Розглянуто задачу визначення усталеного гравітаційного руху зсувного шару зернистого заповнення циліндричної камери, що обертається навколо горизонтальної осі. Поведінку заповнення описано за допомогою осереднених величин. Застосовано пластичну реологічну модель зернистого середовища. На основі розрахунку напружено-деформованого стану визначено розподіл зсувних швидкостей по нормалі до напряму течії шару. Виявлено залежності профілю швидкостей зсувного шару від кінематичного, геометричних та реологічних параметрів системи

Ключові слова: зернисте заповнення, обертова камера, зсувний шар, гравітаційна течія, розподіл швидкостей

Рассмотрена задача определения установившегося гравитационного движения сдвигового слоя зернистого заполнения цилиндрической камеры, вращающейся вокруг горизонтальной оси. Поведение заполнения описано с помощью осредненных величин. Применена пластическая реологическая модель зернистой среды. На основе расчета напряженно-деформированного состояния определено распределение сдвиговых скоростей по нормали к направлению течения слоя. Выявлены зависимости профиля скоростей сдвигового слоя от кинематического, геометрических и реологических параметров системы

Ключевые слова: зернистое заполнение, вращающаяся камера, сдвиговый слой, гравитационное течение, распределение скоростей

UDC 621.926.5 : 539.215
DOI: 10.15587/1729-4061.2017.107242

THE ROTATING CHAMBER GRANULAR FILL SHEAR LAYER FLOW SIMULATION

Yu. Naumenko
Doctor of Technical Sciences, Associate Professor
Department of construction, road, reclamation, agricultural machines and equipment
National University of Water and Environmental Engineering
Soborna str., 11, Rivne, Ukraine, 33028
E-mail: informal9m@i.ua

V. Sivko
Doctor of Technical Sciences, Professor
Department of machinery and equipment manufacturing processes
Kyiv National University of Construction and Architecture
Povitroflotsky ave., 31, Kyiv, Ukraine, 03037
E-mail: motpknuba@ukr.net

1. Introduction

Traditional drum type machines remain the basic equipment of large-tonnage processing of granular materials in many industries due to a number of operational and economic advantages.

However, the main drawback of such equipment is high specific energy costs. The majority of the energy spent in this way is dissipated in the processed material and the environment. Extremely low energy efficiency of drum machines is compounded by the workflow non-selectivity.

At the same time, a paradoxical feature of this equipment is the combination of an extremely simple design and extremely difficult-to-describe behavior of the treated medium.

The traditional workflow theory of drum machines is based on the concept of a separate idealized element of the granular fill of the rotating chamber, isolated from the medium [1]. It is assumed that the flow regime of the whole chamber fill is two-phase waterfall, only with a solid-state zone and a zone of throwing and falling in the chamber cross-section.

Meanwhile, the real flow regime of the fill is three-phase, when the quasi-solid-state zone and the non-free-fall zone in the cross-section of the rotating chamber are supplemented by the shear layer zone. In this zone, the main stages of processing granular materials are implemented.

However, the flow of the shear layer of the rotating chamber fill is characterized by high complexity, which greatly hampers the simulation of its parameters. The lack of a method for predicting the behavior of this layer significantly limits the efficiency of implementing the stages of processing granular materials in drum machines.

The implementation of the workflows of the drum type machines is due to the nature of the flow regime of the shear layer of the granular fill of the rotating chamber. Therefore, the problem of determining the patterns of changes in the velocity parameters of such a flow is rather relevant.

2. Literature review and problem statement

The flow regimes of the granular fill of the rotating cylindrical chamber significantly affect the process implementation and the energy capacity of the drives of drum machines [2]. Simulation of the fluid dynamics of such regimes is of interest in the study of various rotor systems [3].

The application relevance of the problem of forecasting the workflows of such equipment is constantly drawing increased research attention to describing the behavior of the processed granular medium. The considerable complexity of such a problem makes it necessary to improve the traditional and apply new theoretical and experimental methods of research.

There have been several attempts of numerical solution of the problem of determining the parameters of gravitational flow of the shear layer of the granular fill of the cylindrical rotating chamber.

In [4], simulation of the free surface and the velocity profile of the shear layer of the cohesive granular fill at a slow rotation of the chamber using the discrete element method has been performed. Based on this method, in [5], the profiles of tangential velocities of the shear layer of the fill have been determined and the values have been compared with the experimental data of other authors. The velocity parameters of the shear layer have been described in [6] with the help of a numerical model of the kinetic flow theory of granular medium, taking into account frictional interaction and averaging principle.

However, the initial conditions of the problem under consideration are a priori uncertain, and boundary conditions have a non-physical nature. This causes essential limitation of the accuracy of numerical calculations, the results of which do not meet practical needs.

The flow of the shear layer of the granular fill of the rotating chamber has been repeatedly investigated also by various experimental methods.

To determine the geometric and velocity characteristics of the flow, the method of visual analysis of instantaneous flow patterns of the fill in the chamber cross-section has been used.

