На базі методу V-функції здійснюється продовження оптико-механічної аналогії. Розглядається траєкторно-хвильовий рух частинки. Вказується на наявність квантування енергії частинки, рішення без частинки в разі прямолінійного рівномірного руху з постійною швидкістю. Досліджуються властивості хвильової природи руху електрона в водородоподібному атомі як рішення прямої задачі. Показується спосіб знаходження кінцевого рішення стаціонарного хвильового рівняння

Ключові слова: варіаційний принцип, пряма задача динаміки, зворотна задача динаміки, оптико-механічна аналогія, хвильовий рух, траєкторний рух, хвильова функція, хвильове рівняння

На базе метода V-функции осуществляется продолжение оптико-механической аналогии. Рассматривается траекторно-волновое движение частицы. Указывается на наличие квантования энергии частицы, решения без частицы в случае прямолинейного равномерного движения с постоянной скоростью. Исследуются свойства волновой природы движения электрона в водородоподобном атоме как решение прямой задачи. Показывается способ нахождения конечного решения стационарного волнового уравнения

Ключевые слова: вариационный принцип, прямая задача динамики, обратная задача динамики, оптико-механическая аналогия, волновое движение, траекторное движение, волновая функция, волновое уравнение

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A METHOD OF V-FUNCTION: ULTIMATE SOLUTION TO THE DIRECT AND INVERSE PROBLEMS OF DYNAMICS FOR A HYDROGEN-LIKE ATOM

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1. Introduction

Variational principles and optical-mechanical analogy were essential for the development of quantum mechanics. The analogy of motion of the mechanical conservative systems and the propagation of light rays in an optically heterogeneous medium was first paid attention to in paper [1]. Thus, from the stationary Hamilton-Jacobi equation

$$H\left(\frac{\partial W}{\partial q}, q\right) = E,$$

recorded for a single particle:

$$\sum_{i=1}^{3}\left(\frac{\partial W}{\partial q_i}\right)^2 = 2m(E-U), \quad (1)$$

where $W(q)$ is the Hamilton's characteristic function, $U(q)$ is the potential energy of a particle, E is the total energy of a particle, m is the mass of a particle; and the eikonal equation that describes the propagation of a light ray

$$\sum_{i=1}^{3}\left(\frac{\partial \phi}{\partial q_i}\right)^2 = \frac{1}{\lambda^2}, \quad (2)$$

where $\varphi(q)$ is a function of the eikonal – a light wave phase, λ is the light wavelength, it follows that these equations are similar in the general form.

L. de Broglie shed a new light on the optical-mechanical analogy [2–4]. He considered the correspondence between a wave and a particle based on equations (1) and (2), and on the basis of variational principles by Maupertuis and Fermat. It is the very optical-mechanical analogy at the level of geometrical optics that allowed L. de Broglie to establish wave properties of the particle. Thus, if one puts $\varphi = W/h$, in (2), then we obtain from (1) and (2)

$$\frac{1}{\lambda^2} = \frac{2m(E-U)}{h^2} = \frac{p^2}{h^2}, \quad (3)$$

where h is the Planck's constant.

Optical-mechanical analogy and the ratio of Louis de Broglie (3) were consequently employed by Schrodinger to formulate a wave equation [5].

Experimental achievements in the study of the behavior of separate microscopic systems revive in turn sustainable interest in verifying the basic provisions of quantum theory and stimulate a deeper rethinking of its physical principles, a role of information in the theoretical description of the micro-particle behavior [6, 7].

2. Literature review and problem statement

Optical-mechanical analogy is, first of all, a view of the nature of light. Optical-mechanical analogy has remained relevant up to now [8–10]. Study [8] shows the existence of connection between the trajectories of particles under the action of nonholonomic constraint, and the trajectories of light rays with a variable refractive index. Article [9] provides a proof of the existence of a new optical-mechanical analogy between the equation of rotational motion of the body in mechanics (taking into account the principle of relativity) and the first pair of Maxwell's equations. The Hamilton's optical-mechanical analogy between a material's particle trajectory in potential fields and the trajectory of light rays in media with a continuously changeable refractive index has played an important role in the substantiation of Schrödinger's wave mechanics [10]. In this case, based on the existing variational principles, this analogy is drawn only at the level of geometrical optics. In the given paper, the motion of an object is explored using a V-function method, which consists of a local variational principle (LVP), new statement of the direct and inverse problems of dynamics [11, 12].

In some problems, light manifests itself as a particle. In other problems – as a wave. In other words, a dualism of the wave and the particle is detected. The same dualism manifests itself also for the particles of matter. Continuing attempts to comprehend paradoxical manifestations of a corpuscular-wave dualism in the motion of the electron (as well as other micro-particles) do encourage undertaking new research [13–16].

This makes it possible to argue about theoretical interest in the approach based on corpuscular-wave monism to explaining the nature of the particles (the object). In particular, a theory being developed can apply the description of physical reality where the existence of the particle trajectory is taken into account, which reflects the fact of the existence of the particle, while it is also accepted that the motion of a particle is determined by a physical wave $V(x,t)$.

3. Research goal and objectives

The goal of present work is to demonstrate the capabilities of a V-function method to study the motion of an object.

To accomplish the goal, the following tasks have been set:

– to perform optical-mechanical analogy at the level of wave optics based on the local variational principle, taking into account the wave and the trajectory motion of the object;

– to conduct research into the properties of wave nature of the electron motion in the hydrogen-like atom as a solution to the direct problem of dynamics;

– to devise a technique for finding the ultimate solution to the stationary wave equation for a hydrogen-like atom.

4. Research materials and methods

We shall define the content of a V-function method. Let the trajectory motion of an object is assigned by a system of differential equations from classical physics:

$$\frac{d}{dt}x = f(x), \tag{4}$$

where the vector of phase particle coordinates $x(t) = (x_1, x_2, ..., x_n)^{\mathrm{T}}$ is assigned in n-dimensional Euclidean space $(x \in R^n)$, t is the time.

