Розглядається дискретна динамічна модель, яка описує управління інноваційним процесом (ІП) на підприємстві і враховує наявність керуючих впливів і неконтрольованого параметра (вектора ризиків). Розроблено модель адаптивного управління ІП, результати практичного застосування якої дозволяють приймати управлінські рішення щодо адаптації до змін зовнішнього середовища при оперативному управлінні інноваційною діяльністю підприємства

Ключові слова: інноваційний процес, адаптивне управління, минимакс, гарантований результат, моделювання управління інноваційними процесами

Рассматривается дискретная динамическая модель, которая описывает управление инновационным процессом (ИП) на предприятии и учитывает наличие управляющих воздействий и неконтролируемого параметра (вектора рисков). Разработана модель адаптивного управления ИП, результаты практического применения которой позволяют принимать управленческие решения по адаптации к изменениям внешней среды при оперативном управлении инновационной деятельностью предприятия

Ключевые слова: инновационный процесс, адаптивное управление, минимакс, гарантированный результат, моделирование управления инновационными процессами

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DEVELOPMENT OF THE MODEL OF MINIMAX ADAPTIVE MANAGEMENT OF INNOVATIVE PROCESSES AT AN ENTERPRISE WITH CONSIDERATION OF RISKS

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1. Introduction

The current state of Ukraine's economy largely depends on accurate estimation, prediction, effective planning and management of innovation activity of enterprises. A modern enterprise is a complex integrated organizational and production system, the components of which are constantly changing, interacting with each other. Achievement of set goals in the face of increasing competition among enterprises results in an increase in the volume and complexity of processes of production, analysis, planning, management, internal and external relations with suppliers, intermediaries, etc.

However, innovative activity in the process of dynamic development of a company cannot be considered fully justified and adapted without involvement of contemporary approaches of economic-mathematical modelling. This, in turn, is an effective means of theoretical processing and practical generalization of mechanisms and tools for innovative activity of an enterprise. Moreover, production of an enterprise is a complex, open, capable of self-organization and self-development economic system with dynamically changeable non-determined and contradictory characteristics.

We will note that there are scientific studies focusing on problems of management of innovative processes (IP) at an enterprise, various economic-mathematical models and methods for finding managerial decisions. However, the problem of economic-mathematical modeling of adaptive IP management under conditions of uncertainty, taking into consideration the risk factor at enterprises has not been solved yet and is a relevant research topic.

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2. Literature review and problem statement

Let us consider up-to-date approaches to modeling of socio-economic systems, in particular those related to IP management at an enterprise. In this way, the problem of mathematical modeling of innovative development in the agrarian economy was studied [1, 2]. Models for formation and functioning of innovative systems of upgrading of agrarian clusters were proposed, and related models were developed. The methods of the theory of single-criterion optimization on hypergraphs with the help of deterministic and probabilistic automata of Moor, probabilistic-automatic simulation and stochastic Petri nets were used However, preliminary analysis of the exogenous factors proved the put forward hypothesis of determinism of the researched model and unacceptability of using stochastic values in it [3]. That is why solution of the problem of adaptive IP management at an enterprise, set in the work, requires a different mathematical tool set. In addition, according to the author, for a comprehensive study on effectiveness of IP management at an enterprise, it is advisable to use multicriteria optimization models.

Static and dynamic optimization problems of organization of innovation in the system of an enterprise were explored. As an approach in the study of mathematical models for IP management, it was proposed to construct and analyze the regions of values of the models' parameters that determine their type, etc. [4]. But these studies did not pay sufficient attention to IP management in the aspect of relationships with the main production functions during its implementation (in particular, building a portfolio of new product release in accordance with the implemented IP). As a consequence, the problems of dynamic optimization of IP management parameters, relating to characteristics of the production system, resource capabilities and other indicators of activity of an enterprise were not completely explored.

An analysis of literature on economic-mathematical modeling in IP management has shown that there are difficulties associated with the dynamics of a model. This is due to the parameters changing over time, as well as to complexity of the nature of relationship between the components of the model (including existence of delays and "feedback") [5].

Realization of the problems, associated with IP modeling in practice contributed to scientific development and widespread application of hybrid models based on a combination of formal and informal text, verbal and special graphic approaches. These tools offer simplified innovative models and in many cases involve graphic interpretation of [6]. As a result, most models of IP management were implemented by descriptive means or represented fragmentarily, with insufficient analytical formalization [7]. Some of them are characterized by lack of practical focus, integrity, and complexity in IP application [8], they simplify obtaining knowledge and offer opportunity to experiment with IP management. In this case, there is no possibility to assess the impact and consequences of different variants for IP management in the prospects of minimization of risks of innovation activity while making managerial decisions [9]. Thus, innovative management with the use of simulation remains an unresolved problem for researchers. In addition, existing models may have a limited availability and must be adapted to conditions of activity of specific enterprises.

