13. Xie, M.-Q. Forecasting the Short-Term Passenger Flow on High-Speed Railway with Neural Networks [Text] / M.-Q. Xie, X.-M. Li, W.-L. Zhou, Y.-B. Fu // Computational Intelligence and Neuroscience. – 2014. – Vol. 2014. – P. 1–8. doi: 10.1155/2014/375487
14. Tortum, A. The Modelling of Mode Choices of Intercity Freight Transportation with the Artificial Neural Networks and Adaptive Neuro-Fuzzy Inference System [Text] / A. Tortum, N. Yayla, M. Gokdag // Expert Systems with Applications. – 2009. – Vol. 36, Issue 3. – P. 6199–6217. doi: 10.1016/j.eswa.2008.07.032
15. Borovikov, V. P. Programma STATISTICA dlya studentov i inzhenerov [Text] / V. P. Borovikov. – Moscow: Goryachaya liniya – Telekom, 2001. – 301 p.
16. Borovikov, V. P. Prognozirovanie v sisteme Statistica v srede Windows. Osnovyi teorii i intensivnaya praktika na kompyutere [Text]: uch. pos. / V. P. Borovikov, G. I. Ivchenko. – Moscow: Finansyi i statistika, 2000. – 283 p.
17. Vidpravlennya vantazhiv zaliznychnym transportom u sichni-veresni 2016 roku [Electronic resource]. – Available at: http://www.ukrstat.gov.ua/operativ/operativ2016/tr/opr/opr_u/opr0916_u.htm
18. Kruglov, V. V. Iskusstvennyie neyronnyie seti. Teoriya i praktika [Text] / V. V. Kruglov, V. V. Borisov. – 2-e izd. – Moscow: Goryachaya liniya – Telekom, 2002. – 382 p.
19. Haykin, S. Neyronnyie seti [Text] / S. Haykin. – Moscow: OOO «I. D. Vilyams», 2006. – 1104 p.

UDC 656:51-74
DOI: 10.15587/1729-4061.2017.103220

FORMATION OF SEPARATE OPTIMIZATION MODELS FOR THE ANALYSIS OF TRANSPORTATION-LOGISTICS SYSTEMS

A. Bosov
Doctor of technical sciences, Professor
Department of Applied Mathematics
Dnipropetrovsk National University of Railway Transport named after academician V. Lazaryan
Lazaryana str., 2, Dnipro, Ukraine, 49010
E-mail: AABosov@i.ua

N. Khalipova
PhD, Associate Professor
Department of Transport systems and technologies
University of Customs and Finance
Vernadskoho str., 2/4, Dnipro, Ukraine, 49000
E-mail: khalipov@rambler.ru

Для аналізу та удосконалення транспортно-логістичних систем запропоновані підходи векторної оптимізації функціоналів. Для більш повного та об'єктивного відображення властивостей об'єктів транспортно-логістичних систем, поряд із характеристикою стану, враховано динамічні властивості системи та окремих елементів. Сформульовано математичну модель для оптимізації декількох параметрів і функціоналів та формування повної множини ефективних планів. Розглянуто задачу векторної оптимізації функціоналу з квадратичною апроксимацією функцій для аналізу проходження потоків вантажів через склад митниці

Ключові слова: удосконалення транспортно-логістичних систем, векторна оптимізація, функціональні критерії, ефективні розв'язки

Для анализа и усовершенствования транспортно-логистических систем предложены подходы векторной оптимизации функционалов. Для более полного и объективного отображения свойств объектов, наряду с характеристикой состояния системы и отдельных элементов, учтены их динамические свойства. Сформулирована математическая модель для оптимизации нескольких параметров и функционалов и формирования полного множества эффективных планов. Рассмотрена задача векторной оптимизации функционала с квадратичной аппроксимацией функций для анализа прохождения потоков грузов через склад таможни

Ключевые слова: усовершенствование транспортно-логистических систем, векторная оптимизация, функциональные критерии, эффективные решения

1. Introduction

Integration of Ukraine into the global economic system puts forward new requirements to the development and implementation of methods for the protection of economic interests of the state. Absolutely relevant is the need for achieving economic security when carrying out foreign trade operations.

An increase in the competitiveness and quality of transportation services for the economy should be promoted by the creation of sustainable, cost-effective system of transport and logistics.

Effective functioning of transportation sector at present is impossible without implementation of information and communication technologies, development of intelligent transport systems (ITS). ITS provide systemic changes through improving the efficiency, safety and environmental friendliness in transportation. Since 2003, technical and technological problems of improvement of the transportation systems have acquired special relevance. Development of ITS is regularly included in the agenda of the Committee on Inland Transport (CIT) of the UN ECE [1]. Over this time, a considerable progress has been achieved in solving particular technical problems. Discussion of modern problems on the management of ITS takes place within the framework of Task forces:

– on intermodal transportation and logistics;
– on customs issues related to transportation;
– on inter-state infrastructure projects.

A comprehensive analysis of the problems of transportation-logistics systems requires development of scientific approaches and information support in the process of making management decisions. Solving the problems of optimization of transportation routes, the choice of type of transport, designing systems in general requires consideration of different content criteria, that is, solving the problems on vector optimization.

The relevance of application and development of approaches of vector optimization to the analysis of transportation-logistics systems is predetermined by the processes of globalization that take place in the modern world, and which lead to the integration of economies of different countries.

2. Literature review and problem statement

Study of performance of transportation systems implies the application of systematic methodology and should be based on the combination of econometric, kinematic and technological components of the analysis of transportation processes [2]. Quality, efficiency, stability of the transportation process depend on many factors. They take into account volumes of cargo transportation, state of transportation services market, the throughput of transportation system, operational parameters of transportation vehicles, the cost of maintenance and organization of transport flows, etc. When assessing competitiveness of cargo transportation, it is important to take into account the contradictory nature of commercial interests of stakeholders of transportation process, which is characterized by an accepted level of profitability, timeliness, regularity of delivery, a degree of protection of freight. Predicting development of transportation processes is a multi-parametric task and requires a comprehensive approach, interconnected in all aspects, in order to solve it based on a thorough analysis and processing of statistical information.

Paper [3] describes categorization of the models and methods employed in different functional areas of logistics, authors analyzed theoretical and practical aspects of providing an effective solution to the problems on cargo delivery.

An analysis of separate methods of optimization and their practical application to ensure the effectiveness of transportation processes are given in article [4].