Along with a system of equations (4), we shall also introduce a wave function $V(x,t)$. The rate of its change for the system being studied (4) will be determined by expression

$$\frac{d}{dt}V = \frac{\partial}{\partial t}V + \frac{\partial}{\partial x}V^{\mathrm{T}} f.$$

Consider an isochronous variation of the rate in the wave function change

$$\delta\left(\frac{d}{dt}V\right) = \frac{\partial}{\partial t}\delta V + \frac{\partial}{\partial x}\delta V^{\mathrm{T}} f + \frac{\partial}{\partial x}V^{\mathrm{T}}\delta f,$$

here

$$\delta V = \frac{\partial}{\partial x}V^{\mathrm{T}}\delta x, \quad \delta f = \frac{\partial}{\partial x} f \delta x.$$

We accept that at a variation in the rate of change in wave function $\delta\left(\frac{d}{dt}V\right)$, an object from a certain initial state passes into the state that is different by a new spatial coordinate $x + \delta x$. Such a transition will be referred to as a wave transition of the object, at which a magnitude δV assigns the possible wave transition from the initial state to the new state, while δx determines trajectory variations. When implementing a wave transition, the spatial variation takes the form of displacement $\delta x = dx = \dot{x}dt$, implemented in space.

Let us formulate an LV principle: *out of all the possible transitions to a new state, the only one, which is actually carried out, is the one at which in each moment a rate of change in the wave function V(x,t) takes a stationary value*

$$\delta\left(\frac{d}{dt}V\right) = 0. \tag{5}$$

By assuming the feasibility of (5), we also accept that a wave function satisfies additional condition for *a full variation in the change rate of wave function V(x,t):*

$$\Delta\left(\frac{d}{dt}V\right) = 0, \tag{6}$$

where $\Delta(.) = \delta(.) + \frac{d}{dt}(.)\Delta t$.

Once we have classical equations (4) and conditions (5), (6), we shall derive a wave equation for $V(x,t)$, taking into account the implementation of wave transition $(\delta x = dx = \dot{x}dt)$ in (5) and (6):

$$\Delta\left(\frac{dV}{dt}\right) = \left\{\frac{\partial^2 V}{\partial t^2} + 3\frac{\partial^2 V}{\partial t \partial x}^T f + 2 f^T W f + 2\frac{\partial V^T}{\partial x}\frac{df}{dt}\right\}dt =$$
$$= 3\delta\left(\frac{dV}{dt}\right) + \left(\frac{\partial^2 V}{\partial t^2} - f^T W f - \frac{\partial V^T}{\partial x}\frac{df}{dt}\right)dt \rightarrow$$
$$\rightarrow \frac{\partial^2 V}{\partial t^2} - f^T W f - \frac{\partial V^T}{\partial x}\frac{df}{dt} = 0, \tag{7}$$

where $V(x,t)$ is the piecewise continuous, finite, single-valued function, $W = [\partial^2_{x_i x_j} V(x,t)]$ is the function matrix. Equation (7) is a necessary and sufficient condition for the feasibility of (6). We shall demonstrate that there is equality

$$\frac{\partial V}{\partial x}^{\mathrm{T}} \frac{d}{dt}\dot{x} = 0. \tag{8}$$

According to the method of a V-function, particle motion occurs in such a manner that at each point in time, the particle's velocity is co-directed with the wave function gradient, that is

$$\frac{\partial}{\partial x}V^{\mathrm{T}}\dot{x} = \left|\frac{\partial}{\partial x}V\right|\left|\dot{x}\right|.$$

Hence, we obtain $\partial V / \partial x = k_2(x)\dot{x}$. Further, we assume that a velocity field in a three-dimensional space coincides with the field gradient corresponding to it, which occurs at $k_2(x) = k_2$, and, accordingly, we obtain equality

$$\partial V / \partial x = k_2\dot{x}. \tag{9}$$

In the case when a wave transition is implemented, relation (5) takes the form

$$\frac{d}{dt}\left(\frac{\partial V}{\partial x}^{\mathrm{T}} \delta x\right) = \frac{d}{dt}\left(\frac{\partial V}{\partial x}^{\mathrm{T}} \dot{x}dt\right) = 0 \Rightarrow \frac{\partial V}{\partial x}^{\mathrm{T}} \dot{x} = \text{const}. \tag{10}$$

Then, taking into account (9) and (10), the equality (8) should hold, that is,

$$\frac{\partial V}{\partial x}^{\mathrm{T}} \frac{d}{dt}\dot{x} = k_2\dot{x}^{\mathrm{T}}\frac{d}{dt}\dot{x} =$$
$$= \frac{k_2}{2}\frac{d}{dt}\left(\dot{x}^{\mathrm{T}}\dot{x}\right) = \frac{1}{2}\frac{d}{dt}\left(\frac{\partial V}{\partial x}^{\mathrm{T}}\dot{x}\right) = 0.$$

Consequently, equation (7), considering (8), takes the form

$$\partial^2_{tt}V - \dot{x}^{\mathrm{T}}W\dot{x} = 0. \tag{11}$$

Moreover, if the following condition is satisfied

$$\dot{x}_j = \lambda_i \frac{\partial \dot{x}_i}{\partial x_j} \ (i, j = \overline{1, n}),$$

equation (11), considering (9), takes the form:

$$\frac{\partial^2 V}{\partial t^2} - \vartheta^2\nabla^2 V = 0, \tag{12}$$

where

$$\vartheta^2 = \sum_{i=1}^{n} \dot{x}_i^{\,2} = \dot{x}^T\dot{x} \ \text{ and } \ \nabla^2 = \sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2}.$$

Equations (4) and (12) describe trajectory and wave motion of the particle being studied. In order to find a solution to this system of equations, it is required to know the boundary conditions. Note that we shall use as the boundary conditions for (12) the properties of a wave on the trajectory of a particle. The proposed approach to the description of particle behavior includes a system from the trajectory equation (4) and the wave equation (12). Further, we shall find the boundary conditions for wave $V(x, t)$ on the trajectory of a particle.

Condition 1.

We obtain from equality (9) a boundary condition for wave in point $x = x_M$ of the trajectory of a particle

$$\partial V / \partial x\big|_{x=x_M} = k_2\dot{x}\big|_{x=x_M}. \tag{13}$$

Condition 2.

Assuming the implementation of wave transition in (5), we shall obtain

$$\frac{\partial}{\partial x}V^{\mathrm{T}}\dot{x} = \text{const}. \tag{14}$$

Employing condition (14), for a full variation (6), we in turn obtain equality

$$\frac{d}{dt}\left(\frac{\partial}{\partial t}V\right) = 0,$$

applying which we find, respectively, condition 2 for wave behavior on the trajectory of a particle

$$\frac{\partial}{\partial t}V\bigg|_{t=0} = k_1, \tag{15}$$

where $k_{1,2}$ are some constants.

Condition 3.

