

*Розроблено методику розрахунку конструкцій сталевих складених балок із профільованою стінкою. Представлено роботу профільної стінки у вигляді ортотропної пластинки. Виконано розрахунок диференційного рівняння четвертого порядку і отримано функцію згинальної вісі стінки балки, що використовується для знаходження залежностей напружено-деформованого стану даного типу конструкцій. Проаналізовано роботу профільованої стінки трапецеїдального обрису*

*Ключові слова: сталева балка, профільована стінка, коробчастий переріз, методика розрахунку, ортотропна пластинка*

*Разработана методика расчета конструкций стальных составных балок с профилированной стенкой. Представлена работа профильной стенки в виде ортотропной пластинки. Выполнен расчет дифференциального уравнения четвертого порядка и получена функция изогнутой оси стенки балки, которая используется для определения зависимостей напряженно-деформированного состояния данного типа конструкций. Проанализирована работа профилированной стенки трапецеидального очертания*

*Ключевые слова: стальная балка, профилированная стенка, коробчатое сечение, методика расчета, ортотропная пластинка*

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# DERIVING A FUNCTION OF THE BENDING AXIS OF A PROFILED WALL IN THE FORM OF ORTHOTROPIC PLATE

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## 1. Introduction

At the stage of development of new structural forms of light beams, the search for effective structural solutions and a rational technique for their calculation is an important issue for contemporary design.

Studying existing practice of research and application of structures of beams with a corrugated wall, as well as node connections, made it possible to identify a number of advantages of such structures, compared with conventional I-beams. A decrease in material consumption is achieved by excluding stiffeners and wall corrugation since corrugation enables elastic behavior of steel and enhances local stability of a wall.

Provision of reliability with a simultaneous decrease in material resources is an important task for global construction industry, especially when searching for new structural solutions. Therefore, there is a need for obtaining a new effective structural solution for the elements of beams with a profiled wall and methodological approaches for their consequent calculation. Such structures would have to decrease costs of materials, production, mounting and transportation, indicators of high reliability, manufacturability, and corrosion resistance.

The use of beams with (corrugated) profiled walls must be preceded by both experimental and theoretical research into the stressed-strained state. Considering a profiled wall as a body that has electric symmetry, we will subsequently

represent a wall in the form of an orthogonal-anisotropic or orthotropic plate.

Approximated theory of bending thin anisotropic plates rests on the following two assumptions:

1. Straight segments, normal to the middle surface of a non-deformed plate, remain rectilinear and normal to the bent middle surface.

2. Normal stresses to the surface of a plate are very small compared with the stress in cross sections.

Based on these assumptions, we must determine the equation for a deflection function for an orthotropic plate, in which the directions of axes  $x$  and  $y$  are combined with the main directions of elasticity. The relevance of such studies is predetermined by the lack of a universal technique for calculating the beams with a profiled wall of the box cross section. Development of functional dependence based on representation of a profiled wall in the form of an orthotropic plate and reliable numerical description of work of this wall is a relevant issue for contemporary designing.

## 2. Literature review and problem statement

A simplified algorithm for calculation of beams with a profiled wall of the box cross section [1], compared with existing methods [2], was proposed. Researchers proved the possibility to represent a corrugated wall in the form

of a flat orthotropic plate of the same thickness with given elastic characteristics. The work presented the formula for determining the values of normal stresses at any point of the symmetrical cross section of beams of such type. However, the algorithm and features of calculation of a profiled wall was not presented.

Under conditions of promotion of this type of structures, there is a need to develop a universal technique for analytical calculation. This technique should meet experimental parameters of proposed structural solutions. This methodological approach also simplifies calculation at each step of the calculation algorithm.

In order to study the nature of work of thin-walled profiles, the following stages are distinguished [3]:

- 1) simplification of a complex physical problem to such a degree of idealization, which can be expressed mathematically;
- 2) solution of a purely mathematical problem;
- 3) analysis of obtained results.

For calculation of elements of beams with a profiled wall [1], first of all, it is necessary to correctly determine a calculation model and determine dependences, which can describe the operation of models of such structures.