Thus, the thickness of the shear layer has been measured in [7], and generalization of the results has been made on the basis of a simplified similarity model. Similar studies have been performed in [8] for dry and wet half granular fill of the chamber. In [9], the thickness and the velocity profile of the layer for different rotation velocities and chamber diameters, as well as the relative size of the fill element have been measured. The velocity profile of the layer at a low filling of the chamber has been determined using the visual analysis in [10]. Based on the analysis of flow patterns, the velocity profile of the layer has been measured in [11] and the results have been generalized based on a simplified rheological model.

The method of analysis of averaged and dynamic patterns of the steady flow of the chamber fill has been also used.

In [12], the thickness and the velocity profile of the shear layer of the half-fill have been determined by visual analysis of images of line flow trajectories of particles in the chamber cross-section. The velocity characteristics of the layer, under the variation of the chamber filling degree, the Froude number and the particle size, have been obtained in [13] when analyzing the video images of the medium flow. In [14], the velocity profile and the thickness of the shear layer of the fill with elongated spheroidal particles have been investigated by means of high-speed video recording.

The limited possibilities of imaging have led to the extensive involvement of the methods of tomographic analysis of the behavior of the granular fill of the rotating chamber recently.

The application of magnetic resonance velocimetry to determine the velocity profile of the shear layer has been described in [15]. The radioactive particle tracking method has been used in [16] to study the dependence of the velocity profile of the layer on the polydispersity of the fill during slow rotation of the chamber. The effect of density and internal friction on the velocity profile of the wet granular fill layer has been studied in [17] by the particle velocity tracking method. Multiple radioactive particle tracking has been used in [18] to determine the velocity characteristics of the layer of cylindrical elements. In [19], the geometric and velocity characteristics of the shear layer under cascade and cataract regimes of the fill flow, measured by the positron emission particle tracking technique, have been given. The influence of the shape of cylindrical particles on the velocity profile of the layer has been studied in [20] by means of multiple radioactive particle tracking. The dependences of velocities of the shear layer of particles on their roughness during slow rotation of the chamber have been investigated in [21] using the particle image velocimetry method.

At the same time, the technical complexity of instrumental control of the behavior of the granular fill of the rotating chamber, due to the limited resolution, reduces the reliability and accuracy of the results of experimental studies.

In view of the limited possibilities of numerical and experimental methods, a comparison of the application results in determining the velocity characteristics of the shear layer of the fill has been made.

In [22], the results of studies of the velocity field of the fill layer during slow rotation of the chamber, obtained by the numerical discrete element method and the experimental radioactive particle tracking method, have been compared. The comparison of the results of the visual analysis and computation of the layer velocities using the discrete element method at slow rotation has been given in [23]. In [24], the velocity fields of the shear layer obtained numerically using the discrete element method and by means of experimental flow patterns of the fill, have been investigated. The results of determining the velocity and thickness of the shear layer during slow rotation of the chamber by means of high-speed video recording and computational fluid dynamics algorithms has been given in [25]. Similar methods of experimental and numerical studies have been applied in [26] for a comparative estimation of the influence of the flow regimes and the shape of particles on the velocity profile of the shear layer.

Wide application of numerical and experimental methods, however, showed limited possibilities for studying the behavior of the granular fill of the rotating chamber. As a result, it seems appropriate to involve analytical methods that allow obtaining universal results with a high level of generalization. However, application of such methods is significantly complicated by the characteristic features of the problem under consideration. This is due to the flow geometry complexity, large deformation of the free boundary and mobility of the solid wall.

As a result, attempts to involve analytical methods for determining the granular flow were limited only to the problem of simulation of the gravitational flow along fixed straight-line guides.

An attempt to analytically determine the parameters of the gravitational flow of granular material on a fixed rough slope has been made in [27]. In [28], the experimental-analytical method for determining the characteristics of the gravitational flow of granular material on a slope has been proposed.

The complexity of the dynamic behavior of the granular fill of the rotating chamber is complemented by the rheological aspect. In [29], the complexity and practical importance of an adequate determination of rheological properties of the granular media were estimated for the case of solving the problem of determining the flow of the rotating chamber fill by the analytical method. It has been noted that the rheological characteristics of such media vary considerably depending on the type of flow.

However, insurmountable computational difficulties and low reliability of instrumental control limit the effectiveness of the known methods for determining the parameters of the shear flow of the active layer of the granular fill of the chamber. Therefore, the results of numerical calculations and experiments approach the real flow regimes of the investigated medium mainly at slow rotation and low filling of the chamber only in terms of qualitative characteristics and external features. In quantitative indices, they diverge essentially.

In view of the above, currently there are no generalized analytical models of the velocity characteristics of the shear layer, taking into account wide-range variations of geometric and rheological parameters of the system. The lack of such models is particularly characteristic of the case of considerable rotation velocity and filling degree of the chamber.

3. The aim and objectives of the study

The aim of the work is to create a mathematical model of the behavior of the shear layer of the granular fill near the free surface in the cross-section of the cylindrical chamber that stationarily rotates around a horizontal axis. This will enable to determine the dynamic parameters of the steady flow of the shear layer of the fill and predict the efficiency of implementing the stages of processing the granular medium in the rotating chamber.