Problems of modeling of IP management were most fundamentally studied in [10], which dealt with the overall problems of software and adaptive IP management at an enterprise. However, it is not covered by the problem of elaborating a detailed model for practical implementation in the workplace.

3. The aim and objectives of the study

The aim of present research is to model processes in IP management at an industrial enterprise under conditions of risks.

To accomplish the set goal, the following tasks had to be solved:

– to develop a general model of adaptive IP management at an enterprise in the face of risks;

– to form a detailed model of IP management at an enterprise in the face of risks.

4. Development of general model of IP management at an enterprise under conditions of risks

Let us assume that a multi-step dynamic problem, which consists of one controlled object – object I (controlled by player P – the subject of control) – is considered in the assigned integer time interval

$$\overline{0,T} = \{0,1,\ldots,T\}\ (T > 0).$$

Its motion is described by a linear discrete recurrent vector equation of the following form

$$\bar{x}(t+1) = A(t)\bar{x}(t) + B(t)\bar{u}(t) + \\ +C(t)\bar{w}(t) + D(t)\bar{v}(t),\ \bar{x}(0) = \bar{x}_0. \quad (1)$$

Here $t \in \overline{0,T-1}$, $\bar{x} \in \mathbf{R}^{\bar{n}}$ is the phase vector of object I, which for a model of dynamics of IP management at an enterprise [12] consists of $\bar{n} = n + m + 2$ coordinates, i. e.

$$\bar{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t),$$

$$Z(t), k(t))' \in \mathbf{R}^{\bar{n}},$$

where

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbf{R}^n$$

is the vector of residual volumes of finished produce, stored at the warehouse in time period t;

$$y(t) = (y_1(t), y_2(t), \ldots, y_m(t))' \in \mathbf{R}^m$$

is the vector of residual volumes of production resources, stored at the warehouses of an enterprise in time period t; $Z(t)$ is the total costs of an enterprise in time period t; $k(t)$ is the amount of available financial resources, formed by the beginning of period t; n, $m \in \mathbf{N}$; $\mathbf{N}$ is the set of all integers; for $k \in \mathbf{N}$, $\mathbf{R}^k$ is the k-dimensional Euclidean vector space of vectors-columns (even if, for the sake of space economy, they are written down in line);

$$\bar{u}(t) = (\bar{u}_1(t), \bar{u}_2(t), \ldots, \bar{u}_n(t))' \in \mathbf{R}^{\bar{n}}$$

is the vector of innovative management of intensive production of finished produce in time period t $(t \in \overline{0,T-1})$, in which every j-th coordinate $u_j(t)$ has the value of production volume of produce of the j-th form $(j\in\overline{1,\bar{n}})$, limited by assigned constraint

$$\bar{u}(t)\in \boldsymbol{U}_1(t)=U_{N_t}(t)\subset \mathbf{R}^{\bar{p}} \quad (\bar{p}\in\mathbf{N}:\bar{p}=n), \qquad (2)$$

where $U_{N_t}(t)$ for every $t\in\overline{0,T-1}$ is the finite set of vectors, i. e. finite set, consisting of N_t ($N_t\in\mathbf{N}$) vectors in $\mathbf{R}^n$, determining all possible realizations of various management scenarios at time moment t;

$$\bar{w}(t)=(w_1(t),w_2(t),...,w_{\bar{m}}(t))'\in R^{\bar{m}} \ (\bar{m}=m)$$

is the vector of intensity of restocking of warehouse resources at time period t $(t\in\overline{0,T-1})$, which depends on permissible realization of management $\bar{u}(t)\in\boldsymbol{U}_1(t)$ and must meet the following constraint:

$$\bar{w}(t)\in\boldsymbol{W}_1(\bar{u}(t))=W_{M_t}(\bar{u}(t))\subset\mathbf{R}^{\bar{m}} \quad (\bar{m}\in\mathbf{N}:\bar{m}=m), \qquad (3)$$

where $W_{M_t}(\bar{u}(t))$ for each time moment $t\in\overline{0,T-1}$, and management $\bar{u}(t)\in U_{N_t}(t)$ is the finite set of vectors, i. e. a finite set, consisting of $M_t(i)$ ($M_t(i)\in\mathbf{N}$, $i\in\overline{1,N_t}$ vectors in space $\mathbf{R}^{\bar{m}}$, determining all possible realization of various scenarios of restocking of warehouse resources at time period t.

