditional method using ARP A.

As can be seen from Fig. 3, the passing distance is 12 sections×100 m=1200 m, or \( D_{\text{pas}} = 0.65 \) nm, and the passing time \( T_{\text{section}} = 12 \) sections×10 s=120 s=2 min.

Fig. 4 shows the results of the passing of the MSC Container Ship (Dis. 32025t) using ARP A. Shown are the range rings after 0.5 nm, the area of safe passing \( D_s.a. = 0.5 \) nm, the line of relative motion passing through the center of the sweep, the trajectory of the echo signal of the target during passing, the velocity vector of the own ship at the center of the sweep, the vector of the relative velocity and the vector of the target velocity.

As can be seen from Fig. 4, the passing distance between point 4 and point 5 is 1 nautical mile, the relative velocity vector is 28 knots, \( T_{\text{pass}} = 2.1 \) min.

The results of the considered and two other cases of discrepancy are summarized in Table 1. As can be seen from the obtained results, compared to traditional passing methods using ARP A, the relative reduction in trajectory length and fuel consumption with automatic optimal passing is about 30 %, and the reduction in passing time is 4 %.

The workability and efficiency of the developed method, algorithmic and software are verified by mathematical modeling in a closed loop with mathematical models of ships in the MATLAB environment, for different positions of the ship and purposes and different navigational and weather conditions.
The impossibility of using it for manual control should be attributed to the limitations of the developed method.

4. Conclusions

The method of optimal passing in the field of risks has been developed, which allows, in comparison with traditional methods, to minimize the length of the trajectory of passing, provided that the given collision risk is not exceeded. The obtained result is explained by the use of an on-board calculator for constructing the risk field, calculating, at each step of the calculation, the gradient of the risk field at the location of the ship and the direction of movement of the ship, the perpendicular direction of the gradient of the field and the tangent to the ellipse equal to the risk at the location of the ship, the formation of controls that ensure movement of the ship along the given risk ellipse in the process of passing. The developed method can be applied in automatic passing modules of automated ship traffic control systems. This will make it possible to automate the passing processes, reduce crew fatigue, significantly reduce the influence of the human factor on control processes, reduce the length of the passing trajectory and fuel consumption by 30 %, provided that the permissible risk of passing is not exceeded, and generally increase the safety of shipping.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

The manuscript has no associated data.

References

3. Pipchenko, A. (2018). Radar Plotting or... Do we really understand what ARPA does? Available at: https://learnmarine.com/blog/Radar-Plotting-or...-Do-we-really-understand-what-ARPA-does

Paolo Mamenko, Senior Lecturer, Deep Sea Captain, Department of Ship Handling, Kherson State Maritime Academy, Kherson, Ukraine, e-mail: pavlo.mamenko@gmail.com, ORCID: https://orcid.org/0000-0001-7358-9299