

*Визначений коефіцієнт корисної дії диференціалів, коли ведучою ланкою є сонячне колесо першої ступені, веденою – водило останньої, а ланками керування є епіцикли окремих ступеней. Отримані результати мають практичне застосування при проектуванні нових пристроїв керування швидкістю, дозволяють оцінити роботу диференціалів з огляду на енерговитрати і самогальмування та є підґрунтям для подальших досліджень*

*Ключові слова: коефіцієнт корисної дії, зубчастий диференціал, зміна швидкості, сателіт, епіцикл*

*Определен коэффициент полезного действия дифференциалов, когда ведущим звеном является солнечное колесо первой ступени, ведомым – водило последней, а звеньями управления – эцикли отдельных ступеней. Полученные результаты имеют практическое применение при проектировании новых устройств управления скоростью, позволяют оценить работу дифференциалов с точки зрения самоторможения и являются подспорьем для дальнейших исследований*

*Ключевые слова: коэффициент полезного действия, зубчатый дифференциал, изменение скорости, сателлит, эцикл*

# DETERMINING PERFORMANCE EFFICIENCY OF THE DIFFERENTIAL IN A DEVICE FOR SPEED CHANGE THROUGH EPICYCLE

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## 1. Introduction

Performing technological operations by machines in different industries requires control over changes in speed for the magnitude and direction of controlling elements. There are widely known engineering methods and devices for stepwise [1–4] and stepless [5–7] control over speed for the magnitude and direction in the form of step and stepless speed boxes. Known methods and devices for controlling the changes of speed have many drawbacks. The main disadvantages of stepwise speed control for the magnitude and direction are the complexity of device designs, high material consumption, large dynamic loads that occur during transition from one speed to another even in the presence of synchronizers. As far as the stepless speed control is concerned, the main drawback is an intensive wear of parts due to friction caused by the application of friction brakes and blocking friction clutches. The specified drawbacks negatively affect the durability and reliability of parts of drives and machinery in general. Design at the level of inventing a load stopper [8, 9] in the form of hydraulic system and its application in differential gears, which are the subject of special interest, for example [10–12], led to the creation of new devices to control changes in speed [13]. Designed devices eliminate the indicated drawbacks.

In order to conduct research into strength, to design and operate such devices, it is necessary to have a knowledge about performance efficiency (PE). Determining PE for a gear differential, in which speed is controlled by the means of the above-mentioned hydraulic system through epicycles, is an important task. Solving it would make it possible to estimate energy consumption by the toothed differential and the possibility of automatic braking.

## 2. Literature review and problem statement

Toothed differential transmissions were studied by authors of papers [14–22].

In [14], structural changes were proposed to improve the two-stage planetary gears of a large gear ratio without taking performance efficiency into consideration.

In [15], authors analyzed impact of change in the shape of a tooth profile of the planetary transmission on the dynamic without accounting for energy cost.

In [16], a nonlinear dynamic model of the two-stage planetary mechanism was proposed and investigated, based on the analytical solution to dynamic equations in the MATLAB software, but the authors failed to consider the losses expressed by performance efficiency.

In [17], methods for calculating the efficiency of toothed gears, estimated by performance efficiency are addressed, based on the hypothesis of constant friction coefficient and uniform distribution of load along the line of contact in the teeth engagement. None of them, however, is accurate, which is why the derived expressions make it possible to determine approximate losses of friction power.

Authors of [18] study PE in a conical differential, where power is distributed between two driven links. There are proposals to change the design of the conical differential to avoid self-blocking and to improve overall efficiency.

In [19], the formulae of theoretical efficiency are derived for the two-step differential transmissions, which were tested in the course of experimental research.

Paper [20] addresses PE of complex toothed gears based on the graphical and spiral theories that make it possible to obtain approximate values.

In [21], authors considered the process of speed control changes by using a differential transmission with a closed hydraulic system when the drive link is the sun cogwheel, and the driven one – a carrier, or vice versa. Controlling link is the epicycle.

In [22], work of the toothed differential transmission was analyzed in a speed change device when the drive link is the sun cogwheel, and the driven one – a carrier, or vice versa. Controlling link for a change in speed is the epicycle. A theoretical study of PE for such transmission was performed and, by using computer simulation, its dependences on the transmission parameters were obtained in the graphical form.

To expand the range of speed change in machine drives, it is necessary to apply multistage toothed differential transmissions. Such possibilities were addressed at scientific and technical conferences, symposia, and other scientific debates. Based on the analysis of periodic scientific sources, one can conclude that PE of these transmissions has not been given sufficient attention up to now. The issues of determining PE of such transmissions are still awaiting a solution. The devices for controlling changes in speed equipped with such multistage toothed differential transmissions, which were developed at the level of patents of Ukraine for useful models, for example [23, 24] and others, require further theoretical and practical kinematic, power and energy studies.