It follows from the condition of connectedness between a wave and a trajectory (wave amplitude $V(x, t)$ is equal to zero in the point of particle position with coordinate $x = x_M$ in time t)

$$V(x = x_M, t) = V(x, t = 0) = 0. \tag{16}$$

Direct problem of dynamics in the method of V-function is stated as follows:

The differential equations are given that describe the motion of an object (4).

It is required to determine wave function $V(x,t)$ that satisfies equation (12). For the case $\dot{x} = \vartheta$ (n=1), we obtain a solution to equation (12) considering (13)–(16) in the following form:

$$V(x,t) = \pm Ae^{\pm i\left(\frac{\omega}{\vartheta}x - \omega t\right)}. \tag{17}$$

Inverse problem of dynamics is stated as follows:

For a given wave function $V(x,t)$, which satisfies equation (12), it is required to derive differential equations of the motion of an object (4).

At the given wave function, a solution to the inverse problem of dynamics immediately follows from (9):

$$\dot{x}_i = k\frac{\partial V}{\partial x_i}. \tag{18}$$

For the one-dimensional case ($n=1$), equation (11) takes the form:

$$\frac{\partial^2 V(x,t)}{\partial t^2} - \frac{\partial^2 V(x,t)}{\partial x^2}\dot{x}^2 = 0. \tag{19}$$

Assume that the wave function is given in the form of a plane wave equation (17), which propagates in the motion direction of the object. Then (17) will satisfy (19) if $\dot{x} = \vartheta$.

In this case, it follows from equality (15), where the wave function is given in the form (17), that

$$\frac{\partial V(x,t)}{\partial t}=\mp iA\omega e^{\pm i\left(\frac{\omega}{\vartheta}x-\omega t\right)}=\text{const}. \tag{20}$$

The constant in the right part (20) is a real number. Therefore, in order to satisfy condition (20), the phase should take the values:

$$\phi=\left(\frac{\omega}{\vartheta}x-\omega t\right)=\omega\left(\frac{x}{\vartheta}-t\right)=\frac{\pi}{2}+\pi n,\ \ (n=0,\ 1,\ 2,\ 3....). \tag{21}$$

Since $\dot{x}=\vartheta\Rightarrow\frac{x}{\vartheta}-t=C,$ equality (21) takes the form:

$$\omega C=\frac{\pi}{2}+\pi n\Rightarrow\omega=\frac{\pi}{2C}(1+2n)=\frac{\omega_0}{2}(1+2n), \tag{22}$$

that is, in solution (17), natural frequencies can take only certain discrete values. Then (20), considering (21) and (22), will take the form:

$$A\omega=A\frac{\omega_0}{2}(1+2n)=\text{const}. \tag{23}$$

This means that equality (22) also takes only discrete values.

From equality (9), considering (17), it follows that

$$\frac{\partial V(x,t)}{\partial x}=\pm i\frac{A\omega}{\vartheta}e^{\pm i\left(\frac{\omega}{\vartheta}x-\omega t\right)}=k_2\dot{x}=k_2\vartheta. \tag{24}$$

Hence, considering (20), we obtain

$$\frac{\partial V(x,t)}{\partial x}\vartheta=k_2\vartheta^2=\text{const}. \tag{25}$$

Equality (25) is nothing else than the fulfillment of (10) at $n=1$.

5. Results of studies into particle motion

5. 1. Continuation of the optical-mechanical analogy

Let us consider the trajectory motion of a particle, which satisfies equation (18) $\dot{x}=k\frac{\partial V}{\partial x}$. Trajectory motion of the particle, as follows from (18), is matched with the wave motion, which satisfies wave equation (19):

$$\frac{\partial^2 V(x,t)}{\partial t^2}-\left(k\frac{\partial V}{\partial x}\right)^2\frac{\partial^2 V(x,t)}{\partial x^2}=0. \tag{26}$$

Function (17) $V(x,t)=Ae^{i(\frac{\omega}{\vartheta}x-\omega t)}$ will satisfy equation (26), if equality (21) holds. In this case, we obtain

$$|A|=\frac{k_2\vartheta^2}{\omega}.$$

Let here $k_2=m$ be the mass of the particle. Then amplitude $|A|$ takes dimensionality of the action. If we accept $A=\frac{h}{2\pi}=\hbar$, h is the Planck's constant, then the rule of energy quantization follows from (23), similar to that by Schrödinger in the case of Planck oscillator. In this case, we obtain from (24) considering (21)

$$\frac{\hbar\omega}{\vartheta}=m\vartheta. \tag{27}$$

By employing the results obtained, it is possible to draw such a correspondence between the wave and the particle [9]

$$\vartheta=\vartheta,\ \ \omega=\frac{m\vartheta^2}{\hbar}=\frac{2E}{\hbar},$$

$$\lambda=\frac{h}{m\vartheta},\ \ A=\hbar. \tag{28}$$

In this case, the wave and trajectory measurements can be described by a single wave function:

$$V(x,t)=Ae^{\pm i\left(\frac{\omega}{\vartheta}x-\omega t\right)}=\hbar e^{\pm i\frac{1}{\hbar}\left(\frac{\hbar\omega}{\vartheta}x-\hbar\omega t\right)}=\hbar e^{\pm i\frac{1}{\hbar}(m\vartheta x-Et)}. \tag{29}$$

In relations (28), principal is the equality between the wave phase velocity and the particle speed, while in quantum mechanics, the particle's speed equals the group velocity of waves by L. de Broglie. The energy quantization condition (23) is produced naturally as a result of solving the inverse problem. According to the second relation in (28), energy is transferred by a particle. In turn, according to the third relation in (28), the pulse of a particle determines wavelength λ, which coincides with the known formula by Louis de Broglie. In the physical sense, wave $V(x,t)$ characterizes properties of the activity that manifests itself in the motion of a particle. Thus, the wave is connected by its node with the location of the particle and thus guides it, however, the particle (trajectory) generates a wave that propagates with it.

In addition, equation (26) has a solution at $k_2=m\to 0$. In this case, we obtain a wave function in the form of a monochromatic flat wave without the particle, which propagates at a given speed and with a given frequency. This can explain the interference pattern when the particle (photon) passes through two slits.

