Розроблено і проаналізовано економіко-математичну модель дуополії промислових підприємств-виробників продукції з урахуванням інноваційної діяльності в умовах реальної асиметрії підприємств. Припускається, що функція попиту лінійна, а витрати на виробництво продукції залежать від обсягів інвестицій у інноваційну активність. Знайдено рівноважні рішення дуополії по Курно і по Стекельбергу

Ключові слова: інноваційна діяльність, промислове підприємство, ланцюги поставок, дуополія, рівноважне рішення

Разработана и проанализирована экономико-математическая модель дуополии промышленных предприятий-производителей продукции с учетом инновационной деятельности в условиях реальной асимметрии предприятий. Предполагается, что функция спроса линейная, а затраты на производство продукции зависят от объемов инвестиций в инновационную активность. Найдены равновесные решения дуополии по Курно и по Стэкельбергу

Ключевые слова: инновационная деятельность, промышленное предприятие, цепи поставок, дуополия, равновесное решение

UDC 519.865.3:330.341.1
DOI: 10.15587/1729-4061.2017.103989

METHOD OF FINDING EQUILIBRIUM SOLUTIONS FOR DUOPOLY OF SUPPLY CHAINS TAKING INTO ACCOUNT THE INNOVATION ACTIVITY OF ENTERPRISES

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1. Introduction

At present, the concept of supply chain management has been disseminated. This creates many problems associated with modeling the interaction between the elements of the chain. These problems include, for example, a requirement to take account of:

– organization of effective cooperation among partners;

– the impact of factors of external and internal environment;

– competition among suppliers of raw materials, manufacturers of goods and logistics intermediaries.

For example, there are many developed methods and models in microeconomics that are used to study competition among enterprises. Existing mathematical models of monopoly and oligopoly type allow establishing rules determining optimal volumes of output and formulas to calculate the optimal prices in such markets. Different approaches are also known to the problem on modeling a duopoly as a particular case of oligopoly. Principles were developed to construct economic solutions for the companies-duopolies generated by such concepts as the Cournot equilibrium, the Stackelberg equilibrium, the Nash equilibrium, cartel agreements [1].

However, as it is well-known [2], one of the main factors of success of an enterprise in the market is the intention to achieve market advantages based on innovative activity. It is the use of innovations that is a decisive factor for successful commercial activity of any enterprise. Innovation implementation is considered to be one of the most important means to improve competitiveness of manufactured products, to maintain high rates of development and the level of profitability.

That is why it is a relevant task to construct and analyze optimization models, which would take into account simultaneously the impact of innovation activity of enterprises on their gaining competitive advantages and the use of the logistic concept of supply chains.

2. Literature review and problem statement

In the field of studies into competitive struggle between enterprises, economic-mathematical methods and models have been widely applied at present. Thus, articles [3–6] examine competition, in particular, its mathematical modeling from a viewpoint of the classical theory of the company. Monographs [1, 3, 4] addressed the patterns of interaction between enterprises under market conditions and proposed mathematical models for different types of competition. Papers [5, 6] tackle the problems of searching for balance given the price competition of production-transportation systems based on the classical methods of the theory of the company. A problem of the impact of innovative policy of the enterprise, which is a part of the supply chain, was analyzed in articles [7–9]. Collective monograph [7] paid considerable attention to the issue of the transfer of scientific knowledge to logistics business and provided many examples, based on the practice of enterprises in the Ruhr region of Germany. Paper [8] explored the effect of sharing knowledge in the supply chains on the effectiveness of new products on the example of practice of many international companies; emphasis in this case is on the improvement of interaction between suppliers and buyers of products. Article [9] proposed a conceptual model of interaction between processes of maintaining enterprise sustainability and supply chains, innovation activity and market demand, which was verified based on data from 77 Italian manufacturers.

