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IDENTIFICATION OF THE ADDITIONAL EXPOSURE ZONE FOR ENSURING A COMPLETE CONTACT OF THE TWO-LAYERED SYSTEM

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Розв'язана задача ідентифікації величини і зони впливу на верхній шар двошарової конструкції, що знаходиться під дією нормально розподіленого навантаження і власної ваги, для забезпечення повного контакту. Досліджено можливість застосування методу обернених задач, реалізованого за допомогою методу вектора спаду. Зроблено чисельний аналіз збіжності процесу усунення деформації моделі в залежності від механічних і геометричних параметрів системи

Ключові слова: плоска контактна задача, односторонні зв'язки, ідентифікація впливу, метод обернених задач

Решена задача идентификации величины и зоны воздействия на верхний слой двуслойной конструкции, находящейся под действием нормально распределенной нагрузки и собственного веса, для обеспечения полного контакта. Исследована возможность применения метода обратных задач, реализованного с помощью метода вектора спада. Проведен численный анализ сходимости процесса устранения деформации модели в зависимости от механических и геометрических параметров системы

Ключевые слова: плоская контактная задача, односторонние связи, идентификация воздействия, метод обратных задач

1. Introduction

The phenomenon of separation of interlayer bonds is observed in the operation of highways, airfield pavements, foundations of high-rise buildings with supports or isolation joints. The contact between the building sole and the base often leads to emergency situations. Such phenomena are modeled by contact problems with unilateral constraints.

A detailed research of the fundamental laws of contact interaction requires a comprehensive consideration of the geometric features and imperfections of the conjugate surfaces, physical and mechanical phenomena in the areas of their direct attachment (friction, slippage, adhesion, etc.). The numerical solution of contact problems is usually carried out on the basis of the finite element method. For the modeling of unilateral constraints, various physical models are used. The research on the stress-strain state of the model and the state of the contact zone (presence of a friction zone and a separation zone) can be defined as the solution of a direct problem.

Studies of contact problems of mechanics of deformable media are conducted in two directions. Within the framework of the first direction, problems of conjugation of media with sharply differing mechanical properties (layer-inhomogeneous elastic and elastoplastic bodies, conjugation of solids to liquid or gaseous media, etc.) are considered. In such problems, the boundaries of the contact area are, as a rule, specified and not changed in the course of deformation. To solve these problems, the methods of the function theory of a complex variable and the theory of potentials with integral transforms and paired integral equations, paired trigonometric series, integral and integral-differential equations and systems of equations and so forth are used. The second direction of the study of contact problems of mechanics includes contact problems with the unknown in advance boundaries of the contact area. When modeling such contact problems, the conditions imposed on displacements and forces in the contact zone are often presented as inequalities. Problems of this kind are called problems with unilateral constraints. They are characterized by significant changes in system properties when the state of the contact changes and are essentially nonlinear even for linearly elastic media. The configuration of contact or separation zones (and also slippage-adhesion regions when friction is taken into account) is unknown in advance and shall be determined only in the process of problem solving.

Proceeding from this, the research of the problem shall include two main problems – construction of a physical model of unilateral constraint taking into account friction and separation, its algorithmic implementation, and determination of the parameters of exposure needed to prevent separation. Determination of this exposure will allow ensuring the reliability of the corresponding structures using the results obtained, which determines the relevance of the problem. The peculiarity of the statement and solution methodology of the problem, along with the possibilities of wide practical application, can stimulate further development of this direction of solid mechanics.

2. Literature review and problem statement

The development of a physical model is the subject of the research [1–5]. In [1], two types of contact zone models are considered – bilinear and exponential, which describe the damaged bonds leading to stratification (separation). Here, the model parameters are investigated from the point of view of the possibility of separation. The authors of [2] proposed a contact model, which is implemented in two stages – in the first, layers press against each other, and in the second they interact by means of tangential stresses, responsible for contact with friction. The contact zone with separation in [3] is modeled by varying-rigidity rods, and the separation zone – directly by the finite element method. In [4], a synergy approach is used that allows modeling on the basis of finite elements in order to study wear mechanisms for two groups of doped carbide inserts (coated and uncoated). The authors of [5] consider a quasistatic and velocity-independent evolution in small stresses and the concept of the so-called energy solution. This concept is applied to cohesive contact problems and stratification in various regimes.

The algorithms used for numerical analysis are very diverse. Thus, in [6], in the study of a constructively nonlinear problem with unilateral constraints and friction with an unknown contact zone, an iterative approach was applied to model the bonds using special contact elements in a thin friction layer in combination with a finite element model of a plane problem. In [7], on the basis of the domain decomposition method, contact interaction was studied by the method of penalty functions, which are the conditions for kinematic admissibility of displacements. In the case when large deformations are observed [8, 9], a hierarchical algorithm for constructing a binary tree for the current state of the contact surface geometry is proposed. Two procedures are used – global, defining all pairs of candidates, and then local, defining elements with a weakened connection. The paper [10] is based on the application of finite element software to simulate the behavior of laminated composite plates at a low impact speed, and it examines exposure, post-exposure and destruction of these structures.