The necessity of searching for a reliable technique for calculation of a double profiled wall of the trapezoidal configuration determined the subject-matter of this research. To solve the set problem, it is proposed to represent such a wall in the form of an orthotropic plate [4] with the assigned height of profile and a full pitch of a profiled sheet.

It should be noted that a plate bearing in the longitudinal direction is elastic, but in order to simplify calculations, it is taken to be rigid or hinged. In the transverse direction, plate bearing is actually hinged because the sides' length exceeds their width. That is why the plates subsequently can be regarded in calculations as infinite in length and it may be assumed that elastic bearing will not affect considerably the specifics of work.

In general, problems of stability of elastic systems, rods, plates, and shells were explored in papers [5–7]. In particular, paper [5] examined differential equations and formulated conditions for geometric nonlinearity of the theory of orthotropic plates of variable thickness, solved the problems of an orthotropic band-plate under condition of distributed load; article [6] revealed suitability of different two-dimensional theories for analysis of stressed-strained state of plates and proposed a generalized theory of elastodynamics of orthotropic laminated plates; paper [7] studied stressed state of an elastic plate from composite materials.

In [8, 9], the authors explored a corrugated panel. They constructed numerical three-dimensional models of elementary sandwich cells of trapezoidal outline from various materials, including aluminum alloys. It is proved that the model is able to take into account an internal orthotropy of component elements of a sandwich. Considering a trapezoidal shape of a sandwich panel with cross corrugation, it is noted that the properties are constant along axis  $x$ . Because of its geometry, corrugated material has its own character, in particular, properties vary between  $x$  and  $y$  directions in the plane and beyond it. The authors considered the equivalent part of a thin plate that is subject to insignificant deformation. Elastic constants for directions of  $x$  and  $y$  were determined through the orthotropy of material. The results, acquired by analytical deformation method, were compared with data of numerical modeling of three-dimensional models in the ANSYS software. Numerical modeling proved

conformity of data to theoretical results. A number of conducted experimental tests for bending and twisting of panels demonstrated a significant influence of the in-plane shear deformation component. It is believed that shear rigidity outside the plane cannot be neglected [8].

Paper [9] reported results of a series of experimental research and numerical modeling by the method of finite elements of light sandwich panels. Techniques of 3D simulation for composite corrugated material were presented.

We shall additionally consider the studies that address steel beams of a corrugated wall. This type of structures is believed to have a number of technical and economic advantages. Development and numerical studies of new forms of corrugated beams are represented in articles [10–14], where it was found that beams from thin corrugated material allow us to reduce weight considerably compared with hot-rolled or welded ones. The authors cited a number of competitive advantages and outlined the prospects of subsequent development of corrugated beams.

Consider existing methodological approaches to calculation of beams with corrugated walls. According to modern researchers, longitudinal rigidity of a corrugated sheet is considered negligible and deflection moment is accepted only by flanges [12]. Other scientists are guided by assumptions that it is expedient to take into account the impact of work of a corrugated wall [14] and to obtain the equation of interaction between a flange and the wall of a corrugated beam of trapezoidal outline for mixed loading [15]. A modified form of the coefficient of changes in the critical moment of beams with a corrugated wall of sinusoidal configuration was also offered. Comparison of analytical results with results of analysis of linear deformation by the method of finite elements demonstrated that the proposed modifications lead to an accurate prediction of flexural strength [16]. The existence of coefficients of local [12, 18], general and combined [17] bending of a corrugated wall of the beam was experimentally proved. In addition, the authors noted the existence of residual strength that is roughly half of ultimate bearing capacity, regardless of the type of bending. Results of nonlinear analysis of finite elements showed that stress tangents are distributed throughout the whole wall before outpressing, after which it decreases, and distribution becomes non-unified, while full resistance is provided by the increased stress. In [17], formulae for the estimation of shear strength were compared to current and existing experimental results. Comparison of existing analytical models for evaluation of shear strength in relation to the data of the test showed that [18] is accurate for economic modeling and general calculation.