Generalization of some classes of optimization problems allowed the author in paper [5] to highlight main directions in the optimization of transportation systems. To maximize the revenue of a transportation enterprise, it is necessary to route the traffic, to effectively load the vehicles, to rationally use manpower and technical resources in a transport node, to execute production and transportation planning in logistics systems by the criterion of minimal total costs, to determine optimal tariffs, etc.

An analysis of existing approaches to the management of interaction between related enterprises in general transportation nodes is given in [6]. Author considered an adaptation to the conditions of work of transport in a market environment. The search for new solutions to optimally control the process of transshipment corresponds to the procedure of systems approach.

A method of calculation of plan of formation of uniform freight trains is described in [7]. The use of genetic algorithms and capabilities of modern computational systems will contribute to the formation of planning the trains in the polygons of railways.

Scientific approaches to the analysis and modeling of technical and technological properties of transportation processes and systems have been developing in the direction of integration with the tasks of logistics, management of supply chains, project management, quality management. These areas rapidly evolved as scientific disciplines in the second half of the 20th century.

Study of the problems of logistics is reflected in articles [8–10]. The analysis, given in these articles, indicates the evolutionary status of logistics, which, as a science, undergoes the state of development and formation of new approaches to modeling the systems and evaluation of effectiveness.

The last decade of the 20th century saw development of the concept of supply chain management (SCM) [11].

Supply chain management (SCM) includes control over all types of logistics activity, manufacturing operations, sales, product design, finance and information technologies. Articles [12, 13] consider SCM as continuation and development of the concept of integrated logistics in terms of inter-functional and inter-organizational logistical coordination. Supply chain management is represented by an integrated functional that is responsible for uniting the key business functions and business processes within and between companies into a perfect business model. It is important to solve the problems related to making effective solutions at the operational level, and to the issues of integration of partners, modeling of key business processes in a supply chain.

Success of the integrated cooperation in a supply chain is associated with the highest quality of design, production, supply and logistics. In order to achieve synchronization inside the firm and across the entire supply chain network, a strategy is required to use the information systems [14–16].

At present, an understanding of the concept of SCM as a new ideology is being expanded. In particular, it is regarded as "planning and control over all activities in a supply chain, including sourcing and management of procurement, transformation (processing) of products and management of all types of logistics activities" [17].

Dynamic development of distribution systems under conditions of globalization of economy will contribute to the development of logistics networks and the use of different types of transport [18].

An analysis of the problems on optimization of providing service to customers in a supply chain reveals that competition takes place in at least two planes: costs and additional benefit in the form of a high-level logistics customer service [19].

The problem of choosing the best alternatives in the process of designing a transportation-technological scheme of cargo delivery based on the formation of a set of alternative delivery systems is tackled in articles [20, 21]. To design an optimal transportation-technological systems of cargo delivery, a procedure is proposed to form alternative combinations of different types of transport for each order.

Given the fact that a large number of applied tasks come down to the optimization problems whose effective solution requires a description of the purpose by multiple criteria, the theory of decision making in the presence of many criteria has been gaining traction of late.

The lack of scientific knowledge in the field of theory and methodology of solving vector problems of mathematical programming and urgent need for the application of modeling algorithms have predetermined the relevance of development of theoretical bases and methods of vector optimization in the management of economic systems [22].

Article [23] addresses the issues related to the choice of solution in the presence of several criteria. The Pareto principle is strictly stated and the requirements for a justified application are set.

An analysis of the conceptual and mathematical features of decision making in the problems of a multi-criterion selection and optimization under conditions of deterministic, stochastic and fuzzy data is given in [24].

An application of vector optimization for the functions of sets is outlined in [25].

Separate mathematical aspects of the problem of vector optimization, substantiation of the existence of generalized effective solutions using the scalarization, are described in [26].

A method for the formation of a system of information and computational support of making management decision, based on results of decomposition by the module principle, is proposed in [27].

Papers [2–7] examined the impact of various factors on the processes that take place in transportation systems. Adaptation to market conditions [2, 6], the use of information technologies and intelligent models [3, 7], development of logistic concepts to control flows in transportation-logistics systems [8–19], formation of alternative variants of delivery systems [20, 21] contributes to improved efficiency. A comprehensive approach to solving the problems [2], the application of resulting [3] or total [5] criteria of costs are important; however, the discrepancies between separate criteria requires the application of vector optimization. Articles [22–27] considered theoretical bases of vector optimization [22, 23] and applied problems of a multi-criterion analysis of technical systems [24–28]. In papers [25, 28], analysis is based on the vector optimization of functions, which makes it possible to more fully characterize the state of the system, however, it does not characterize the dynamic processes sufficiently enough.

Development and complication of transportation-logistics systems requires improvement of scientific approaches to mathematical representation and construction of models for the optimization of transportation and logistics processes.

The effectiveness of functioning of the transportation-logistics systems and separate elements of the systems is evaluated by the whole tuple of technical and economic indicators, which necessitates solving the task of multi-purpose optimization.

In order to provide a more objective representation of dynamical properties of the processes, it is required to develop a multi-criterion analysis of transportation- logistics systems using the vector optimization of functionals.

3. The aim and objectives of the study

The goal of present research is to develop approaches of a multi-criterion analysis of transportation-logistics systems using the vector optimization of functionals.

To accomplish the goal, the following tasks have been set:

– an analysis of the development of a systems approach and the application of methods of vector optimization in the study into international systems of transportation;

– statement of particular mathematical models for simultaneous and independent optimization of several parameters and functionals and the formation of a complete set of effective plans, which a decision maker (DM) has to analyze.

4. Analysis of transportation-logistics systems based on the vector optimization of functionals

4. 1. Development of a systems approach to studying international systems of transportation

Given the development of complex technologies of production and management processes, systems of information, applied objects of systems theory, it is a relevant task to form and develop scientific approaches to the representation of complex systems of international transportation. Practical optimization problems, due to the presence of many criteria, significant constraints, algorithmic representation of functions, require unconventional methods in order to solve them. This necessitates the formation of new approaches to the management, analysis and assessment of effectiveness of transportation-logistics systems and the substantiation of making multi-criteria decisions.

At the present stage of development of scientific approaches to the analysis and improvement of transportation-logistics systems, we should note the impact of the concept of supply chain management (SCM), which manifests itself:

– in the evolutionary development of the very concept of SCM: from internal distribution chains to the supply chains, and hence to the analysis of network structures;

– in the increased attention to the issues of safety and reliability of functioning of logistics systems, supply chains, to studying the impact of stochastic factors and uncertainty;

– in the transition to the analysis of complex socio-economic systems involving transportation process and logistics.