### 3. The aim and objectives of the study

The aim of present work is to conduct theoretical-computer study into PE of multistage toothed differentials in the devices for changing speed through epicycles in order to assess their energy efficiency and the possibility of automatic braking.

To accomplish the aim, the following tasks have been set:

- to derive analytical expressions for determining PE;
- to derive graphical dependences of PE on angular velocity of the epicycle, transfer ratio and number of stages;
- to draw conclusions on the possibility and feasibility of practical application of the two- and three-stage toothed differentials.

### 4. Research methods

Analytical expressions for determining PE are derived in the following way. For the toothed differential

transmissions, PE in practice is determined based on the losses for friction in each clutch of wheels. These losses are proportional to the product of circular force on teeth and circular velocity of the point of initial circle of the satellite relative to the carrier, or the product of torque of this force by angular velocity. This product is called potential power. By applying this method, analytical expressions were derived in [22] for determining PE of a single-stage toothed differential transmission when a sun cogwheel is the drive wheel, and the carrier is the driven wheel, or vice versa. Based on a given procedure [22] and the relation between speeds [21] of the drive and the driven link of stages, we derived for the present work analytical expressions for PE of the multistage toothed differential.

Graphical dependences of PE on transfer ratio, angular velocity of the epicycle and the number of stages were obtained using computer simulation that employed software package MS Excel. As a result, based on the graphical dependences, we draw conclusions on the possibility and feasibility of practical application of the multistage toothed differentials.

## 5. Results of research into PE of the multistage toothed differential

Fig. 1 shows a schematic of the multistage toothed differential.

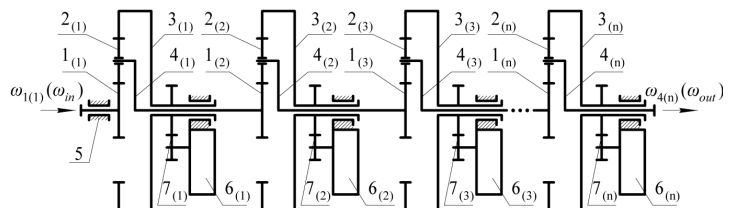


Fig. 1. Schematic of the multistage toothed differential transmission with closed-loop hydraulic systems with a drive sun cogwheel and the driven carrier with control over speed via epicycles

In a given differential, carrier of the first stage  $4_{(1)}$  is connected to sun cogwheel  $1_{(2)}$  of the second stage, carrier of the second stage  $4_{(2)}$  is connected to sun cogwheel  $1_{(3)}$  of the third stage, etc. Control over speed is executed through epicycles  $3_{(1)}$  of the first,  $3_{(2)}$  of the second,  $3_{(3)}$  of the third, ...,  $3_{(n)}$  of the  $n$ -th, stages, using closed hydraulic systems  $6_{(1)}$ ,  $6_{(2)}$ ,  $6_{(3)}$ , ...,  $6_{(n)}$ , mounted onto them.

The drive wheel in such a transmission is the sun cogwheel  $1_{(1)}$  of the first stage, and the driven link is carrier  $4_{(n)}$  of the  $n$ -th stage. Closed hydraulic systems  $6_{(1)}$ ,  $6_{(2)}$ ,  $6_{(3)}$ , ...,  $6_{(n)}$  are of identical design, arranged inside housing 5 and connected to the epicycles through toothed gears  $7_{(1)}$ ,  $7_{(2)}$ ,  $7_{(3)}$ , ...,  $7_{(n)}$ . Design of closed hydraulic systems  $6_{(1)}$ ,  $6_{(2)}$ ,  $6_{(3)}$ , ...,  $6_{(n)}$  are identical, they are arranged inside housing 5 and connected to the epicycles through toothed gears  $7_{(1)}$ ,  $7_{(2)}$ ,  $7_{(3)}$ , ...,  $7_{(n)}$ . A closed hydraulic system whose structure and work are described in more detail, for example in [13], includes a geared hydraulic pump, short pipes, control valve, check valve and a container for a liquid. Control over change in speed is executed by a control valve through which a fluid moves in a closed hydraulic system.

The relationship between speeds of the driven link (carrier  $4_{(1)}$ )  $\omega_{out}$  and the drive link (sun cogwheel  $1_{(1)}$ )  $\omega_{in}$ , or vice

versa, for each stage of the toothed differential transmission, is described in [21].