Analysis of work of beams with corrugated walls, in particular the influence of bending moment, is considered in more detail in [19]. Paper [20] presents the solution that is based on the theory of thin-walled plates, which is used for calculation of stressed-strained state of beams with corrugated walls. The authors acquired characteristics of bend and torsion of cross section at an increase in torsional moment with the help of introduction to calculations of equivalent flange thickness. To verify effectiveness of the introduced method, the authors calculate five beams with sinusoidal corrugation for two concentrated influence of forces. The paper presents results of these five methods for calculation of beams, including the method, developed by the authors. Obtained results showed that the magnitude of stresses in each case is almost the same (with maximum difference of up to 7 %).

A review of the scientific literature showed a lack of technique for calculation of the developed beams with a profiled wall of the box cross section [1]. Since such structural forms are new, there is a need to present a universal technique for analytical calculation, which would best meet experimental parameters of this type of structures and would not cause significant computation difficulties at each step of the calculation algorithm.

### 3. The aim and objectives of the study

The aim of present work is to obtain a dependence, which is described by a displacement function and depends on the parameters of a profiled wall (ordinates of determining point, height of the wall, number of half-waves of stability loss). Provided displacements are found, there is a possibility to obtain the maximum close values of stresses in the cross section. This will make it possible to form reliable analytical methods for the calculation of bending moment and normal stresses of the profiled wall of a beam.

To accomplish the set goal, the following tasks had to be solved:

- to confirm the accepted calculation model of a profiled wall;
- to make up a mathematical (differential) dependence of work of the profiled wall, the box cross section under load, and to obtain a displacement function based on its solution;
- to use a function of displacement of the profiled wall in cross section and to compare with calculation of a beam with the profiled wall using the method of finite elements.

### 4. Technique for the calculation of a profiled wall in the form of an orthotropic plate

The basis for calculation of a profiled wall is replacement of the actual deformed form with approximated one. The main feature of a simplified equivalent system is that it operates under load similarly to the original one and is limited to the same sizes.

As a result of solution of a differential equation, we represent the original function for the generalized model of the wall of a beam of box cross section with a double profiled wall of trapezoidal outline, which, compared to the obtained function, has a rather small divergence. Further, the possibility of using the presented dependences for analytical calculation of characteristics of the beam's wall is shown.

In the course of the conducted research, we selected from literary sources the most common functions for description of transverse plate deflection. Their enumeration was long enough, so we shall cite only some of them:

$$\omega = \sin \frac{n\pi x}{s} \sin \frac{n\pi y}{h}$$

and

$$\omega = \cos \frac{m\pi x}{s} \sin \frac{n\pi y}{h} \quad [21];$$

$$\omega = \left(1 - \cos \frac{m\pi x}{s}\right) \cdot \left(1 - \cos \frac{n\pi y}{h}\right)$$

and

$$\omega = \sin \frac{m\pi x}{s} \left(1 - \cos \frac{n\pi y}{h}\right) \quad [22, 23].$$

We shall note that using the above approximating functions and other functions similar to them, by changing the coefficients, we derived curves of different nature (sine wave, cosine wave, hyperboles, etc.), but none of the functions yielded a positive result. Based on the conducted analysis, we revealed the necessity and set the task to determine the course of subsequent calculations and to search for the optimal solution to this problem.

For calculation of the elements of beams with a profiled wall, first of all, it is necessary to determine correctly the calculation model and define dependences, which can describe the operation of models having such structures.

The problem about bending of an extended plate by the cylindrical surface was first solved on the example of a separated band-beam [24] and subsequently will be analyzed and supplemented in [25]. The differential equation that was written down as follows (1), given that stress  $\sigma$  is constant for the whole area of the band:

$$D \cdot \frac{d^4 \omega}{dy^4} - H \cdot \sigma \frac{d^2 \omega}{dy^2} = q. \quad (1)$$

Employing [20], which examines a plate, reinforced with straight-standing parallel elements of rigidity in one direction, we shall write the equation for an anisotropic plate (material possesses excellent properties of medium in different directions) at bending:

$$D_x \cdot \frac{d^4 \omega}{dx^4} + 2 \cdot H \cdot \frac{d^4 \omega}{dx^2 \cdot dy^2} + D_y \cdot \frac{d^4 \omega}{dy^4} = q. \quad (2)$$