The processes that take place in international transportation-logistics systems in the process of delivery of cargoes, goods, people, are diverse in character and complex in content. Therefore, such systems can be characterized as conditionally discrete, mixed, open, dynamic.

A scheme of delivery of cargo by successive stages W_J, $J=\overline{1,N}$, which represents the structuring of a logistics system at the macro level, is shown in Fig. 1. Each of the modules pictured in the diagram characterizes successive stages of cargo delivery and is represented at the micro level by a technological facility where the process of conversion of the material flow takes place.

Block "t", denoted in Fig. 1 by the dotted line, indicates the use of an intermediary at stage J, as well as cargo delivery to an intermediary ((J-1)-th stage) and then the supply chain ((J+1)-th stage). Suppliers (S_i) can be represented by warehouses at bases, enterprises, ports, customs, etc. Intermediaries (IM_i) may include at different stages the regional warehouses, logistics centers, in the international delivery – customs border crossing points for different types of transport, etc. Delivery at different stages (CD_i^J) involves the motion of cargoes using the means of different types of transport (road, sea, rail, etc.). Cargoes in a general case may possess different properties and can be transported both in containers and without them. Depending on the type of cargo transported, the type of packaging and the type of vehicle is selected.

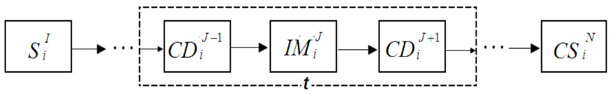

Fig. 1. Scheme of cargo delivery with successive stages (S_i – suppliers; CS_i –consumers; CD_i^J – cargo delivery at the J-th stage; IM_i – intermediaries involved in the supply chain)

Logistics in international trade activity has its own feature in operations related to the customs procedures, to the regulation of terms of goods delivery in line with purchase and sale agreements based on Incoterms rules and international rules of transportation, to filling in transportation documents and those that accompany goods, insurance, etc. In the international transportation-logistics systems, at the stage of the customs procedures, there forms a gap in the chain, which affects the efficiency of the whole system. The customs is one of the state institutions and performs control functions and provides services, controls and influences both the flows of goods (cargoes) and of transportation vehicles and people. This control is executed when crossing the customs border and inside the customs territory. In order to better understand its role in the management of material flows, a task of customs logistics should be considered holistically with other spheres of international activity.

Problems that arise at each stage of the progress of material flows belong to different functional areas and require comprehensive consideration based on many criteria for the rational and optimal use of resources over the entire logistics system. That is why the task on analysis of international logistics system can be stated as a problem of vector optimization.

Problem of decision making is one of the most complex categories in the systems analysis and all subsequent operations and actions. Integrity and emergence are important properties of large and complex systems, which include, in particular, transportation-logistics systems. Integrity indicates orientation of all elements of the system towards a common goal and contributes to the formation of optimal results by the accepted criteria of effectiveness.

Emergence implies that the investigated systems have properties not inherent to separate elements. Thus, the occurrence of an extreme characteristic in the joint functioning of individual elements that do not have an extremum is a manifestation of emergence. The process of making logistics solutions by the criterion of general logistic costs can be affected by the level of stocks, transportation service, product characteristics – price, product range, overall dimensions, weight, density of the goods, etc. [29].

This predetermines the need to develop approaches, underlying which is the optimization based on a vector criterion and the use of functionals for the analysis of these systems.

4. 2. Formation of modern approaches to the solution of problems on the improvement of transportation-logistics systems based on vector optimization

The problems of optimal control solve the tasks on allocating limited resources to achieve a complex of competing goals over a certain period of time from the starting point to the end. Among the main directions in the theory of solution of the management problems, article [30] highlighted the following: methods of variational calculus, dynamic programming, the principle of maximum and differential games. Author analyzed the present status of the theory of solution of extreme problems, systemized basic directions in the theory of optimization and outlined the basics of the theory of optimal control.

Solving optimization problems requires mathematical and algorithmic support. The tasks on making decisions are divided in paper [31] into two classes – the choice of optimal variants and the choice of the best variants. Substantiation of design solutions can occur both under conditions of complete certainty and of uncertainty in purpose and environment. By a degree of certainty and completeness of source data, the tasks are divided into three classes.

1. The problems of optimization and control. These are the problems of complete certainty. To solve them, they use methods of optimization and optimal control.

2. The problems on decision-making under conditions of uncertainty. This class includes the problems of qualitative nature. To obtain the solution, the methods of expert assessments are commonly used.

3. The problems on decision-making under conditions of partial uncertainty. To solve the problems of the third class, they use both classic techniques with a well developed theory and the heuristic ones.

Due to the high complexity of transportation-logistics systems, solving the existing problems requires the application of systems analysis, formation of a comprehensive approach based on the integration of different techniques. To increase efficiency of the process of transportation, important is a statistical analysis of transportation and freight flows, using data of chronometric observations [32–35].

When studying the objects that provide the functioning of transportation-logistics systems, and especially when designing the systems and their separate elements, effective is the use of optimization methods. The choice of mathematical optimization method is determined by the statement of a problem and requires a mathematical description of the object, that is, construction of a mathematical model [22, 30, 31, 35].

It is impossible to define the only method to solve all the possible optimization problems, some methods are more general, while others are less general. The problems of variational calculus and the problems of optimal control accept the functional as a criteria while the extremum is determined on a set of functions, assigned in functional spaces.

Most of the methods that solve optimization problems require the application of analytic approach at the first stage.

Variational problems and the problems on optimal control belong to a class of optimization problems in infinite functional spaces, and functions serve as elements, based on which the minimum or maximum of the functional is searched for.

For the quantitative estimation of the required properties of the examined object, they formulate an optimality criterion whose form is determined by particular content of the optimization problem and can be affected by the choice of method of solution. One can use as optimality criterion: objective function of one or several variables, several objective functions, functional. In the problems of multi-criteria, or the so-called vector optimization, we have several such functions.

When solving multi-criteria (vector) problems, the very concept of optimal solution changes, since it is required to find a reasonable compromise among several criteria. Typically, before one determines the extremum of a vector function, one assigns the rule of comparison of two vectors, which allows drawing a conclusion about which of the vectors is the best.