Nevertheless, articles [7–9] apply either empirical or pure theoretical analysis of the problem of impact of innovation activity of an enterprise on the improvement of its competitive position. Examining a problem of practical implementation of theoretical studies in the given field of economic science is still at the embryonic stage, although their role in the development of competitive strategy of both a separate enterprise and of relevant supply chains is rather significant. In this respect, papers [10–14] may be considered promising as they propose a combination of methods of the theory of the firm and classical problems on optimal planning of production and delivery of finished products based on the use of linear programming. This approach made it possible to develop the economic-mathematical model, which is closer to practice of planning the work of enterprises, to analyze duopoly of the type "industrial enterprises – distribution network". In addition, it allows examining the impact of competitive environment on the optimal distribution of material flows that move from competing manufacturers of products to the places of their consumption. Articles [12, 13] proposed a methodical approach to the optimization of plans of production and transportation of products, taking into account innovative technological activity of enterprises-manufacturers, however, without considering possible competition among them.

That is why of interest is the study into economic-mathematical models of competition, which would be a synthesis of innovative policies of enterprises and logistics management concept.

3. The aim and objectives of the study

The aim of present study is to develop a method of finding equilibrium solutions for competing enterprises-manufacturers involved in supply chains, taking into account innovative activities of competitors.

To accomplish the set aim, the following tasks had to be solved:

– to develop economic-mathematical model of duopoly considering the innovation activity of enterprises-manufacturers of products;

– to determine equilibrium solutions of duopoly by Cournot and Stackelberg;

– to investigate the effect of amount of deductions for innovation activity of enterprises on their attaining competitive advantages.

4. Construction and analysis of economic and mathematical model of duopoly considering innovation activity of enterprises-manufacturers

4. 1. Modeling of duopoly considering innovation activity of enterprises

As a basis for the construction of economic-mathematical model of duopoly, we will consider the optimization model for planning production and delivery of multi-item products [12].

We will assume that there are two enterprises in the market, which produce similar products of K types using the same resources of R types and which deliver produced goods to M points of consumption.

Assume that plant with number i, i=1,2, in order to produce one unit of product of the k-th type (k=1,2,....,K) uses $a_{rk}^{(i)}$ units of the r-type resource, which they have on stock in quantity $b_r^{(i)}$, r=1, 2,..., R. Produced goods arrive to the warehouse of the enterprise, from where they should be delivered to destination points D_1, D_2,..., D_M. Demand on the k type product in the point of consumption m is denoted as d_{km}, $d_{km}>0$. We shall introduce the following designations: $x_k^{(i)}$ is the number of the k-type products, which the i-th plant is planning to produce, $y_{km}^{(i)}$ is the number of finished goods of the k-type that is planned to be delivered to destination point $D_{m.}$

First, we shall record constraints for the optimization models of plans for production and transportation of goods relative to duopoly [12]:

$$\sum_{k=1}^{K} a_{kr}^{(i)} x_k^{(i)} \le b_r^{(i)}, \quad r=1,\dots,R, \quad i=1,2, \tag{1}$$

$$y_{km}^{(1)} + y_{km}^{(2)} \le d_{km}, \quad k=1,\dots,K, \quad m=1,\dots,M, \tag{2}$$

$$x_k^{(i)} = \sum_{m=1}^{M} y_{km}^{(i)}, \quad k=1,\dots,K, \quad i=1,2, \tag{3}$$

$$x_k^{(1)}, x_k^{(2)}, y_{km}^{(1)}, y_{km}^{(2)} \ge 0 \ \forall k, m.$$

Assume that a demand function for the k-type goods in the m-th destination point takes the following form:

$$p_{km}\left(y_{km}^{(1)}, y_{km}^{(2)}\right) = p_{km} - g_{km}\left(y_{km}^{(1)} + y_{km}^{(2)}\right), \tag{4}$$

where p_{km} is the maximally possible price for the product of k-type in the m-th point of consumption, g_{km} is the parameter that defines elasticity of demand, that is, price reduction at an increase in the volume of sold products by one unity.

This assumption is based on the fact that when modeling competition between enterprises it can be assumed that the prices for products of any of the companies- competitors that produce the same goods depend on the volume of products sold by all companies and decrease when there is an increase in the goods that arrive to the market. Since a demand function cannot be negative, the conditions must hold:

$$y_{km}^{(1)} + y_{km}^{(2)} \le p_{km}/g_{km},$$

that is, we shall consider that demand d_{km} is equal to ratio p_{km}/g_{km} (index k runs over values from 1 to K; index m – from 1 to M). We shall also consider that the delivery of finished product is conducted at the expense of its buyer.