Thus, the presented sources consider only the direct problem – determination of the stress-strain state of the two-layered system and evaluation of possible development of the separation zone. Meanwhile, with combinations of loads and geometric parameters, it is necessary to ensure the interaction of layers, which can be achieved by additional mechanical exposure, rigid inclusions, etc.

3. The aim and objectives of the study

In the paper, the statement of the problem of determining additional exposure with the aim of eliminating the separation zone (identification of the parameters of its location and magnitude) according to the known characteristics in the contact zone obtained from the solution of the direct problem is defined as the statement of the inverse problem.

The aim of the paper is to investigate the possibility of using the inverse problem method to identify the magnitude and location of exposure on the upper layer of the two-layered base that is under the action of a normally distributed load and its own weight.

To achieve this aim, the following objectives were set:

– to perform parameterization of the system under study and construct its finite element model;

– to determine the stress-strain state of the system under consideration on the basis of the finite element method and the algorithm allowing to take into account the contact zone variability and the presence of friction at fixed values of exposure parameters;

– to determine the values and location of exposure, ensuring the presence of complete contact between the two infinite layers in question by the inverse problem method;

– to investigate the convergence of the process of eliminating the model deformation depending on the friction force, height and rigidity of the upper layer.

4. A mathematical model of the inverse problem

We consider the problem of determining the value and location of exposure p (Fig. 1), providing a complete contact of the two infinite layers considered in the regions

$$\Omega_k = \begin{cases} x^k,\ x^k = \left\{x_1^{(k)}, x_2^{(k)}\right\} \in R^2, \\ -\infty \le x_1^{(k)} \le \infty, 0 \le x_2^{(1)} \le h_1 - h_2 \le x_2^{(2)} \le 0, \end{cases}$$

which are under the action of normal pressure $q(x_1)$, $0 \le q(x_1) \le q^*$, where q^* is the limit load value, k is the layer number.

The resolving system of equations of the plane elasticity theory in the regions Ω_k given $h_2/h_1 >> 1$ has the form

$$(\lambda_k + \mu_k)\,\mathrm{grad}\,\mathrm{div}\,u^k + \mu_k \Delta u^k + Q = 0, \quad (1)$$

where $u^k = \left\{u_1^k, u_2^k\right\}^T$ is the displacement vector of the k-th layer,

$$\lambda_k = E_k \frac{\nu_k}{(1+\nu_k)(1-2\nu_k)}, \quad \mu_k = \frac{E_k}{2(1+\nu_k)},$$

λ_k, μ_k are the Lame coefficients, E_k, ν_k is the elastic modulus and Poisson's ratio (k=1,2) for the upper layer (k=1) and the base (k=2), respectively; Q is the load including its own weight.

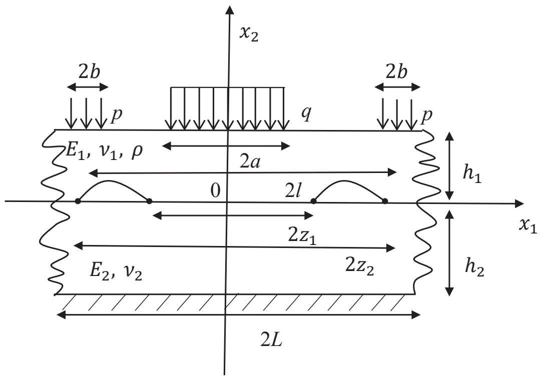

Fig. 1. Loading diagram of the two-layered structure

We denote the sections of the boundary B_{top} ($x_2=h_1$) as

$$B_p = \{x_1, -(l+b) \le x_1 \le -(l-b), l-b \le x_1 \le l+b\},$$

$$B_q = \{x_1, -a \le x_1 \le a\}.$$

Then it is necessary that

$$\sigma_{22}^{(1)}(u) = q, \quad x_1 \in B_q,$$

$$\sigma_{22}^{(1)}(u) = p, \quad x_1 \in B_p,$$

$$\sigma_{22}^{(1)}(u) = 0, \quad x_1 \notin B_q, \quad x_1 \notin B_p,$$

$$\sigma_{12}^{(1)}(u) = 0, \quad -\infty \le x_1 \le \infty, \qquad (2)$$

where $\sigma_{ij}^{(k)}(u)$, i, j=1,2 are the stresses expressed through displacements.