It is possible to determine PE  $\eta_{i(1)-4(n)}$  of the transmission (Fig. 1) from expression

$$\eta_{i(1)-4(n)} = \eta_{i4(1)}\eta_{i4(2)}\eta_{i4(3)} \cdots \eta_{i4(n)}, \quad (1)$$

where  $\eta_{i4(1)}, \eta_{i4(2)}, \eta_{i4(3)}, \dots, \eta_{i4(n)}$  are, respectively, performance efficiency of the first, second, third, ...,  $n$ -th stages, derived from [22]:

$$\begin{aligned} \eta_{i4(1)} &= \frac{(1+u_{13(1)}^{(4)}\eta_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)})}{(1+u_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}\eta_{13(1)}^{(4)})}, \\ \eta_{i4(2)} &= \frac{(1+u_{13(2)}^{(4)}\eta_{13(2)}^{(4)}) (\omega_{1(2)} + \omega_{3(2)}u_{13(2)}^{(4)})}{(1+u_{13(2)}^{(4)}) (\omega_{1(2)} + \omega_{3(2)}u_{13(2)}^{(4)}\eta_{13(2)}^{(4)})}, \\ \eta_{i4(3)} &= \frac{(1+u_{13(3)}^{(4)}\eta_{13(3)}^{(4)}) (\omega_{1(3)} + \omega_{3(3)}u_{13(3)}^{(4)})}{(1+u_{13(3)}^{(4)}) (\omega_{1(3)} + \omega_{3(3)}u_{13(3)}^{(4)}\eta_{13(3)}^{(4)})}, \\ &\dots; \\ \eta_{i4(n)} &= \frac{(1+u_{13(n)}^{(4)}\eta_{13(n)}^{(4)}) (\omega_{1(n)} + \omega_{3(n)}u_{13(n)}^{(4)})}{(1+u_{13(n)}^{(4)}) (\omega_{1(n)} + \omega_{3(n)}u_{13(n)}^{(4)}\eta_{13(n)}^{(4)})}, \end{aligned} \quad (2)$$

where  $\omega_{1(1)}, \omega_{1(2)}, \omega_{1(3)}, \dots, \omega_{1(n)}$  are the angular velocities of sun cogwheels of the first, second, third, ...,  $n$ -th stages, respectively;  $\omega_{4(1)}, \omega_{4(2)}, \omega_{4(3)}, \dots, \omega_{4(n)}$  are the angular velocities of epicycles of the first, second, third, ...,  $n$ -th stages, respectively;  $u_{13(1)}^{(4)}, u_{13(2)}^{(4)}, u_{13(3)}^{(4)}, \dots, u_{13(n)}^{(4)}$  are the transfer ratios of transmissions at stopped carrier, of the first, second, third, ...,  $n$ -th stages, respectively;  $\eta_{13(1)}^{(4)}, \eta_{13(2)}^{(4)}, \eta_{13(3)}^{(4)}, \dots, \eta_{13(n)}^{(4)}$  are their PE at stopped carrier.

We shall consider the case when the drive link of such a transmission is carrier  $4_{(1)}$  of the first stage, and the driven link is sun cogwheel  $1_{(n)}$  of the  $n$ -th stage (Fig. 2).

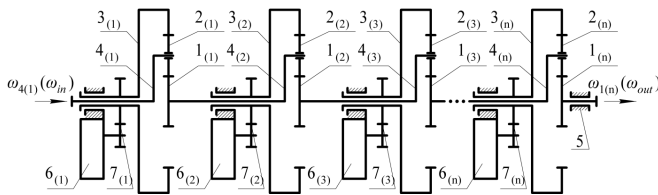


Fig. 2. Schematic of the multistage toothed differential transmission with closed-loop hydraulic systems with a drive carrier and the driven sun cogwheel with control over speed via epicycles

Expression of its PE  $\eta_{4(1)-1(n)}$  takes the form

$$\eta_{4(1)-1(n)} = \eta_{41(1)}\eta_{41(2)}\eta_{41(3)} \cdots \eta_{41(n)}, \quad (3)$$

where  $\eta_{41(1)}, \eta_{41(2)}, \eta_{41(3)}, \dots, \eta_{41(n)}$  are PE of the first, second, third, ...,  $n$ -th stages, respectively, which are derived from [22]:

$$\begin{aligned} \eta_{41(1)} &= \frac{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}] \eta_{13(1)}^{(4)}}{\omega_{4(1)}(\eta_{13(1)}^{(4)} + u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}}, \\ \eta_{41(2)} &= \frac{[\omega_{4(2)}(1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}] \eta_{13(2)}^{(4)}}{\omega_{4(2)}(\eta_{13(2)}^{(4)} + u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}}, \end{aligned}$$

