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DEVELOPMENT OF THE METHOD FOR CREATING EXPLICIT INTEGRAL DYNAMIC MODELS OF MEASURING TRANSDUCERS

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Розглянутий в статті метод полягає у визначенні імпульсної перехідної характеристики за заданими диференціальними рівняннями, яка є ядром явної інтегральної динамічної моделі вимірювальних перетворювачів, як з розподіленими, так і з зосередженими параметрами. Інтегральні динамічні моделі дозволяють розширити алгоритмічні основи комп'ютерного моделювання у задачах дослідження вимірювальних перетворювачів та надають досліднику можливість вибору оптимального виду математичної моделі

Ключові слова: інтегральна динамічна модель, імпульсна перехідна функція, вимірювальний перетворювач, диференціальне рівняння

Рассмотренный в статье метод заключается в определении импульсной переходной характеристики по заданным дифференциальным уравнениям, которая является ядром явной интегральной динамической модели измерительных преобразователей, как с распределенными, так и с сосредоточенными параметрами. Интегральные динамические модели позволяют расширить алгоритмические основы компьютерного моделирования в задачах исследования измерительных преобразователей, а также предоставляют исследователю возможность выбора оптимального вида математической модели

Ключевые слова: интегральная динамическая модель, импульсная переходная функция, измерительный преобразователь, дифференциальное уравнение

1. Introduction

According to the systems approach, widely applied for examining many classes of objects, measuring transducers (MT) are the devices that serve for the perception and primary conversion of information on a physical magnitude to be measured [1–3].

Dynamic models of stationary and non-stationary measuring transducers are typically very different. At the same time, each MT has its own operator that describes it. Such an operator can be the law according to which each given function that describes an input signal is associated with the function that describes the output signal. These are the operators that represent all the variety of dynamic properties inherent to MT, if, of course, this operator is composed taking into consideration all important factors and regularities that accompany MT operation [4].

We shall note that MT is stationary if its dynamic properties do not change over time. If such a change does occur, MT is called a non-stationary. A consequence of constancy in time of dynamic properties of MT is that the process of conversion of the measured magnitudes (impacts) has the property of invariance relative to the time-shift of the measured impacts. Thus, the reaction of stationary MT does not depend on the moment of application of the measured impacts, and depends only on the difference between the current time and the moment of application of the measured impacts. Non-stationary MT do not possess the specified property of invariance and reaction of these MT depends on both the current time and the moment of application of the measured impacts. The inputs of measuring transducers with lumped parameters can be represented by points. Dynamic properties of these MT are typically described by ordinary differential equations (DE) [5, 6]. The inputs of measuring transducers with distributed parameters are continuously distributed along a certain line or a surface. Dynamic properties of such MT are most often described by DO with partial derivatives or, in a generalized form, by functional equations. We shall note a rather important circumstance on that solving DO is a quite complicated computational task [7]. A special feature is those cases when it is required to determine a reaction of MT under a change in the influence of external medium.

One of the ways to successfully tackle the above-indicated problem is the extension of the class of mathematical models to account for the peculiarities of modeling tasks of the examined devices, specifically, through the application of integral dynamic models (IDM) [8]. We shall note that IDM have a number of such positive attributes as high universality (the structure of the model is unchanged for different classes of MT; the properties that are assigned by a single function   the core of the integral operator). IDM also possess a potentially high adequacy of modeling processes, a smoothing property when performing computations and processing signals with high-frequency noise, high convergence of iterative processes for solving computational problems [9]. Thus, one can consider it a relevant task to undertake research into formation of explicit integral dynamic models of MT.

2. Literature review and problem statement

One should point to a certain conditionality of dividing MT into different groups, which can be illustrated using a temperature MT as an example [10]. One of the main facts in the field of thermometry is that at the values of so-called Biot criterion less than 0.02, temperature gradients within the homogeneous thermal receivers are negligible. Therefore, in order to describe dynamic properties of these MTs with a high degree of accuracy, it is possible to confine ourselves to normal first order DE. Accordingly, at the values of Biot criterion larger than 0.02, temperature gradients within thermal receivers are significant, which is why when describing the behavior of these MTs over time, it is required to pass over to the equation of heat conduction in partial derivatives. Under a given geometrical size and thermal conductivity coefficient of the material of a thermal receiver, the magnitude of Biot criterion is fully determined by the value of a convective heat transfer coefficient. In turn, the coefficient of convective heat transfer to a very great extent depends directly on the external conditions that accompany the temperature measurement. It depends on the character of change in the measured temperature and on the flow rate of a liquid or gas whose temperatures must be measured. Therefore, one and the same thermal receiver depending on the measurement conditions can be interpreted as MT with lumped parameters or MT with distributed parameters [11–13].

At the same time, any MT, having essentially distributed parameters, depending on the required accuracy of analysis of dynamic properties, can be described by a regular DE. That is, it can be assigned within the range of required accuracy to MT with lumped parameters [14, 15].

Therefore, the fact of assigning a particular MT to any of the groups is also determined by required accuracy of analysis of dynamic properties of MT [16, 17].

In addition, the next postulate important for modeling is that a certain MT, as an object with distributed parameters, can be described by models completely different in form and structure [18].

