20. Zaets, V. P. Noise reduction with soundproof screens [Text] / V. P. Zaets // Eastern-European Journal of Enterprise Technologies. – 2012. – Vol. 6, Issue 10 (60). – P. 25–33. – Available at: http://journals.uran.ua/eejet/article/view/5605/5047
21. Zaets, V. P. Shumozakhysni ekrany dlia znyzhennia rivniv zvukovoho tysku vid rukhomykh dzherel zvuku [Text]: dys. ... kand. tekhn. nauk / V. P. Zaets. – Kyiv, 2013. – 182 p.
22. Trokhymenko, M. P. Vplyv parametriv shumozakhysnoho ekranu na yoho efektyvnist [Text] / M. P. Trokhymenko, V. P. Zaets // Budivelni materialy, vyroby ta sanitarna tekhnika. – 2010. – Issue 36. – P. 71–76.
23. Shenderov, E. L. Volnovye zadachi gidroakustiki [Text] / E. L. Shenderov. – Leningrad: Sudostroenie, 1972. – 347 p.
24. Spravochnik po spetsial'nym funktsiyam [Text] / M. Abramovits, I. Stigan (Eds.). – Moscow: Nauka, 1979. – 832 p.

Розглянуто задачу визначення нестаціонарного температурного поля в захищеній металевій конструкції в умовах пожежі. Визначено вогнезахисну ефективність покриття, нанесеного на металеву пластину, нагріту під час нестандартного температурного режиму за умови неідеального теплового контакту. Використовуючи розроблену математичну модель розрахунку температури та експериментальні дані, визначено вогнезахисну ефективність покриття

Ключові слова: нестаціонарне температурне поле, металева конструкція, конвекційний теплообмін, вогнезахисна ефективність покриття

Рассмотрена задача определения нестационарного температурного поля в защищенной металлической конструкции в условиях пожара. Определена огнезащитная эффективность покрытия, нанесенного на металлическую пластину, нагретую во время нестандартного температурного режима при условии неидеального теплового контакта. Используя разработанную математическую модель расчета температуры и экспериментальные данные, определена огнезащитная эффективность покрытия

Ключевые слова: нестационарное температурное поле, металлическая конструкция, конвективный теплообмен, огнезащитная эффективность покрытия

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RESEARCH INTO NON-STATIONARY TEMPERATURE FIELD IN THE PROTECTED METALLIC STRUCTURE UNDER CONDITIONS OF FIRE

V. Loik
PhD*
E-mail: v.loik1984@gmail.com
O. Lazarenko
PhD*
E-mail: lazarenkoolexandr@gmail.com
T. Bojko
PhD, Deputy Head of Institute
Educational and scientific institute of fire and technogenic safety**
E-mail: boykotaras@gmail.com
S. Vovk
PhD
Department of Supervision and Preventive Work**
E-mail: sergiy_vovk@ukr.net
*Department of Fire Tactic and Rescue Works**
**Lviv State University of Life Safety
Kleparivska str., 35, Lviv, Ukraine, 79007

1. Introduction

At all stages of capital construction or reconstruction of objects of any designation it is required to take into account the ability of metallic building structures to maintain their properties under fire conditions, particularly, to ensure the necessary fire resistance limit.

It is known that in order to improve the limit of fire resistance of structures, they are treated with fire-retarding substances. One of the means to protect metallic structures from fire is the application of special coatings on their surfaces, bloating under the influence of high temperature of fire with the formation of a heat insulation layer.

To analyze the processes and phenomena that occur during fire, it is necessary to apply methods of general physics, chemistry, thermophysics, thermodynamics, mechanics of solids, mathematics, etc. By employing a complex of these disciplines, it is possible to study and describe complex phenomena that accompany the start and development of fire, that is, to model a process that takes place there [1].

In the practice of calculating a temperature problem during fire, it is as a rule necessary to solve the problem of nonstationary thermal conductivity. Under conditions of fire, a situation that most often occurs involves heating of the wall on the one hand, and the presence of convective heat exchange with the surrounding medium, on the other hand.

Under actual fire conditions, there is a nonstationary temperature regime, which in turn leads to a non-ideal thermal contact at the border of conjugation "metallic wall – flame-retardant coating". The task is considerably simplified if one sets the conditions for a perfect thermal contact; the solutions created in this case are well-known. Therefore, examining temperature regimes in a two-layer structure, considering conditions of a non-ideal thermal contact, remains a relevant task.

2. Literature review and problem statement

The task of studying a non-stationary temperature field has quite often been the subject of research in many scientific papers. But if carefully considered, we can see that the set challenge was solved on its own, and in most cases it has no experimental confirmation. In addition, each study employed different mathematical apparatus, as well as methods for solving differential equations.