5. 2. Motion of the electron in a hydrogen-like atom

Let us consider the motion of an object (a particle) in a 3-dimensional potential force field in the Cartesian coordinate system. Let the trajectory equations of the object (the particle) (4) allow the first integral of motion in the form of the law of conservation of energy of a particle, that is,

$$\frac{m(\dot{x}^2+\dot{y}^2+\dot{z}^2)}{2}+U(x,y,z)=E, \tag{30}$$

where m is the mass of a particle, E is the total energy of a particle, U is the potential energy of a particle. Then the object motion (of the particle) is fully described by the following system of equations (30) and (12):

$$\begin{cases}\frac{m\vartheta^2}{2}+U=E,\\ \frac{\partial^2 V}{\partial t^2}-\vartheta^2\nabla^2 V=0,\end{cases} \tag{31}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operator, $\vartheta^2=$ $=\dot{x}^2+\dot{u}^2+\dot{z}^2$ is the square of the particle velocity. Hence, the second equation, considering the first one, takes the form:

$$\frac{\partial^2 V}{\partial t^2}-\frac{2(E-U)}{m}\nabla^2 V=0. \tag{32}$$

We shall apply a method of separation of variables in equation (32) $\left(V=X(x,y,z)T(t)\right)$,

$$\frac{\frac{d^2T(t)}{dt^2}}{T(t)}=\frac{2(E-U)\nabla^2 X(x,y,z)}{mX(x,y,z)}=-\omega^2. \tag{33}$$

Consequently, we obtain the following stationary equation

$$\frac{2(E-U)}{m}\nabla^2 X+\omega^2 X=0. \tag{34}$$

As is known, potential energy of a hydrogen-like atom is equal to

$$U(r)=-Ze^2/r. \tag{35}$$

Then equation (34), considering (35), takes the form

$$\left(-\beta_0^2+\frac{\alpha}{r}\right)\Delta X+\omega^2 X=0, \tag{36}$$

where

$$\beta_0^2=-\frac{2E}{m},\quad \alpha=\frac{2Ze^2}{m}.$$

In equation (36), we shall proceed to a spherical coordinate system

$$\left(-\beta_0{}^2+\frac{\alpha}{r}\right)\times$$
$$\times\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]X+$$
$$+\omega^2 X=0. \tag{37}$$

where

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+$$
$$+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}=\Delta$$

is the Laplace operator in a spherical coordinate system.

We shall search only for the spherically symmetric solutions. Then $X=R(r)$,

$$\Delta=\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)=$$
$$=\frac{1}{r^2}\left(2r\frac{d}{dr}+r^2\frac{d^2}{dr^2}\right)=\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}$$

and equation (37) takes the form

$$\left(-\beta_0{}^2+\frac{\alpha}{r}\right)\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\omega^2 R=0. \tag{38}$$

We shall replace $R=\frac{u}{r}$ in equation (38) to obtain

$$\left(-\beta_0{}^2+\frac{\alpha}{r}\right)\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\frac{u}{r}}{dr}\right)+\omega^2\frac{u}{r}=0\Rightarrow$$
$$\Rightarrow\left(-\beta_0{}^2+\frac{\alpha}{r}\right)\frac{1}{r}\frac{d^2u}{dr^2}+\omega^2\frac{u}{r}=0\Rightarrow$$
$$\Rightarrow\frac{d^2u}{dr^2}+\frac{\omega^2 r}{\alpha-\beta^2 r}=0. \tag{39}$$

Represent equation (39) in the following form

$$\frac{d^2u}{dr^2}+\left(\frac{k_0{}^2\alpha}{\alpha-\beta_0{}^2 r}-k_0{}^2\right)u=0, \tag{40}$$

where

$$k_0^2=\frac{\omega^2}{\beta_0^2}=-\frac{\omega^2 m}{2E}.$$

The solution to be obtained to the direct problem of dynamics for equation (40) must satisfy natural condition $u(r=r_0)=0$, (here $r_0=\alpha/\beta_0^2=-Ze^2/E=Ze^2/|E|$), which corresponds to the implementation of boundary conditions (16), at which amplitude of the wave becomes zero at $r=r_0$, where, accordingly, as a solution to the inverse problem, the particle acquires a trajectory. Find the asymptotic solution to equation (40) at $r\to\infty$.

$$\frac{d^2u}{dr^2}-k_0{}^2u=0. \tag{41}$$

We shall record a general solution to equation (41) in the form

$$u_\infty=u_-(r)+u_+(r)=e^{-k_0 r}f_-(r)+e^{k_0 r}f_+(r).$$

Then

$$u'=\pm k_0{}^2e^{\pm k_0 r}f_\pm(r)+e^{\pm k_0 r}f'_\pm(r),$$
$$u''=e^{\pm k_0 r}(f''_\pm(r)\pm 2k_0 f'_\pm(r)+k_0{}^2 f_\pm(r))$$

and equation (40) takes the form

$$f_\pm''(r)\pm 2k_0 f_\pm'(r)+\frac{\beta_1}{r_0-r}f_\pm(r)=0, \tag{42}$$

where

$$\beta_1=k_0^2\alpha/\beta_0^2=\tfrac{1}{2}Ze^2\omega^2 m_e/E^2,$$
$$r_0=\alpha/\beta_0^2=-Ze^2/E=Ze^2/|E|. \tag{43}$$

Solution to equation (42) will be searched for in the form of the following power series

$$f_\pm(r)=\sum_{m=0}^{\infty}a_m^{(\pm)}(r_0-r)^m, \tag{44}$$

where a particle trajectory actually becomes localized on the surface of radius $r=r_0$. Equation (42) after the given substitution of (44) takes the form

$$\sum_{m=0}^{\infty} m(m-1)a_m^{(\pm)}(r_0-r)^{m-2} \pm$$
$$\pm 2k_0\sum_{m=0}^{\infty} -ma_m^{(\pm)}(r_0-r)^{m-1}+\beta_1\sum_{m=0}^{\infty} a_m^{(\pm)}(r_0-r)^{m-1}=0 \Rightarrow$$
$$\Rightarrow \sum_{n=0}^{\infty}[(n+1)na_{n+1}^{(\pm)} \mp 2k_0 na_n^{(\pm)}+\beta_1 a_n^{(\pm)}](r_0-r)^{n-1}=0. \quad (45)$$

Equality (45) is identically fulfilled only when $r=r_0$, or when all coefficients of the obtained series are equal to zero, that is,

$$(n+1)na_{n+1}^{(\pm)} \mp 2k_0 na_n^{(\pm)}+\beta_1 a_n^{(\pm)}=0.$$

Hence, it follows that $a_0=0$, while coefficients $a_{n\geq 1}^{(\pm)}$ satisfy recurrent relation

$$a_{n+1}^{(\pm)}=\frac{\pm 2k_0 n-\beta_1}{(n+1)n}a_n^{(\pm)}. \quad (46)$$

Since, based on the inverse problem of dynamics, we search for the trajectory of a particle that holds provided

$$\beta_1=2k_0 n \ \ (\beta_1=\tfrac{1}{2}Ze^2\omega^2 m_e/E^2,$$
$$k_0^2=-\frac{\omega^2 m}{2E}, \ \ \beta_1>0, k_0>0). \quad (47)$$

The given condition is satisfied only when series

$$f_+(r)=\sum\nolimits_{m=1}^{\infty} a_m^{(+)}(r_0-r)^m$$

is discontinued, that is, $a_m^{(+)}=0$ at $m\geq n+1$, which leads to the following solution

$$u_{+,n}(r)=Ce^{k_{0,n}r}\sum\nolimits_{m=1}^{n} a_m^{(+)}(r_{0,n}-r)^m, \quad (48)$$

where C is the constant.