$$\begin{aligned} \eta_{41(3)} &= \frac{[\omega_{4(3)}(1+u_{13(3)}^{(4)}) - \omega_{3(3)}u_{13(3)}^{(4)}] \eta_{13(3)}^{(4)}}{\omega_{4(3)}(\eta_{13(3)}^{(4)} + u_{13(3)}^{(4)}) - \omega_{3(3)}u_{13(3)}^{(4)}}, \\ &\dots; \\ \eta_{41(n)} &= \frac{[\omega_{4(n)}(1+u_{13(n)}^{(4)}) - \omega_{3(n)}u_{13(n)}^{(4)}] \eta_{13(n)}^{(4)}}{\omega_{4(n)}(\eta_{13(n)}^{(4)} + u_{13(n)}^{(4)}) - \omega_{3(n)}u_{13(n)}^{(4)}}, \end{aligned} \quad (4)$$

where  $\omega_{4(1)}, \omega_{4(2)}, \omega_{4(3)}, \dots, \omega_{4(n)}$  are the angular speeds of carriers of the first, second, third, ...,  $n$ -th stages, respectively;  $\omega_{3(1)}, \omega_{3(2)}, \omega_{3(3)}, \dots, \omega_{3(n)}$  are the angular speeds of their epicycles, respectively;  $u_{13(1)}^{(4)}, u_{13(2)}^{(4)}, u_{13(3)}^{(4)}, \dots, u_{13(n)}^{(4)}$  are the transfer ratios of gears at stopped carrier of the first, second, third, ...,  $n$ -th stages, respectively;  $\eta_{13(1)}^{(4)}, \eta_{13(2)}^{(4)}, \eta_{13(3)}^{(4)}, \dots, \eta_{13(n)}^{(4)}$  are PE of gears at stopped carrier of the first, second, third, ...,  $n$ -th stages, respectively.

Given the complexity of the task, we performed here theoretical-computer research into PE of the multistage toothed differential transmissions in the devices of speed change with the closed-loop hydraulic systems, using two- and three-stage transmission as examples.

Fig. 3 shows a two-stage toothed differential.

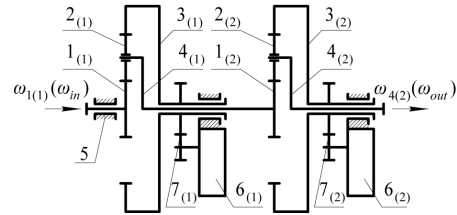


Fig. 3. Schematic of a two-stage toothed differential transmission with closed-loop hydraulic systems with a drive sun cogwheel and the driven carrier with control over speed via epicycles

In a given transmission, carrier of the first stage  $4_{(1)}$  is connected to sun cogwheel  $1_{(2)}$  of the second stage. Speed control is executed through the epicycles of the first  $3_{(1)}$  and the second  $3_{(2)}$  stages using closed hydraulic systems  $6_{(1)}$  and  $6_{(2)}$ , mounted onto them. The drive link of such a two-stage differential transmission is sun cogwheel  $1_{(1)}$  of the first stage, and the driven link is carrier  $4_{(2)}$  of the second stage.

Expression for determining PE of the two-stage toothed differential transmission takes the form

$$\eta_{i(1)-4(2)} = \eta_{i4(1)}\eta_{i4(2)}, \quad (5)$$

where

$$\eta_{i4(1)} = \frac{(1+u_{13(1)}^{(4)}\eta_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)})}{(1+u_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}\eta_{13(1)}^{(4)})}, \quad (6)$$

$$\eta_{i4(2)} = \frac{(1+u_{13(2)}^{(4)}\eta_{13(2)}^{(4)}) (\omega_{1(2)} + \omega_{3(2)}u_{13(2)}^{(4)})}{(1+u_{13(2)}^{(4)}) (\omega_{1(2)} + \omega_{3(2)}u_{13(2)}^{(4)}\eta_{13(2)}^{(4)})}. \quad (7)$$

With respect to [21]:

$$\omega_{1(2)} = \omega_{4(1)} = \frac{\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}}{1+u_{13(1)}^{(4)}} \quad (8)$$

and by substituting in (7), and by substituting (6) and (7) in (5), we obtain the expression for determining PE of the two-stage toothed differential transmission in a device for speed

change through the epicycle, when the drive wheel is the sun cogwheel, and the carrier is the driven wheel

$$\eta_{1(1)-4(2)} = \frac{(1+u_{13(1)}^{(4)})\eta_{13(1)}^{(4)}(\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)})}{(1+u_{13(1)}^{(4)})\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}\eta_{13(1)}^{(4)}} \times \frac{(1+u_{13(2)}^{(4)})\eta_{13(2)}^{(4)}[\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}(1+u_{13(1)}^{(4)})]}{(1+u_{13(2)}^{(4)})[\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}\eta_{13(2)}^{(4)}(1+u_{13(1)}^{(4)})]} \quad (9)$$