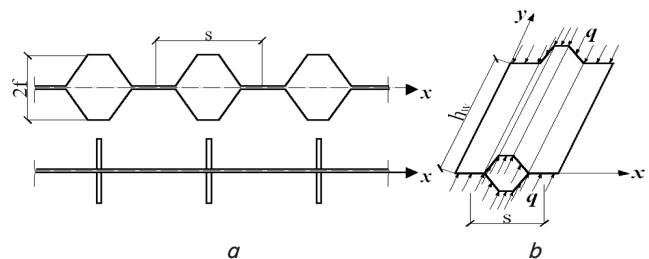


Fig. 1. Profiled wall of trapezoidal outline:

- a – geometrical form of generalized representation;
- b – calculation model in the form of orthotropic plate

Similar to it, we shall present a profiled wall of a beam as an orthotropic plate.

One of elastic properties of a plate is parallel to medium plane (a particular case of anisotropy) with trapezoidal corrugation (Fig. 1, a, b). These corrugations increase rigidity in one direction  $Y$ .

It should be noted that rigidity in direction  $X$  can be neglected as a result of great deformability in this plane for corrugated sheet material. Thus, equation (2) can be transformed, in this case, rigidity of a corrugated wall in the direction, perpendicular to corrugations, it accepted as minimal, so these values in the numerical form can be neglected.

We shall determine transverse rigidity of a double profiled wall at bending relative to axis  $Y$ :

$$D_y = \frac{2 \cdot E \cdot t_w^3}{12(1-\nu^2)} + \frac{E \cdot I}{h_w}, \quad (3)$$

where  $E$  is the modulus of steel elasticity;  $\nu$  is the coefficient of transverse deformation (of Poisson);  $I$  is the moment of inertia of a double profiled wall relative to the axis of symmetry of transverse cross section of a plate;  $h_w$  is the height of the profiled wall of a beam;  $2 \cdot t_w$  is the thickness of the double profiled wall.

After designating overall transverse rigidity of a profiled wall in equation (3)

$$H = \frac{2 \cdot E \cdot t_w^3}{12 \cdot (1-\nu^2)},$$

we shall write a fourth-order differential equation of equilibrium (4). This mathematical expression displays deflection of a rectangular orthotropic plate (loaded with distributed load) in one direction and is written down as follows:

$$D_y \cdot \frac{d^4}{dy^4} \omega(y) - 2 \cdot H \cdot \frac{d^2}{dy^2} \omega(y) = q, \quad (4)$$

where  $q$  is the distributed load, applied to the element of a wall;  $\omega(y)$  is the resulting function for description of transverse deflections.

Using the MatCAD programming environment, we shall find a solution to a fourth-order equation (4) that most accurately meets the original conditions and tendencies of changing the values of transverse deformations for the height of the wall. Solving this equation makes it possible to derive a function of bending axis of the wall of a beam, which subsequently can be used for finding dependences of the stressed-strained state of this type of structures.

In order to examine required function in a general form, we shall represent boundary conditions: first derivative:

$$z(y) = \frac{d}{dy} \omega(y);$$

– second derivative:

$$k(y) = \frac{d^2}{dy^2} \omega(y);$$

– third derivative:

$$r(y) = \frac{d^3}{dx^3} \omega(y);$$

– fourth derivative:

$$p(y) = \frac{d^4}{dx^4} \omega(y).$$

Taking into account such boundary conditions, we shall search for the original feature, which displays elastic surface of a plate at stability loss.

Through numerous iterations in the MatCAD environment using elementary functions, listed earlier, and considering the practice of differentiation of such equations [22, 23], it is possible to represent the original function in the following form

$$\omega_1(y) = \exp \left[ k_w \cdot \ln \left( n \cdot \frac{y \cdot \pi}{h_w} \right) \right]$$

or

$$\omega_1(y) = \operatorname{sh} \left( \frac{k_w \cdot n \cdot \pi \cdot y}{h_w} \right) + \operatorname{ch} \left( \frac{k_w \cdot n \cdot \pi \cdot y}{h_w} \right), \quad (5)$$

where  $n$  is the number of half-waves of stability loss on the surface of the plate;  $k_w$  is the coefficient that depends on load (represents power of argument of function  $\omega_1(y)$  of deformation of the beams' middle axis);  $y$  is the argument that is assigned within the limits of the height of a wall;

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}, \quad \operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

are the hyperbolic sine and cosine, respectively.