Considering equality (47) and

$$k_0^2=\frac{\omega^2}{\beta_0^2}=-\frac{\omega^2 m}{2E},$$

we shall obtain $k_0^2=\dfrac{\beta^2}{4n^2}$,

$$-\frac{\omega^2 m}{2E}=\frac{Z^2e^4\omega^4 m_e{}^2}{16E^4n^2} \Rightarrow$$
$$\Rightarrow -\frac{E^4\omega^2}{E\omega^4}=\frac{2Z^2e^4m_e{}^2}{16n^2m_e} \Rightarrow \frac{E^3}{\omega^2}=-\frac{Z^2e^4m_e}{8}\frac{1}{n^2}. \quad (49)$$

Since, from the results of optical-mechanical analogy (28) we have $\omega^2=\left(\dfrac{2E}{\hbar}\right)^2$, then we shall obtain, from relation (49), the energy value of the n-th state of the particle (a rule of energy quantization)

$$E_n=-\frac{Z^2e^4m_e}{2\hbar^2}\frac{1}{n^2}, \quad (50)$$

which exactly coincides with the solution obtained in the Bohr model [17], or based on the stationary Schrödinger equation in paper [5].

We shall record a radius of the n-th state of the particles considering (50)

$$r_{0,n}=-\frac{Ze^2}{E_n}=\frac{2\hbar^2n^2}{Ze^2m_e}. \quad (51)$$

Next, we shall search for the ultimate solution to equation (40), because solution (48) approaches infinity $u(r)\to\infty$ at $r\to\infty$. For this purpose, consider a general solution

$$u=u_-(r)+u_+(r)=e^{-k_{0,n}r}f_-(r)+e^{k_{0,n}r}f_+(r)=$$
$$C_1e^{-k_{0,n}r}\sum\nolimits_{m=1}^{\infty}a_m^{(-)}(r_{0,n}-r)^m+C_2e^{k_{0,n}r}\sum\nolimits_{m=1}^{n}a_m^{(+)}(r_{0,n}-r)^m.$$

Here solution $u_-(r)$ is considered also in the form of a power series, but a series, as it follows from relation (46), is not discontinued.

It follows from (46) that for sufficiently large values of n, a relation of two coefficients of series (44) takes the form

$$\frac{a_{n+1}^{(\pm)}}{a_n^{(\pm)}}=\frac{\pm 2k_0}{n+1}.$$

But it is the very relationship that exists between two adjacent terms of a series

$$e^{2k_0 r}=1+\frac{2k_0r}{1!}+\ldots+\frac{(2k_0)^n r^n}{n!}+\frac{(2k_0)^{n+1}r^{n+1}}{(n+1)!},$$

$$\frac{(2k_0)^{n+1}n!}{(n+1)!(2k_0)^n}=\frac{2k_0}{n+1}.$$

Therefore, at $r\to\infty$, there are asymptotics $u_\pm(r)\to e^{k_{0,n}r}$, which is why, in order $u_\infty\to 0$, the ultimate solution will be sought for in the form of $u=u_-(r)-u_+(r)$. For this purpose, we shall consider the form that functions $u_{-,n}(r)$ and $u_{+,n}(r)$ take at m=1, 2, 3...n.

Because, at $m=1$, functions $u_-(r), u_+(r)$ take the following form:

$$u_{+,1}=e^{k_{0,1}r}a_1^{(+)}(r_{0,1}-r)$$

$$u_{-,1}=e^{-k_{0,1}r}\sum\nolimits_{m=1}^{\infty}a_m^{(-)}(r_{0,1}-r)^m=$$
$$=e^{-k_{0,1}r}\sum\nolimits_{m=1}^{\infty}\frac{(-1)^{m-1}(2k_{0,1})^{m-1}}{(m-1)!}a_1^{(-)}(-1)^m(r-r_{0,1})^m=$$
$$=-e^{-k_{0,1}r}\sum\nolimits_{m=1}^{\infty}\frac{(2k_{0,1})^{m-1}}{(m-1)!}a_1^{(-)}(r-r_{0,1})^m=$$
$$=-e^{-k_{0,1}r}(r-r_{0,1})a_1^{(-)}\sum\nolimits_{m=1}^{\infty}\frac{(2k_{0,1})^{m-1}}{(m-1)!}(r-r_{0,1})^{m-1}=$$
$$=-e^{-k_{0,1}r}(r-r_{0,1})a_1^{(-)}e^{2k_{0,1}(r-r_{0,1})}=\alpha_1 e^{k_{0,1}r}(r_{0,1}-r),$$

where equality (47) is taken into account, which takes the form $\beta_{1,1}=2k_{0,1}$ and the recurrent relationship (46) for $a_m^{(-)}$ in the following form

$$a_m^{(-)}=\frac{(-1)^{m-1}(2k_{0,1})^{m-1}}{(m-1)!}a_1^{(-)}.$$

Hence, it is clear that if $a_1^{(+)}=\alpha_1=a_1^{(-)}e^{-2k_{0,1}r_{0,1}}$, then $u_{+,1}(r)=u_{-,1}(r)$.