To achieve this aim, the following objectives were set:

– to perform an analytical simulation of the field stresses and velocities in the mass of the shear layer of the granular fill in the chamber cross-section;

– to determine the conditions of the steady gravitational flow of the shear layer in conditions of significant previous growth of its kinetic energy in the non-free-fall zone of the fill of the fast-rotating chamber;

– to determine the mean value and the distribution of shear velocities along the normal to the flow of the fill layer;

– to determine the influence of system parameters on the velocity distribution of the shear layer.

4. Method of studying the strain-strain state of the shear layer of the chamber fill

4. 1. General conceptual approach to the research methodology

It was considered that the extent of the inhomogeneities of the rotating chamber fill is much less than the distances at which the averaged parameters of the granular medium are significantly altered. The granular fill was considered as a continuous medium with parameters that are averaged by volume and continuously distributed in space. The mathematical description of the fill flow was carried out using such averaged variables. The method of calculating the stress-strain state of the fill was used. A plastic rheological model of the granular fill medium was adopted.

4. 2. Applied methods for simulation of the behavior of the granular fill medium

Further numerical solution of the problem of determining the steady-state gravitational flow of the shear layer of the volume-averaged granular fill of the chamber is sought in a plastic formulation. The experimental data allowed taking a plastic rheological model as a special case of manifestation of viscoplastic properties of the rheologically complex fill.

The shear resistance at the point for one-dimensional flow of the fill consists of the resistance to internal friction and cohesion and is expressed by the dependence that occurs when the equilibrium is violated

$$\left|\tau_n\right| = \sigma_n tg\phi + k,$$

where σ_n and τ_n are normal and tangential pressure components, n is the normal to the sliding surface, ϕ is the angle of internal friction of the granular medium, k is the cohesion coefficient of the medium.

To determine the flow of the shear layer under a strict approach, it is possible to use the system of equations (1)–(5) of the two-dimensional state of the flowing granular medium [30]

$$F_x - \frac{g}{\gamma}\left(\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y}\right) = \frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y}, \quad (1)$$

$$F_y - \frac{g}{\gamma}\left(\frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{yx}}{\partial x}\right) = \frac{\partial V_y}{\partial t} + V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y}, \quad (2)$$

$$\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2 = \left(\sigma_x + \sigma_y + 2k\cdot ctg\phi\right)^2 \sin^2\phi, \quad (3)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0, \quad (4)$$

$$\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\frac{1}{2}\left(\frac{V_y}{V_x} - \frac{V_x}{V_y}\right) \pm tg\phi}{1 \mp \frac{1}{2}\left(\frac{V_y}{V_x} - \frac{V_x}{V_y}\right) tg\phi}, \quad (5)$$

where σ_x, σ_y and $\tau_{xy}=\tau_{yx}$ are the components of the stress tensor; V_x and V_y are two projections of the velocity vector; F_x and F_y are the projections of mass forces; x and y are the coordinates; γ is the volumetric specific weight of the medium; g is the gravitational acceleration.

The first two equations of the system (1)–(5) are the flow equations of the medium. The third equation is the condition of the limit equilibrium. The fourth one represents the continuity condition of the medium. The fifth equation expresses the condition of coincidence of the direction of the maximum shear strain rate with one of the families of slip lines (active family).

5. Results of flow simulation of the shear layer of the chamber fill

The pattern of the three-phase steady flow regime of the granular fill in the cross-section of the cylindrical chamber that stationarily rotates around a horizontal axis with an angular velocity ω can be divided into three zones (Fig. 1).

Zone 1 (*EABE*) of quasi-solid-state flow, without the relative displacement of the fill particles and slipping on the chamber surface, occurs near the cylindrical chamber wall. Zone 2 (*BACB*) of non-free fall, associated with separation from zone 1 when sheared along the *AB* surface and subsequent fall with the interaction of particles, is realized in the upper part of the chamber. Zone 3 (*BCDEB*) of the shear layer, in the form of a fast, gravitational shear flow, occurs near the free surface of the fill in the lower part of the chamber.

The rotation velocity of the chamber significantly affects the implementation of the fill flow zones. At slow rotation, the quasi-solid-state zone 1 prevails, the non-free-fall zone 2 is weakly pronounced and the shear layer zone 3 has a small thickness. As the rotation velocity increases, the mass fractions of zones 2 and 3 increase at the expense of zone 1. As the rotation velocity approaches the critical value, a part of zone 2 reaches the maximum, and a part of zone 3 tends to zero.

In a simplified formulation, it is possible to consider a steady, additionally accelerated, gravitational flow of the granular layer of thickness *h* without slipping along a fragment, with the length *l*, of a flat rough surface that is inclined to a horizontal by the angle α and shifted up at a constant rate (Fig. 2).