The obvious usefulness of various forms of description is in that, for a specific case of measurements, there is some well-chosen model that can ensure the best effectiveness of analysis of dynamic properties of MT [12, 16, 17].

Traditional approach to solving the problems of dynamics is typically based on the use of DE [5, 15]. Theoretical positions and the practice of solving many problems show that in some cases, specifically when building models based on experimental data, it is appropriate to consider, instead of models of the DE type, more common IDM [10, 12].

It is obvious that in a general case, when choosing methods for constructing models of dynamic objects in advance, regardless of the subject of the task, it is difficult to compare approaches based on models of DE or IDM [4, 5, 19]. On can note such advantages of IDM as its great generality and the absence of a differentiation operation.

In addition, it is known that for quite broad classes of MT the use of IDM and, in particular, the models of equivalent DE, makes it possible to receive the basis for building highly stable numerical algorithms for analysis and calculation of their parameters [8, 20, 21].

At the same time, the application of IDM has its peculiarities, consideration of which requires conducting a series of theoretical studies and practical developments [14, 16, 19]. Specifically, it concerns the problems of formation of IDM by the characteristics of MT based on known differential models, since a significant portion of the physical laws are described by differential equations, as well as based on experimental data [12, 14, 19].

3. The aim and objectives of the study

The aim of present study is to improve effectiveness of the methods and tools for mathematical modeling of processes in measuring transducers by devising alternative approaches to the formation of dynamic models of MT in the form of explicit integral models.

To achieve the set aim, the following tasks had to be solved:

– further development and improvement of the theory of measuring transducers, including devising the methods for compiling a mathematical description of dynamic characteristics of measuring transducers;

– design of analytical methods for obtaining pulse transition functions of measuring transducers with both lumped and distributed parameters obtained by the means of *a priori* data;

– verification of the theoretical results obtained by research using specific transducers as an example.

4. Integral modeling method in the problems of dynamics during formation of integral dynamic models of measuring transducers

When modeling problems of dynamics and creating integral dynamic models of MT, as well as in analysis of the dynamics of any other systems, a determining role is played by the impulse transition function of the modeled object [17].

A dynamic model of the measuring transducer can be represented in the form of some operator $A(t)$, which sets the dependence of scalar input data $X(t)$ on the output data $Y(t)$ at any given time t [8]. This dependence, input → transformation operator, which is a scalar function → output, typically represented symbolically in the form

$$Y(t)=AX(t). \quad (1)$$

If one knows the pulse transition function $g(t, \tau)$ from linear MT, then expression (1) takes the form

$$Y(t)=\int_{-\infty}^{\infty} g(t, \tau) X(\tau) \mathrm{d}\tau, \quad (2)$$

where $g(t, \tau)=A\delta(t-\tau)$, $\delta(t-\tau)$ is the delta function, which is the core of IDM.

A pulse transition function of MT is the reaction of MT, not excited in advance, on the input signal that takes the form of pulse δ-function, and is essentially a universal dynamic characteristic of the device.

In a general case, the pulse transition function depends on two arguments: τ is the moment of application of pulse

δ-function, and t – current monitoring moment of MT indications. Based on the principle of causality, the reaction of MT can occur only after the application of an input impact, which is why

$$g(t,\ \tau)=0 \ \text{at} \ \tau>t \tag{3}$$

a condition for the physical possibility of the system's existence. If a transducer starts operation at some time moment t_0, the integrand expression in (2) is zero also at $\tau<t$.

Thus, for physically possible MT, relationship between measured signal $X(t)$ and indications $Y(t)$ of MT is determined from ratio

$$Y(t)=\int_{t_0}^{t} g(t,\ \tau)X(\tau)\mathrm{d}\tau. \tag{4}$$

If the moment of the beginning of the measurement is taken as $t=0$, then it is possible to apply expression

$$Y(t)=\int_{0}^{t} g(t,\ \tau)X(\tau)\mathrm{d}\tau, \tag{5}$$

which in a general case is the integrated dynamic model.

4. 1. Formation of explicit integral dynamic MT models with distributed parameters

We shall note a very important circumstance in the described process of formation of the mathematical description of a dynamic system. Its essence is that it is not required to specify what type of dynamic objects is being considered – with lumped or distributed parameters. Suffice it to say that we considered measuring transformations with lumped parameters, since behavior of the indicated MT, as it followed from relation (1), was characterized by only one independent variable t. However, the pulse transition function is a universal dynamic characteristic also for PIs with distributed parameters. In a general case, the concept of pulse transition function for MT with distributed parameters can be illustrated as follows. We shall introduce a multidimensional pulse δ-function, specifically, four-dimensional pulse δ-function

$$\delta(t,\ x_1,\ x_2,\ x_3)=\delta(t)\delta(x_1)\delta(x_2)\delta(x_3),$$

where spatial coordinates are denoted as x_1, x_2, x_3.