Let $m=2$, then $\beta_{1,2}=4k_{0,2}$,

$$a_2^{(+)}=\frac{2k_{0,2}-\beta_{1,2}}{2\cdot 1}a_1^{(+)}=\frac{-2k_{0,2}}{2\cdot 1}a_1^{(+)}=-k_{0,2}a_1^{(+)},$$

$$u_{+,2}=e^{k_{0,2}r}a_1^{(+)}\left(\left(r_{0,2}-r\right)-k_{0,2}\left(r_{0,2}-r\right)^2\right),$$

$$a_2^{(-)}=\frac{-2k_{0,2}-\beta_{1,2}}{2\cdot 1}a_1^{(-)}=\frac{(-1)^1(2k_{0,2})3}{2\cdot 1}a_1^{(-)},$$

$$a_3^{(-)}=\frac{-2k_{0,2}\cdot 2-\beta_{1,2}}{3\cdot 2}a_2^{(-)}=\frac{(-8\cdot k_{0,2})}{3\cdot 2}\frac{(-6\cdot k_{0,2})}{2\cdot 1}a_1^{(-)}=$$
$$=\frac{(-1)^2(2k_{0,2})^2 4\cdot 3}{(3\cdot 2)(2\cdot 1)}a_1^{(-)},...,$$
$$a_m^{(-)}=\frac{(-1)^{m-1}(2k_{0,2})^{m-1}((m+1)\cdot m\cdot(m-1)\cdots 4\cdot 3)}{(m\cdot(m-1)\cdots 3\cdot 2)((m-1)\cdots 3\cdot 2\cdot 1)}a_1^{(-)}=$$
$$=(-1)^{m-1}(2k_{0,2})^{m-1}\frac{(m+1)}{2\cdot(m-1)!}a_1^{(-)},$$

$$u_{-,2}=e^{-k_{0,2}r}a_1^{(-)}\sum_{m=1}^{\infty}\frac{(-1)^{m-1}(2k_{0,2})^{m-1}}{(m-1)!}\frac{m+1}{2}\left(r_{0,2}-r\right)^m=$$
$$=e^{-k_{0,2}r}a_1^{(-)}\sum_{m=1}^{\infty}\frac{(-1)^{m-1}(2k_{0,2})^{m-1}}{(m-1)!}\frac{m+1}{2}(-1)^m\left(r-r_{0,2}\right)^m=$$
$$=-e^{-k_{0,2}r}a_1^{(-)}\sum_{m=1}^{\infty}\frac{(2k_{0,2})^{m-1}}{(m-1)!}\frac{m+1}{2}\left(r-r_{0,2}\right)^m=$$
$$=\frac{-e^{-k_{0,2}r}a_1^{(-)}}{2}\left((r-r_{0,2})^2e^{2k_{0,2}(r-r_{0,2})}\right)'=$$
$$=-e^{-k_{0,2}r}a_1^{(-)}e^{2k_{0,2}(r-r_{0,2})}\left((r-r_{0,2})+k_{0,2}(r-r_{0,2})^2\right)=$$
$$e^{k_{0,2}r}\alpha_2\left((r_{0,2}-r)-k_{0,2}(r_{0,2}-r)^2\right).$$

If $a_1^{(+)}=\alpha_2=a_1^{(-)}e^{-2k_{0,2}r_{0,2}}$, then $u_{+,2}(r)=u_{-,2}(r)$.
Let *m=3,* then $\beta_{1,3}=6k_{0,3}$,

$$a_2^{(+)}=\frac{2k_{0,3}-\beta_{1,3}}{2\cdot 1}a_1^{(+)}=\frac{-4k_{0,3}}{2\cdot 1}a_1^{(+)},$$

$$a_3^{(+)}=\frac{-2k_{0,3}}{3\cdot 2}a_2^{(+)}=\frac{(-2k_{0,3})}{3\cdot 2}\frac{(-4k_{0,3})}{2\cdot 1}a_1^{(+)},$$

$$u_{+,3}=\frac{e^{k_{0,3}r}a_1^{(+)}}{3!}\times$$
$$\times\left(6\left(r_{0,3}-r\right)-6(2k_{0,3})\left(r_{0,3}-r\right)^2+(2k_{0,3})^2\left(r_{0,3}-r\right)^3\right),$$

$$a_2^{(-)}=\frac{-2k_{0,3}-\beta_{1,3}}{2\cdot 1}a_1^{(-)}=\frac{(-1)^1(2k_{0,3})4}{2\cdot 1}a_1^{(-)},$$

$$a_3^{(-)}=\frac{-2k_{0,3}\cdot 2-\beta_{1,3}}{3\cdot 2}a_2^{(-)}=$$
$$=\frac{(-10\cdot k_{0,3})}{3\cdot 2}\frac{(-8\cdot k_{0,3})}{2\cdot 1}a_1^{(-)}=\frac{(-1)^2(2k_{0,3})^2 5\cdot 4}{(3\cdot 2)(2\cdot 1)}a_1^{(-)},...,$$

$$a_m^{(-)}=\frac{(-1)^{m-1}(2k_{0,3})^{m-1}((m+2)\cdot(m+1)\cdot m\cdots 5\cdot 4)}{(m\cdot(m-1)\cdots 3\cdot 2)((m-1)\cdots 3\cdot 2\cdot 1)}a_1^{(-)}=$$
$$=(-1)^{m-1}(2k_{0,3})^{m-1}\frac{(m+2)(m+1)}{3\cdot 2\cdot(m-1)!}a_1^{(-)},$$

$$u_{-,3}=e^{-k_{0,3}r}a_1^{(-)}\sum_{m=1}^{\infty}\frac{(-1)^{m-1}(2k_{0,3})^{m-1}}{(m-1)!}\frac{(m+2)(m+1)}{3\cdot 2}(r_{0,3}-r)^m=$$
$$=\frac{-e^{-k_{0,3}r}a_1^{(-)}}{3!}\left((r-r_{0,3})^3e^{2k_{0,3}(r-r_{0,3})}\right)''=$$
$$=-\frac{e^{-k_{0,3}r}a_1^{(-)}e^{2k_{0,3}(r-r_{0,3})}}{3!}\times$$
$$\times\left(6(r-r_{0,3})+6(2k_{0,3})(r-r_{0,3})^2-(2k_{0,3})^2(r-r_{0,3})^3\right)=$$
$$=\frac{e^{k_{0,2}r}\alpha_3}{3!}\left(6(r_{0,3}-r)-6(2k_{0,3})(r_{0,3}-r)^2+(2k_{0,3})^2(r_{0,3}-r)^3\right).$$