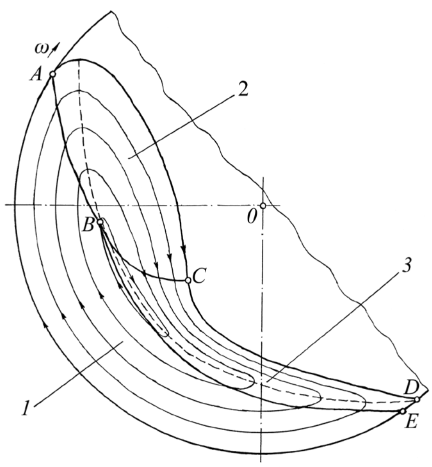

Fig. 1. The diagram of the pattern of the three-phase flow regime of the granular fill in the cross-section of the rotating chamber: 1 – quasi-solid-state flow zone, 2 – non-free-fall zone, 3 – shear layer zone

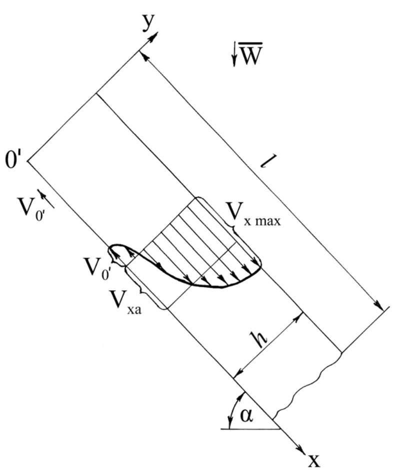

Fig. 2. The simplified calculation scheme of the accelerated gravitational flow of the granular layer on an inclined rough flat surface that is shifted up

In this case, the projections of mass forces on the coordinate axis take the values

$$F_x = W\sin\alpha,$$

$$F_y = W\cos\alpha,$$

where $W=g+a$ is the conditional total vertical acceleration of the layer, which determines its kinematic parameters; a is the conditional additional vertical inertial acceleration due to the growth of kinetic energy of the layer when falling with the interaction of particles after being thrown in the non-free-fall zone 2.

It is possible to introduce new dimensionless parameters that are marked with dashes

$$\bar{\sigma}_x = \frac{\sigma_x}{p},\quad \bar{\sigma}_y = \frac{\sigma_y}{p},\quad \bar{\tau}_{xy} = \frac{\tau_{xy}}{p},$$

$$\bar{V}_x = \frac{V_x}{\sqrt{\dfrac{pg}{\gamma}}},\quad \bar{V}_y = \frac{V_y}{\sqrt{\dfrac{pg}{\gamma}}},$$

$$\bar{F}_x = \frac{F_x}{\dfrac{pg}{\gamma h}},\quad \bar{F}_y = \frac{F_y}{\dfrac{pg}{\gamma h}},$$

$$\bar{\chi} = \frac{h}{l},\quad \bar{x} = \frac{x}{l},\quad \bar{y} = \frac{y}{h}.$$

Then the equations (1)–(5) take the form

$$\bar{F}_x - \left(\bar{\chi}\frac{\partial\bar{\sigma}_x}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{xy}}{\partial\bar{y}}\right) = \bar{\chi}\bar{V}_x\frac{\partial\bar{V}_x}{\partial\bar{x}} + \bar{V}_y\frac{\partial\bar{V}_x}{\partial\bar{y}}, \tag{6}$$

$$\bar{F}_y - \left(\bar{\chi}\frac{\partial\bar{\sigma}_y}{\partial\bar{y}} + \frac{\partial\bar{\tau}_{yx}}{\partial\bar{x}}\right) = \bar{\chi}\bar{V}_x\frac{\partial\bar{V}_y}{\partial\bar{x}} + \bar{V}_y\frac{\partial\bar{V}_y}{\partial\bar{y}}, \tag{7}$$

$$\left(\bar{\sigma}_x - \bar{\sigma}_y\right)^2 + 4\bar{\tau}_{xy}^2 = \left(\bar{\sigma}_x + \bar{\sigma}_y + 2\frac{k}{p}ctg\phi\right)^2 \sin^2\phi, \tag{8}$$

$$\bar{\chi}\frac{\partial\bar{V}_x}{\partial\bar{x}} + \frac{\partial\bar{V}_y}{\partial\bar{y}} = 0, \tag{9}$$

$$\frac{2\bar{\tau}_{xy}}{\bar{\sigma}_x - \bar{\sigma}_y} = \frac{\dfrac{1}{2}\left(\dfrac{\bar{V}_y}{\bar{V}_x} - \dfrac{\bar{V}_x}{\bar{V}_y}\right) \pm tg\phi}{1 \mp \dfrac{1}{2}\left(\dfrac{\bar{V}_y}{\bar{V}_x} - \dfrac{\bar{V}_x}{\bar{V}_y}\right) tg\phi}. \tag{10}$$

The boundary conditions on the moving supporting surface of the layer, formed by a part of the fill in the quasi-solid-state flow, lie in the fact that there is no slipping of the layer relative to the surface, and the velocity vector is directed in parallel to it. Then

$$\bar{V}_x(0) = \bar{V}_{o'}. \tag{11}$$

The boundary conditions on the free surface are the absence of cohesion between the particles of the medium – it is considered ideal granular material with $k=0$.