It is also assumed that for all $t\in\overline{0,T-1}$, every permissible realization of phase vector

$$\bar{x}(t)=(x_1(t),x_2(t),...,x_n(t),y_1(t),y_2(t),..., y_m(t),Z(t),k(t))\in\mathbf{R}^{\bar{n}}$$

meets the following phase constraint:

$$\bar{x}(t)=(x_1(t),x_2(t),...,x_n(t),y_1(t),y_2(t),..., y_m(t),Z(t),k(t))\in\boldsymbol{X}_1(t)$$

$$\begin{cases} x_j(t)\geq 0,\ x_j(0)=0,\ j\in\overline{1,n}; \\ y_i(t)\geq 0,\ y_i(0)=b_i,\ i\in\overline{1,m}; \\ k(t)\geq 0,\ k(0)=G+G_0\geq 0; \\ Z(t)\geq 0,\ Z(0)=0\cdot \end{cases} \qquad (4)$$

where G is the volume of financial means of bank loan, intended for investment in extension of production in the initial period of management (at $t=0$); G_0 is the volume of the own financial means, deducted from net profit and directed to extension of production (at $t=0$);

$$\bar{v}(t)=(v(t),v'(t),v''(t))' \in\mathbf{R}^q\times\mathbf{R}^l\times\mathbf{R}^r$$

is the generalized vector of risks;

$$v(t)=(v_1(t),v_2(t),\ldots,v_q(t))' \in\mathbf{R}^q$$

is the vector of risks, describing possible unfavorable realizations of vector a priori uncertain factors or the vector, uniting all errors of modeling of the considered process, influencing production of a unit of manufactured produce of each kind in time period t;

$$v'(t)=(v'_1(t),v'_2(t),\ldots,v'_l(t))' \in R^l$$

is the vector of risks, influencing the state of a unit of available resources of each kind in time period t;

$$v''(t)=(v''_1(t),v''_2(t),\ldots,v''_r(t))' \in\mathbf{R}^r$$

is the vector of financial risks, influencing a unit of total costs in time period t; q, l, $r\in\mathbf{N}$), which during IP management at an enterprise in time period t $(t\in\overline{0,T-1})$ depends on permissible realization of management $\bar{u}(t)\in\boldsymbol{U}_1(t)$ and must meet the following constraint:

$$\bar{v}(t)\in \boldsymbol{V}_1(\bar{u}(t))\subset\mathbf{R}^{\bar{q}} \quad (\bar{q}\in\mathbf{N}:\bar{q}=q+l+r). \qquad (5)$$

Matrices $\bar{A}(t)$, $\bar{B}(t)$, $\bar{C}(t)$ and $\bar{D}(t)$ in vector equation (1) for economic-mathematical model, describing dynamics of IP management at an enterprise, are actual matrices of orders $(\bar{n}\times\bar{n})$, $(\bar{n}\times\bar{p})$, $(\bar{n}\times\bar{m})$ and $(\bar{n}\times\bar{q})$ respectively. They are matrices that for all $t\in\overline{0,T-1}$ matrix $\bar{A}(t)$ is non-degenerated, that is, for it, there exists a correspondent invertible matrix $\bar{A}^{-1}(t)$, and the rank of matrix $\bar{B}(t)$ is equal to $\bar{p}$ (dimensionality of vector $\bar{u}(t)$).

For the considered process of IP management at an enterprise [13], data of the matrix have the following particular form:

$$\bar{A}(t)=\begin{pmatrix} a_{11}(t) & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & a_{22}(t) & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & a_{nn}(t) & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & r_{11}(t) & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & r_{22}(t) & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & r_{mm}(t) & 0 & 0 \\ z_1(t) & z_2(t) & \ldots & z_n(t) & p_1(t) & p_2(t) & \ldots & p_m(t) & 1 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 1 \end{pmatrix};$$

$$\bar{B}(t)=\begin{pmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & 1 \\ -b_{11} & -b_{12} & \ldots & -b_{(1,(n-1))} & -b_{1n} \\ -b_{21} & -b_{22} & \ldots & -b_{(2,(n-1))} & -b_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -b_{m1} & -b_{m2} & \ldots & -b_{(m,(n-1))} & -b_{mn} \\ 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & \ldots & 0 & 1 \end{pmatrix};$$

$$\bar{C}(t)=\begin{pmatrix} 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \end{pmatrix};$$

$$\bar{D}(t)=\begin{pmatrix} -c_{11} & -c_{12} & \dots & -c_{1q} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ -c_{11} & -c_{12} & \dots & -c_{1q} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ -c_{n1} & -c_{n2} & \dots & -c_{nq} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & -c'_{11} & -c'_{12} & \dots & -c'_{1l} & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & -c'_{21} & -c'_{22} & \dots & -c'_{2l} & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & -c'_{m1} & -c'_{m2} & \dots & -c'_{ml} & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & -c''_1 & -c''_2 & \dots & -c''_r \end{pmatrix}.$$