Let the behavior of MT with distributed parameters be described by the operator equation

$$A\cdot X(x_1,\ x_2,\ x_3,\ t)=X(x_1,\ x_2,\ x_3,\ t). \tag{6}$$

In this case, by analogy with a one-dimensional ratio, we have

$$X(x_1,\ x_2,\ x_3,\ t)= \\ =\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(x_1-x_1')\cdot\delta(x_2-x_2')\cdot\delta(x_3-x_3')\cdot\delta(t-\tau)\times \\ \times X(x_1',\ x_2',\ x_3',\ \tau)\mathrm{d}x_1'\mathrm{d}x_2'\mathrm{d}x_3'\mathrm{d}\tau, \tag{7}$$

where x_1', x_2', x_3' are the points of application of pulse δ-functions relative to spatial coordinates x_1, x_2, x_3.

Substituting (7) in (6) and considering it possible to change a sequence of activities on integration and transformation, we shall obtain

$$X(x_1,\ x_2,\ x_3,\ t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}X(x_1',\ x_2',\ x_3',\ \tau)\times \\ \times A\left[\delta(x_1-x_1')\delta(x_2-x_2')\cdot\delta(x_3-x_3')\cdot\delta(t-\tau)\right]\mathrm{d}x_1'\mathrm{d}x_2'\mathrm{d}x_3'\mathrm{d}\tau,$$

hence

$$X(x_1,\ x_2,\ x_3,\ t)= \\ =\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x_1,\ x_1',\ x_2,\ x_2',\ x_3,\ x_3',\ t,\ \tau)\times \\ \times X(x_1',\ x_2',\ x_3',\ \tau)\mathrm{d}x_1'\mathrm{d}x_2'\mathrm{d}x_3'\mathrm{d}\tau,$$

where function

$$g(x_1,\ x_1',\ x_2,\ x_2',\ x_3,\ x_3',\ t,\ \tau)= \\ =A\left[\delta(x_1-x_1')\cdot\delta(x_2-x_2')\cdot\delta(x_3-x_3')\cdot\delta(t-\tau)\right] \tag{8}$$

is the pulse transitional function of the measuring transducer with distributed parameters.

For MT with distributed parameters, to be considered below, as follows from the equations and boundary conditions given to them, it is possible to confine ourselves to a one-dimensional pulse δ-function. Consequently, also the pulse transition function of the form $g(t,\ \tau)$. However, if need be, for MT with distributed parameters, it is expedient consider a local pulse transition function $g(t,\ \tau,\ x_1,\ x_2,\ x_3)$, which shows a reaction of each point $(x_1,\ x_2,\ x_3)$ of the MT, not excited in advance, on the input signal of the δ-function type.

4. 2. Formation of explicit integral dynamic models of stationary measuring transducers

Since the reaction of stationary MT does not depend on the moment of impact application, the pulse transitional function of stationary MT also does not depend on the moment of impact application and depends only on the difference of the arguments, that is

$$g(t,\ \tau)=g(t-\tau), \tag{9}$$

$g(t,\ \tau)$ is the function of two parameters; expression (5) takes the form

$$Y(t)=\int_{0}^{t} g(t-\tau)X(\tau)\mathrm{d}\tau \tag{10}$$

or

$$Y(t)=\int_{0}^{t} g(\tau)X(t-\tau)\mathrm{d}\tau. \tag{11}$$

In the analysis of dynamic properties of linear MT, along with pulse transition function $g(t,\ \tau)$, they often use the so-called singular transition function $h_0(t,\ \tau)$ [17], interconnected by ratio

$$h_0(t)=\int_{0}^{t} g(t-\tau)\mathrm{d}\tau.$$

When one sends a signal to the MT input, which the form of a singular function, one will receive at the MT output a transition function, by differentiating which one will find the pulse transition function.

Thus, the dynamic characteristics considered fully determine dynamic properties of linear MT and are the basis for obtaining integral dynamic models for different types of measuring transducers.

4. 3. Relationship between a pulse transition function and the Green's function

Since each characteristic unambiguously describes a model of MT, these characteristics need to be interconnected and expressed one through the other.

We shall consider a linear ordinary differential equation of the n-order [15]

$$L\left[Y(t)\right]=X(t), \tag{12}$$

under initial conditions

$$\left.\frac{d^iY(t)}{dt^i}\right|_{t=t_0}=0, \quad i=0,\ 1,\ 2,\ \ldots,\ n-1, \tag{13}$$

that is, a linear model of MT with lumped parameters.

Linear differential operator $L[Y(t)]$ takes the form

$$L\left[Y(t)\right]=a_n(t)\cdot\frac{d^nY}{dt^n}+a_{n-1}(t)\cdot\frac{d^{n-1}Y}{dt^{n-1}}+$$
$$+a_{n-2}(t)\cdot\frac{d^{n-2}Y}{dt^{n-2}}+\ \cdots\ +a_1(t)\cdot\frac{dY}{dt}+a_0(t)\cdot Y.$$

A general solution to equation (12) is determined from equation

$$Y(t)=\int_{t_0}^{t}G_0(t,\ \tau)\cdot X(\tau)\mathrm{d}\tau, \tag{14}$$

which satisfies sufficient conditions for the existence of $Y(t)$. $G_0(t,\ \tau)$ is called a one-side Green's function for differential operator $L[Y(t)]$.