An equation is considered solved only if all boundary conditions are satisfied. Functions (5) at differentiation in MatCAD give the same results and take into account basic parameters of a corrugated wall. The listed power and parabolic functions are basic, not obtained earlier solutions of the differential equation (4), adapted to this problem.

If there is an original equation and functions of deflection axis of a beam that is its solution, such mathematical algorithm can be applied in calculations of corrugated structures.

## 5. Research results on the example of a steel beam with a double profiled wall

Let us specify the above calculation algorithm on the example of a steel beam with a double profiled wall of trapezoidal outline with the aim of obtaining calculation formulae of bending moments and normal stresses in the wall. Assigning argument  $y$  (0; 50), we shall write down the original function:

$$\omega_1(y) = \exp \left[ 4,5 \cdot \ln \left( 1 \cdot \frac{y \cdot \pi}{50} \right) \right]$$

or

$$\omega_1(y) = \operatorname{sh} \left( \frac{4,5 \cdot n \cdot \pi \cdot y}{50} \right) + \operatorname{ch} \left( \frac{4,5 \cdot 1 \cdot \pi \cdot y}{50} \right). \quad (6)$$

For dependence (6), using the method of argument substitution  $y=0$ ;  $y=10$ , we shall acquire boundary conditions for the differential equation for two characteristic points of the original functions:  $\omega(0)=0$ ;  $\omega'(0)=0$ ;  $\omega''(0)=0$ ;  $\omega'''(0)=0$ ;  $\omega(10)=0.124$ ;  $\omega'(10)=0.056$ ;  $\omega''(10)=0.019$ ;  $\omega'''(10)=0.004864$ .

We shall graphically represent the original function and solution of the differential equation, calculated using the MatCAD software (Fig. 2, a).

Original function  $\omega_1(y)$  and dependence, derived while solving equation  $\omega(y)$ , coincide and can be used to obtain calculation parameters of work of the corrugated wall of a beam. Approximating function must as exactly as possible correspond to tendencies of change of transverse deformations by the height of a wall in order to subsequently compare the nature of resulting diagrams in the course of calculations with the use of the method of finite elements.