We will note that for all $t\in\overline{0,T-1}$, set $\boldsymbol{U}_1(t)$ in constraint (2) is not empty and is not a finite set, consisting of N_t ($N_t\in\mathbf{N}$) vectors of space $\mathbf{R}^{\bar{p}}$; for all $t\in\overline{0,T-1}$ and vectors $\bar{u}(t)\in\boldsymbol{U}_1(t)$. Set $\boldsymbol{W}_1(\bar{u}(t))$ in constraint (3) is not empty and is a finite set, consisting of $M_t(i)$ ($M_t(i)\in\mathbf{N}$ $i\in\overline{1,N_t}$) vectors of space $\mathbf{R}^{\bar{m}}$. Set $X_1(t)$, in accordance with its definition (4), is not empty and is a convex, closed and limited polyhedron (with a finite number of vertices) in space $\mathbf{R}^{\bar{n}}$. It is assumed that set $\boldsymbol{V}_1(\bar{u}(t))$ in constraint (5), is not empty either and is a convex, closed and limited polyhedron (with a finite number of vertices) in space $\mathbf{R}^{\bar{q}}$.

Let us describe information capabilities of player P in the process of minimax adaptive (by feedback principle) IP management at an enterprise for a discrete dynamical system (1)–(5).

It is assumed that, for any moment of time $\tau\in\overline{1,T}$ and correspondent integer time interval $\overline{0,\tau}\subseteq\overline{0,T}$ ($0<\tau$) by time moment τ in the process of IP management at an enterprise by player P, the following magnitudes are measured and memorized: $\bar{x}(0)=\bar{x}_0$ is the initial phase state of object I;

$$\bar{u}(\cdot)=\{\bar{u}(t)\}_{t\in\overline{0,\tau-1}}$$

is the history of implementation of management of player P in interval $\overline{0,\tau}$;

$$\bar{w}(\cdot)=\{\bar{w}(t)\}_{t\in\overline{0,\tau-1}}$$

is the history of implementation of the vector of intensity of restocking warehouse resources in interval $\overline{0,\tau}$;

$$\bar{v}(\cdot)=\{\bar{v}(t)\}_{t\in\overline{0,\tau-1}}$$

is the history of implementation of the risk vector in interval $\overline{0,\tau}$. Equation (1) and constraints (2)–(5) for it are also known.

The considered IP management process at an enterprise is assessed by value of convex functional $\tilde{F}:R^{\bar{n}}\to R^1$, defined on the possible implementations of phase vector $\bar{x}(T)\in\mathbf{R}^{\bar{n}}$ of system (1)–(5) in the final moment of time T.

Then for system (1)–(5), objective of optimal adaptive management from the position of player P can be stated as follows. In the assigned interval of time $\overline{0,T}$, it is required that player P should organize his management $\bar{u}(\cdot)=\{\bar{u}(t)\}_{t\in\overline{0,T-1}}$ (for all $t\in\overline{0,T-1}:\bar{u}(t)\in\boldsymbol{U}_1(t)$) by the feedback principle (as implementation of minimax adaptive strategy [14–16] for the selected class of permissible adaptive strategies). In this case, it can use all available for him information about its process so that the largest possible value of functional $\tilde{\boldsymbol{F}}$, determined for implementation of vector $\bar{x}(T)\in\mathbf{R}^{\bar{n}}$ (where $\bar{x}(T)$ is implementation of phase vector of object I at time moment T, corresponding to implementation of management $\bar{u}(\cdot)$), should be minimal. In this case, it is assumed that the worst (the largest) values of functional $\tilde{\boldsymbol{F}}$ could be implemented due to possible unfavorable implementations

$$\bar{v}(\cdot)=\{\bar{v}(t)\}_{t\in\overline{0,T-1}}$$

(for all $t\in\overline{0,T-1}:\bar{v}(t)\in\boldsymbol{V}_1(\bar{u}(t))$ of the generalized risk vector, and realizations

$$\bar{w}(\cdot)=\{\bar{w}(t)\}_{t\in\overline{0,T-1}}$$

(for all $t\in\overline{0,T-1}:\bar{w}(t)\in\boldsymbol{W}_1(\bar{u}(t))$) of the vector of intensity of restocking of warehouse resources contributed to accomplishments of objectives of player P. In other words, their selection (assigned by player P) is directed at minimization of functional $\tilde{\boldsymbol{F}}$ in accordance with the strategy, chosen by him.

5. Formation of a detailed model of IP management at an enterprise under conditions of risks

A detailed model of multicriteria optimization of software IP control at an enterprise in the face of risks was developed for practical application at enterprises [17].

For formation of economic-mathematical model of IP management process at an enterprise, the following designations were introduced:

n is the total number of kinds of finished produce of an enterprise;

m is the total number of types of resources, used for manufacturing of this produce;

q is the total number of factor-risks that influence manufacturing of produce;

$x(t)\in\mathbf{R}^n$ is the vector of residual volume of finished products, stored at company's warehouses within period of time t ($t\in\overline{0,T-1}$), in which every j-coordinate $x_j(t)$ is the value of volume of produce of the j-th kind ($j\in\overline{1,n}$).