Green's function $G_0(t,\ \tau)$ is expressed through the fundamental system of solutions to equation $L[Y(t)]=0$ using ratio

$$G_o(t,\ \tau)=\frac{(-1)^{n-1}}{u_n(\tau)\Delta_0(\tau)}\times\begin{vmatrix}\varphi_1(t) & \varphi_2(t) & \ldots & \varphi_n(t)\\ \varphi_1(\tau) & \varphi_2(\tau) & \ldots & \varphi_n(\tau)\\ \varphi_1'(\tau) & \varphi_2'(\tau) & \ldots & \varphi_n'(\tau)\\ \varphi_1''(\tau) & \varphi_2''(\tau) & \ldots & \varphi_n''(\tau)\\ \ldots & \ldots & \ldots & \ldots\\ \varphi_1^{(n-2)}(\tau) & \varphi_2^{(n-2)}(\tau) & \ldots & \varphi_n^{(n-2)}(\tau)\end{vmatrix}, \tag{15}$$

where functions $\varphi_1(t)$, $\varphi_2(t)$,..., $\varphi_n(t)$ form a fundamental system of solutions to equation $L[Y(t)]=0$; $\Delta_0(\tau)$ is the Wronskian, which is determined from formula

$$\Delta_0(\tau)=\begin{vmatrix}\varphi_1(\tau) & \varphi_2(\tau) & \ldots & \varphi_n(\tau)\\ \varphi_1'(\tau) & \varphi_2'(\tau) & \ldots & \varphi_n'(\tau)\\ \ldots & \ldots & \ldots & \ldots\\ \varphi_1^{(n-1)}(\tau) & \varphi_2^{(n-1)}(\tau) & \ldots & \varphi_n^{(n-1)}(\tau)\end{vmatrix}.$$

Green's function satisfies homogeneous equation

$L[G_0(t)]=0$

and initial conditions

$$\left.\frac{d^iG_0(t,\ \tau)}{dt^i}\right|_{t=\tau}=0, \quad i=0,\ 1,\ 2,\ \ldots,\ n-2;$$

$$\left.\frac{d^{n-1}G_0(t,\ \tau)}{dt^{n-1}}\right|_{t=\tau}=\frac{1}{a_n(\tau)}. \tag{16}$$

As it follows from (15), Green's function can be represented in the form

$$G_o(t,\ \tau)=\sum_{i=1}^{n}\psi_i(\tau)\varphi_i(t), \tag{17}$$

where $\psi_i(\tau)$ is derived by substituting (17) in the initial conditions (16).

By comparing (14) with the previously described relation

$$Y(t)=\int_{t_0}^{t}g(t,\ \tau)X(\tau)\mathrm{d}\tau,$$

it can be concluded that Green's function $G_0(t,\ \tau)$ coincides with the pulse transition function only at $t>\tau$, that is exactly in the region where the pulse transition function, being different from zero, makes sense and describes the physically possible systems. As far as the region $t>\tau$ is concerned, here the pulse transition function is identically equal to zero whereas the Green's function can vary from zero.

Now, after establishing a relation between the Green's function and the impulse transition function, it can be argued that Green's function at $t>\tau$ is also a solution to the inhomogeneous equation

$$L\left[G_o(t,\ \tau)\right]=\delta(t-\tau),$$

under initial conditions

$$\left.\frac{\partial^iG_0(t,\ \tau)}{\partial t^i}\right|_{t=t_0}=0, \quad i=0,\ 1,\ 2,\ \ldots,\ n-1.$$

On the other hand, it can be assumed that the pilse transition function pulse $g(t,\ \tau)$ is a solution to homogeneous equation $L[g(t,\ \tau)]=0$, under conditions

$$\left.\frac{d^ig(t,\ \tau)}{dt^i}\right|_{t=\tau}=0, \quad i=0,\ 1,\ 2,\ \ldots,\ n-2;$$

$$\left.\frac{d^{n-1}g(t,\ \tau)}{dt^{n-1}}\right|_{t=\tau}=\frac{1}{a_n(\tau)}. \tag{18}$$

Finally, we shall note that as it follows from relation (17), a pulse transition function can be always represented in the form

$$g(t,\ \tau)=\sum_{i=1}^{n}\psi_i(\tau)\cdot\varphi_i(t), \tag{19}$$

in this case, the number of terms in the expansions of this type coincides with the order of the appropriate linear differential equation that describes behavior of the measuring transducer.

One of the methods for determining pulse transition function of MT follows from the results of this comparison. Upon finding a solution to the homogeneous equation that corresponds to the non-homogeneous one, we shall obtain a general solution with arbitrary constants. Upon deriving the arbitrary constants from conditions (18), we shall obtain the desired pulse transition function. The result will be the same as when solving a non-homogeneous equation with the right-hand side in the form of pulse n-function and under zero initial conditions.

5. Analytical procedures of obtaining pulse transition functions of measuring transducers

5. 1. Obtaining a pulse transition function of measuring transducers with lumped parameters

We shall consider analytical procedures to build pulse transition functions of MT, that is, the cores of explicit integral dynamic models.

The method formulated above makes it possible to find the cores of integral models (dynamic characteristics) by the assigned differential equations. The easiest way is to obtain indicated functions in an analytical form for MT of the first and second order, which is possible to perform using specific transducers as an example.