If $a_1^{(+)}=\alpha_3=a_1^{(-)}e^{-2k_{0,3}r_{0,3}}$, then $u_{+,3}(r)=u_{-,3}(r)$.
Let *m=n*, then $\beta_{1,n}=2nk_{0,n}$,

$$u_{+,n}(r)=e^{k_{0,n}r}\sum_{m=1}^{n}a_m^{(+)}\left(r_{0,n}-r\right)^m=$$
$$=e^{k_{0,n}r}a_1^{(+)}\left((r_{0,n}-r)+\frac{2k_{0,n}(1-n)}{1\cdot 2}(r_{0,n}-r)^2+\right.$$
$$+\frac{(2k_{0,n})^2(1-n)(2-n)}{(3\cdot 2)(1\cdot 2)}(r_{0,n}-r)^3+$$
$$\left.+\cdots+\frac{(2k_{0,n})^{n-1}(1-n)(2-n)\cdots 1}{n!(n-1)!}(r_{0,n}-r)^n\right),$$

$$u_{-,n}(r)=\frac{e^{-k_{0,n}r}}{n!}\alpha_n\left(\left(r-r_{0,n}\right)^n e^{2k_{0,n}(r-r_{0,n})}\right)^{(n-1)}.$$

If $a_1^{(+)}=\alpha_n=a_1^{(-)}e^{-2k_{0,n}r_{0,n}}$, then $u_{+,n}(r)=u_{-,n}(r)$. Hence, it follows that the solutions $u_-(r)$, $u_+(r)$ are linearly dependent.

Let us find a second linearly independent solution.

$$u''+p_1(r)u'+p_2(r)u=f(r).$$

$u_+(r)$ and $u_-(r)$ – are the solutions to this equation, then

$$W=\begin{vmatrix}u_+(r) & u_-(r)\\ u_+'(r) & u_-'(r)\end{vmatrix}=u_+(r)u_-'(r)-u_-(r)u_+'(r).$$

$$u_+^2(r)\left(\frac{u_+(r)u_-'(r)-u_-(r)u_+'(r)}{u_+^2(r)}\right)=u_+^2(r)\left(\frac{u_-(r)}{u_+(r)}\right)'=W.$$

$$\int\left(\frac{u_-(r)}{u_+(r)}\right)'dr=C\int\frac{e^{-\int p_1(r)dr}}{u_+^2(r)}dr.$$

Since $p_1(r)=0$, then

$$\frac{u_-(r)}{u_+(r)}=C\int\frac{e^0}{u_+^2(r)}dr.$$

Therefore, the second linearly independent solution will take the form:

$$u_-(r)=Cu_+(r)\int\frac{1}{u_+^2(r)}dr$$

and, considering solution (48), we shall obtain a solution that falls exponentially with distance $u_{-,n}(r\to\infty)\sim e^{-k_{0,n}r}$, that is,

$$u_{-,n}(r)= =Ce^{k_{0,n}r}\sum_{m=1}^{n}a_m^{(+)}\left(r_{0,n}-r\right)^m\int\frac{e^{-2k_{0,n}r}}{\left(\sum_{m=1}^{n}a_m^{(+)}\left(r_{0,n}-r\right)^m\right)^2}dr \quad (52)$$

Note that wave $u_{-,n}(r)$ changes sign during transition r through point $r_{o,n}$, which, in accordance with conditions (3) and (4), indicates the existence of a particle's trajectory at this point. Since $R_{-,n}=\frac{u_{-,n}}{r}$, then, in accordance with (52), we shall obtain solutions to $R_{-,n}$, (n=1, 2, 3...) (Fig. 1–3).

At $n=1$, $u_{-,1}(r)$ is equal to:

$$u_{-,1}(r)=e^{k_{0,1}r}\left(r_{0,1}-r\right)\int\frac{e^{-2k_{0,1}r}}{\left(r_{0,1}-r\right)^2}dr.$$

Considering that

$$k_{0,1}=\frac{\beta_1}{2n}=\frac{1}{2}\frac{Ze^2\omega^2m_e}{2nE^2}= =\frac{1}{2}\frac{Ze^2 4E^2m_e}{2nE^2\hbar^2}=\frac{Ze^2m_e{}^2}{n\hbar^2}$$

and

$$r_{0,1}=\frac{2n^2\hbar^2}{Ze^2m_e},\quad k_{0,1}\cdot r_{0,1}=2n=2\quad\Rightarrow$$

$$u_{-,1}(r)=e^{\frac{2r}{r_{0,1}}}\cdot r_{0,1}\left(1-\frac{r}{r_{0,1}}\right)\int\frac{e^{-\frac{4r}{r_{0,1}}}}{r_{0,1}{}^2\left(1-\frac{r}{r_{0,1}}\right)^2}dr.$$

We shall replace:

$$\frac{r}{r_{0,1}}=z,\quad dr=r_{0,1}dz\quad\Rightarrow u_{-,1}(r)=e^{2z}\cdot(1-z)\int\frac{e^{-4z}}{(1-z)^2}dz.$$

Therefore,

$$R_{-,1}=\frac{u_{-,1}(r)}{r}=\frac{e^{2z}\cdot(1-z)\int\frac{e^{-4t}}{(1-t)^2}dt}{r}.$$

At $n=2$, $u_{-,2}(r)$ is equal to:

$$u_{-,2}(r)= =e^{k_{0,2}r}\left(\left(r_{0,2}-r\right)-k_{0,2}\left(r_{0,2}-r\right)^2\right)\times \times\int\frac{e^{-2k_{0,2}r}}{\left(r_{0,2}-r\right)^2\left(1-k_{0,2}\left(r_{0,2}-r\right)\right)^2}dr,$$

$$k_{0,2}\cdot r_{0,2}=2n=4\quad\Rightarrow$$

$$u_{-,2}(r)=e^{\frac{4r}{r_0}}\left(r_{0,2}\left(1-\frac{r}{r_{0,2}}\right)\left(1-k_{0,2}\left(r_{0,2}-r\right)\right)\right)\times \times\int\frac{e^{-\frac{8r}{r_0}}}{r_{0,2}{}^2\left(1-\frac{r}{r_{0,2}}\right)^2\left(1-k_{0,2}r_{0,2}\left(1-\frac{r}{r_{0,2}}\right)\right)^2}d\frac{r}{r_{0,2}}= =e^{4z}(1-z)(1-4(1-z))\times \times\int\frac{e^{-8z}}{(1-z)^2(1-4(1-z))^2}dz.$$

Therefore,

$$R_{-,2}=\frac{u_{-,2}(r)}{r}=\frac{-e^{4z}(1-z)(3-4z)\int\frac{e^{-8z}}{(1-t)^2(3-4t)^2}dt}{r}.$$