At the interface between the upper layer and the base (x_2=0), in the contact zone B_{con} there are the boundary conditions

$$\sigma_{22}^{(1)} = \sigma_{22}^{(2)}, \quad u_2^{(k)} \le 0 \text{ or } \sigma_{22}^{(k)} \le 0, \quad u_2^{(k)} \cdot \sigma_{22}^{(k)} = 0, \quad k = 1,2. \qquad (3)$$

It is assumed that there are boundaries B_A, B_{Sl}, B_{Sep}, corresponding to the adhesion, slippage and separation zone, respectively, such that $B_A \cup B_{Sl} \cup B_{Sep} = B_{con}$, $B_A \cap B_{Sl} \cap B_{Sep} = \varnothing$.

In the adhesion zone $x_1 \in B_A$, the conditions are satisfied

$$u_1^{(1)} = u_1^{(2)}, \; u_2^{(1)} = u_2^{(2)},$$

$$\left|\sigma_{12}^{(1)}\right| \le K \left|\sigma_{22}^{(1)}\right|, \qquad (4)$$

where K is the coefficient of friction.

In the slippage zone for all $x_1 \in B_{Sl}$

$$u_2^{(1)}\left(x_1 + u_1^{(1)}\right) = u_2^{(2)}\left(x_1 + u_1^{(2)}\right),$$

$$\left|\sigma_{12}^{(k)}\right| - K\left|\sigma_{22}^{(k)}\right| \ge 0, \quad u_1^{(1)} \ne u_1^{(2)}. \qquad (5)$$

In the separation zone for all $x_1 \in B_{Sep}$,

$$\sigma_{22}^{(k)} = 0, \quad \sigma_{12}^{(k)} = 0. \qquad (6)$$

Here, $\sigma_{ij}^{(k)}$, $u_i^{(k)}$, i, j=1,2 are the components of the stress and displacement tensor in the k-th layer, k=1, 2.

Further, it is assumed that for the loading values $0 \le q \le q_{cr}$, the layers are in contact through complete adhesion or slippage, separation zones appear at a load value $q=q_{cr}$ with further development of these zones $q>q_{cr}$ (Fig. 2).

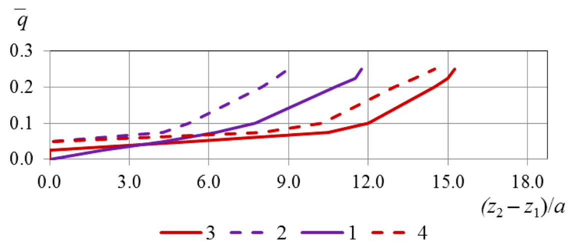

Fig. 2. Dependence of the relative separation zone on load at η=0.2: 1 – K=0, χ=0.01; 2 – K=0.3, χ=0.01; 3 – K=0, χ=2; 4 – K=0.3, χ=2

To determine the value and location of exposure p, ensuring the absence of separation zones $z_1 \le x_1 \le z_2$, $-z_2 \le x_1 \le -z_1$, x_2=0, where σ_{22}=0, we formulate the problem of identification of the parameters $V=\{l, b, p\}^T$ as inverse. Then the value of the vector V is defined as

$$V = \arg\min_{V \in \tilde{V}} J(V), \qquad (7)$$

where

$$J = \int_{-L}^{L} \left[\left(\tilde{u}_2^{(1)}(V) - \tilde{u}_2^{(2)}(V)\right)^2 - \varepsilon^2\right] \mathrm{d}x_1, \qquad (8)$$

$\varepsilon \ll 1$ is a small value, $\tilde{V}$ is the domain of the vector V, $\tilde{u}_2^{(i)}$ is the value of the function $u_2^{(i)}(x_1, 0)$ for a fixed vector V on the contact line B_{con}.

5. Method and algorithms for solving the identification problem

Let us formulate the method for solving a direct problem – determination of the stress-strain state of the system under consideration at a fixed vector V [11].

To describe unknown areas of the boundaries, we introduce the characteristic functions for the points of the boundaries B_A, B_{Sl}, B_{Sep} in the form

$$\gamma_1(x) = \begin{cases} 1 \text{ at } |\sigma_{12}| \ge K|\sigma_{22}|, \; x \in B_{Sl}, \\ 0 \text{ at } |\sigma_{12}| < K|\sigma_{22}|, \; x \in B_A; \end{cases}$$

$$\gamma_2(x) = \begin{cases} 0 \text{ at } \sigma_{22} < 0, \; x \notin B_{Sep}, \\ 1 \text{ at } \sigma_{22} \ge 0, \; x \in B_{Sep}. \end{cases} \qquad (9)$$