For a group of measuring transducers with lumped parameters of the first order described by equation [15]

$$T\frac{\partial Y}{\partial t}+Y=kX(t),$$

the pulse transition function takes the form

$$g(t-\tau)=\frac{k}{T}e^{-\frac{t-\tau}{T}}. \quad (20)$$

Equation for MT with lumped parameters of the second order of the aperiodic type

$$T_1T_2Y''+(T_1+T_2)Y'+Y=kX(t),$$

T_1, T_2 are some parameters of the transducer) makes it possible to obtain a pulse transition function of the form

$$g(t-\tau)=k\left(\frac{1}{T_1-T_2}e^{-\frac{t-\tau}{T_1}}+\frac{1}{T_2-T_1}e^{-\frac{t-\tau}{T_2}}\right). \quad (21)$$

For measuring transducers with lumped parameters of the second order of the oscillatory type, we have equation

$$T_0^2Y''+2\varepsilon_0T_0Y'+Y=kX(t),$$

that is equivalent to the previously described (if one accepts $T=1/\omega_0$, at $k=T_0^2$ for accelerometers, $k=T_0^2\cdot H_0/J$ for angular velocity MT, etc.) and makes it possible to obtain a pulse transition function

$$g(t-\tau)=\frac{k}{T_0\sqrt{1-\varepsilon_0^2}}e^{-\frac{\varepsilon_0}{T_0}(t-\tau)}\sin\left[\frac{\sqrt{1-\varepsilon_0^2}}{T_0}(t-\tau)\right], \quad (22)$$

that is the core of the integral dynamic model with lumped parameters.

5. 2. Obtaining a pulse transition function of MT with distributed parameters

5. 2. 1. Pressure MT

Under actual conditions, widespread are the pressure MT that employ a circular membrane [1]. The equation for these MT [10] will be given in the form

$$\frac{\partial^2W(r,t)}{\partial t^2}+2\delta_0\frac{\partial W(r,t)}{\partial t}=$$
$$=a_0^2\left[\frac{\partial^2W(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial W(r,t)}{\partial r}\right]+\frac{1}{\rho_s}P(t). \quad (23)$$

We shall accept zero initial conditions with boundary

$$W(r,t)\big|_{r=R}=0.$$

We shall determine relationship between $W(r,t)$ and $P(t)$. A solution to equation (23) will be searched for in the form

$$W(r,t)=\sum_{n=1}^{\infty}T_n(t)\cdot J_0(k_nr), \quad (24)$$

where J_0 is the Bessel function of the first kind of zero order; $k_n=\mu_n/R$, μ_n is the roots of equation

$$J_0(\mu)=0. \quad (25)$$

Substituting (24) in (23), by multiplying all terms of received ratio by $r{\cdot}J_0(k_mr)$, and upon integration in the range from 0 to R, and considering that

$$\int_0^R r\cdot J_0(k_nr)\mathrm{d}r=\frac{J_1(\mu_n)}{\mu_n}\cdot R^2,$$

$$\int_0^R r\cdot J_0(k_nr)\cdot J_0(k_mr)dr=\begin{cases}0 & \text{at } k_n\neq k_m,\\ \frac{k^2}{2}\cdot J_1^2(\mu_n) & \text{at } k_n=k_m.\end{cases}$$

We shall obtain

$$T_n''+2\delta_0T_n'+a_0^2k_n^2T_n=\frac{2}{\rho_s\mu_n}\frac{1}{J_1(\mu_n)}P(t). \quad (26)$$

Solving this equation, we shall come to expression

$$T_n(t)=\frac{2}{\rho_s\cdot\eta_n\cdot\mu_n}\frac{1}{J_1(\mu_n)}\int_0^t e^{-\delta_0(t-\tau)}\sin\eta_n(t-\tau)P(\tau)\mathrm{d}\tau, \quad (27)$$

Therefore, the solution to the original equation takes the form

$$W(r,t)=\frac{2}{\rho_s}\sum_{n=1}^{\infty}\frac{1}{\eta_n\mu_n}\frac{J_0(k_nr)}{J_1(\mu_n)}\times$$
$$\times\int_0^t e^{-\delta_0(t-\tau)}\sin\eta_n(t-\tau)P(\tau)\mathrm{d}\tau, \quad (28)$$

where

$$\eta_n=\sqrt{a_0^2k_n^2-\delta_0^2}.$$

We obtain a local pulse transition function from relation (28)

$$g(r,t-\tau)=\frac{2}{\rho_s}e^{-\delta_0(t-\tau)}\sum_{n=1}^{\infty}\frac{1}{\eta_n\mu_n}\frac{J_0(k_nr)}{J_1(\mu_n)}\sin\eta_n(t-\tau). \quad (29)$$

The pulse transition function, corresponding to the indications of MT, is derived from the expression for $g(r,t-\tau)$ at $r=0$, that is

$$g(t-\tau)=\frac{2}{\rho_s}e^{-\delta_0(t-\tau)}\sum_{n=1}^{\infty}\frac{1}{\eta_n\mu_n}\frac{1}{J_1(\mu_n)}\sin\eta_n(t-\tau). \quad (30)$$

In practice, the magnitude δ_0 is often neglected, assuming damping to be small, in this case, we have from (30)

$$g(t-\tau)=\frac{2\cdot R}{a_0\cdot\rho_s}\cdot\sum_{n=1}^{\infty}\frac{1}{\mu_n^2}\cdot\frac{1}{J_1(\mu_n)}\cdot\sin\alpha_0 k_n(t-\tau). \quad (31)$$

To resolve the issue of how many terms in a series should be left during specific calculations, it is necessary to know the roots of equation (25). The first roots of the equation: μ_1=2.4048, μ_2=5.5201, μ_3=8.6537. For subsequent roots, the approximated relation $\mu_{n+1}-\mu_n=\pi$ holds.