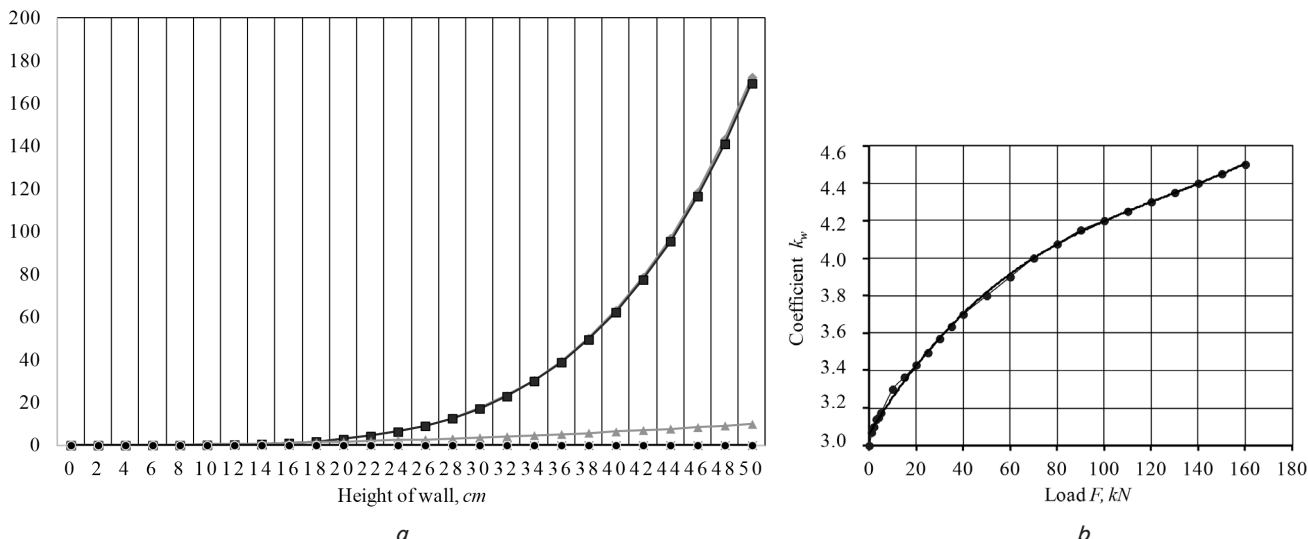


Fig. 2. Parameters of stressed state of profiled wall: *a* – parameters of differential equation (4): ▲▲▲ – *z*(*y*); +++ – *k*(*y*); ●●● – *r*(*y*); \*\*\* – *p*(*y*); ■■■ – *ω*(*y*); ◆◆◆ – *ω*<sub>1</sub>(*y*); *b* – diagram of coefficient *k<sub>w</sub>*; – dependence of coefficient *k<sub>w</sub>* on load; — – polynomial dependence

Let us explore in detail dependence of coefficient *k<sub>w</sub>* (displays power of argument of function ω<sub>1</sub>(*y*) of deformation of the medium beam’s axis) on applied load *F*(kN) by constructing a graph (Fig. 2, *b*). It was graphically shown that is possible to calculate coefficient *k<sub>w</sub>* by the obtained polynomial expression:

$$k_w = -5 \cdot 10^{-5} \cdot F^2 + 0,0163 \cdot F + 3,0932. \quad (7)$$

A guarantee of obtaining polynomial (7) is 0.9952 (is assigned in the MatCAD environment).

Presented parameters of the stressed state of a profiled wall (Fig. 2, *a*) were obtained as one of the calculation results of the software, written in the MatCAD environment. Graphical representation of dependence ω<sub>1</sub>(*y*) and ω(*y*) indicates coincidence of these functions.

Calculation of values of bending moments and normal stresses in a wall was carried out from the following formulae:

$$\frac{d^2\omega}{dy^2} = -\frac{M}{EI},$$

provided *EI*=*D<sub>y</sub>* (for corrugated sheet material), we shall obtain:

$$M = -D_y \cdot \frac{d^2}{dy^2} \omega(y), \quad (8)$$

$$\sigma = \frac{M}{I_x} \cdot y, \quad (9)$$

where *I<sub>x</sub>* is the moment of inertia of cross section of the wall relatively to neutral axis; *y* is the distance from axis *X* to fibrous points of cross section of the wall.

Guided by assumptions that on the section of the flange, the values of normal stresses increase monotonously and do not undergo significant changes, the resulting dependence for the wall of a beam was extended to the thickness of a flange.

The nature of the diagram and the values of stresses for the most stressed sections of beams are illustrated in Fig. 3, *a*.