In particular, paper [2] proposes an algorithm for the calculation of non-stationary temperature fields at friction of bodies taking into account behavior peculiarities of uneven contact surface. Heat conduction equation was solved by the method of integral Laplace and Hankel transforms. Numerical results were obtained for the average dimensionless temperature for different Fourier parameters and roughness of the surface, as well as for different loads.

Article [3] proposed a procedure for calculating the class of problems on heat exchange in multilayer structures with a generalized non-ideal contact. The procedure is illustrated by calculating heat transfer in a pile of plates with fluids that move in the gaps between plates.

Authors of paper [4] show the structure of unique mathematical solutions to causal linear systems, invariant over time, applying time-domain methods and generalized Laplace and Fourier transforms. In particular, the authors correct a widespread inconsistency in using the Laplace transform.

Scientific study [5] addresses the problem of determining a non-stationary temperature field of a multi-layer flat structure, heated by convective heat exchange with the surrounding environment whose temperature changes over time in line with the logarithmic law. A solution to the corresponding problem on thermal conductivity was obtained using the method of Green's function under conditions of perfect contact.

Authors of scientific research [6] developed a method of analytical solutions to the problems of thermal conduction of conductivity in the one-dimensional multilayer composite bodies. The resulting method enables determining the exact transition temperature and heat flow. The proposed method is numerically effective, as it requires simple operations.

In paper [7], authors obtained analytical solutions to the problems on non-stationary heat conduction under conditions of a non-ideal frictional contact between two solid isotropic semi-limited bodies.

The purpose of study [8] was to test numerical inverse Laplace transformation methods using a thermal characteristic within the framework of experiment. As a result, the work proved the reliability of methods based on the Fourier series methods.

Paper [9] considered a nonlinear boundary problem of thermal conductivity for an isotropic infinite thermosensitive layered plate with thermally insulated face surfaces and a foreign through heat-producing inclusion. By applying the proposed transform, a partial linearization was performed of the original heat conductivity equation; a numerical-analytical solution to this equation was found with boundary conditions of the second kind using an integral Fourier transform.

Based on the theoretical, as well as known, experimental results, authors of scientific study [10] constructed an improved mathematical model of thermo-chemical destruction of a multi-layered heat insulation coating. Consideration of flow through the body made it possible to predict the state of a protected structure under fire conditions more accurately. Results of numerical calculations were compared with known data.

Scientific study [11] tackles a one-dimensional multilayer diffusion problem. The multi-layer diffusion problem is represented as a sequence of single-layer tasks.

Paper [12] proposed and justified design scheme for building a solution to the mixed problem for the heat conductivity equation with piecewise continuous coefficients that depend on a coordinate at the finite interval. The basis for the scheme is formed by the method of reduction, the concept of quasi-derivatives, the modern theory of linear differential equations, the Fourier method, and the modified method of native functions. The results obtained could be used in the study of heat transfer process in a multi-layer plate under conditions of perfect thermal contact between the layers.

Article [13] reported experimental results of testing fire resistance of unprotected steel beams and their comparison with the estimated models. The comparison reveals differences between experimental temperature data and the calculations carried out in accordance with standards.

Thus, to date, there is a relevant task on creating a mathematical model, which would make it possible to adequately perform analytical calculations for determining the effectiveness of a fire-retardant coating as there are no practical results of research into non-stationary temperature fields for layered structures, considering conditions for a non-ideal heat contact on the conjugating surfaces of their non-uniform elements.

3. The aim and objectives of the study

The aim of present study is to devise a mathematical model of temperature field calculation in the structure "metallic wall – flame-retardant coating" that would make it possible to determine the effectiveness of a fire-retardant coating under fire conditions, taking into account a non-ideal thermal contact on the conjugating surfaces of non-uniform layers. The result will be determining the time of occurrence of the critical value of surface temperature of the metallic plate, which is not heated, without conducting experimental studies.

To achieve the set aim, the following tasks must be solved:

– to solve a non-stationary task of thermal conductivity for a two-layer structure "metallic wall – flame-retardant coating" considering the conditions for a non-ideal thermal contact at the conjugating surfaces;

– applying the obtained analytical solution of a boundary-value problem, to calculate a non-stationary temperature field of the examined structure for specific geometrical and thermophysical parameters;

– to compare results of experimental studies, conducted previously on determining the efficiency of a fire-retardant

coating, with the numerical results obtained when employing the devised mathematical model.