The principle of optimality by Pareto appears to be the most natural in the sense that it corresponds to the intuitive understanding about the best values of these functions. Improvement by Pareto can be formulated as follows: one should assume that any change that does not cause any damage but does someone a favor (in their own estimates), is the improvement. These so-called optimal compromise plans are called the Pareto set, or a set of effective plans. Important properties of the Pareto-solutions are the impossibility of improving any decision using any of the criteria without compromising other criteria.

4. 3. Procedure of determining an optimal solution to the problems of improvement of transportation-logistics systems based on the vector optimization of functionals

An analysis that employ modern mathematical models is required to effectively manage the design and improvement of transportation-logistics systems. In particular, the formation of models based on mathematical apparatus of optimal control with functional criteria whose extremum is defined on the set of functions, assigned in functional spaces.

In order to solve optimization problems with an optimality criterion in the form of functional, where the variables that are needed to be defined are the unknown functions, the use the techniques of variational calculus. This approach makes it possible to reduce a solution of the original optimal problem to the integration of some system of second-order nonlinear differential equations with boundary conditions assigned on both ends of the interval of integration (Euler equations).

Euler equations are the necessary conditions for the extremum of a functional, which is why the functions, obtained by integrating a systems of differential equations, should be checked on the extremum of a functional.

When there are constraints on the type of equalities that take the form of functionals, they use the Lagrange multipliers, which provides a possibility to pass from a conditional problem to the unconditional one.

The largest difficulties in the application of variational methods occur in the case of solving the problems with constraints of the inequality type.

When solving variational problems with movable boundaries, they use the so-called conditions of transversality.

In a general form, a management problem is stated in the following form [31]: it is required to determine

$$\int_{t_0}^{t_1} F(x,u,t) + l(x_1,t_1) \to \max_{\{u(t)\}} \tag{1}$$

under condition $\dot{x} = f(x,u,t)$

t_0 and $x(t_0) = x_0$ are fixed, (2)

$(x(t),t) \in T$ at $t = t_1$,

$\{u(t)\} \in U$.

Every possible phase trajectory x(t) is realized when using some permissible control {u(t)}. From a set of permissible phase trajectories whose beginning corresponds to the assigned initial state x_0 in the initial moment t_0, it is required to select the optimal phase trajectory.

Optimal trajectory $\{x^*(t)\}$ is such a permissible phase trajectory that terminates on the end surface on which the objective functional reaches a maximum.

A differential of functional is understood as a linear part of functional increment, that is, the trajectory that is close to x(t).

$$\delta I = \delta I \varepsilon + o(\varepsilon).$$

Assume that φ is an arbitrary function. Then

$$\Delta I = I[x + \varepsilon\phi] - I[x];$$

$$I[x+\varepsilon\phi] = \\ = \int_{t_0}^{t_1} F(t, x+\varepsilon\phi, \dot{x}+\varepsilon\dot{\phi})dt + l(x(t_0)+\varepsilon\phi(t_0), x(t_1)+\varepsilon\phi(t_1));$$

$$\Delta I = \int_{t_0}^{t_1}\left(\frac{\partial F}{\partial x}\varepsilon\phi + \frac{\partial F}{\partial \dot{x}}\varepsilon\dot{\phi}\right)dt + \\ + \frac{\partial l}{\partial x(t_0)}\varepsilon\phi(t_0) + \frac{\partial l}{\partial x(t_1)}\varepsilon\phi(t_1) + o(\varepsilon).$$

We shall separate a linear part

$$\delta I = \varepsilon\int_{t_0}^{t_1}\left(\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right)\right)\phi(t)dt + \\ + \frac{\partial F}{\partial \dot{x}}\varepsilon\phi(t)\Big|_{t_0}^{t_1} + \frac{\partial l}{\partial x(t_0)}\varepsilon\phi(t_0) + \frac{\partial l}{\partial x(t_1)}\varepsilon\phi(t_1).$$

Since ε is arbitrary, all terms must equal 0:

$$\int_{t_0}^{t_1}\left(\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right)\right)\phi(t)dt = 0,$$

$$\left(\frac{\partial F}{\partial \dot{x}}\phi(t)\Big|_{t_1} + \frac{\partial l}{\partial x(t_1)}\right)\phi(t_1) = 0,$$

$$\left(-\frac{\partial F}{\partial \dot{x}}\phi(t)\Big|_{t_0} + \frac{\partial l}{\partial x(t_0)}\right)\phi(t_0) = 0.$$

Then it is possible to write for the total of Euler equation:

$$\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}}\right) = 0, \quad \begin{cases} \frac{\partial F}{\partial \dot{x}}\Big|_{t_1} + \frac{\partial l}{\partial x(t_1)} = 0, \\ \frac{\partial F}{\partial \dot{x}}\Big|_{t_0} + \frac{\partial l}{\partial x(t_0)} = 0, \end{cases} \tag{3}$$

where (3) represent the condition of transversality.

If there are two criteria of optimization that are described by functionals I_1, I_2, then it is necessary to determine

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \to \min \qquad (4)$$

under condition (2).

Definition 1. Assume function x(t) belongs to D_1, that is, to the class of continuous functions with continuous derivatives. Then function x(t) is effective if differentials of the functionals have opposite sign.

Definition 2. Functions $x_1(t)$, $x_2(t)$ are called incomparable when at one function one functional turns out to be the best, while at another one is another.

Definition 3. Set X with D_1 is termed a solution to the problem of vector optimization, if $\forall x \in X$ is effective, and two functions $x_1(t)$, $x_2(t)$ from X are incomparable to each other.

From definition 1 we obtain, since x(t) is effective, that $\exists a$ is so that the condition holds

$$\delta I_1 + a\delta I_2 = 0,$$

$$\delta I = \int_{t_0}^{t_1} \left(\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}} \right) \right) \phi(t) dt +$$

$$+ \frac{\partial F}{\partial \dot{x}} \phi(t) \Big|_{t_0}^{t_1} + \frac{\partial l}{\partial x(t_0)} \phi(t_0) + \frac{\partial l}{\partial x(t_1)} \phi(t_1) = 0.$$

Then

$$F = F_1 + aF_2,$$

$$l = l_1 + al_2.$$

A management problem in the classic variational calculus implies that among a set of functions of time, that is, the phase trajectories that connect two fixed points in the initial and end periods, it is necessary to select a function that depends on the phase coordinate, rate of change in the phase coordinate and time, that is, takes the form:

$$I = \int_{t_0}^{t_1} F(x(t), \dot{x}(t), t) dt \to \min, \qquad (5)$$

$$x(t_0) = x_0,$$

$$x(t_1) = x_1. \qquad (6)$$

This problem can be considered as a particular case of a general problem on management, which does not include the function of finite parameters (Lagrange problem). In such statement, solution to the problem depends on one phase coordinate and on one controlling parameter – a speed of change in the phase coordinate. That is, the equation takes the form $\dot{x} = u$, therefore, $\dot{x}$ in F(...) replaces u, and the control parameter may take any value.