The shear layer is considered thin – $l>>h$. Then $\sigma_y>>\sigma_x$ and $\sigma_x>>\tau_{xy}$, and the system of equations (6)–(10) takes the form

$$\bar{F}_x = \bar{V}_y\frac{\partial\bar{V}_x}{\partial\bar{y}}, \tag{12}$$

$$\frac{\bar{V}_y}{\bar{V}_x} - \frac{\bar{V}_x}{\bar{V}_y} = -2tg\phi. \tag{13}$$

The solution of (13) with respect to $\bar{V}_y$ has the form

$$\bar{V}_y = \bar{V}_x \frac{1-\sin\phi}{\cos\phi}. \quad (14)$$

From the equation (12), taking into account (14) and (11), it is possible to obtain the expression for $\bar{V}_x$

$$\bar{V}_x = \sqrt{\left|2\bar{F}_x \frac{\cos\phi}{1-\sin\phi}\bar{y} - \bar{V}_{o'}^2\right|}, \quad (15)$$

where

$$\bar{V}_x \le 0 \text{ with } \left(2\bar{F}_x \frac{\cos\phi}{1-\sin\phi}\bar{y} - \bar{V}_{o'}^2\right) \le 0,$$

$$\bar{V}_x \ge 0 \text{ with } \left(2\bar{F}_x \frac{\cos\phi}{1-\sin\phi}\bar{y} - \bar{V}_{o'}^2\right) \ge 0.$$

From (15) we can determine the average flow velocity of the fill in the shear layer

$$\bar{V}_{xa} = \int_0^h \bar{V}_x(\bar{y})\mathrm{d}\bar{y}. \quad (16)$$

The solution of (16) has the form

$$\bar{V}_{xa} = \frac{1}{3}\cdot\frac{1-\sin\phi}{\cos\phi}\left[\left(\left|\frac{2\bar{F}_x\cos\phi}{1-\sin\phi} - \bar{V}_{o'}^2\right|\right)^{1,5} + \bar{V}_{o'}^3\right]. \quad (17)$$

The expressions (15) and (17) in absolute values take the form

$$V_x = \sqrt{\left|2W\sin\alpha\frac{\cos\phi}{1-\sin\phi}y - V_{o'}^2\right|}, \quad (18)$$

where

$$V_x \le 0 \text{ with } \left(2W\sin\alpha\frac{\cos\phi}{1-\sin\phi}y - V_{o'}^2\right) \le 0$$

and

$$V_x \ge 0 \text{ with } \left(2W\sin\alpha\frac{\cos\phi}{1-\sin\phi}y - V_{o'}^2\right) \ge 0,$$

$$V_{xa} = \frac{\left|V_{o'}^3\right|(1-\sin\phi)}{3hW\sin\alpha\cos\phi}\left\{\left[\frac{2hW\sin\alpha\cos\phi}{V_{o'}^2(1-\sin\phi)} - 1\right]^{1,5} + 1\right\}. \quad (19)$$

The conditional acceleration W is an initially unknown value, which may vary along the layer depending on its geometric and kinematic parameters. To determine W, it is possible to transform (19)

$$V_{xa} = \frac{c}{W}\left[\left(d\cdot W - V_{o'}^2\right)^{1,5} + \left|V_{o'}^3\right|\right], \quad (20)$$

where

$$c = \frac{1-\sin\phi}{3h\sin\alpha\cos\phi}, \quad d = \frac{2h\sin\alpha\cos\phi}{1-\sin\phi}.$$

The solution of (20) with respect to W has the form

$$W = \sqrt[3]{-\frac{L}{2} + \sqrt{f}} + \sqrt[3]{-\frac{L}{2} - \sqrt{f}} - \frac{q}{3}, \quad (21)$$

where

$$f = \left(-\frac{q^2}{9} + \frac{m}{3}\right)^3 + \left(\frac{L}{2}\right)^2, \quad L = 2\left(\frac{q}{3}\right)^3 - \frac{qm}{3} - 2\frac{V_{o'}^6}{d^3},$$

$$q = -\frac{1}{d^3}\left(\frac{V_{xa}^2}{c^2} + 3d^2V_{o'}^2\right), \quad m = \frac{1}{d^3}\left(2\frac{V_{xa}}{c}\left|V_{o'}^3\right| + 3dV_{o'}^4\right).$$

Based on the simplified calculation scheme (Fig. 2) of the layer flow along an inclined flat surface, it is possible to proceed to the refined scheme (Fig. 3). The stabilized, previously additionally accelerated in the non-free-fall zone II, gravitational flow of the shear layer zone III along the supporting surface of the quasi-solid-state zone I of the granular fill of the cylindrical chamber of radius R, which rotates stationarily around the horizontal axis, is assumed.