We shall introduce to our considerations magnitudes $v_k^{(i)}$, which will represent the amount of investment into implementation of technological innovations at the i-th plant for the production of the k-type products. An innovative project for a manufacturing enterprise may include, for example, the introduction of new advanced transportation and transshipment technologies, more fuel-efficient engines in vehicles, replacement of vehicles with the newer ones.

Assume that the costs of production $s_k^{(i)}\left(v_k^{(i)}\right)$ of one unit of the k-type product at the enterprise with number i are decreasing functions of the amount of investment into implementation of the innovative project [13]. For example, one can assume that:

$$s_k^{(i)}\left(v_k^{(i)}\right)=s_{0k}^{(i)}/\left(1+\gamma_k^{(i)}v_k^{(i)}\right)$$

or

$$s_k^{(i)}\left(v_k^{(i)}\right)=s_{0k}^{(i)}e^{-\gamma_k^{(i)}v_k^{(i)}},\quad k=1,\dots,K,\quad i=1,2, \tag{5}$$

where $s_{0k}^{(i)}$ is the value of costs for obsolete technology; $\gamma_k^{(i)}$ is the coefficient characterizing degree of efficiency of innovation in the production of the k-type product at the i-th plant. We shall also assume that the following conditions are satisfied:

$$\left[s_k^{(i)}\left(v_k^{(i)}\right)\right]''>0. \tag{6}$$

In particular, these conditions hold for functions (5).

Considering (4) and (5), plants will receive profit $\Pi^{(1)}$ and $\Pi^{(2)}$:

$$\Pi^{(1)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(1)}\right]-$$
$$-\sum_{k=1}^{K}s_k^{(1)}\left(v_k^{(1)}\right)\cdot x_k^{(1)}-\sum_{k=1}^{K}v_k^{(1)},$$

$$\Pi^{(2)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(2)}\right]-$$
$$-\sum_{k=1}^{K}s_k^{(2)}\left(v_k^{(2)}\right)\cdot x_k^{(2)}-\sum_{k=1}^{K}v_k^{(2)}.$$

By excluding variables $x_k^{(1)}$, $x_k^{(2)}$, using the condition (3), we will receive the following expression for the profit of first enterprise:

$$\Pi^{(1)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(1)}\right]-$$
$$-\sum_{k=1}^{K}\left(s_k^{(1)}\left(v_k^{(1)}\right)\cdot\sum_{m=1}^{M}y_{km}^{(1)}\right)-\sum_{k=1}^{K}v_k^{(1)}. \tag{7}$$

First enterprise-manufacturer wants to optimize this profit by variables

$$y_{km}^{(2)},\ v_k^{(2)}\ge 0\ \forall k,m.$$

Similarly, second plant will strive to maximize its profit by variables

$$y_{km}^{(2)},\ v_k^{(2)}\ge 0\ \forall k,m,$$

that is, function

$$\Pi^{(2)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(2)}\right]-$$
$$-\sum_{k=1}^{K}\left(s_k^{(2)}\left(v_k^{(2)}\right)\cdot\sum_{m=1}^{M}y_{km}^{(2)}\right)-\sum_{k=1}^{K}v_k^{(2)}. \tag{8}$$

Constraints for two optimization problems will take the form:

$$\sum_{k=1}^{K}a_{kr}^{(i)}\sum_{m=1}^{M}y_{km}^{(i)}\le b_r^{(i)},\quad r=1,\dots,R,\ i=1,2,$$

$$y_{km}^{(1)}+y_{km}^{(2)}\le p_{km}/g_{km},\quad k=1,\dots,K,\quad m=1,\dots,M. \tag{9}$$

Certain dependency may exist between variables $x_k^{(1)}, y_{km}^{(1)}$ and $x_k^{(2)}, y_{km}^{(2)}$ as a result of competition between plants. This should be taken into account when recording the necessary conditions of optimality for each function $\Pi^{(1)}$ and $\Pi^{(2)}$ with appropriate constraints. In the simplest case, this dependence can be neglected.