The solution to the problem of variational calculus is the trajectory {x(t)}, along which we achieve extreme value by the integral functional (the minimum). Under conditions of the existence of a solution, the solution must satisfy the required condition that is described by the Euler equation

$$\frac{\partial F}{\partial x} - \frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}} \right) = 0. \qquad (7)$$

If we have two functionals, for example, cost and time

$$I_1 = \int_{t_0}^{t_1} F_1(x(t), \dot{x}(t), t) dt,$$

$$I_2 = \int_{t_0}^{t_1} F_2(x(t), \dot{x}(t), t) dt,$$

to solve problem (4) under conditions (6), we shall introduce functional

$$I = I_1 + aI_2 \to \min, \quad a \geq 0.$$

Represent $F=F_1+aF_2$ and then substitute in (7).

As a result of the solution of the optimization problem of several functionals, there forms a complete set of effective plans that are provided to a DM for further analysis.

5. Results of examining the indicators of functioning of transportation-logistics systems based on the vector optimization of functionals

In order to study indicators of functioning of transportation-logistics systems, we shall consider a progress of the flows of cargoes through the customs warehouse. Preparation of model and conducting analysis based on the vector optimization of functionals for obtaining a region of efficient solutions is performed in several stages.

At the first stage, it is necessary to collect statistical data on the examined indicators, in particular on the volumes of cargoes processed at the customs office over a certain period. An analysis of freight traffic in a foreign-economic activity can be used to develop infrastructure of the customs body, planning the places of arrival and handling of flows, efficient distribution of queue, etc.

At the second stage, the problem is to choose a method for the approximation of data and determining an approximating dependence, which describes empirical data most accurately. Dependences that describe behavior of the system are of non-linear, often extreme, character. Thus, considering simultaneously the functions of cost of delivery and storage of material resources in the problems of inventory management shows that though each of these functions has no extremum, the function of total cost has a minimum. Similar systemic effects can be seen when exploring food, technological, logistics and other components by their combined influence on the behavior of the system as a whole [3, 29].

The obtained results of an analysis of freight traffic of a foreign economic activity using different methods of approaching the curves showed the validity of the use of a quadratic model [31]. An analysis of region of the application of various methods of prediction and the use of the method of exponential smoothing in the case when a trend is described by a quadratic predicting polynomial that is most often used, is given in article [32].

At the third stage, based on the analyzed data, we form a model of the vector optimization of functionals.

In the mathematical representation of transportation-logistic processes, functionals can characterize the costs, time, energy of displacement, etc. As a phase coordinate, we can accept, for example, a volume of the cargo transported from a warehouse, spent on their displacement or on the execution of final and initial operations. Dependences that describe one or another indicator can be obtained empirically, by the methods of statistical or correlation-regression analysis, methods of factor analysis, analysis of hierarchies, etc.

The process of cargo transportation is affected by a whole range of factors – total volume of transportation, time, size of the batch, type of transport, type of rolling stock. The functions of quality (criteria) are defined for a particular problem statement, using the methods of estimation of indicators of economic processes, analysis of technological and logistical costs, etc.

Substantiation of the analysis of functioning of the customs units based on the optimization of functionals predetermines the impact on the indicators from both the volumes and speed of cargo processing. This makes it possible to more accurately describe the state of the system and its dynamic properties.

In order to analyze the movement of flows of goods through the customs warehouse, we shall accept as constraints particular equations that contain a quadratic approximation of data on the volumes of cargo and the speed of their processing. This example does not limit a general approach.

The constraints are accepted in terms of the cost and time of processing the cargoes.

When defining initial conditions, we accept that in the beginning of the period volume of the processed goods is zero. At the end of the period we accept 1 as the fulfillment of the whole volume of the planned work.

The fourth stage includes analytical work on the model.

At the fifth stage, we solved a problem of the vector optimization of functionals using the applied computer programs to determine a complete set of effective plans.

Assign two functions:

$$F_1 = x^2 + \dot{x}^2,$$

$$F_2 = \dot{x}^2.$$

Initial conditions:

$$x(0) = 0, x(1) = 1.$$

1. Solve a Lagrange problem. Assume

$$F = x^2 + \dot{x}^2 + a\dot{x}^2.$$

Construct an Euler equation by (7)

$$2x - \frac{d}{dt}\left(2(1+a)\dot{x}\right) = 0 \text{ or } x - (1+a)\ddot{x} = 0.$$

General solution to the problem takes the form

$$x = C_1 e^{t/\sqrt{1+a}} + C_2 e^{t/-\sqrt{1+a}}.$$

Determine C_1, C_2 from the initial conditions:

$$\begin{cases} C_1 + C_2 = 0, \\ C_1 e^{1/\sqrt{1+a}} + C_2 e^{1/-\sqrt{1+a}} = 1. \end{cases}$$

$$C_2 = 1/\left(e^{-b} - e^{b}\right), \quad b = \frac{1}{\sqrt{1+a}}.$$

Define the limits within which a change in indicator b occurs:

$$a = 0; x(t,a)\big|_{a=0} = \frac{\left(e^{-t} - e^{t}\right)}{\left(e^{-1} - e^{1}\right)},$$

$$a \to \infty; x(t,a)\big|_{a=\infty} = \lim_{a\to\infty} \frac{\left(e^{\frac{-t}{\sqrt{1+a}}} - e^{\frac{t}{\sqrt{1+a}}}\right)}{\left(e^{\frac{-1}{\sqrt{1+a}}} - e^{\frac{1}{\sqrt{1+a}}}\right)}; \ \frac{0}{0}.$$

$$\lim_{b\to 0} \frac{\left(e^{-bt} - e^{bt}\right)}{\left(e^{-b} - e^{b}\right)} = \lim_{b\to 0} \frac{\left(-te^{-bt} - te^{bt}\right)}{\left(-e^{-b} - e^{b}\right)} = \lim_{b\to 0} \frac{t\left(e^{-bt} + e^{bt}\right)}{\left(e^{-b} + e^{b}\right)} = t,$$

$$x(t,a)\big|_{a\to\infty} = t; \ a = 0; b = 1; a = \infty; b = 0.$$

Then

$$x = \frac{1}{e^{-b} - e^{b}}\left(e^{-bt} - e^{bt}\right), \text{ where } 0 \le b \le 1. \qquad (8)$$

The region of efficient solutions for (8) is defined in the Maple programming environment, by changing variable b from 0 to 1 at step 0.1. A set of effective functions that reflect a dependence of phase coordinate x on time t is shown in Fig. 2.