To perform a quantitative analysis of change in PE, expression (9) is programmed on a computer for the assigned parameters. Thus, angular speed of the drive link, sun cogwheel, is accepted to be  $\omega_{1(1)} = 75; 100; 150; 300$  rad/s. Transfer ratios of stages are taken at stopped carrier and vary within the range of  $u_{13(i)}^{(4)} = u_{13(2)}^{(4)} = 1,0...10$ . Angular speeds of control links, the epicycles, are  $\omega_{3(1)} = \omega_{3(2)} = 0...25$  rad/s. PE of toothed gears at stopped carrier is  $\eta_{13(i)}^{(4)} = \eta_{13(2)}^{(4)} = 0,97$ . Based on these data, we constructed graphical dependences of performance efficiency as a function of  $\eta_{1(1)-4(2)} = f(\omega_{4(1)}, \omega_{1(i)}, u_{13(i)}^{(4)}, \eta_{13(i)}^{(4)})$ . One of them is shown in Fig. 4.

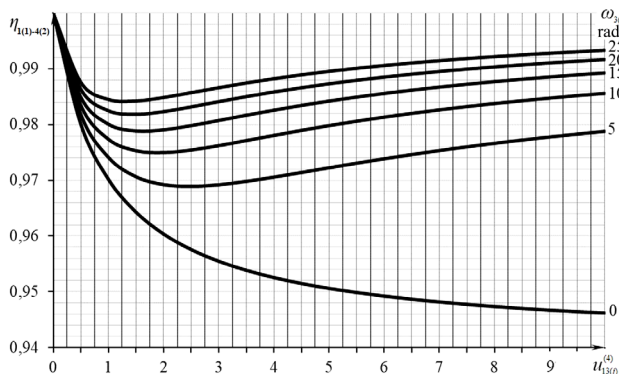


Fig. 4. Graphical dependences of PE of the two-stage toothed differential transmission with a sun cogwheel as the drive wheel and a carrier as the driven wheel; control is executed via epicycles  $\eta_{1(1)-4(2)} = f(\omega_{4(1)}, \omega_{3(i)}, u_{13(i)}^{(4)}, \eta_{13(i)}^{(4)})$  for  $\omega_{1(1)} = 100$  rad/s

If the drive link in a two-stage toothed differential transmission is carrier 4<sub>(1)</sub> of the first stage, and the driven link is sun cogwheel 1<sub>(2)</sub> of the second stage, we obtain the transmission shown in Fig. 5.

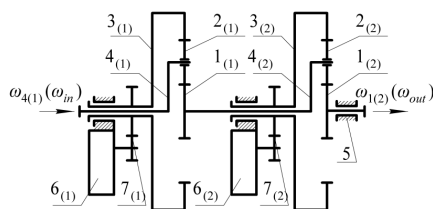


Fig. 5. Schematic of the two-stage toothed differential transmission with closed-loop hydraulic systems with the drive carrier, the driven sun cogwheel, and control via epicycles

Expression for PE takes the form

$$\eta_{4(1)-1(2)} = \eta_{41(1)}\eta_{41(2)}, \quad (10)$$

where

$$\eta_{41(1)} = \frac{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}]\eta_{13(1)}^{(4)}}{\omega_{4(1)}(\eta_{13(1)}^{(4)} + u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}}, \quad (11)$$

$$\eta_{41(2)} = \frac{[\omega_{4(2)}(1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}]\eta_{13(2)}^{(4)}}{\omega_{4(2)}(\eta_{13(2)}^{(4)} + u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}}. \quad (12)$$

If we accept from [22]

$$\omega_{4(2)} = \omega_{1(1)} = \omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)} \quad (13)$$

and substitute (13) in (12), and substitute (11) and (12) in (10), we obtain the expression for determining PE for the two-stage toothed differential transmission in a device for speed change through the epicycle, when the drive wheel is a carrier, and the driven wheel is a sun cogwheel

$$\eta_{4(1)-1(2)} = \frac{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}]\eta_{13(1)}^{(4)}}{\omega_{4(1)}(\eta_{13(1)}^{(4)} + u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}} \times \frac{\{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}\}\eta_{13(2)}^{(4)}}{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](\eta_{13(2)}^{(4)} + u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}}. \quad (14)$$

Similar to the first case, expression (14) was programmed for the same parameters; we constructed graphical dependences of performance efficiency, one of which is shown in Fig. 6.