Taking into account the relation (9), a variational statement of the boundary value problem (1)–(6) will have the form

$$W = \arg\min_{W \in \tilde{W}} G(u, u^*), \qquad (10)$$

under the preliminary fulfillment of the condition (4), where $u=\{u^k\}^{\mathrm{T}}$, k=1,2, $W=\{u, u^*\}^T$,

$$G(u,u)=\sum_{k=1}^{2}\left\{\int_{\Omega_k}\frac{1}{2}C_k^{ijlm}\varepsilon_{ij}^k(u)\varepsilon_{lm}^k(u)\mathrm{d}x_1\mathrm{d}x_2+\left[\int_{-a}^{a}u_2^{(2)}qdx_1+\int_{l-b}^{l+b}u_2^{(2)}pdx_1+\int_{-(l+b)}^{-(l-b)}u_2^{(2)}pdx_1\right]\Bigg|_{x_2=h_1}+\right.$$

$$\left.+\int_{B_{con}}\left\{\gamma_1K\sigma_{22}^{(k)}(u)u_1^{(k)}+\gamma_2\left[\sigma_{22}^{(k)}(u)\left(u_2^{(k)}-u_{2k}^*\right)+\sigma_{12}^{(k)}(u)\left(u_1^{(k)}-u_{1k}^*\right)\right]\right\}\right\}\mathrm{d}B_{con};\qquad(11)$$

$\varepsilon_{ij}=\dfrac{\partial u_i}{\partial x_j}$, i,j,l,m=1,2, u_{ik}^* are the variable values of displacements on the contact line, $\sigma_{ij}^{(k)}, u^{(k)}$ are the values of stresses and displacements from the region Ω_k on its boundary.

To construct the solution of the problem (10), the transition to a discrete model is performed using finite element approximation is carried out. To this end, a grid with nodes at the points with coordinates X_s, $s=\overline{1,N}$, where $X_s=\{x_{1s}, x_{2s}\}$ is introduced in the region Ω_k, then the unknown functions $u(x)$, $\sigma_{ij}(x)$, $u^*(x)$ are presented as vectors whose components are the values of the problem functions at the grid nodes.

$$u=\{u_i\}^T,\sigma_{ij}=\{\sigma_{ijs}\}^T,\quad u^*=\{u_i^*\},u_i=\{u_{ik}\},u_{ik}=\{u_{iks}\},$$
$$u_i^*=\{u_{ik}^*\}^T,u_{ik}^*=\{u_{iks}^*\},\gamma=\{\gamma_k\}^T,\ \gamma_k=\{\gamma_{ks}\}^T,$$
$$i,j=1,2,\ s=\overline{1,N},\ k=1,2.\qquad(12)$$

The nodes lying on the boundary B_{con} are numbered as $P=\{p_1, p_2,..., p_M\}$, the number M is chosen by a numerical experiment. The elements of the vector P can form the vectors

$$P^A=\{p_1^A,p_2^A,...,p_{r_1}^A\},$$
$$P^{Sl}=\{p_1^{Sl},p_2^{Sl},...,p_{r_2}^{Sl}\},$$
$$P^{Sep}=\{p_1^{Sep},p_2^{Sep},...,p_{r_3}^{Sep}\},$$
$$r_1+r_2+r_3=M,\qquad(13)$$

which define the grid nodes corresponding to adhesion, slippage and separation zones by calculating the values of γ_k in accordance with the conditions (9).

After substituting the finite element approximation into the functional (11), integrating and performing the procedure (10), we obtain a resolving system of the N-th order linear equations

$$Au=R,\qquad(14)$$

equivalent to the condition $\partial\tilde{G}^*/\partial u_{is}=0$, i=1,2, $s=\overline{1,N}$, $\tilde{G}^*$ are the values of the functional $\tilde{G}$ after substituting the finite element approximation and performing the integration procedure, A is the stiffness matrix depending on the values of the vectors γ, R is the vector depending on u^*.

When constructing the stiffness matrix, it is taken into account that

$$\sigma=Du,\qquad(15)$$

where

$$\sigma=\{\sigma_{ijs},\ i,j=1,2,\ s=\overline{1,N}\}^{\mathrm{T}},$$

D is the functional matrix of the coefficients obtained from physical and geometric relationships.

To fulfill the minimum condition $\tilde{G}$ by u_{is}^*, $s=\overline{1,N}$, the gradient descent method is used

$$u^{*(n)}=u^{*(n-1)}-\alpha^{(n-1)}\sigma^{(n-1)}.\qquad(16)$$

Here

$$\sigma^{(n-1)}=\{\sigma_s^{(n-1)}\}^{\mathrm{T}},$$
$$\sigma_s^{(n-1)}=\{\sigma_{12s}^{(n-1)}\sigma_{22s}^{(n-1)}\}^{\mathrm{T}},\quad s=\overline{1,N},$$

where n is the iteration number.

The coefficient $\alpha^{(n)}$ is determined from the condition

$$\alpha^{(n)}=\arg\min_{\alpha}\tilde{G}\left(u^{*(n-1)}\right)\qquad(17)$$

using the half division method.

The value of the function γ_{ks} is determined in accordance with the condition (9). The solution of the problem is carried out using algorithm 1.