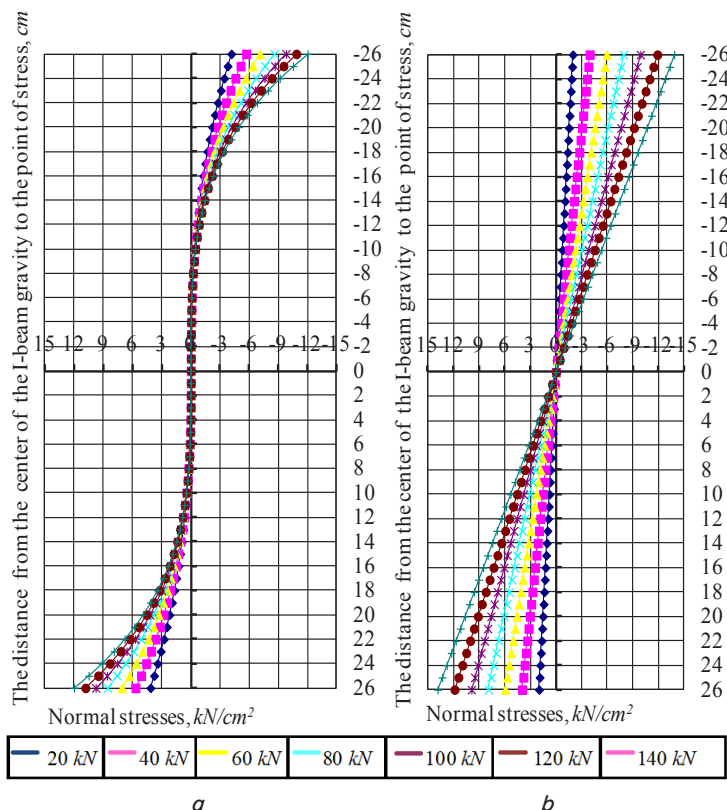


Fig. 3. Diagram of normal stresses of beams with a profiled wall: *a* – taking into account proposed function; *b* – by traditional calculation

The values of normal stresses by the form of dependence are in line with previous calculations (parabolic dependences) and subsequent experimental data. The conducted calculations are consistent with the results of earlier studies [18, 26–28]. It should be noted that an increase in stresses in the wall of beams, in particular in the lower and upper parts are caused by the influence of work of the flanges,



that accept a much larger part of deflection moment. In the middle third of a beam, normal stresses decrease, thus, the influence of deflection moment on stressed-strained state of the wall is minimal.

In accordance with traditional calculation, flanges of a beam are calculated for compression and stretching for the uniaxial stressed state. For elements with a transversally corrugated wall, bent in one of the major planes, verification of maximum stresses in belts is performed by formula:

$$\sigma_f = \frac{M_x}{A_f \cdot h} \leq R_y \gamma_c, \tag{10}$$

where  $\sigma_f$  is the normal stresses in belts of a beam with a transversally corrugated wall;  $A_f$  is the area of the belt of a beam;  $h=h_w+t_f$  is the distance between the centers of belts' weight;  $t_f$  is the thickness of the belt of a beam;  $R_y$  is the calculation resistance of steel of stretching, compression, and deflection beyond fluidity measures.

Using (10), if we substitute the moment of inertia of cross section of beams relative to the neutral axis (a wall and a flange), it is possible to find the values of maximum normal stresses with the help of the traditional method (Fig. 3, b).

**6. Discussion of results of comparative calculation of a beam with a profiled wall**

It is generally accepted that a corrugated wall is not able to accept longitudinal forces, so in [12, 29], it is noted that deflection moment in a beam is completely accepted only by flanges, whereas a wall works for transverse forces approximately to an ideal double tee. But there is a possible impact of a part of the wall on resistance of deflection compared to the ideal shape of a beam because corrugation gives a wall significant transverse deflection rigidity. This issue has not been studied in detail and still remains relevant.

The problem of stability of a wall was not considered and it is believed to be provided by corrugation. Only evenly distributed external load under condition of using stiffeners for each local action was explored.

In the discussion of the obtained results, we shall note that authors addressed the solution of such problems, but only for flat plates, describing the nature of work by means of differential equations [5–7]. That is why, based on the already existing practice of searching for methods to calculate beams with corrugated walls of different configuration [12–17, 19], there was a need to address this task for the profiled plates, in particular, box cross section.

These results were verified when calculating the examined structures using the method of finite elements (the software complex SCAD), (Fig. 4), and require further experimental research that would confirm them. We shall note that during computer simulation of work, the calculation model only partially meets the characteristics of a real object. The idealization of structures was performed in a form,

adapted to the application of the method of finite elements, in other words, the system was represented in the form of a set of standard bodies (rods, plates, membranes, etc.). Such bodies are the finite elements (Fig. 4, a). We also constructed a deformation scheme of the compartment (Fig. 4, b).

In general, the final result depended on the accuracy of construction and simplification of the estimated model. When considering a beam compartment separately, it is necessary to note a large influence of local stresses (Fig. 4, c). Stress jump occurred in the range of force application in the upper part of the wall. It was found that with the gradual movement downwards the structure, the diagrams obtain the traditional form.