5. 2. 2. Chemotronic measuring transducers

For the first model of a chemotronic MT with flat electrodes [10], we have the original equation in the form

$$\frac{\partial C(x,y,t)}{\partial t}+\upsilon(t)\frac{\partial C(x,y,t)}{\partial y}=D_0\frac{\partial^2 C(x,y,t)}{\partial x^2}, \quad (32)$$

$$\frac{\partial Q(x,t)}{\partial t}-D_0\frac{\partial^2 Q(x,t)}{\partial x^2}=\upsilon(t)(C_0-C_1), \quad (33)$$

where

$$Q(x,t)=\int_0^{t_0}C(x,y,t)\mathrm{d}y,$$

$$Q(x,t)\big|_{t=0}=0.$$

Let

$$Q(x,t)=\sum_{n=0}^{\infty}T_n(t)\cos k_n x, \quad (34)$$

where

$$k_n=\left(n+\frac{1}{2}\right)\frac{\pi}{r_0},\quad n=0,\ 1,\ 2,\ \ldots,$$

μ_n are the roots of equation $J_0(\mu)=0$ for flat thermal receivers, given in Table 1.

Here the flow velocity profile in a channel of the transducer is accepted to uniform.

By integrating all terms (32) by y within a range from zero to l_0 and by considering the boundary conditions described above, we shall obtain

$$Q(x,t)\big|_{x=\pm r_0}=0.$$

Substituting (34) in (33), we multiply the result by $\cos k_m x$ and integrate by x with a range of $-r_0$ to r_0, it will yield

$$T_n'+D_0k_n^2T_n=(C_0-C_1)\frac{2}{r_0k_n}(-1)^n\upsilon(t),$$

hence

$$T_n=(C_0-C_1)\frac{2}{r_0k_n}(-1)^n e^{-D_0\cdot k_n^2\cdot t}\int_0^t e^{D_0\cdot k_n^2\cdot\tau}\upsilon(\tau)d\tau,$$

here we took into consideration zero initial conditions.

Therefore, we have

$$Q(x,t)=\frac{2(C_0-C_1)}{r_0}\times$$
$$\times\sum_{n=0}^{\infty}(-1)^n\frac{1}{k_n}\cos k_n x e^{-D_0\cdot k_n^2\cdot t}\int_0^t e^{D_0\cdot k_n^2\cdot\tau}\upsilon(\tau)\mathrm{d}\tau. \quad (35)$$

Table 1

Values of roots μ_n for flat thermal receivers

B_i	μ_1	μ_2	μ_3	μ_4
0.000	0.0000	3.1416	6.2832	9.4248
0.002	0.0447	3.1422	6.2835	9.4250
0.004	0.0632	3.1429	6.2838	9.4252
0.006	0.0774	3.1435	6.2841	9.4254
0.008	0.0893	3.1441	6.2845	9.4256
0.010	0.0998	3.1448	6.2848	4.4258
0.020	0.1410	3.1479	6.2864	9.4269
0.040	0.1987	3.1543	6.2895	9.4290
0.060	0.2425	3.1606	6.2927	9.4311
0.080	0.2791	3.1668	6.2959	9.4333
0.100	0.3111	3.1731	6.2991	9.4354
0.200	0.4328	3.2039	6.3148	9.4459
0.300	0.5218	3.2341	6.3305	9.4565
0.400	0.5932	3.2636	6.3461	9.4670
0.500	0.6533	3.2923	6.3616	9.4775
0.600	0.7051	3.3204	6.3770	9.4879
0.700	0.7506	3.3477	6.3923	9.4983
0.800	0.7910	3.3744	6.4074	8.5087
0.900	0.8274	3.4003	6.4224	9.5190
1.000	0.8603	3.4256	6.4373	9.5293

Now we shall find diffusion flow $I(t)$ arriving onto both plates of the electrode. Because

$$I(t)=2\bar{b}_0\int_0^{l_0}jdy=-2\bar{b}_0D_0\frac{\partial Q}{\partial x}\bigg|_{x=r_0},$$

where

$$j=-D_0\frac{\partial C}{\partial x}\bigg|_{x=r_0}$$

is the density of the diffusion flow on the electrode,

$$I(t)=\frac{4\bar{b}_0D_0(C_0-C_1)}{r_0}\sum_{n=0}^{\infty}e^{-D_0\cdot k_n^2\cdot t}\int_0^t e^{D_0\cdot k_n^2\cdot\tau}\upsilon(\tau)\mathrm{d}\tau. \quad (36)$$

It follows from equation (36) that the pulse transition function, corresponding to the diffusion flow, takes the form

$$g(t-\tau)=\frac{4\bar{b}_0D_0(C_0-C_1)}{r_0}\sum_{n=0}^{\infty}\exp\left[-D_0k_n^2(t-\tau)\right]. \quad (37)$$

(37) shows that the character of the transition process of MT is fully determined by criterion $F=D_0t/r_0^2$. However, if the value of criterion is $F\geq0.5$, it is possible, with a high degree of accuracy, to neglect all terms, except for the first, since the second term turns out to be vanishingly small. It is therefore possible to confine ourselves to only the first term at $F\geq0.5$.