The time derivative equals

$$\dot{x}(x,t) = \frac{e^{b}\left(-be^{bt} - be^{-bt}\right)}{1 - e^{2b}}.$$

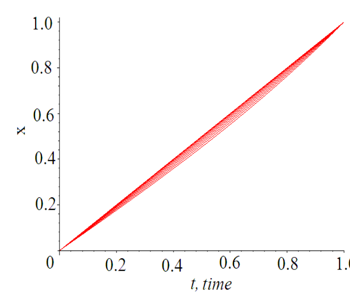

Fig. 2. A set of effective functions

Determine the value of the first functional.

Integral function that is equal to

$$I_1(x,\dot{x},t) = \frac{1}{2}\frac{\left(e^{4b} + 4e^{2b}b^3 + e^{4b}b^2 - 4e^{2b}b - 1 - b^2\right)}{\left(-1 + e^{2b}\right)^2 b}$$

is shown in Fig. 3 as a function of b.

Determine the value of the second functional. Integral function equals

$$I_2(x,\dot{x},t) = \frac{1}{2}\frac{b\left(e^{4b} + 4e^{2b}b - 1\right)}{\left(-1 + e^{2b}\right)^2}$$

and is shown in Fig. 4 as a function of b.

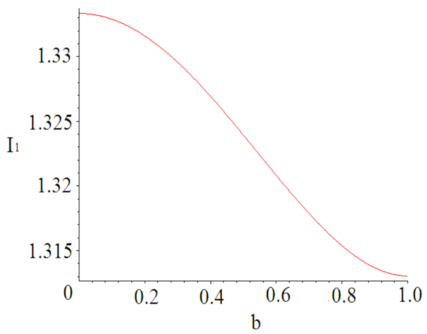

Fig. 3. Integral function for functional I_1

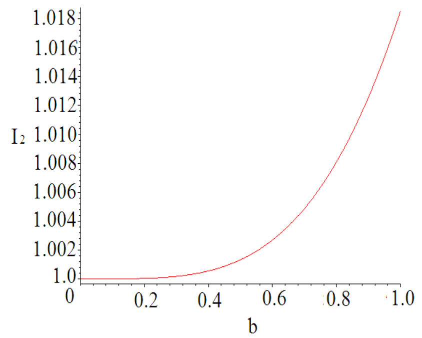

Fig. 4. Integral function for functional I_2

Dependence curve between I_1 and I_2 in the space of functionals is shown in Fig. 5.

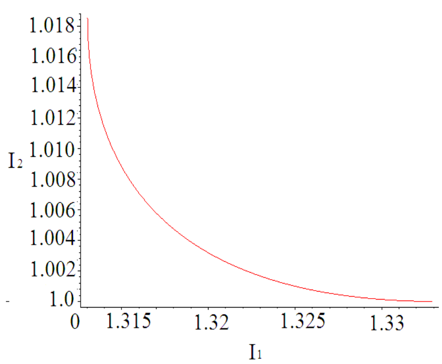

Fig. 5. Solution to the problem of vector optimization in the space of functionals

The resulting dependence allows us under particular conditions to determine a rational value of one of functionals I_1 or I_2, if another one has a particular fixed value.

2. Let is consider a problem of vector optimization of functionals in a general form, taking into account the functions of initial and final operations (the Boltz problem). Functionals are assigned in the following form:

$$I_1 = \int_0^1 \left(\dot{x}^2 + x^2\right) dt - 2x(1)\mathrm{sh}(1),$$

$$I_2 = \int_0^1 \dot{x}^2 dt - 4x^2(0) - 5x^2(1).$$

Define:

$$F = x^2 + \dot{x}^2 + a\dot{x}^2,$$

$$l = -2x(1)\mathrm{sh}(1) + a\left(4x^2(0) - 5x^2(1)\right),$$

$$\frac{\partial F}{\partial x} = 2x;\ \frac{\partial F}{\partial \dot{x}} = 2\dot{x} + 2a\dot{x},$$

$$\frac{\partial l}{\partial x(0)} = 8ax(0);\ \frac{\partial l}{\partial x(1)} = -2\mathrm{sh}(1) - 10ax(1).$$

Find a general solution to the Euler equation in the form $x=e^{\lambda t}$.

$$x - \ddot{x}(1+a) = 0;$$

$$e^{\lambda t} - \lambda^2(1+a)e^{\lambda t} = 0;\ \ 1 - \lambda^2(1+a) = 0;$$

$$\lambda_{1,2} = \frac{1}{\pm\sqrt{1+a}} = b.$$

$$x = C_1 e^{bt} + C_2 e^{-bt};\ \ \dot{x} = C_1 b e^{bt} - C_2 b e^{-bt}.$$

Constants C_1 and C_2 are found from initial conditions (3)

$$\left.\frac{\partial F}{\partial \dot{x}}\right|_{t_1} + \frac{\partial l}{\partial x(t_1)} = 0;\ 2\dot{x}(t_1)(1+a) - 2\mathrm{sh}(1) - 10ax(1) = 0.$$

$$\left.\frac{\partial F}{\partial \dot{x}}\right|_{t_0} + \frac{\partial l}{\partial x(t_0)} = 0;\ \ 2\dot{x}(t_0)(1+a) + 8ax(0) = 0,$$

$$\begin{cases} 2(1+a)\left(C_1 b e^{bt_1} - C_2 b e^{-bt_1}\right) - 2\mathrm{sh}(1) - 10ax(1) = 0, \\ 2(1+a)\left(C_1 b e^{bt_0} - C_2 b e^{-bt_0}\right) + 8ax(C_1 + C_2) = 0 \end{cases}$$

or

$$\begin{cases} 2b(1+a)\left(C_1 e^{b} - C_2 e^{-b}\right) - 2\mathrm{sh}(1) - 10a\left(C_1 e^{b} + C_2 e^{-b}\right) = 0, \\ 2b(1+a)(C_1 - C_2) + 8ax(C_1 + C_2) = 0. \end{cases} \quad (9)$$

Changing parameter a, we obtain a solution to system (9) in the form of a family of curves that form the region of effective solutions. Based on subsequent analysis of the complete set of effective plans, a DM can make a justified decision.