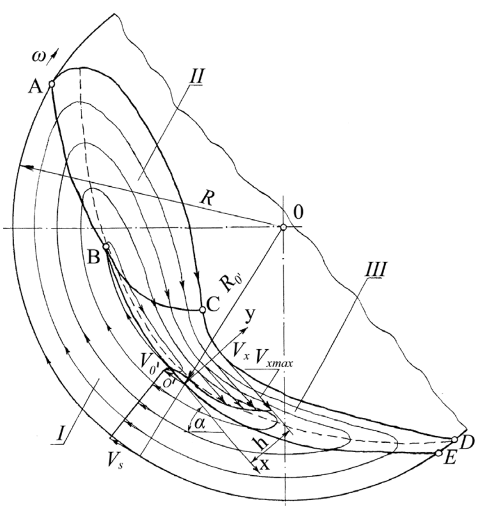

Fig. 3. The refined calculation scheme of the stabilized, additionally accelerated in the non-free-fall zone, gravitational flow of the shear layer on the surface of the quasi-solid-state zone of the fill of the rotating chamber

To perform calculation of the desired velocity profile in a certain section of the shear layer of the fill using the refined scheme (Fig. 3), the position of the origin point O' of the coordinates on the surface BE is selected. The direction of the axes of these coordinates corresponds to the tangent and normal to the supporting surface of the layer. The position of the axes defines the value of the layer thickness h and the angle of inclination of its supporting surface to the horizontal α. Having determined the value of the conditional additional acceleration W from (21), it is possible to calculate the distribution of shear velocities by the normal to the direction of the shear layer flow using the expression (18).

The value of the velocity of the point O' in (18)–(21) is determined by its radial coordinate $R_{o'}$.

The value of the average velocity of the fill flow in the shear layer in (21) is given by the condition of equality of the flow rate in the normal cross-section of the shear layer and in the radial cross-section of the solid-state zone between the point O' and the chamber surface, in the linear range from $V_{o'}$ to $V_s=\omega R$,

$$V_{xa}=\frac{\omega\left(R^2-R_{o'}^2\right)}{2h}. \qquad (22)$$

The equations (18), (21) and (22) allow determining and analyzing the velocity distribution in the cross-section of the shear layer of the fill, depending on the variation of parameters of the system under consideration.

6. Discussion of the results of the research on the influence of the parameters of the filled rotating chamber on the velocity distribution in the cross-section of the shear layer of the fill

The mathematical description of the distribution of shear velocities along the normal to the flow of the shear layer of the granular fill of the rotating chamber was performed according to the classical scheme using the volume-averaged quantities. The assumption was adopted as to the slowness of the flow of the cohesionless granular medium with the predominance of plastic forces over viscous inertial forces. This allowed obtaining the equations that approximately define the dependences of the velocity profile on a number of parameters of the system under consideration:

– geometric: the chamber radius R, the radial coordinate of the basis of the normal section of the shear layer $R_{o'}$, the thickness of this layer h and the filling degree of the chamber κ;

– kinematic: the angular rotation velocity of the chamber ω;

– rheological: the angle of internal friction of the fill φ and the angle of inclination of the supporting surface of the shear layer to the horizontal α.

The advantage of the proposed approach, in comparison with the traditional hypothesis [1], is the possibility to determine such a velocity profile, depending on the values of the parameters R, $R_{o'}$, h, κ, ω, φ and α. However, the traditional model of the two-phase flow regime of the granular fill of the rotating chamber does not provide for the flow simulation of the shear layer.

The analysis of (18), (19) and (22) shows that with increasing values of the parameters ω, R, $R_{o'}$ and α and decreasing values of h and φ, the values of the average V_{xa} and maximum $V_{x\max}$ increase, with $y=h$, the flow velocities of the shear layer of the fill increase. In addition, the values of V_{xa} and $V_{x\max}$ increase with decreasing κ, due to the increase in $R_{o'}$ and decrease of h.

The disadvantages of the developed algorithm of numerical calculation of the velocity profile include the need for a preliminary experimental determination of a number of geometric flow parameters. Such parameters are the radial coordinate of the basis of the chosen normal section of the shear layer $R_{o'}$, the thickness h and the angle of inclination of the layer in this section to the horizontal α. For this purpose, for example, a method of visual analysis of the flow patterns in the chamber cross-section can be applied. In addition, the distribution of shear velocities significantly depends on the choice of the position of the section of the fill layer.

In the future, it will be appropriate to calculate the flow patterns of the fill in the chamber section, taking into account the characteristics of the velocity profile of the shear layer determined in the work, as well as the characteristics of the boundary of destruction of the quasi-solid-state zone obtained in [31]. It also seems advisable to perform a comparative analysis of the flow patterns obtained by calculation and experimental methods in a wide variation range of system parameters. This will allow clarifying the mechanism of appearance of the three-phase flow regime of the granular fill of the rotating chamber.

7. Conclusions

1. The fields of stresses and velocities in the fill mass in the cross-section of the rotating chamber are formalized using the system of differential equations of the two-dimensional state of the flowing granular medium. The equation that approximates the velocity distribution in the cross-section of the shear layer, which is normal to the flow direction, depending on the rotation velocity, geometric and rheological parameters of the system are obtained.

2. The condition of the steady gravitational flow of the shear layer of the fill of the rotating chamber is obtained. It was revealed that such gravitational flow arises under the action of a conditional, additional to gravitational, vertical inertial acceleration, which is due to the previous growth of kinetic energy of the layer in the non-free-fall zone of the fill. The flow patterns of the shear layer of the granular fill in the cross-section of the cylindrical chamber rotating around the horizontal axis are determined.