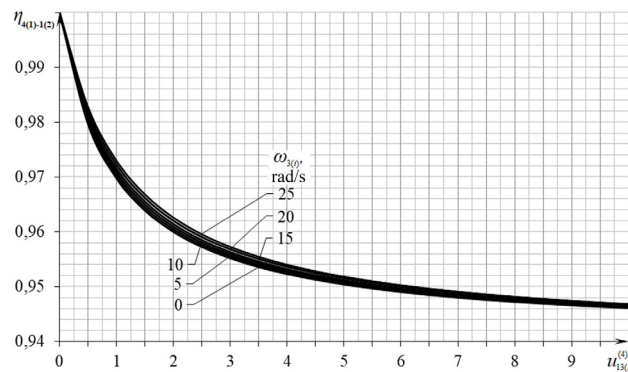


Fig. 6. Graphical dependences of PE of the two-stage toothed differential transmission with a carrier as the drive wheel and a sun cogwheel as the driven wheel; control is executed via epicycles  $\eta_{4(1)-1(2)} = f(\omega_{4(1)}, \omega_{3(i)}, u_{13(i)}^{(4)}, \eta_{13(i)}^{(4)})$  for  $\omega_{4(1)} = 100$  rad/s

Fig. 7 shows a three-stage toothed differential transmission.

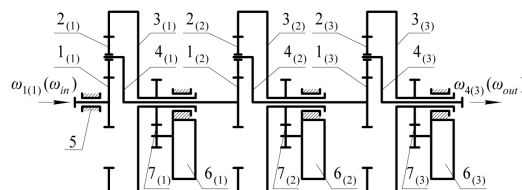


Fig. 7. Schematic of the three-stage toothed differential transmission with closed-loop hydraulic systems with the drive sun cogwheel, the driven carrier, and control via epicycles

In a given transmission, carrier of the first stage 4<sub>(1)</sub> is connected to sun cogwheel 1<sub>(2)</sub> of the second stage, carrier of the second stage 4<sub>(2)</sub> is connected to sun cogwheel 1<sub>(3)</sub> of the third stage. Speed control is executed by toothed gears, the epicycles of the first 3<sub>(1)</sub>, second 3<sub>(2)</sub> and third 3<sub>(3)</sub> stages using closed hydraulic systems 6<sub>(1)</sub>, 6<sub>(2)</sub> and 6<sub>(3)</sub>, mounted onto them. The drive link of such a transmission is sun cogwheel 1<sub>(1)</sub> of the first stage, and the driven link is carrier 4<sub>(3)</sub> of the third stage.

Expression for PE of such a transmission takes the form

$$\eta_{1(1)-4(3)} = \eta_{14(1)}\eta_{14(2)}\eta_{14(3)}, \quad (15)$$

where

$$\eta_{14(1)} = \frac{(1+u_{13(1)}^{(4)}\eta_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)})}{(1+u_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}\eta_{13(1)}^{(4)})}, \quad (16)$$

$$\eta_{14(2)} = \frac{(1+u_{13(2)}^{(4)}\eta_{13(2)}^{(4)}) (\omega_{1(2)} + \omega_{3(2)}u_{13(2)}^{(4)})}{(1+u_{13(2)}^{(4)}) (\omega_{1(2)} + \omega_{3(2)}u_{13(2)}^{(4)}\eta_{13(2)}^{(4)})}, \quad (17)$$

$$\eta_{14(3)} = \frac{(1+u_{13(3)}^{(4)}\eta_{13(3)}^{(4)}) (\omega_{1(3)} + \omega_{3(3)}u_{13(3)}^{(4)})}{(1+u_{13(3)}^{(4)}) (\omega_{1(3)} + \omega_{3(3)}u_{13(3)}^{(4)}\eta_{13(3)}^{(4)})}. \quad (18)$$

If we again accept from [21]

$$\omega_{1(2)} = \omega_{4(1)} = \frac{\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}}{1 + u_{13(1)}^{(4)}}; \quad (19)$$

$$\omega_{1(3)} = \omega_{4(2)} = \frac{\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}(1+u_{13(1)}^{(4)})}{(1+u_{13(2)}^{(4)})(1+u_{13(1)}^{(4)})} \quad (20)$$

and substitute in (17) and (18), and substitute (16), (17) and (18) in (15), we obtain the expression for determining PE of the three-stage toothed differential transmission in a device for speed change through the epicycles when the drive wheel is a sun cogwheel and the driven wheel is a carrier

$$\eta_{1(1)-4(2)} = \frac{(1+u_{13(1)}^{(4)}\eta_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)})}{(1+u_{13(1)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)}\eta_{13(1)}^{(4)})} \times \frac{(1+u_{13(2)}^{(4)}\eta_{13(2)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}(1+u_{13(1)}^{(4)}))}{(1+u_{13(2)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}\eta_{13(2)}^{(4)}(1+u_{13(1)}^{(4)}))} \times \frac{(1+u_{13(3)}^{(4)}\eta_{13(3)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}(1+u_{13(1)}^{(4)}) + \omega_{3(3)}u_{13(3)}^{(4)}(1+u_{13(2)}^{(4)})(1+u_{13(1)}^{(4)}))}{(1+u_{13(3)}^{(4)}) (\omega_{1(1)} + \omega_{3(1)}u_{13(1)}^{(4)} + \omega_{3(2)}u_{13(2)}^{(4)}(1+u_{13(1)}^{(4)}) + \omega_{3(3)}u_{13(3)}^{(4)}\eta_{13(3)}^{(4)}(1+u_{13(2)}^{(4)})(1+u_{13(1)}^{(4)}))}. \quad (21)$$