4. Analytical solutions to the boundary value problem of heat conduction for a two-layered structure

4. 1. Mathematical model of temperature field calculation

We shall determine a distribution of temperature field in a double-layered structure with a non-ideal thermal contact between the layers and in the presence of conditions for convective heat exchange with the external environment. The heat conduction equations for each of the layers take the form:

$$\frac{\partial^2 t_1(x,\tau)}{\partial x^2}=\frac{1}{a_1}\frac{\partial t_1(x,\tau)}{\partial \tau},\ \ (0<x<l_1;\ \tau>0), \tag{1}$$

$$\frac{\partial^2 t_2(x,\tau)}{\partial x^2}=\frac{1}{a_2}\frac{\partial t_2(x,\tau)}{\partial \tau},\ \ (l_1<x<l_2;\ \tau>0), \tag{2}$$

where l_1, l_2 are the thickness of the first and second layers, respectively; a_1, a_2 are the thermal diffusivity coefficients; τ is the time; x is the spatial coordinate; t_1, t_2 is the temperature of the first and second layer, respectively.

Initial temperature distribution through the thickness of the two-layered system will be assigned in the following form:

$$t(x,0)=f(x),\ \ (0\le x\le l_2). \tag{3}$$

We assume that there is a convective heat exchange with the surrounding environment on the external surfaces of a double-layered wall, in line with the law of Newton

$$\lambda_1\frac{\partial t_1(0,\tau)}{\partial x}=-\alpha_0(t_{p0}(\tau)-t_1(0,\tau)) \tag{4}$$

and

$$\lambda_2\frac{\partial t_2(l_2,\tau)}{\partial x}=\alpha_2(t_{p2}(\tau)-t_2(l_2,\tau)), \tag{5}$$

where λ_1, λ_2 are the coefficients of thermal conductivity of materials of the first and second layers, respectively; α_0, α_2 are the heat release coefficients form the boundary surfaces of the structure; $t_{p0}(\tau)$ is the change in fluid temperature outside the borders of a thermal near-surface layer in the vicinity of surface $x=0$; $t_{p2}(\tau)$ is the change in fluid temperature outside the borders of a thermal near-surface layer in the vicinity of surface $x=l_2-l_1$.

There is a non-ideal thermal contact between the layers, that is, the following conditions are satisfied

$$\lambda_1\frac{\partial t_1(l_1,\tau)}{\partial x}=\lambda_2\frac{\partial t_2(l_1,\tau)}{\partial x}, \tag{6}$$

$$\lambda_1\frac{\partial t_1(l_1,\tau)}{\partial x}=\alpha_1(t_2(l_1,\tau)-t_1(l_1,\tau)). \tag{7}$$

To solve a heat conduction problem (1)–(7), we shall apply the Laplace transform for parameter, then equations (1) and (2) will take the form:

$$\frac{d^2 t_1(x,\omega)}{dx^2}-\frac{\omega}{a_1}t_1(x,\omega)=-\frac{1}{a_1}f(x),\ \ (0<x<l_1), \tag{7}$$

$$\frac{d^2 t_2(x,\omega)}{dx^2}-\frac{\omega}{a_2}t_2(x,\omega)=-\frac{1}{a_2}f(x),\ \ (l_1<x<l_2), \tag{8}$$

where

$$t_i(x,\omega)=\int_0^\infty t_i(x,\tau)e^{-\omega\tau}d\tau,\ \ (i=1,2). \tag{9}$$

Solutions to differential equations (7) and (8) are:

$$t_1(x,\omega)=C_{11}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}x+C_{21}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}x+$$
$$+\frac{1}{\sqrt{\omega a_1}}\int_0^x f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-x)d\xi,$$
$$(0\le x\le l_1), \tag{10}$$

$$t_2(x,\omega)=C_{12}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}x+C_{22}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}x+$$
$$+\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^x f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-x)d\xi,$$
$$(l_1\le x\le l_2). \tag{11}$$

where C_{11}, C_{12}, C_{21}, C_{22} are the integration constants.

We shall determine integration constants by using boundary conditions (4), (5) and conditions for a non-ideal thermal contact between layers (6), (7), which, after performing the Laplace transform, will take the form:

$$\lambda_1\frac{dt_1(0,\omega)}{dx}=-\alpha_0(t_{p0}(\omega)-t_1(0,\omega)), \tag{12}$$

$$\lambda_2\frac{dt_2(l_2,\omega)}{dx}=\alpha_2(t_{p2}(\omega)-t_2(l_2,\omega)), \tag{13}$$

$$\lambda_1\frac{dt_1(l_1,\omega)}{dx}=\lambda_2\frac{dt_2(l_1,\omega)}{dx}, \tag{14}$$

$$\lambda_1\frac{dt_1(l_1,\omega)}{dx}=\alpha_1(t_2(l_1,\omega)-t_1(l_1,\omega)). \tag{15}$$

By substituting (11) into (14) and (15), we shall obtain a system of equations:

$$\sqrt{\frac{\omega}{a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}l_1C_{12}+\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}l_1C_{22}=\frac{\lambda_1}{\lambda_2}\frac{dt_1(l_1,\omega)}{dx};$$

$$\mathrm{sh}\sqrt{\frac{\omega}{a_2}}l_1C_{12}+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}l_1C_{22}=t_1(l_1,\omega)+\frac{\lambda_1}{\alpha_1}\frac{dt_1(l_1,\omega)}{dx}.$$