4. 2. Determining the equilibrium solution by the Cournot

An analysis of duopoly as the simplest form of oligopoly was first implemented by the French economist Augustin Cournot. This model describes market equilibrium under conditions of non-cooperated oligopoly. Cournot assumed that competitors manufactured similar products, and make decisions about production independently. Competitor's output was also considered constant and a function of market demand was assumed to be known.

Let us determine a duopoly solution, equilibrium in the sense of Cournot, at which the profits of the enterprises are maximally possible.

Necessary conditions of extremum of profit functions (7), (8) take the form:

$$\frac{\partial\Pi^{(1)}}{\partial y_{km}^{(1)}}=p_{km}-g_{km}y_{km}^{(2)}-2g_{km}y_{km}^{(1)}-s_k^{(1)}\left(v_k^{(1)}\right)=0,$$

$$\frac{\partial\Pi^{(1)}}{\partial v_k^{(1)}}=-\sum_{m=1}^{M}y_{km}^{(1)}\frac{\partial s_k^{(1)}\left(v_k^{(1)}\right)}{\partial v_k^{(1)}}-1=0,$$

$$\frac{\partial\Pi^{(2)}}{\partial y_{km}^{(2)}}=p_{km}-2g_{km}y_{km}^{(1)}-g_{km}y_{km}^{(2)}-s_k^{(2)}\left(v_k^{(2)}\right)=0,$$

$$\frac{\partial\Pi^{(2)}}{\partial v_k^{(2)}}=-\sum_{m=1}^{M}y_{km}^{(2)}\frac{\partial s_k^{(2)}\left(v_k^{(2)}\right)}{\partial v_k^{(2)}}-1=0\quad\forall k,m.$$

Hence, we find the equations that will reflect the optimal level of output of a duopolist production through the optimum of output of its competitor. Then the solution of duopoly, equilibrium in the sense of Cournot, is determined by formulas:

$$y_{km}^{(1)}=\frac{p_{km}-2s_k^{(1)}\left(v_k^{(1)}\right)+s_k^{(2)}\left(v_k^{(2)}\right)}{3g_{km}},$$

$$y_{km}^{(2)}=\frac{p_{km}-2s_k^{(2)}\left(v_k^{(2)}\right)+s_k^{(1)}\left(v_k^{(1)}\right)}{3g_{km}}, \tag{10}$$

$$\sum_{m=1}^{M} y_{km}^{(i)} = -1\Big/\left[s_k^{(i)}\left(v_k^{(i)}\right)\right]', \quad i=1,2.$$

In this case, the following conditions must be satisfied:

$$\sum_{k=1}^{K} a_{kr}^{(1)} \sum_{m=1}^{M} \frac{p_{km} - 2s_k^{(1)}\left(v_k^{(1)}\right) + s_k^{(2)}\left(v_k^{(2)}\right)}{3g_{km}} \le b_r^{(1)}, \ r=1,\dots,R,$$

$$\sum_{k=1}^{K} a_{kr}^{(2)} \sum_{m=1}^{M} \frac{p_{km} - 2s_k^{(2)}\left(v_k^{(2)}\right) + s_k^{(1)}\left(v_k^{(1)}\right)}{3g_{km}} \le b_r^{(2)}, \ r=1,\dots,R. \quad (11)$$

If the conditions (11) do not hold, then in order to find the equilibrium solution to the pair of problems (7), (9) and (8), (9), general optimization algorithms [15] should be applied considering

$$\frac{\partial y_{km}^{(1)}}{\partial y_{km}^{(2)}} = 0, \quad \frac{\partial y_{km}^{(2)}}{\partial y_{km}^{(1)}} = 0, \quad k=1,\dots,K, \quad m=1,\dots,M,$$

that is, the Cournot's conditions are satisfied for possible variations.