It should be noted that if at the beginning of period of time t ($t\in\overline{0,T-1}$) there was a stock of finished produce in amount of $x(t)$ at the warehouses, by the end of this period only a part of them, equal to $A(t)x(t)$, will be suitable to usage (sale), where

$$A(t)=\left\|a_{jj}(t)\right\|_{j\in\overline{1,n}}$$

is the diagonal matrix of order n, characterizing the “aging” of produce within this period, $a_{jj}(t)$ is the coefficient, characterizing storage conditions of produce of the j-th kind; $j\in\overline{1,n}$, $a_{jj}(t)\in[0, 1]$);

$u(t)\in\mathbf{R}^n$ is the vector of innovative management of intensity of production of finished produce within period of time t ($t\in\overline{0,T-1}$), in which every j-th coordinate $u_j(t)$ is the value of volume of output of production of the j-th form ($j\in\overline{1,n}$);

$s(t)\in\mathbf{R}^n$ is the vector of volume of demand for finished produce, manufactured within period of time t ($t\in\overline{0,T-1}$), in which every j-th coordinate $s_j(t)$ is the value of magnitude of demand for finished produce of the j-th kind ($j\in\overline{1,n}$) at moment t;

$y(t)\in\mathbf{R}^m$ is the vector of residual volume of production resources, stored at company's warehouses within time period t $(t\in\overline{0,T-1})$, in which every i-th coordinate $y_i(t)$ is the value of volume of resource of the i-th form $(i\in\overline{1,n})$;

$w(t)\in\mathbf{R}^m$ is the vector of intensity of restocking warehouse resources within period of time t $(t\in\overline{0,T-1})$, in which every i-th coordinate $w_i(t)$ is the value of volume of resource of the i-th kind $(i\in\overline{1,n})$, in this case, implementation of management $u(t)\in\mathbf{R}^n$ determines realization

$$w(t)=W(u(t))=(W_1(u(t)),W_2(u(t)),...,W_m(u(t)))'\in\mathbf{R}^m,$$

where $W:\mathbf{R}^n\to\mathbf{R}^m$ is the vector-function of intensity of restocking of warehouse resources;

$$B(t)=B(u(t),w(t))\left\|b_{ij}(t)\right\|_{i\in\overline{1,m},j\in\overline{1,n}}$$

is the technological matrix of the process of considered correspondent fixed implementation of management influence $u(t)\in\mathbf{R}^n$ in the period of time t $(t\in\overline{0,T-1})$; $b_{ij}(t)$ are the norms of consumption of resource of the i-th type, necessary for manufacturing a unit volume of produce of the j-th kind, corresponding to implementation $u(t)$ $(i\in\overline{1,m};\ j\in\overline{1,n})$;

$v(t)\in\mathbf{R}^q$ is the risk vector, influencing output of a unit of produce of each kind within period of time t $(t\in\overline{0,T-1})$, ($q\in\mathbf{N}$). This vector describes possible unfavorable implementations of the vector of *a priori* undetermined factors. It can also include errors of modeling of the process under consideration.

$$C(T)=\left\|c_{jk}(t)\right\|_{j\in\overline{1,n},\,k\in\overline{1,q}}$$

is the matrix, consisting of factors of conversion of the level of influence of the risks' vector per unit of produce of every kind within period of time t $(t\in\overline{0,T-1})$;

$v'(t)\in\mathbf{R}^l$ is the vector of risks, influencing the state of a unit of available resources of each kind within period of time t $(t\in\overline{0,T-1})$, ($l\in\mathbf{N}$);

$$C'(t)=\left\|c'_{ih}(t)\right\|_{i\in\overline{1,m},\,h\in\overline{1,l}}$$

is the matrix, consisting of factors of conversion of the level of influence of the vector of risks per state of a resource unit of every kind within period of time t $(t\in\overline{0,T-1})$;

$v''(t)\in\mathbf{R}^r$ is the vector of financial risks, influencing the unit of financial resources of an enterprise within period of time t $(t\in\overline{0,T-1}),\ r\in\mathbf{N}$;

$c''(t)\in\mathbf{R}^r$ is the vector, consisting of factors of conversion of the level of influence of the vector of financial risks per unit of total financial costs of an enterprise within period of time t $(t\in\overline{0,T-1})$.

Similar to the stocks of finished produce, by the end of period t, only part of stocks of production resources, equal to $R(t)y(t)$, will be suitable for using in production, where

$$R(t)=\left\|r_{ii}(t)\right\|_{i\in\overline{1,m}}$$

is the diagonal matrix of order m, characterizing the "aging" of production resources within the correspondent period.