Next, we shall obtain the required ratios for a model of chemotronic MT, having a cylindrical electrode. Upon performing a similar transform, similar to the previous case, we shall obtain the following equation:

$$\frac{\partial Q(r,\,t)}{\partial t}-D_0\left[\frac{\partial^2 Q(r,\,t)}{\partial r^2}+\frac{1}{r}\frac{\partial Q(r,\,t)}{\partial r}\right]=\upsilon(t)(C_0-C_1),$$

$$Q(r,\,t)\big|_{t=0}=0,\;\; Q(r,\,t)\big|_{r=r_0}=0. \qquad (38)$$

Solution to this equation will be searched for in the form

$$Q(r,\,t)=\sum_{n=1}^{\infty}T_n(t)J_0(k_n r), \qquad (39)$$

where $k_n=\mu_n/r_0$, μ_n; μ_n are the roots of equation $J_0(\mu)=0$ given in Table 2.

Table 2

Values of roots μ_n for cylindrical thermal receivers

B_i	μ_1	μ_2	μ_3	μ_4
0.00	0.0000	3.8317	7.0156	10.1736
0.01	0.1412	3.8343	7.0170	10.1745
0.02	0.1995	3.8369	7.0184	10.1754
0.04	0.2814	3.8421	7.0213	10.1774
0.06	0.3438	3.8473	7.0241	10.1794
0.08	0.3960	3.8525	7.0270	10.1813
0.10	0.4417	3.8577	7.0298	10.1833
0.15	0.5376	3.8706	7.0369	10.1882
0.20	0.6170	3.8835	7.0440	10.1931
0.30	0.7465	3.9091	7.0582	10.2029
0.40	0.8516	3.9344	7.0723	10.2127
0.50	0.9408	3.9594	7.0864	10.2225
0.60	1.0184	3.9841	7.1004	10.2322
0.70	1.0873	4.0085	7.1143	10.2419
0.80	1.1490	4.0325	7.1282	10.2519
0.90	1.2048	4.0562	7.1421	10.2613
1.00	1.2558	4.0795	7.1558	10.2710

Substituting (39) in (38), by multiplying the expression obtained by $rJ_0(rk_m)$ and by integrating within the range from zero to r_0, we shall obtain as the result

$$T_n'+D_0k_n^2T_n=2\upsilon(t)(C_0-C_1)\frac{1}{\mu_n J_1(\mu_n)}.$$

A solution to this equation is the expression

$$T_n(t)=\frac{2(C_0-C_1)}{\mu_n J_1(\mu_n)}e^{-D_0\cdot k_n^2\cdot t}\int_0^t e^{D_0\cdot k_n^2\cdot\tau}\upsilon(\tau)\mathrm{d}\tau.$$

Therefore, the solution to equation (38) is represented in the form

$$Q(r,\,t)=$$

$$=2(C_0-C_1)\sum_{n=1}^{\infty}\frac{J_0\left(\mu_n\frac{r}{r_0}\right)}{\mu_n J_1(\mu_n)}e^{-D_0\cdot k_n^2\cdot t}\int_0^t e^{D_0\cdot k_n^2\cdot\tau}\upsilon(\tau)\mathrm{d}\tau. \qquad (40)$$

The diffusion flow arriving onto a cylindrical electrode is

$$I(t)=-2\pi r_0 D_0\frac{\partial Q}{\partial r}\bigg|_{r=r_0}=$$

$$=4\pi D_0(C_0-C_1)\sum_{n=1}^{\infty}e^{-D_0k_n^2t}\int_0^t e^{D_0k_n^2\tau}\upsilon(t)\mathrm{d}\tau, \qquad (41)$$

hence, for the pulse transition function of the diffusion flow we have

$$g(t-\tau)=4\pi D_0(C_0-C_1)\sum_{n=1}^{\infty}\exp\left[-D_0k_n^2(t-\tau)\right]. \qquad (42)$$

As follows from the equations for a chemotronic MT given above, flow velocity profile, generally speaking, is different from the uniform and for the sake of accuracy it should be accepted that

$$\upsilon(x,\,t)=\upsilon_0(t)\left(1-\frac{x^2}{r_0^2}\right),$$

for a channel of the flat shape and

$$\upsilon(r,\,t)=\upsilon_0(t)\left(1-\frac{r^2}{r_0^2}\right),$$

for a channel of the cylindrical shape.

This would mean that we accepted a Poiseille profile of hydrodynamic flow velocities. Consideration of the non-uniform distribution of the flow velocity profile does not lead to additional difficulties.