4. 3. Determining the equilibrium solution by the Stackelberg

Within the framework of the proposed approach, it is possible to determine a solution, equilibrium in the sense of Stackelberg, when one or both of the plants consider that the competitor will behave as a Cournot duopolist. The follower adapts its output in accordance with the output of the leader, giving a chance to the competitor to be the first in the market to offer a desired amount of goods. The follower assumes that the leader would not react to its action. The leader adheres to the opposite point of view; its choice leads to a change of the follower's expectations. The enterprise-leader takes into account, when making its decisions, that the follower would react to its behavior.

Let us assume that plant 1 is the leader and believes that plant 2 will react according to direct reaction of Cournot, that is,

$$y_{km}^{(2)} = \frac{p_{km} - g_{km} y_{km}^{(1)} - s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}}. \quad (12)$$

Then a possible variation

$$\frac{\partial y_{km}^{(2)}}{\partial y_{km}^{(1)}} = -\frac{1}{2}.$$

Considering

$$\frac{\partial \Pi^{(1)}}{\partial y_{km}^{(1)}} = p_{km} - g_{km} y_{km}^{(2)} - s_k^{(1)}\left(v_k^{(1)}\right) - \frac{3}{2} g_{km} y_{km}^{(1)}.$$

Equating $\frac{\partial \Pi^{(1)}}{\partial y_{km}^{(1)}}$ to zero, we shall receive the equation of direct reaction from the plant:

$$y_{km}^{(1)} = \frac{2\left(p_{km} - g_{km} y_{km}^{(2)} - s_k^{(1)}\left(v_k^{(1)}\right)\right)}{3g_{km}}. \quad (13)$$

Let plant 1 assumes that plant 2 employs a Cournot reaction (12).

Then, considering (12), (13), as well as equality to zero of $\frac{\partial \Pi^{(1)}}{\partial v_k^{(1)}}$ and $\frac{\partial \Pi^{(2)}}{\partial v_k^{(2)}}$, the duopoly solution will be the Stackelberg equilibrium for plant 1:

$$y_{km}^{(1)} = \frac{p_{km} - 2s_k^{(1)}\left(v_k^{(1)}\right) + s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}},$$

$$y_{km}^{(2)} = \frac{p_{km} + 2s_k^{(1)}\left(v_k^{(1)}\right) - 3s_k^{(2)}\left(v_k^{(2)}\right)}{4g_{km}},$$

$$\sum_{m=1}^{M} y_{km}^{(i)} = -1\Big/\left[s_k^{(i)}\left(v_k^{(i)}\right)\right]',$$

$$i=1,2.$$

For these solutions to satisfy conditions (9), the following constraints must hold:

$$\sum_{k=1}^{K} a_{kr}^{(1)} \sum_{m=1}^{M} \frac{p_{km} - 2s_k^{(1)}\left(v_k^{(1)}\right) + s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}} \le b_r^{(1)}, \ r=1,\dots,R,$$

$$\sum_{k=1}^{K} a_{kr}^{(2)} \sum_{m=1}^{M} \frac{p_{km} - 3s_k^{(2)}\left(v_k^{(2)}\right) + 2s_k^{(1)}\left(v_k^{(1)}\right)}{4g_{km}} \le b_r^{(2)}, \ r=1,\dots,R. \quad (14)$$

In a situation when plant 2 assumes that plant 1 will react according to a Cournot reaction:

$$y_{km}^{(1)} = \frac{p_{km} - g_{km} y_{km}^{(2)} - s_k^{(1)}\left(v_k^{(1)}\right)}{2g_{km}},$$

$$y_{km}^{(2)} = \frac{2\left(p_{km} - g_{km} y_{km}^{(1)} - s_k^{(2)}\left(v_k^{(2)}\right)\right)}{3g_{km}}, \quad (15)$$

and the duopoly solution will be the Stackelberg equilibrium for plant 2:

$$y_{km}^{(1)} = \frac{p_{km} - 3s_k^{(1)}\left(v_k^{(1)}\right) + 2s_k^{(2)}\left(v_k^{(2)}\right)}{4g_{km}},$$

$$y_{km}^{(2)} = \frac{p_{km} + s_k^{(1)}\left(v_k^{(1)}\right) - 2s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}},$$

$$\sum_{m=1}^{M} y_{km}^{(i)} = -1\Big/\left[s_k^{(i)}\left(v_k^{(i)}\right)\right]', \ i=1,2. \quad (16)$$