At $n=3$, $u_{-,3}(r)$ is equal to:

$$u_{-,3}(r)=e^{k_{0,3}r}\times \times\left(6\left(r_{0,3}-r\right)-6(2k_{0,3})k_{0,3}\left(r_{0,3}-r\right)^2+(2k_{0,3})^2\left(r_{0,3}-r\right)^2\right)\times \times\int\frac{e^{-2k_{0,3}r}}{\left(r_{0,3}-r\right)^2\left(6-12k_{0,3}\left(r_{0,3}-r\right)^2+4k_{0,3}{}^2\left(r_{0,3}-r\right)^2\right)^2}dr$$

$$k_{0,2}\cdot r_{0,2}=2n=6\ \Rightarrow$$

$$u_{-,3}(r)=e^{\frac{6r}{r_0}}\left(r_{0,3}\left(1-\frac{r}{r_{0,3}}\right)6-12k_{0,3}r_{0,3}\left(1-\frac{r}{r_{0,3}}\right)+4k_{0,3}{}^2r_{0,3}{}^2\left(1-\frac{r}{r_{0,3}}\right)^2\right)\times \times\int\frac{e^{-\frac{12r}{r_0}}}{r_{0,3}{}^2\left(1-\frac{r}{r_{0,3}}\right)^2\left(6-12k_{0,3}r_{0,3}\left(1-\frac{r}{r_{0,3}}\right)+4k_{0,3}{}^2r_{0,3}{}^2\left(1-\frac{r}{r_{0,3}}\right)^2\right)^2}d\frac{r}{r_{0,3}}= =e^{6z}(1-z)\left(6-72(1-z)+144(1-z)^2\right)\times \int\frac{e^{-12z}}{(1-z)^2\left(6-72(1-z)+144(1-z)^2\right)^2}dz.$$

Therefore,

$$R_{-,3}=\frac{u_{-,3}(r)}{r}=\frac{e^{6z}(1-z)\left(6-72z(1-z)+144(1-z)^2\right)}{r}\times \times\int\frac{e^{-12t}}{(1-t)^2\left(6-72(1-t)+144(1-t)^2\right)^2}dt.$$

We shall construct charts of functions $R_{-,n}$, using the Maple programming complex.

The charts show that starting from second lower state, amplitude of the wave crosses zero more than once, but only at $r=r_{0,n}$ the derivative of wave $\frac{\partial}{\partial r}V_n(r,t)$ changes sign in this point, which according to (13), indicates the existence of the electron trajectory only on the surface with radius $r_{0,n}$.

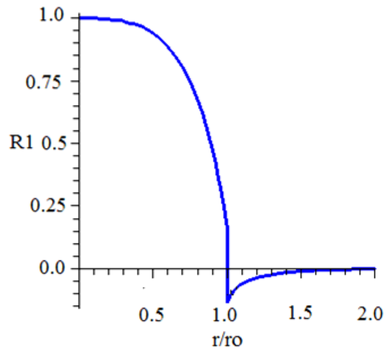

Fig. 1. Stationary solution for a wave of the particle (electron) to first lower stationary state (n=1)

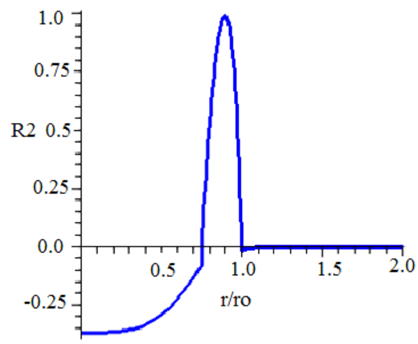

Fig. 2. Stationary solution for a wave of the particle (electron) to second lower stationary state (n=2)

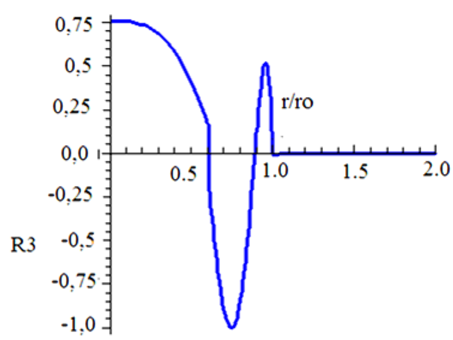

Fig. 3. Stationary solution for a wave of the particle (electron) to third lower stationary state (n=3)

The properties of the trajectory and wave $V_n(r,t)$, described above, indicate a different spatial arrangement of the electron in the hydrogen atom compared to the known pattern, described by the Schrödinger's wave function.

6. Discussion of results of the conducted research

The research undertaken indicates that the desire of L. de Broglie to overcome a wave-particle dualism through the concept of wave-pilot is substantiated here via a continuation of the optical-mechanical analogy, which is solved at the level of wave optics. In this case, the wave function $V(x, t)$ itself is not only connected to the motion of the particle in some way, but directly expresses the motion itself, which is always of a wave nature, be it light, or any other object.

When modeling an electron motion in the Coulomb field, the V-function method makes it possible to establish a rule of energy quantization of a hydrogen-like atom, which fully coincides with the classical results by Schrödinger and Bohr. In this case, discreteness of energy arises from satisfying the conditions following from the V-function method. The trajectory and the electron wave are *interconnected*, this relation is described by the method of V-function based on the local variational principle and solution to the direct and inverse problems of dynamics. According to the given approach, stationary behavior of the electron on the n-th stable state is described by wave R_n, which subsides exponentially to zero at $r \to \infty$. In this case, the amplitude of the wave passes zero on the sphere with a Bohr radius $r_{o,n}$, which means the existence of the electron trajectory on the sphere of the given radius.

A benefit of the given method is that when simulating the motion of an object, one takes into account its wave motion and the trajectory motion at the same time. Reliability of the results is achieved by confirming the known results of quantum mechanics. In this case, however, the inevitability should be noted of the emergence of difficulties for experimental confirmation of the new results. It should also be noted that the trajectory motion of an object is described by the method of V-function only with a system of stationary differential equations, which can be regarded a constraint of the performed research.

7. Conclusions

1. Based on the method of V-function, we have drawn an optical-mechanical analogy, which thus gained a new continuation. Wave function $V(x, t)$ directly expresses the motion itself, which is always of a wave nature, be it light, or any other object.

2. We obtained a solution to the direct and inverse problems of dynamics in a new statement for a hydrogen-like atom. The method of V-function makes it possible to establish a rule for the energy quantization of a hydrogen-like atom, which fully coincides with the classical results.

3. The ultimate solution to the stationary wave equation for a hydrogen-like atom was obtained. Stationary behavior of the electron is described by wave R_n, subsiding exponentially to zero at unlimited distance from the nucleus, and whose amplitude passes through zero on the sphere with a Bohr radius.

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