We will introduce vector

$$l(t)=\left(\left\langle\overline{b_1}(t),u(t)\right\rangle_n,\left\langle\overline{b_2}(t),u(t)\right\rangle_n,...,\left\langle\overline{b_m}(t),u(t)\right\rangle_n\right)'\in\mathbf{R}^m,$$

where $\overline{b_i}(t)$ is the vector of correspondent lines of matrix $B(t)$ $(i\in\overline{1,m})$.

We will designate as $Z(t)$ the total costs of an enterprise within period of time t; $k(t)$ is the amount of available financial means, formed by the beginning of period t.

We will form a system of discrete linear recurrent equations that describes in full the dynamics of the considered process of IP management at an enterprise:

$$\begin{cases} x(t+1)=A(t)x(t)+u(t)-s(t)-C(t)v(t),\\ x(0)=0_n,\ s(0)=s,\\ y(t+1)=R(t)y(t)+W(u(t))-B(t)u(t)-C'(t)v'(t),\\ y(0)=b,\\ Z(t+1)=Z(t)+\left\langle q(t),l(t)\right\rangle_m+\left\langle z(t),x(t)\right\rangle_n+\left\langle p(t),y(t)\right\rangle_m,\\ Z(0)=\left\langle z(0),x(0)\right\rangle_n+\left\langle p(0),y(0)\right\rangle_m,\\ k(t+1)=k(t)+\left\langle g(t),s(t)\right\rangle_n-\alpha\cdot\left\langle g(t),s(t)\right\rangle_n-\\ -Z(t)-\beta(t)-\gamma(t)-\left\langle c''(t)v''(t)\right\rangle_r,\\ k(0)=G+G_0,\quad t\in\overline{0,T-1}, \end{cases} \tag{11}$$

where $q(t)\in\mathbf{R}^m$ is the vector of actual prices for used production resources, necessary for an enterprise for manufacturing products within period of time t $(t\in\overline{0,T-1})$, in which each i-th coordinate $q_i(t)$ is the value of price for a unit of produce of the production resource of the i-th kind $(i\in\overline{1,m})$;

$z(t)\in\mathbf{R}^n$ is the vector of costs of an enterprise for storage residues of finished products at the warehouse within period of time t $(t\in\overline{0,T-1})$, in which each j-th coordinate $z_j(t)$ is the value of volume of costs per unit of produce of the j-th kind $(j\in\overline{1,n})$;

$p(t)\in\mathbf{R}^m$ is the vector of costs of an enterprise for storage of residual production resources at the warehouse within period of time t $(t\in\overline{0,T-1})$, in which each i-th coordinate $p_i(t)$ is the value of volume of costs per resource unit of the i-th kind $(i\in\overline{1,m})$;

$s\in\mathbf{R}^n$ is the vector of initial volume of demand for finished produce during implementation of the management process at the initial moment (at t=0), in which each j-th coordinate s_j is the value of magnitude of demand for finished production of the j-th kind $(j\in\overline{1,n})$ at the initial moment of time;

$b\in\mathbf{R}^m$ is the vector of initial volume of production resources during implementation of management process at the initial moment of time (at t=0), in which each i-th coordinate b_i is the value of production resource of the i-th kind $(i\in\overline{1,m})$, which is consumed at initial moment of time;

G is the volume of bank loan, G_0 is the own financial resources of an enterprise, directed at expansion of production;

α is the coefficient, considering a share of tax deductions from selling of goods (services);

$g(t)\in\mathbf{R}^n$ is the vector of actual purchasing prices fo sold produce, manufactured by an enterprise within period of time $(t\in\overline{0,T-1})$, in which each j-th coordinate $g_j(t)$ is the value of price per unit of produce of the j-th kind $(j\in\overline{1,n})$;

$\gamma(t)$ are the other taxes, paid within period of time t $(t\in\overline{0,T-1})$;

$\beta(t)$ is the volume of loan-related deductions.

We should note that the resulting system allows us simulate dynamics of the multi-step process of IP management at an enterprise depending on assigned initial conditions and selection of specific implementations of managerial influences.

It should be mentioned that in the formed discrete dynamical system (11), technological matrix

$$B(t)=B(u(t))=\left\|b_{ij}(t)\right\|_{i\in\overline{1,m},\ j\in\overline{1,n}}$$

and the vector of intensity of restocking of warehouse resources $w(t)=W(u(t))$ clearly depend on implementation of managerial influence $u(t)\in\mathbf{R}^n$ in the period of time t $(t\in\overline{0,T-1})$.