Algorithm 1

0. To set t=1, n=1, $\sigma_{22s}=q_s^0$ for $\leftrightarrow x_s\in B_q$, $\sigma_{22r}=p_r^0$ for $\forall x_r\in B_p$, $r=\overline{1,m}$, σ_{12s}=0 for $\forall x_s\in B_{top}$, $u_{is}^{*(1,1)}=0$, $\gamma_{ks}^{(1,1)}=0$, k=1, 2, ε is a small number $\alpha^{(1,1)}$, q^*, p_{cur}, $\Delta q=q^*/n^*$, $\Delta p=p_{cur}/n^*$, n^* is the number of steps, $q^{(1)}=\Delta q$, $p^{(1)}=\Delta p$.

1. To construct a grid with the nodes X_s, $s=\overline{1,N}$.

2. To form a vector of unknowns u, taking into account the conditions $u_{1s}^{(1)}=u_{1s}^{(2)}$, $u_{2s}^{(1)}=u_{2s}^{(2)}$ for the nodes X_s such that $X_s\in B_A$.

3. To form a system of equations (14).

4. To determine the vector $u^{(n,t)}$, $\sigma^{(n,t)}$ by the equations (15).

5. To determine the vector $\gamma_{ks}^{(n,t)}$, $X_s\in B_{cont}$, k=1, 2 in accordance with the conditions (9).

6. If $\exists\gamma_{ks}^{(n,t)}\neq0$, then go on, otherwise proceed to step 12.

7. To determine $u_{is}^{*(n,t)}$ by the formula (16).

8. To determine the numbers of the nodes $P^{Sep(n,t)}$, $P^{Sl(n,t)}$, $P^{A(n,t)}$, based on the values of the vector $\gamma_{ks}^{(n,t)}$.

9. To create the nodes $P_i^{Sep(n,t)}$, i=1, 2, with the corresponding displacements $u_2^{(1)(n,t)}\neq u_2^{(2)(n,t)}$, and the nodes $P_i^{Sl(n,t)}$, i=1, 2, with the corresponding displacements $u_1^{(1)(n,t)}\neq u_1^{(2)(n,t)}$, to form a vector of unknowns u.

10. To determine $\alpha^{(n,t)}$ from the condition (17), $t=t+1$.

11. If $\left\|u_{is}^{*(n,t)}-u_{is}^{*(n,t-1)}\right\|\leq\varepsilon$, then go on, otherwise proceed to step 3.

12. $q^{(n)}=q^{(n-1)}+\Delta q$, $p^{(n)}=p^{(n-1)}+\Delta p$, $n=n+1$.

13. $n\leq n^*$ go to step 3, otherwise the end.

The inverse problem will be solved by the recession vector method. Let us denote the nodes lying on the boundary B_p as

$$C=\{x_{j1},x_{j2},...,x_{jk}\}$$

and introduce the vector $F=\{\tilde{C},p\}$, where

$$\tilde{C}=\{x_{k1},x_{k2},...,x_{kp}\}$$

is the vector of coordinate values of the points of load application.

The functional J after sampling takes the form

$$\tilde{J}=(\tilde{u}_2^{(1)}(\tilde{F})-\tilde{u}_2^{(2)}(\tilde{F}))^T(\tilde{u}_2^{(1)}(\tilde{F})-\tilde{u}_2^{(2)}(\tilde{F}))-\varepsilon^2,\qquad(18)$$

and the condition (7) is written in the form

$$F=\arg\min_{F\in\bar{F}}J(F).\qquad(19)$$

Let W be a discrete point space, $\bar{F}$ – a set of admissible solutions, $\bar{F}\in W$, and W –a metric space with a metric

$$\rho^2(F_1,F_2)=\sum_{i=1}^{M+1}(\bar{\bar{f}}_{1i}-\bar{\bar{f}}_{2i})^2, \quad (20)$$

where f_{1i}, f_{2i} are the coordinates of the points F_1, F_2 in the space W.

Let F' be some admissible solution of the inverse problem (7). We define the vicinity $\tilde{W}$ of the point F' with the radius r_1 as a set of possible solutions F_k', $k=\overline{1,12}$, obtained by addition (removal) of the elements of the vector $\tilde{C}$ to the left and right from the points $s=x_{k1}$, $s=x_{kp}$ and change of the corresponding values of p.

The recession vector of the function J in the vicinity $\tilde{W}$ of an arbitrary point F' is defined as a vector with components

$$\Delta_k=J(F_k')-J(F'),$$

where F_k', $k=\overline{1,12}$, are possible solutions of the inverse problem that belong to the vicinity $\tilde{W}$, and $J(F_k')$, $J(F')$ are calculated by solving problems using algorithm 1. Obviously, for all $\Delta_k\geq 0$, in the vicinity of the point F', this point is a local minimum of the function $J(F)$. If some $\Delta_k<0$, and $\Delta_{k^*}=\min\Delta_k$, then the point F_k' is a point of the speedy recession of the function $J(F)$.