For a three-stage toothed differential transmission, when the motion is transferred from a sun cogwheel to the carrier, expression (21) is programmed. Angular velocity of the drive link, the carrier, is accepted to be  $\omega_{1(1)} = 75; 100; 150; 300$  rad/s. Transfer ratios of stages at stopped carrier are the same and change within the range of  $u_{13(1)}^{(4)} = u_{13(2)}^{(4)} = u_{13(3)}^{(4)} = 1, 0 \dots 10$ . Angular velocities of control links, the epicycles, are  $\omega_{3(1)} = \omega_{3(2)} = \omega_{3(3)} = 0 \dots 25$  rad/s. PE of toothed transmissions at stopped carrier is  $\eta_{13(1)}^{(4)} = \eta_{13(2)}^{(4)} = \eta_{13(3)}^{(4)} = 0,97$ . Based on these data, we constructed graphical dependences for performance efficiency, shown in Fig. 8.

A drive link of such a three-stage toothed differential transmission can be carrier  $4_{(1)}$  of the first stage, and a driven link – sun cogwheel  $1_{(3)}$  of the third stage (Fig. 9).

Expression for PE takes the form

$$\eta_{4(1)-1(3)} = \eta_{41(1)}\eta_{41(2)}\eta_{41(3)}; \quad (22)$$

$$\eta_{41(1)} = \frac{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}]\eta_{13(1)}^{(4)}}{\omega_{4(1)}(\eta_{13(1)}^{(4)} + u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}}; \quad (23)$$

$$\eta_{41(2)} = \frac{[\omega_{4(2)}(1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}]\eta_{13(2)}^{(4)}}{\omega_{4(2)}(\eta_{13(2)}^{(4)} + u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}}; \quad (24)$$

$$\eta_{41(3)} = \frac{[\omega_{4(3)}(1+u_{13(3)}^{(4)}) - \omega_{3(3)}u_{13(3)}^{(4)}]\eta_{13(3)}^{(4)}}{\omega_{4(3)}(\eta_{13(3)}^{(4)} + u_{13(3)}^{(4)}) - \omega_{3(3)}u_{13(3)}^{(4)}}. \quad (25)$$

Here we again accept from [22]:

$$\omega_{4(2)} = \omega_{1(1)} = \omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}; \quad (26)$$

$$\omega_{4(3)} = \omega_{1(2)} = [\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)} \quad (27)$$

and substituted in (24) and (25), substituted (23), (24) and (25) in (22); we derived the expression for determining PE of the three-stage toothed differential transmission in a device for speed change through the epicycle when the drive wheel is a carrier, and the driven wheel is a sun cogwheel

$$\eta_{4(1)-1(2)} = \frac{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}]\eta_{13(1)}^{(4)}}{\omega_{4(1)}(\eta_{13(1)}^{(4)} + u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}} \times \frac{\{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}\}\eta_{13(2)}^{(4)}}{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](\eta_{13(2)}^{(4)} + u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}} \times \frac{\{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}\}(1+u_{13(3)}^{(4)}) - \omega_{3(3)}u_{13(3)}^{(4)}}{\{[\omega_{4(1)}(1+u_{13(1)}^{(4)}) - \omega_{3(1)}u_{13(1)}^{(4)}](1+u_{13(2)}^{(4)}) - \omega_{3(2)}u_{13(2)}^{(4)}\}(\eta_{13(3)}^{(4)} + u_{13(3)}^{(4)}) - \omega_{3(3)}u_{13(3)}^{(4)}}}. \quad (28)$$

Similar to the first case for a three-stage toothed differential transmission, but from the carrier to the sun cogwheel, expression (28) is programmed for the same parameters; we constructed graphical dependences for PE, shown in Fig. 10.

Fig. 4, 6, 8, 10 show graphical dependences of PE for a sun cogwheel and a carrier, or vice versa, for the multistage toothed differential with control over speed via epicycles. They clearly demonstrate a change in the magnitude of PE numbers on the transfer ratio, angular velocity of the epicycle, and the number of stages. This allows us to estimate effectiveness of a multistage toothed differential in terms of energy costs and automatic braking.