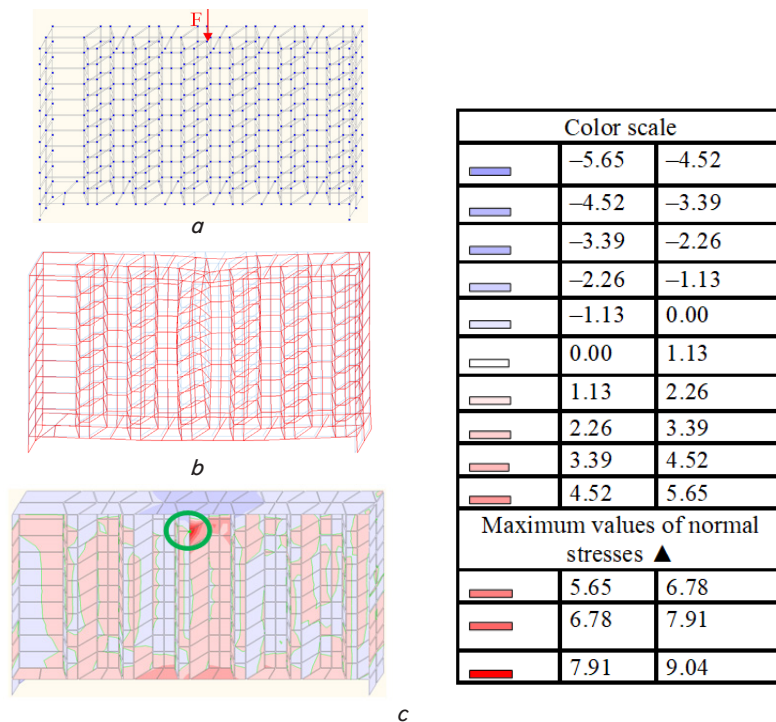


Fig. 4. Parameters of stressed-strained state of compartment of beam with profiled wall at load of 120 kN: a – finite element model of a compartment of a beam; b – joint representation of original (—) deformed scheme; c – distribution of normal stresses, kN/cm<sup>2</sup>

To provide uniformed distribution of local stresses, it can be an appropriate structural solution to mount a stiffener under the place of concentrated load application.

Advantages of the conducted research include deriving the function of bending axis of the wall of a beam, which is described rather reliably. We shall note that in polynomial approximation, the error approaches 0.5%. Considering shortcomings of the research, it is possible to mention insufficiency of conducting a thorough experiment for the structures, proposed earlier. The presence of a number of experimental data will make it possible in the future to conduct comparative analysis of the proposed techniques and an experiment. According to comparison of results of the conducted trial experiment with the presented technique, deflection moment is accepted by flanges and the wall sections that are close to the flanges of the designed beams within (0.1÷0.2)  $h_w$  [30].

The present research, in particular the developed technique for calculating beams with a profiled wall, could be used for corrugations with different outlines (wavy, trap-

ezoidal). Resulting dependences for normal stresses of the cross section might also be used in the design of profiled (corrugated) structures, which are becoming more widely applied under present conditions.

Research work is a continuation of a series of previous studies, but methodical approaches to calculation of profiled structures and nodes require further improvement. Continuation of the search for methods of calculation concerns consideration of influence of local and especially complex loads. As well as for all thin-walled structures, there is a need to subsequently address the problem of providing stability of the wall and prevention of cross section deplanation.

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## 7. Conclusions

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1. A reliable model of a profiled wall of a new beam, box cross section in the form of an orthotropic plate with distributed load was accepted.

2. We proposed generalized original power and hyperbolic functions of bending axis of a profiled plate  $\omega_1(y)$  in the form of solution to a fourth-order differential equation of equilibrium in the MatCAD environment, which describes its work under load. The reduced functions were found to take into account characteristics of the profiled cross section of a wall.

3. Methods of evaluation of stressed-strained state of beams with a profiled wall, which make up determining of parameters  $M$  and  $\sigma$ , were applied. A specific feature of these parameters, which implies using deformation function  $\omega(y)$ , derived when solving differential equation, was revealed.

4. The obtained values of normal stresses on the boundary of a profiled wall in accordance with the developed calculation algorithm are about  $\pm 9$  kN/cm<sup>2</sup>. This is proved by the values of maximum stresses in cross section of a beam according to calculation method of finite elements in the SCAD program. In the proposed structure of the beams with a profiled wall of box cross section, distribution of normal stresses by the parabolic law was proved.

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