6. Discussion of results of applying the vector optimization of functionals in the problems on improvement of transportation-logistics systems

In the present study we proposed an algorithm and procedure of a multi-criterion analysis of transportation-logistics systems using the vector optimization of functionals, which make it possible to take into account dynamic properties of transportation and logistics processes.

For a more accurate representation of behavior of the system, we propose at the first stages to analyze influential factors and qualitative characteristics of the processes that define the form of a quality function (criteria) for each separate problem, to define approximating dependences, which describe empirical data most accurately. This is predetermined by complex interrelations between separate elements and subsystems, and, as a result, by a nonlinear, often extreme, character of dependences that describe behavior of the system.

In the given example of an analysis of freight traffic that is cleared at customs in international transportation-logistics systems, we adopted a quadratic model, the validity of whose application is analyzed in [31]. Results of the analysis of statistical data, obtained at this stage, on the volumes of cargoes can be used to predict and plan the work of the customs units. In order to analyze other subsystems, it is necessary to run an analysis of the region of application of the methods of forecasting and approximation in each particular case [32].

At the next stage, during formation of the model, in order to represent transportation-logistics systems in a more detailed fashion, we considered characteristics of rate of change in the examined processes that will make it possible to take into account dynamic properties of the objects.

When analyzing the functioning of customs units, performance indicators are affected by both the volumes and speed of cargo processing. We examined the impact of the state of the system and its dynamic properties on the indicators of cost and time to processing the cargoes. During formation of constraints, effectiveness can be assessed by other indicators, such as conditions to provide services, consumed energy, payments to the budget, etc.

Using as an example the movement of cargo flows through the customs warehouse, we have shown the formation of separate mathematical models for simultaneous and independent optimization of several parameters and functionals and to obtain a complete set of effective plans for subsequent analysis by a decision-maker. Based on the vector optimization of functional with a quadratic approximation of functions, we received a region of efficient solutions to the Lagrange problem in the Maple programming environment and explored a problem of the vector optimization of the Boltz functional.

Thus, each object that participates in the processing of cargo flows at certain stages is complicated and requires systems analysis.

The proposed approach to modeling will make it possible to receive a complete set of compromise solutions to the problem of vector optimization of functionals for a decision-maker with regard to the state of the system and dynamic processes.

In the future, the approach can be used in the tasks on planning rational distribution and efficient servicing of freight traffic by the objects of transportation-logistic network.

7. Conclusions

The analysis we conducted showed that the existing approaches to the study of international systems of transportation do not fully reflect the properties of the object. For the transportation-logistics systems, it is necessary, together with a characteristic of the state, to take into account dynamic properties of the system and separate elements. It is shown that for a more complete and objective representation of dynamic properties of the processes, it is required to develop a systems approach based on the vector optimization and the use of functionals for the analysis of these systems.

We proposed an algorithm and a procedure of a multi-criterion analysis of objects of transportation-logistics systems using the vector optimization of functionals. It is demonstrated that in order to receive a more accurate representation of behavior of the system, it is necessary at the first stages to analyze statistical data on the examined indicators and to define approximating dependences that describe empirical data most accurately. When forming a model based on the vector optimization of functionals, the equation was introduced with characteristics of rate of change in the examined processes, which will make it possible to take into account dynamic properties of the object.

Using as an example the movement of cargo flows through the customs warehouse, we have shown the formation of separate mathematical models for simultaneous and independent optimization of several parameters and functionals and the formation of a complete set of effective plans for subsequent analysis by a decision-maker.

References

1. Strategicheskie voprosy gorizontal'noy politiki: intellektual'nye transportnye sistemy [Electronic resource]. – Evropeyskaya ehkonomicheskaya komissiya. Komitet po vnutrennemu transportu. Sem'desyat vos'maya sessiya. – Available at: https://www.unece.org/trans/main/itc/itc_doc_2016.html
2. Kokaev, O. G. O tekhnologii analiza transportnyh processov v sovremennyh usloviyah hozyaystvovaniya [Text] / O. G. Kokaev, O. Yu. Lukomskaya, S. A. Seliverstov // Transport Rossiyskoy Federacii. – 2012. – Issue 2 (39). – P. 32–36.
3. Modeli i metody teorii logistiki [Text]: uch. pos. / V. S. Lukinskiy (Ed.). – 2-e izd. – Sankt-Peterburg: Piter, 2008. – 448 p.
4. Kunda, N. T. Doslidzhennya operatsiy u transportnykh systemakh [Text]: navch. pos. / N. T. Kunda. – Kyiv: Slovo, 2008. – 400 p.
5. Gorev, A. E. Osnovy teorii transportnyh sistem [Text]: uch. pos. / A. E. Gorev. – Sankt-Peterburg: SPbGASU, 2010. – 214 p.
6. Murad'yan, A. O. Optimizaciya processa perevalki gruzov v obshchetransportnyh uzlah [Text] / A. O. Murad'yan // Visnik NTU "KhPI». – 2014. – Issue 26 (1069). – P. 64–73.
7. Butko, T. Devising a method for the automated calculation of train formation plan by employing genetic algorithms [Text] / T. Butko, V. Prokhorov, D. Chekhunov // Eastern-European Journal of Enterprise Technologies. – 2017. – Vol. 1, Issue 3 (85). – P. 55–61. doi: 10.15587/1729-4061.2017.93276
8. Bauehrsoks, D. Dzh. Logistika: integrirovannaya cep' postavok [Text] / D. Dzh. Bauehrsoks, D. Dzh. Kloss. – Moscow: ZAO Olimp-Biznes, 2008. – 640 p.
9. Krykavskyy, Ye. V. Lohistyka. Osnovy teoriyi: pidruchnyk [Text] / Ye. V. Krykavskyy. – 2-he vyd., dop. i pererobl. – Lviv: Intelekt-Zakhid, 2006. – 456 p.
10. Mirotin, L. B. Logistika integrirovannyh cepochek postavok [Text]: uch. / L. B. Mirotin, A. G. Nekrasov. – Moscow: Ekzamen, 2008. – 256 p.