We will accept as optimization criteria:

– maximum revenues of products, manufactured on the basis of the relevant innovative technology (f_1);

– maximum gross profit of products, manufactured on the basis of the relevant innovative technology (f_2);

– maximum of value, opposite to cost of products, manufactured on the basis of the relevant innovative technology (f_3), which coincides with maximum of product costs;

– maximum profit gains of products, manufactured based on the relevant innovative technology (f_4), which is an indicator of innovative process efficiency:

$$\begin{cases} f_1=\langle pr,x\rangle_n\to\max,\\ f_2=\langle pr-c,x\rangle_n\to\max,\\ f_3=-\langle c,x\rangle_n\to\max,\\ f_4=\langle p,x\rangle_n-P_l\to\max, \end{cases} \qquad (12)$$

where $x(T)\in\mathbf{R}^n$, and x_i is the planned volume of products of the i-th name, produced based on the innovative technology $(i\in\overline{1,n})$ by the final moment of T;

$pr\in\mathbf{R}^n$ is the price vector, pr_i is the price per unit of product of i-th name, produced based on the innovative technology;

$c\in\mathbf{R}^n$ is the cost vector, and c_i is the cost of a unit of product of the i-th name, produced based on the innovative technology;

$p\in\mathbf{R}^n$ is the profit, and p_i is the profit for a unit of product of the i-th name, produced based on the innovative technology;

P_l is the profit of manufacturing products, made on the basis of the old technology within the similar previous period $(i\in\overline{1,n})$.

Vector of management $u(t)\in\mathbf{R}^n$ of intensity of production of finished produce in the dynamic system (10) must meet the following assigned constraint:

$$\begin{aligned} &u(t)\in U_{N_t}(t)=\\ &=\left\{u(t):u(t)\in\left\{u^{(1)}(t),u^{(2)}(t),...,u^{(N_t)}(t)\right\}\subset\mathbf{R}^n,\forall i\in\overline{1,N},\right.\\ &U_{\min}(t)\le u^{(i)}(t)\le U_{\max}(t),\\ &\langle q(t),w(t)\rangle_m\le k(t),B(t)u^{(i)}(t)\le y(t)+w(t),\\ &U_{\min}(t)\le u(t)\le U_{\max}(t),\\ &\left.\langle q(t),W(u(t))\rangle_m\le k(t),B(t)u(t)\le y(t)+W(u(t))\right\}, \end{aligned} \qquad (13)$$

where $U_{N_t}(t)$ for each $t\in\overline{0,T-1}$ is a finite set of vectors, i. e. a finite set, consisting of N_t ($N_t\in\mathbf{N}$) vectors in $\mathbf{R}^n$, which determine all possible implementations of various management scenarios at the moment of time t; $n\in\mathbf{N}$;

$U_{\min}(t)\in\mathbf{R}^n$ is the vector of lower constraint of produce output (of minimal acceptable volume of manufacturing finished produce), in which each j-th coordinate $U_{min\,j}(t)$ is the value of minimum acceptable output of produce of the j-th kind $(j\in\overline{1,n})$ (for example, the loss-free point for each kind of produce); $U_{\max}(t)\in\mathbf{R}^n$ is the vector of upper constraint of produce output, in which each j-th cordinate $U_{\max}(t)$ is the value of maximum acceptable output of produce of the j-th kind $(j\in\overline{1,n})$ (for example, maximum market capacity for each kind of produce, maximum production capacity, etc.).

Vector $w(t)\in\mathbf{R}^m$ of intensity of warehouse resources restocking within period of time t $(t\in\overline{0,T-1})$ depends of permissible implementation of management $u(t)\in U_N(t)$ and must meet the following assigned restraint:

$$\begin{aligned} &w(t)\in W_{M_t}(u(t))=\\ &=\left\{w(t):w(t)\in\left\{w^{(1)}(t),w^{(2)}(t),...,w^{(M_t(i))}(t)\right\}\subset\mathbf{R}^m,\right.\\ &\forall j\in\overline{1,M_t(i)},\\ &\left.\langle q(t),w^{(i)}(t)\rangle_m\le k(t),B(t)u(t)\le y(t)+w^{(i)}(t)\right\}, \end{aligned} \qquad (14)$$

where $W_{M_t}(u(t))$ for each moment of time $t\in\overline{0,T-1}$ and management $u(t)\in U_N(t)$ is the finite set of vectors, i. e. a finite set, consisting of $M_t(i)$ ($M_t(i)\in\mathbf{N}$, $i\in\overline{1,N_t}$) vectors in space $\mathbf{R}^m$, determining various scenarios of restocking of warehouse resources at the moment of time t.

We should note that vector $w(t)$ can be considered as another managerial influence, the selection of which depends on implementation of management $u(t)$.