If the following constraints are satisfied:

$$\sum_{k=1}^{K} a_{kr}^{(1)} \sum_{m=1}^{M} \frac{p_{km} + s_k^{(1)}\left(v_k^{(1)}\right) - 2s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}} \le b_r^{(1)}, \ r=1,\dots,R,$$

$$\sum_{k=1}^{K} a_{kr}^{(2)} \sum_{m=1}^{M} \frac{p_{km} - 3s_k^{(1)}\left(v_k^{(1)}\right) + 2s_k^{(2)}\left(v_k^{(2)}\right)}{4g_{km}} \le b_r^{(2)}, \ r=1,\dots,R, \quad (17)$$

then conditions (9) will be satisfied, that is, the constraints for the optimization problem are fulfilled.

5. Application of the developed models to analyze behavior of duopoly participants considering innovation activity of enterprises

We shall perform calculations to find the transportation plans of two types of finished products of plants ($y_{km}^{(1)}$, $y_{km}^{(2)}$, K=2) to destination points D_1 and D_2 (M=2) and investment plans ($v_k^{(1)}$, $v_k^{(2)}$) for different variants: according to Cournot and Stackelberg. Assume that costs for the production of a product unit are the decreasing functions from the amount of investments, that is,

$$s_k^{(i)}\left(v_k^{(i)}\right)=s_{0k}^{(i)}/\left(1+\gamma_k^{(i)}v_k^{(i)}\right).$$

The initial data for the problem are the values of maximally possible product prices (p_{km}), parameters that define elasticity of demand (g_{km}), production coefficients ($a_{rk}^{(i)}$), amount of inventory of materials for production ($b_r^{(i)}$, R=2), cost values for outdated technology ($s_{0k}^{(i)}$), and coefficients that characterize a degree of innovation efficiency ($\gamma_k^{(i)}$).

Values required for calculation are given in Table 1. Tables 2, 3 give results of the calculations performed using the software MS Excel, version 2003. Data of Tables 2, 3 allow the enterprises to make decisions on the volumes of output and amount of investment funding in cases of different behavior models of competitors.

Table 1

Initial data for finding the equilibrium solution of duopoly

Designations	Parameter values	Designations	Parameter values
p_{11}	30	$a_{21}^{(2)}$	0.15
p_{12}	35	$a_{22}^{(2)}$	0.1
p_{21}	25	b_{11}	80
p_{22}	40	b_{12}	80
g_{11}	0.2	b_{21}	60
g_{12}	0.15	b_{22}	50
g_{21}	0.1	$\gamma_1^{(1)}$	0.2
g_{22}	0.3	$\gamma_2^{(1)}$	0.3
$a_{11}^{(1)}$	0.2	$\gamma_1^{(2)}$	0.5
$a_{12}^{(1)}$	0.15	$\gamma_2^{(2)}$	0.25
$a_{21}^{(1)}$	0.1	$s_{01}^{(1)}$	8
$a_{22}^{(1)}$	0.3	$s_{02}^{(1)}$	5
$a_{11}^{(2)}$	0.5	$s_{01}^{(2)}$	10
$a_{12}^{(2)}$	0.1	$s_{02}^{(2)}$	15

Table 2

Calculation results of optimal management parameters

Designations	Equilibrium by Cournot		Equilibrium by Stackelberg (leader – plant No. 1)		Equilibrium by Stackelberg (leader – plant No. 2)	
	plant 1	plant 2	plant 1	plant 2	plant 1	plant 2
y_{11}	48.78	49.62	73.85	36.92	35.83	75.03
y_{12}	76.15	77.27	115.13	57.57	56.11	116.71
y_{21}	81.24	81.86	123.34	60.40	59.60	124.20
y_{22}	43.75	43.95	66.11	32.63	32.37	66.40
v_1	65.69	48.38	81.94	41.47	55.65	59.93
v_2	67.37	59.45	83.72	50.56	57.32	74.10