3. It was found that the flow of the shear layer near the free fill surface is realized in the form of a gravitational flow without slipping along the supporting boundary surface of the quasi-solid-state fill zone that is shifted up. The mean value and the distribution of shear velocities along the normal to the flow direction of the layer in the variation range from the minimum velocity of the supporting surface of the quasi-solid-state zone that is shifted up to the maximum velocity of the free surface of the layer shifted downwards are determined.

4. It was found that the average value and velocity distribution in the normal cross-section of the shear layer depend on the camera radius R, the radial coordinate of the basis of the normal cross-section of the shear layer $R_{o'}$, the layer thickness h, the filling degree κ and the angular rotation velocity of the chamber ω, the angle of internal friction of the fill φ and the angle of inclination of the shear layer to the horizontal α. It was revealed that with increasing values of the parameters ω, R, $R_{o'}$ and α and decreasing values of h, κ and φ, the average and maximum velocity of the shear layer of the fill increase.

References

1. Andreev, S. E. Droblenie, izmel'chenie i grohochenie poleznyh iskopaemyh [Text] / S. E. Andreev, V. A. Perov, V. V. Zverevich. – Moscow: Nedra, 1980. – 415 p.
2. Naumenko, Yu. V. The antitorque moment in a partially filled horizontal cylinder [Text] / Yu. V. Naumenko // Theoretical Foundations of Chemical Engineering. – 1999. – Vol. 33, Issue 1. – P. 91–95.

3. Naumenko, Yu. V. Determination of rational rotation speeds of horizontal drum machines [Text] / Yu. V. Naumenko // Metallurgical and Mining Industry. – 2000. – Issue 5. – P. 89–92.
4. Brewster, R. Effects of cohesion on the surface angle and velocity profiles of granular material in a rotating drum [Text] / R. Brewster, G. S. Grest, A. J. Levine // Physical Review E. – 2009. – Vol. 79, Issue 1. doi: 10.1103/physreve.79.011305
5. Third, J. R. Tangential velocity profiles of granular material within horizontal rotating cylinders modelled using the DEM [Text] / J. R. Third, D. M. Scott, S. A. Scott, C. R. Muller // Granular Matter. – 2010. – Vol. 12, Issue 6. – P. 587–595. doi: 10.1007/s10035-010-0199-2
6. Demagh, Y. Surface particle motions in rotating cylinders: Validation and similarity for an industrial scale kiln [Text] / Y. Demagh, H. Ben Moussa, M. Lachi, S. Noui, L. Bordja // Powder Technology. – 2012. – Vol. 224. – P. 260–272. doi: 10.1016/j.powtec.2012.03.002
7. Cheng, N.-S. Application of incomplete similarity theory for estimating maximum shear layer thickness of granular flows in rotating drums [Text] / N.-S. Cheng, Q. Zhou, S. Keat Tan, K. Zhao // Chemical Engineering Science. – 2011. – Vol. 66, Issue 12. – P. 2872–2878. doi: 10.1016/j.ces.2011.03.050
8. Pignatel, F. Parameters and scalings for dry and immersed granular flowing layers in rotating tumblers [Text] / F. Pignatel, C. Asselin, L. Krieger, I. C. Christov, J. M. Ottino, R. M. Lueptow // Physical Review E. – 2012. – Vol. 86, Issue 1. doi: 10.1103/physreve.86.011304
9. Felix, G. Granular flows in a rotating drum: the scaling law between velocity and thickness of the flow [Text] / G. Felix, V. Falk, U. D'Ortona // The European Physical Journal E. – 2007. – Vol. 22, Issue 1. – P. 25–31. doi: 10.1140/epje/e2007-00002-5
10. Hsu, L. Experimental study of bedrock erosion by granular flows [Text] / L. Hsu, W. E. Dietrich, L. S. Sklar // Journal of Geophysical Research. – 2008. – Vol. 113, Issue F2. doi: 10.1029/2007jf000778
11. Cheng, N.-S. Scaling law for velocity profiles of surface granular flows observed in rotating cylinders [Text] / N.-S. Cheng // Powder Technology. – 2012. – Vol. 218. – P. 11–17. doi: 10.1016/j.powtec.2011.11.017
12. Orpe, A. V. Rheology of surface granular flows [Text] / A. V. Orpe, D. V. Khakhar // Journal of Fluid Mechanics. – 2007. – Vol. 571. – P. 1. doi: 10.1017/s002211200600320x
13. Aissa, A. A. Transverse mixing of polymer powders in a rotary cylinder part I: Active layer characterization [Text] / A. A. Aissa, C. Duchesne, D. Rodrigue // Powder Technology. – 2012. – Vol. 219. – P. 193–201. doi: 10.1016/j.powtec.2011.12.040
14. Mandal, S. An experimental study of the flow of nonspherical grains in a rotating cylinder [Text] / S. Mandal, D. V. Khakhar // AIChE Journal. – 2017. doi: 10.1002/aic.15772
15. Sanfratello, L. Velocity depth profile of granular matter in a horizontal rotating drum [Text] / L. Sanfratello, A. Caprihan, E. Fukushima // Granular Matter. – 2006. – Vol. 9, Issue 1-2. – P. 1–6. doi: 10.1007/s10035-006-0023-1
16. Alizadeh, E. Characterization of Mixing and Size Segregation in a Rotating Drum by a Particle Tracking Method [Text] / E. Alizadeh, O. Dube, F. Bertrand, J. Chaouki // AIChE Journal. – 2013. – Vol. 59, Issue 6. – P. 1894–1905. doi: 10.1002/aic.13982
17. Chou, H. T. The effects of particle density and interstitial fluid viscosity on the dynamic properties of granular slurries in a rotating drum [Text] / H. T. Chou, S. H. Chou, S. S. Hsiau // Powder Technology. – 2014. – Vol. 252. – P. 42–50. doi: 10.1016/j.powtec.2013.10.034
18. Rasouli, M. A multiple radioactive particle tracking technique to investigate particulate flows [Text] / M. Rasouli, F. Bertrand, J. Chaouki // AIChE Journal. – 2014. – Vol. 61, Issue 2. – P. 384–394. doi: 10.1002/aic.14644
19. Govender, I. A positron emission particle tracking investigation of the flow regimes in tumbling mills [Text] / I. Govender, T. Pathmathas // Journal of Physics D: Applied Physics. – 2016. – Vol. 50, Issue 3. – P. 035601. doi: 10.1088/1361-6463/aa5125
20. Rasouli, M. Investigating the dynamics of cylindrical particles in a rotating drum using multiple radioactive particle tracking [Text] / M. Rasouli, O. Dube, F. Bertrand, J. Chaouki // AIChE Journal. – 2016. – Vol. 62, Issue 8. – P. 2622–2634. doi: 10.1002/aic.15235
21. Sheng, L.-T. Influence of particle surface roughness on creeping granular motion [Text] / L.-T. Sheng, W.-C. Chang, S.-S. Hsiau // Physical Review E. – 2016. – Vol. 94, Issue 1. doi: 10.1103/physreve.94.012903
22. Alizadeh, E. Comparison of DEM results and Lagrangian experimental data for the flow and mixing of granules in a rotating drum [Text] / E. Alizadeh, F. Bertrand, J. Chaouki // AIChE Journal. – 2013. – Vol. 60, Issue 1. – P. 60–75. doi: 10.1002/aic.14259
23. Komossa, H. Transversal bed motion in rotating drums using spherical particles: Comparison of experiments with DEM simulations [Text] / H. Komossa, S. Wirtz, V. Scherer, F. Herz, E. Specht // Powder Technology. – 2014. – Vol. 264. – P. 96–104. doi: 10.1016/j.powtec.2014.05.021