In this case, for all periods of time t $(t\in\overline{0,T-1})$, the following phase constraints should be satisfied:

$$\begin{cases} x_j(t)\ge 0, x_j(0)=0, j\in\overline{1,n}; y_i(t)\ge 0, y_i(0)=b_i, i\in\overline{1,m};\\ k(t)\ge 0, k(0)=G+G_0\ge 0; Z(t)\ge 0, Z(0)=0. \end{cases} \qquad (15)$$

Based on introduced to system (10) vectors of risks $v(t)\in\mathbf{R}^q$, $v'(t)\in\mathbf{R}^l$ and $v''(t)\in\mathbf{R}^r$, corresponding to period of time t $(t\in\overline{0,T-1})$, the generalized vector of risks was introduced

$$\bar{v}(t)=(v(t),v'(t),v''(t))\in\mathbf{R}^q\times\mathbf{R}^l\times\mathbf{R}^r.$$

This vector in process of IP management at an enterprise depends on permissible implementation of management $u(t)\in U_N(t)$ and must satisfy this specified constraint:

$$\begin{aligned} &\bar{v}(t)=\bar{V}(u(t))=\\ &=\left\{\bar{v}(t):\bar{v}(t)=(v(t),v'(t),v''(t))\in\mathbf{R}^{\bar{q}}\times\mathbf{R}^q\times\mathbf{R}^l\times\mathbf{R}^r,\right.\\ &V_*(u(t))\le v(t)\le V^*(u(t)),\\ &V_*'(u(t))\le v'(t)\le V'^*(u(t)),\\ &\left.V_*''(u(t))\le v''(t)\le V''^*(u(t))\right\}, \end{aligned} \qquad (16)$$

where

$$V_*(u(t)),\ \ V_*'(u(t)),\ \ V_*''(u(t))$$

and

$$V^*(u(t)),\ \ V'^*(u(t)),\ \ V''^*(u(t))$$

are respectively lower and upper constraints for permissible implementations of the vectors of risks within a period of time t $(t\in\overline{0,T-1})$, corresponding to implementation of managerial influence $u(t)$, which are determined based on the history of implementation of IP management process at an enterprise.

$$\bar{q} = (q + l + r) \in \mathbf{N}.$$

It should be noted that in the process of IP management, taking into consideration constraints (13)–(16) is an essential condition. In a discrete dynamical system (11), it must be met by the optimal managerial influences, generated by them parameters of the state of the considered system, implementation of the vector of intensities of restocking of warehouse resources, as well as permissible implementations of risk vectors.

We should also mention that parameters:

$$A(t), B(t), C(t), C'(t), R(t), s(t), s, c(t), c''(t), q(t),$$
$$z(t), w(t), \alpha,, \beta(t), \gamma(t), b, G, G_0, U_{\min}(t), U_{\max}(t), V_*(u(t)),$$
$$V_*(u(t)), V_*(u(t)), V^*(u(t)), V'^*(u(t)), V''^*(u(t))$$

in system (11) for all $t \in \overline{0, T-1}$ must be known before (for example, to be formed based on existing statistical data of the process under consideration, technical and economic forecasts and other data sources) or found with the help of solutions of identification, proposed in this research.

6. Discussion of results of research into modeling the minimax adaptive management

Thus, for organization of minimax adaptive IP management at an enterprise in the selected class of permissible strategies for adaptive management, the model of adaptive management was developed. The proposed recurrent algorithm reduces an original multi-step problem to implementation of the ultimate sequence of problems of minimax software IP management at an enterprise. In turn, solution of each of these problems is reduced to implementation of the finite sequence of only one-step optimization operations in the form of solutions of linear and convex mathematical programming and discrete optimization. Then it is possible to obtain a solution of the considered model of adaptive IP management at an enterprise in the face of risks. Solution is reduced to implementation of solution of a finite sequence of problems of linear convex mathematical programming and discrete optimization.

Presented results can be used for economic-mathematical modeling and solution of other problems of management processes optimization. Economic-mathematical models for such problems are presented, for example, in papers [18–23]. Prospects of the conducted research are associated with possibility of introduction of parameters of the vector of non-determined risks into a model of IP management at an enterprise.

The developed economic-mathematical models are an effective tool of modeling of optimization of the guaranteed outcome of IP management considering the risk factor for enterprises. The results obtained can be used for economic-mathematical modeling and solution of other problems of optimization of processes of data forecasting and management. This takes into consideration conditions of information deficit and uncertainty in the face of risks. The developed mathematical apparatus can be used for implementation of the relevant program-technical complexes for supporting effective managerial decision making in practice.

Developed tools of modeling in IP management at an enterprise make it possible to solve the problem of generation of optimal production programs and pricing during implementation of innovative activity of enterprises.

7. Conclusions

1. A general model of adaptive IP management at an enterprise for organization of minimax adaptive IP management in the selected class of permissible adaptive management strategies was formalized. Solution of the stated problem of minimax IP management makes it possible to obtain optimal guaranteed (minimax) result of IP management at an enterprise.

2. We developed a detailed model of multicriteria optimization of IP management at an enterprise in the face of risks, which fully describes dynamics of the analyzed process and offers an opportunity of effective implementation of the resulting mathematical apparatus into practice of enterprises' operation. The generated optimization criteria and the system of phase constraints of the model take into consideration possibilities of production capacities, as well as meet the requirements for IP.

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