6. Discussion of results of the proposed method for the formation of explicit integral dynamic models of measuring transducers

We shall note that for measuring transducers with distributed parameters we used series by certain functions of spatial coordinates. The choice of these functions is far from arbitrary, specifically, they must be eigenfunctions of the corresponding problems. In addition, we found in the above chapters a general correlation between the measured magnitude and the indications of the measuring transducer, which was followed by one or another structure of pulse transition function. Naturally, it was possible to not search for these relationships but rather build a pulse transition function. To do this, it would be sufficient to find a solution to the homogeneous equations corresponding to the original non-homogeneous equations with the arbitrary constants to be found from the conditions that must be fulfilled by the pulse transition function, namely, from conditions (18). We shall illustrate this using a chemotronic MT with flat electrodes as an example. Solve the homogeneous equation corresponding to equation (33). Because the input magnitude is $\upsilon(t)$, rather than $(C_0-C_1)\upsilon(t)$, the received pulse transition function must be multiplied by a scale multiplier (C_0-C_1). Local pulse transition function will be searched for in the form

$$g(x,\,t-\tau)=\sum_{n=0}^{\infty}T_n(t-\tau)\cos k_n x.$$

As before, we obtain from a homogeneous equation

$$\frac{dT_n}{dt}+D_0k_n^2T_n=0.$$

Hence

$$T_n(t-\tau)=A_n(\tau)e^{-D_0k_n^2t}$$

and

$$g(x,\ t-\tau)=\sum_{n=0}^{\infty}A_n(\tau)e^{-D_0k_n^2t}\cos k_nx.$$

In this case, the condition $g(x, t-\tau)=1$ must hold, which is why the derived conditions for $A(\tau)$ will take the form

$$A_n(\tau)=\frac{2}{r_0k_n}(-1)^n e^{D_0k_n^2\tau}.$$

Now, given the multiplier (C_0-C_1), we shall obtain the following expression for $g(x, t-\tau)$:

$$g(x,\ t-\tau)=\frac{2}{r_0}(C_0-C_1)\sum_{n=0}^{\infty}\frac{1}{k_n}(-1)^n e^{-D_0\cdot k_n^2\cdot(t-\tau)}\cos k_nx.$$

We have for the pulse transition function by diffusion flow

$$g(t-\tau)=\frac{4\bar{b}_0D_0}{r_0}(C_0-C_1)\sum_{n=0}^{\infty}\exp\left[-D_0k_n^2(t-\tau)\right],$$

which implies the identity of results.

Thus, the basis for solving the problems of dynamics of measuring transducers over a temporal domain is the dynamic characteristics that display the physical principle of aftereffect of dynamic objects and the patterns arising therefrom.

Constructive concepts of the pulse transition function and the transition function lead to the formation of MT operators in the form of integral mathematical dependences, that is, integral dynamic models.

An advantage of this type of models is a single structure for describing the dynamic properties of MT with lumped and distributed parameters. The presence of transient characteristics, obtained analytically or experimentally, unambiguously leads to the formation of models in the form of integral dependences (operators). The concept of the pulse transition function of MT is inextricably linked to the concept of the Green's function in the theory and practical methods for solving differential equations.

A technique to obtain analytical expression for the pulse transition function of MT with lumped parameters is solving a homogeneous differential equation corresponding to the assigned non-homogeneous differential equation. This technique is easily illustrated on the example of MT of the first and second order. The principle of determining a pulse transient characteristic for MT with distributed parameters by the assigned equations in partial derivatives remains the same as in the case of MT with lumped parameters. The difference is in the more complex analytical notations, corresponding to the method of integral representations to solve equations in partial derivatives. According to a given approach, transformations imply reducing a problem to the ordinary differential equations whose analytical solution is represented in the form of the integral operator that connects the desired function to the right-hand side of the original equation. The specified operator thus represents the integral, essentially explicit, dynamic model of MT, based on which we determined the pulse transition function. The technique was applied to specific, widespread pressure MT, and to a chemotronic measuring transducer (with flat and cylindrical electrodes).

7. Conclusions

1. We analyzed current state of the problems on the dynamics of measuring transducers. An analysis reveals alternative descriptions, that is, the possibility of applying various forms of mathematical models. Given this, there is a choice, in a sense, of the "best model", providing, for example, the most beneficial ratio between complexity and quality. This position is a factor for the rationale of employing various alternative forms of dynamic modeling, including the integral dynamic models. In this regard, constructively relevant is the method for creating explicit integral models of measuring transducers, as well as, in the future, methods and means of their numerical implementation.

2. The method designed in present paper is suitable for practical application when creating explicit integral dynamic models of MT. When analyzing dynamics of the systems, a determining role is played by dynamic characteristics (pulse transition and transition functions) of the modeled object. The principle of formation is to determine a pulse transient characteristic (function), which is the core of an explicit integral model of MT with distributed parameters and lumped parameters by the assigned differential equations.

The dynamic characteristics obtained fully determine dynamic properties of MT and could form the basis for obtaining integral dynamic models for different types of measuring transducers.

Thus, integral dynamic models make it possible to expand the tools of computer simulation in the tasks on studying measuring transducers based on their numerical implementation.

3. The relevance and effectiveness of the technique for the formation of explicit integral dynamic models of MT are demonstrated by applying them to specific widespread pressure MT and a chemotronic transducer (with flat and cylindrical electrodes).

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