The algorithm that implements the recession method has the form:

Algorithm 2

1. To select the starting point F_0, form the points F_k', $k=\overline{1,12}$.

2. To determine the components of the recession vector for the point F_0 in the directions F_k' by calculating $J(F)$ using algorithm 1. If all $\Delta_k\geq 0$, $k=\overline{1,12}$, then $J(F_0)=\min J(F)$.

3. If $\exists\Delta_k<0$ for $k=\overline{1,12}$, then choose F_{k^*}, corresponding to $\min\Delta_k$, which becomes the center of the new vicinity $F_{k^*}=F_0$.

4. Go to point 2. The process continues as long as $\exists\Delta_k<0$, $k=\overline{1,12}$.

6. Numerical analysis of the identification problem

With the help of the proposed algorithm, the analysis of behavior of the two-layered system having the following characteristics is performed: for the first option, the specific weight $\rho=2.76\cdot10^{-3}$ (kg/cm^3), Young's modulus $E_1=7.6\cdot10^4$ (kg/cm^2) and Poisson's ratio $\nu_1=0.4$, for the lower layer – $E_2=3.8\cdot10^4$ (kg/cm^2), $\nu_2=0.35$, for the second option – $\rho=2.72\cdot10^{-3}$ (kg/cm^3), $E_1=7.6\cdot10^3$ (kg/cm^2), $\nu_1=0.4$, $E_2=7.6\cdot10^5$ (kg/cm^2), $\nu_2=0.2$.

The dimensions of the modeled semi-infinite base were chosen from the condition of the solution damping in case of complete adhesion ($h_2=50$ cm, $L=150$ cm, $a=8$ cm – the zone of distributed surface load q). The solutions of the problem (10) were carried out with the help of the Cosmos application package with automatic preliminary "merging" and "disjoining" of the nodes corresponding to adhesion, slippage and separation zones. The plane finite element was used.

The solutions were carried out sequentially by fragmenting the final element size to obtain the specified accuracy. To describe the solution of the problem, we introduced dimensionless designations of the parameters – $\chi=E_1/E_2$, $\eta=h_1/h_2$, $\bar{q}=q/q^*$, where $q^*=40$ (kg/cm) is the maximum load acting on the upper boundary of the layer, and $b^*=b/a$, $l^*=l/a$, $p^*=p/q^*$ are the values of the vector F at which the function $J(F)$ reaches its minimum.

Fig. 2 shows the values of the relative separation zone at the corresponding load $\bar{q}$, depending on the coefficient of friction K and the ratio of Young's moduli χ.

Fig. 3, 4 (curve 1) show the values of the relative opening of contact surfaces, from which it follows that the size of the separation zone depends on the parameters under study only in the zone of large loads. At the same time, opening degree changes nonlinearly not only with load variations, but also depending on the relative thickness of the layers and their modules over the entire range of loads. Fig. 5 illustrates the stress values depending on the parameters under study at fixed $q=0.25$. Note that the effect of friction increases substantially with increasing relative thickness of the layers. Fig. 3, 4 (curves 2–5) present the results of the iterative process of the recession vector method.

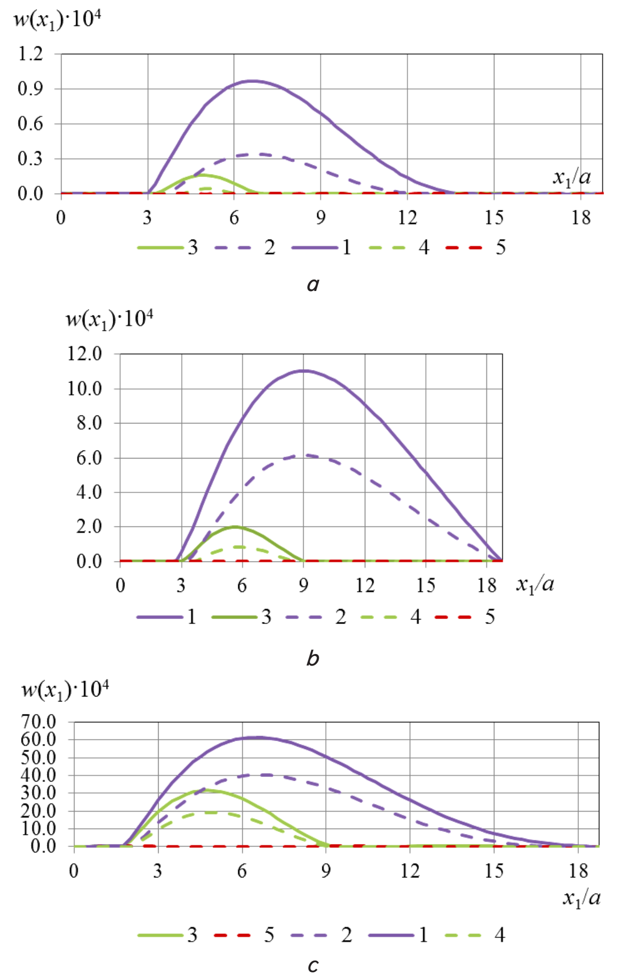

Fig. 3. Relative displacement of the layer and base points at the interface given $\chi=2$: *a* – $\bar{q}=0.075$, $\eta=0.2$; *b* – $\bar{q}=0.25$, $\eta=0.2$, *c* – $\bar{q}=0.525$, $\eta=0.1$, where 1 – $K=0$, $p=0$; 2 – $K=0.3$, $p=0$; 3 – $K=0$, p_0, l_0, b_0; 4 – $K=0.3$, p_0, l_0, b_0; 5 – p^*, l^*, b^*