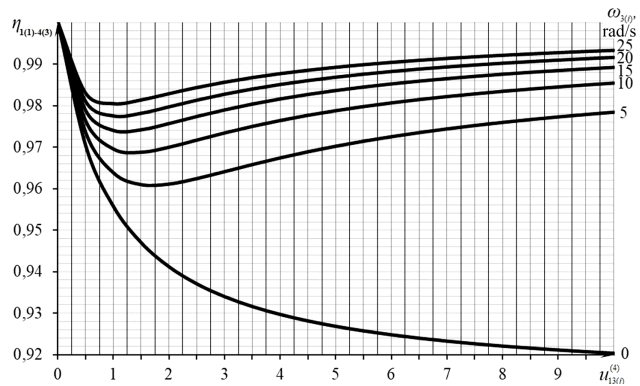


Fig. 8. Graphical dependences of PE of the three-stage toothed differential transmission with a sun cogwheel as the drive wheel and a carrier as the driven wheel; control is executed via epicycles for  $\omega_{1(1)} = 100$  rad/s

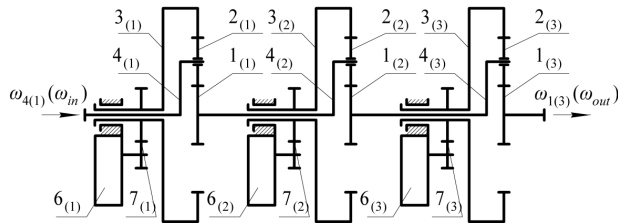


Fig. 9. Schematic of the three-stage toothed differential transmission with closed-loop hydraulic systems with the drive carrier and control via epicycles

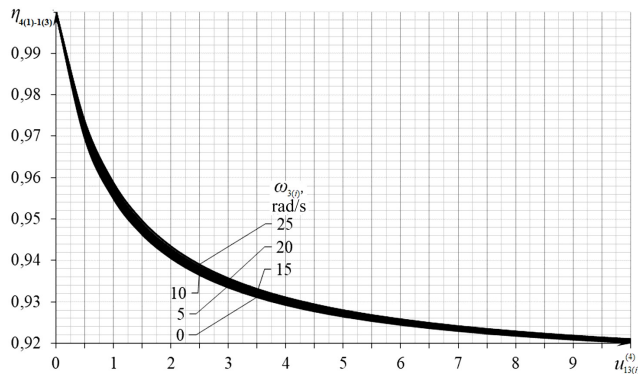


Fig. 10. Graphical dependences of PE of the three-stage toothed differential transmission with a carrier as the drive wheel and a sun cogwheel as the driven wheel; control is executed via epicycles for  $\omega_{4(1)} = 100$  rad/s

## 6. Discussion of results of determining PE of the multistage toothed differential in a device for speed change through the epicycle

We derived analytical expressions for determining PE of multistage toothed differentials in the devices to control speed change through epicycles. By using these expressions, it is possible to accurately determine PE of a toothed differential at known speed of the drive link, speed of the driven link, and gear transfer ratio.

The expressions for PE that we derived could be applied in the case of any number of stages in toothed differentials, but, since they are rather cumbersome for further analysis, we specified them for two- and three-stage transmissions. Estimating PE for differentials with a bigger quantity

of stages using a given methodology could be simplified through the application of computer technologies.

Employing computer simulation, we obtained graphical dependences for PE of two- and three-stage toothed differentials, shown in Fig. 4, 6, 8, 10. Based on these dependences, we have proven the possibility and feasibility of practical application of two- and three-stage toothed differentials. By using a given procedure, it is possible to determine PE of the four- and multistage transmissions, and to draw conclusions on their application in terms of energy costs and automatic braking.

Results of computer simulation are convenient to use, however, for specific numerical parameters of the multi-stage toothed differentials. To solve the problem, it is required to know: speed of the drive link, speed of the driven links, and stage transfer ratios.

Theoretical study that we conducted is the foundation for further power calculations and experimental research into multistage toothed differentials in devices for speed change when the drive link is a sun cogwheel of the first stage, the driven link is a carrier, and control links control are the epicycles of separate stages.

## 7. Conclusions

1. We derived analytical expressions for determining performance efficiency of the multistage toothed differential in a device for speed change with the drive sun cogwheel and the driven carrier, or vice versa. The values of PE could be used at the stage of development and design of such devices in order to estimate their energy efficiency.

2. The obtained graphical dependences for PE of two- and three-stage toothed differentials enable visual tracking of change in the value of PE depending on angular velocity of the epicycle, transfer ratio and the number of stages. This makes it possible to estimate effectiveness of a multistage toothed differential in terms of energy costs and possible automatic braking.

3. In all cases for the two- and three-stage toothed differentials, the condition of automatic braking is not applicable as PE is far greater than zero. This proved the possibility and feasibility of practical application of the two- and three-stage toothed differentials. With an increase in the number of stages in a toothed differential, PE decreases, which confirms general patterns.

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