Table 3

Optimal profit of enterprises at equilibrium solution of duopoly

Calculation variants	Profit values		
	plant 1	plant 2	Total
Equilibrium by Cournot	2313.59	2422.47	4736.07
Equilibrium by Stackelberg (leader – plant No. 1)	2624.46	1270.34	3894.80
Equilibrium by Stackelberg (leader – plant No. 2)	1172.71	2749.87	3922.58

6. Discussion of results of modeling a duopoly considering innovation activity of enterprises

Tables 2, 3 show that position of the leader is more preferable for the enterprise than in the Cournot model, while optimal production and profit of the follower is less than that of the leader. For example, plant 2 as a leader will gain a profit of 2624.46 monetary units, which is by 311.87 monetary units larger than the value of its profit according to Cournot. Nevertheless, if plant 1 will act as a follower, its profit will decrease by 1140.88 monetary units.

However, in this case, the total volume of production of the two enterprises in the case of Cournot model (502.62 monetary units) is less than the total output according to Stackelberg (565.95 monetary units and 566.25 monetary units), while the total profit is larger (4736.04 monetary units versus 3894.8 and 3922.58 monetary units, respectively). The amount of investment in the Cournot model (240.89 monetary units) is less than according to Stackelberg (257.69 and 247.0 monetary units).

It is also instructive to compare the values of profit of the enterprises, taking into account investment in technology and without it.

Table 4 gives data on the amount of profits of the two plants before and after investing into implementation of technological innovations.

Table 4

Profit of enterprises with and without investment funding

Calculation variants	Profit value			
	plant 1		plant 2	
	without investments	with investments	without investments	with investment
Equilibrium by Cournot	1108.52	2313.59	1504.07	2422.47
Equilibrium by Stackelberg (leader – plant No. 1)	1195.42	2624.46	932.71	1270.34
Equilibrium by Stackelberg (leader – plant No. 2)	553.96	1172.71	1677.92	2749.87

Table 4 shows that under the accepted initial data enterprise profit for each of the calculation variants (by Cournot and Stackelberg) increased by 1.4–2.2 times after implementation of innovations.

Thus, given calculations show that the above developed method of finding equilibrium solutions in a duopoly, taking into account innovative activity of competing enterprises, when devising optimal plans for production, for product delivery to points of consumption and for investment in innovation, yields applicable results. The novelty of the proposed method of analysis of a duopoly is that here we take into account competition not at the level of separate enterprises but rather between supply chains. The method makes it possible to develop feasible production programs, considering the influence of innovation activity of each of the competing enterprises in supply chains on the market position. In other words, the proposed method enables aligning of marketing, logistics and innovation strategies of enterprises. In this case, obtaining a synergetic effect is possible not only through coordination of optimal plans for production and delivery of goods, but also due to the consistency with a plan of innovation activity of enterprises.

7. Conclusions

1. We developed and examined an economic-mathematical model of duopoly, which takes into account a possibility to deduct a portion of the profit of competing enterprises-manufacturers on innovative activity. The model represents a pair of problems on nonlinear programming, with non-linearity manifested not only in the introduction of a demand function, linearly decreasing with increased production, but in the assumption on the inverse relationship between production costs and the amount of investment into technological innovation of enterprises.

2. Based on the developed model of duopoly, we identified optimal plans for production by each of the enterprises in the duopoly, for delivery of finished products to the points of consumption and optimal levels of investment into innovative technologies. They define equilibrium solutions according to Cournot (when enterprises decide to release products simultaneously and independently of each other) and according to Stackelberg (when one manufacturer believes that the competitor will behave as a Cournot duopolist).

3. We performed a quantitative analysis of the impact of optimal amount of deductions for technological innovations on the strengthening of competitive positions of enterprises-manufacturers; based on this analysis we found that at certain initial data, investment funding can increase profits and competitiveness of industrial enterprises.

In the future, it is possible to perform different generalizations of results, given in the present article, for example, to study oligopolies for dynamic models of optimization of production plans and innovative activity of enterprises-manufacturers [12, 13].

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