The results of solving the identification problem are shown in Table 1. In the solution of the problem, the initial approximations are chosen: $p_0=0.05q$, $b_0=a/4$, $l_0=5a$; $10a$ depending on χ and η.

From the above data (Table 1), it can be seen that the parameters of additional exposure p^*, l^*, b^* essentially depend on the loading level, geometric and physical properties of the system. Note that taking friction into account reduces the need for additional exposure.

Table 1

Dependence of identification parameters on system parameters

$\bar{q}$	K	χ	η	p^*	l^*	b^*	$\min J(F)$
0.075	0.0	2.0	0.2	0.03	7.25	1.75	9.13E-11
0.075	0.3	2.0	0.2	0.0225	7.75	0.5	1.56E-12
0.25	0.0	2.0	0.2	0.125	8.25	2.0	5.33E-09
0.25	0.3	2.0	0.2	0.1	9.0	1.5	1.02E-10
0.25	0.0	0.01	0.1	0.1	3.25	1	1.95E-08
0.25	0.3	0.01	0.1	0.0875	3.75	0.25	1.21E-09
0.25	0.0	0.01	0.2	0.025	3.25	0.75	4.23E-09
0.25	0.3	0.01	0.2	0.0188	3.5	0.25	1.14E-10
0.525	0.0	2.0	0.1	0.0525	5.25	1.75	9.55E-08
0.525	0.3	2.0	0.1	0.05	5.75	1.5	2.54E-09
1.0	0.0	0.01	0.2	0.1	3.5	1.0	3.02E-08
1.0	0.3	0.01	0.2	0.075	3.75	0.5	1.15E-09

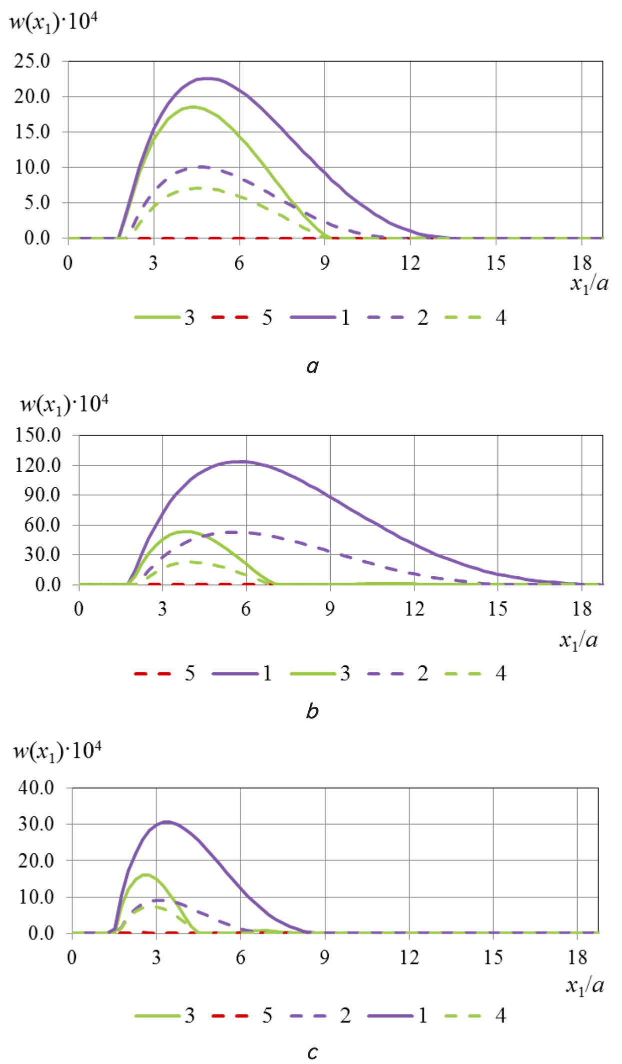

Fig. 4. Relative displacement of the layer and base points at the interface given χ=0.01: *a* – $\bar{q}=0.25$, η=0.2; *b* – $\bar{q}=1$, η=0.2; *c* – $\bar{q}=0.25$, η=0.1, where 1 – K=0, p=0; 2 – K=0.3, p=0; 3 – K=0, p_0, l_0, b_0; 4 – K=0.3, p_0, l_0, b_0; 5 – p^*, l^*, b^*