11. Oliver, K. Supply chain management: Logistics Catches up with Strategy [Text] / K. Oliver, M. Webber; M. Christopher (Ed.) // Logistics: The Strategy Issues. – London: Chapman and Hall, 1982. – P. 61–75.
12. Mentzer, J. T. Defining Supply Chain Management [Text] / J. T. Mentzer, W. DeWitt, J. S. Keebler, S. Min, N. W. Nix, C. D. Smith, Z. G. Zacharia // Journal of Business Logistics. – 2001. – Vol. 22, Issue 2. – P. 1–25. doi: 10.1002/j.2158-1592.2001.tb00001.x
13. Lambert, D. M. Supply Chain Management: Processes, Partnerships, Performance [Text] / D. M. Lambert. – 4th ed. – Ponte Vedra Beach, Fla.: Supply Chain Management Institute, 2014.
14. Upravlenie cepyami postavok: spravochnik izdatel'stva Gower [Text] / Dzh. Gattorn (Ed.). – Moscow: INFRA-M, 2008. – 670 p.
15. Chaberek, M. Makro- i mikroekonomiczne aspekty wsparcia logistycznego [Text] / M. Chaberek. – Gdansk: Wydawnictwo uniwersytetu Gdariskiego, 2005. – 205 p.
16. Simchi-Levi, D. Designing and managing the supply chain: concepts, strategies, and case studies [Text] / D. Simchi-Levi, P. Kaminsky. – N.-Y.: McGraw-Hill Companies, 2008. – 496 p.
17. Supply Chain and Logistics Terms and Glossary [Text]. – Council of Supply Chain Management Professionals, 2005. – 97 p.
18. Szyszka, G. Sieci logistyczne – nowy wymiar logistyky [Text] / G. Szyszka // Logistyki. – 2004. – Issue 3. – P. 5–7.
19. Chukhray, N. Formuvannya lantsyuha postavok: pytannya teoriyi ta praktyky [Text]: monohrafiya / N. Chukhray, O. Hirna. – Lviv: Intelekt-Zakhid, 2007. – 232 p.
20. Naumov, V. S. Forming method of alternative transport technological cargo delivery systems [Text] / V. S. Naumov, N. S. Veter // Eastern-European Journal of Enterprise Technologies. – 2011. – Vol. 5, Issue 4 (53). – P. 16–19. – Available at: http://journals.uran.ua/eejet/article/view/1203/1107
21. Kengpol, A. The development of a framework for route selection in multimodal transportation [Text] / A. Kengpol, S. Tuammee, M. Tuominen // The International Journal of Logistics Management. – 2014. – Vol. 25, Issue 3. – P. 581–610. doi: 10.1108/ijlm-05-2013-0064
22. Mashunin, Yu. K. Teoreticheskie osnovy i metody vektornoy optimizacii v upravlenii ehkonomicheskimi sistemami [Text] / Yu. K. Mashunin. – Moscow: Logos, 2001. – 256 p.
23. Nogin, V. D. Prinyatie resheniy v mnogokriterial'noy srede: kolichestvennyy podhod [Text] / V. D. Nogin. – 2-e izd., ispr. i dop. – Moscow: FIZMATLIT, 2004. – 176 p.
24. Zak, Yu. A. Prikladnye zadachi mnogokriterial'noy optimizacii [Text] / Yu. A. Zak. – Moscow: Ekonomika, 2014. – 455 p.
25. Bosov, A. A. Funkcii mnozhestva i ih primenenie [Text]: monografiya / A. A. Bosov. – Dneprodzerzhinsk: Andriy, 2007. – 182 p.
26. Kogut, P. I. Topological Aspects of Scalarization in Vector Optimization Problems [Text] / P. I. Kogut, R. Manzo, I. V. Nechay // Australian Journal of Mathematical Analysis and Applications. – 2010. – Vol. 7, Issue 2. – P. 25–49.
27. Zaycev, E. H. Formirovanie sistemy podderzhki prinyatiya resheniy v upravlenii transportnoy deyatel'nost'yu na principah mnogomernogo kuba [Text] / E. H. Zaycev // Nauchno-tekhnicheskie vedomosti MGTUGA. – 2004. – Issue 88. – P. 40–47.
28. Bosov, A. A. Pidvyshchennya efektyvnosti roboty transportnoyi systemy na osnovi strukturnoho analizu [Text] / A. A. Bosov, N. A. Mukhina, B. P. Pikh. – Dnipropetrovs'k: Vydavnytstvo Dnipropetr. nats. un-tu zalizn. transp. im. akad. V. Lazaryana, 2005. – 200 p.
29. Denysenko, M. P. Orhanizatsiya ta proektuvannya lohistychnykh system [Text]: pidr. / M. P. Denysenko; M. P. Denysenko, P. R. Levkovets', L. I. Mykhaylov (Eds.). – Kyiv: Tsentr uchbovoyi literatury, 2010. – 336 p.
30. Intriligator, M. Matematicheskie metody optimizacii i ehkonomicheskaya teoriya [Text] / M. Intriligator; A. A. Konyus (Ed.). – Moscow: «Progress», 1975. – 597 p.
31. Muromcev, D. Yu. Metody optimizacii i prinyatie proektnyh resheniy [Text]: uch. pos. / D. Yu. Muromcev, V. N. Shamkin. – Tambov: Izd-vo FGBOU VPO «TGTU», 2015. – 80 p.
32. Lesnikova, I. Yu. Prohnozuvannya okremykh pokaznykiv diyal'nosti mytnykh orhaniv [Text] / I. Yu. Lesnikova, N. V. Khalipova, I. V. Svoroba // Visnyk Akademiyi mytnoyi sluzhby Ukrayiny. Seriya: Tekhnichni nauky. – 2012. – Issue 2. – P. 41–47. – Available at: http://nbuv.gov.ua/UJRN/vamsutn_2012_2_8
33. Lesnikova, I. Yu. Porivnyal'nyy analiz prohnoznykh modeley vantazhopotokiv zovnishn'oekonomichnoyi diyal'nosti [Text] / I. Yu. Lesnikova, N. V. Khalipova // Visnyk AMSU. Seriya: Tekhnichni nauky. – 2010. – Issue 1 (43). – P. 75–85.
34. Kuz'menko, A. I. Pidvyshchennya efektyvnosti funktsionuvannya prykordonnykh perevantazhuval'nykh stantsiy [Text] / A. I. Kuz'menko // Transportni systemy ta tekhnolohiyi perevezen'. – 2015. – Issue 9. – P. 35–41.
35. Khalipova, N. V. Vektorna optymizatsiya v zadachakh udoskonalennya mizhnarodnykh transportnykh system [Text]: monohrafiya / N. V. Khalipova. – Dnipropetrovsk: AMSU, 2016. – 350 p.