24. Schlick, C. P. Granular segregation in circular tumblers: theoretical model and scaling laws [Text] / C. P. Schlick, Y. Fan, P. B. Umbanhowar, J. M. Ottino, R. M. Lueptow // Journal of Fluid Mechanics. – 2015. – Vol. 765. – P. 632–656. doi: 10.1017/jfm.2015.4
25. Santos, D. A. A hydrodynamic analysis of a rotating drum operating in the rolling regime [Text] / D. A. Santos, F. O. Dadalto, R. Scatena, C. R. Duarte, M. A. S. Barrozo // Chemical Engineering Research and Design. – 2015. – Vol. 94. – P. 204–212. doi: 10.1016/j.cherd.2014.07.028
26. Santos, D. A. Investigation of particle dynamics in a rotary drum by means of experiments and numerical simulations using DEM [Text] / D. A. Santos, M. A. S. Barrozo, C. R. Duarte, F. Weigler, J. Mellmann // Advanced Powder Technology. – 2016. – Vol. 27, Issue 2. – P. 692–703. doi: 10.1016/j.apt.2016.02.027
27. Ryazhskih, V. I. Stacionarnoe gravitacionnoe dvizhenie sypuchey sredy [Text] / V. I. Ryazhskih, Yu. V. Chernuhin // Teoret. osnovy him. tekhnologii. – 2000. – Vol. 34, Issue 5. – P. 553–554.
28. Dolgunin, V. N. Bystrye gravitacionnye techeniya zernistyh materialov: tekhnika izmereniya, zakonomernosti, tekhnologicheskoe primenenie [Text] / V. N. Dolgunin, V. Ya. Borshchev. – Moscow: Izd-vo Mashinostroenie-1, 2005. – 112 p.
29. Govender, I. Granular flows in rotating drums: A rheological perspective [Text] / I. Govender // Minerals Engineering. – 2016. – Vol. 92. – P. 168–175. doi: 10.1016/j.mineng.2016.03.021
30. Geniev, G. A. Voprosy dinamiki sypuchey sredy [Text] / G. A. Geniev. – Moscow: Gosstroyizdat, 1958. – 122 p.
31. Naumenko, Y. Modeling of fracture surface of the quasi solid-body zone of motion of the granular fill in a rotating chamber [Text] / Y. Naumenko // Eastern-European Journal of Enterprise Technologies. – 2017. – Vol. 2, Issue 1. – P. 50–57. doi: 10.15587/1729-4061.2017.96447