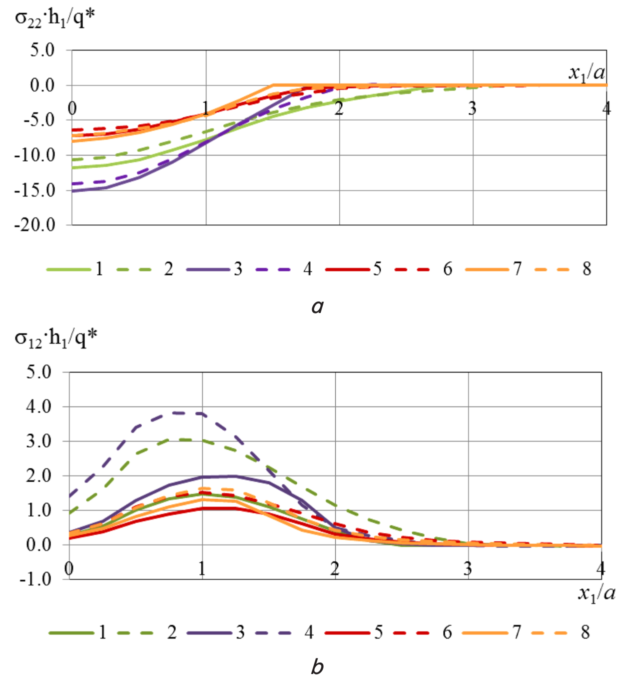

Fig. 5. Dependence of relative stresses on load given q=0.25: *a* – normal; *b* – tangential: 1 – given K=0, χ=2, η=0.2; 2 – given K=0.3, χ=2, η=0.2; 3 – given K=0, χ=0.01, η=0.2; 4 – given K=0.3, χ=0.01, η=0.2; 5 – given K=0, χ=2, η=0.1; 6 – given K=0.3, χ=2, η=0.1; 7 – given K=0, χ=0.01, η=0.1; 8 – given K=0.3, χ=0.01, η=0.1

7. Discussion of the results of solving the identification problem

The computational experiments showed that for a certain ratio of the values of layer thickness and the values of actual loads, the deformation is possible in the presence of slippage, normal contact and separation zones. It can be noted that the existence of the specified behavior depends on system parameters and can be found in a sequential calculation with a change in the load value from zero to the limit value. The analysis of Fig. 5 shows that stress values depend significantly on the type of model (considering the presence of friction, the height of the upper layer and the layer stiffness) and may differ several times in the case of both more rigid and less rigid upper layer.

When determining the effectiveness of the method for solving the problem of identification of the magnitude and zone of exposure on the upper layer of the two-layered structure, as follows from the results obtained (Table 1), the significant influence of system parameters is natural. In the presence of friction, relative displacements of the layer and base points at the interface decrease approximately two-fold. Obviously, this mechanism of influence leads to the following results of solving the problem of eliminating the defect obtained: a decrease in the parameter b^* and an increase in the parameter l^*. It should be noted that with a thinner upper layer, a decrease in the parameters of the identification problem l^* and b^* is observed both for relatively more rigid and relatively softer coatings. In this case, the minimum of the function $J(F)$ is reached at a value of the parameter p not exceeding 10 % of the load q.

The results obtained allow us to conclude that the problem of identification of the location and magnitude of the layer pressing to prevent its separation can be solved by the inverse problem method in combination with the finite element method and the recession method. The pressing zone is separated from the loading zone, the parameters of the zone being dependent on the properties of the layers, the height of the upper layer, the loading magnitude and the coefficient of friction.

The main advantage of the proposed method is its algorithmic nature, which determines the possibility of creating a cost-effective integrated algorithm. This algorithm allows solving practical problems of designing two-layered systems with unilateral contacts.

8. Conclusions

1. Parameterization of the model of the system under consideration has allowed developing an algorithm that implements the conditions of contact or separation, as well as the presence of friction. Its feature is the combination of the finite element method and the method of characteristic functions, determined on the basis of the values of variational inequalities.

2. The stress-strain state of the system is studied at various values of the model parameters, the dependence of the presence and size of the separation zone on them is determined. It is a function of the kind of model (considering the presence of friction, the height of the top layer and the stiffness of the layers), its values may differ several times in case of both more rigid and less rigid upper layer.

3. To determine the value and location of exposure, ensuring a complete contact of the layers, the problem is formulated as inverse in a variational statement and the algorithm of the recession vector method is applied to solve it. It is found that the convergence of the identification process by the recession vector method depends on the loading magnitude and the presence of friction.

4. The influence of physical and geometric properties of the system on the parameters of additional exposure, ensuring the absence of a separation zone was investigated. It is shown that taking friction into account has little effect on exposure parameters, which is determined by the geometric parameters and the level of the main loading.

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