We shall obtain from this system of equations:

$$C_{12}=\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\frac{dt_1(l_1,\omega)}{dx}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}l_1-$$
$$-\frac{\lambda_1}{\alpha_1}\frac{dt_1(l_1,\omega)}{dx}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}l_1-t_1(l_1,\omega)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}l_1;$$

$$C_{22}=-\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\frac{dt_1(l_1,\omega)}{dx}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}l_1+$$
$$+\frac{\lambda_1}{\alpha_1}\frac{dt_1(l_1,\omega)}{dx}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}l_1+t_1(l_1,\omega)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}l_1.$$

Considering these dependences, relation (11) will take the form:

$$t_2(x,\omega)=\left(\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(x-l_1)++\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\right)\frac{dt_1(l_1,\omega)}{dx}+$$
$$+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)t_1(l_1,\omega)+\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{x}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-x)\mathrm{d}\xi, \qquad (16)$$

$$\frac{dt_2(x,\omega)}{dx}=\left(\frac{\lambda_1}{\lambda_2}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(x-l_1)\right)\frac{dt_1(l_1,\omega)}{dx}$$
$$+\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(x-l_1)t_1(l_1,\omega)-\frac{1}{a_2}\int_{l_1}^{x}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(\xi-x)\mathrm{d}\xi. \qquad (17)$$

Boundary values (12) and (13), considering (10), (16) and (17), will take the form:

$$\frac{\lambda_1}{\alpha_0}\sqrt{\frac{\omega}{a_1}}C_{11}-C_{21}=-t_{p0}(\omega);$$

$$\left(\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$+\frac{\lambda_1\lambda_2\omega}{\alpha_1\alpha_2\sqrt{a_1a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\times$$
$$\times\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\times$$
$$\left.\times\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)C_{11}+$$
$$+\left(\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1\lambda_2\omega}{\alpha_1\alpha_2\sqrt{a_1a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$\left.+\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)C_{21}=$$
$$=t_{p2}(\omega)+\left(\frac{\lambda_1}{\alpha_2}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1\lambda_2}{\alpha_1\alpha_2}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$\left.+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\times$$
$$\times\frac{1}{a_1}\int_0^{l_1}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi-$$
$$-\left(\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\times$$
$$\times\frac{1}{\sqrt{\omega a_1}}\int_0^{l_1}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi+$$
$$+\frac{\lambda_2}{\alpha_2a_2}\int_{l_1}^{l_2}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi-\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{l_2}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi.$$

We shall obtain from this system of equations:

$$\Delta C_{11}=-t_{p0}(\omega)\left(\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$+\frac{\lambda_1\lambda_2\omega}{\alpha_1\alpha_2\sqrt{a_1a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\times$$
$$\left.\times\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)+$$
$$+t_{p2}(\omega)+\left(\frac{\lambda_1}{\alpha_2}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$+\frac{\lambda_1\lambda_2}{\alpha_1\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$\left.+\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{a_1}\times\int_0^{l_1}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi-$$
$$-\left(\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{\sqrt{\omega a_1}}\times$$
$$\times\int_0^{l_1}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi+\frac{\lambda_2}{\alpha_2a_2}\int_{l_1}^{l_2}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi-$$
$$-\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{l_2}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi,$$

$$\Delta C_{21}=t_{p0}(\omega)\left(\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}ch\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$+\frac{\lambda_1\lambda_2\omega}{\alpha_1\alpha_2\sqrt{a_1a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\times$$
$$\left.\times sh\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)+$$
$$+\frac{\lambda_1}{\alpha_0}\sqrt{\frac{\omega}{a_1}}\left(t_{p2}(\omega)+\left(\frac{\lambda_1}{\alpha_2}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1\lambda_2}{\alpha_1\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.\right.$$
$$\left.\left.+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{a_1}\times\right.$$
$$\times\int_0^{l_1}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi-$$
$$-\left(\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{\sqrt{\omega a_1}}\times$$
$$\times\int_0^{l_1}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi+\frac{\lambda_2}{\alpha_2a_2}\int_{l_1}^{l_2}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi-$$
$$-\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{l_2}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi,$$

$$\Delta(\omega)=\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1\lambda_2\omega}{\alpha_1\alpha_2\sqrt{a_1a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1^2\omega}{\alpha_0\alpha_2a_1}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\times$$
$$\times\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1^2\lambda_2\omega\sqrt{\omega}}{\alpha_0\alpha_1\alpha_2a_1\sqrt{a_2}}\times$$
$$\times\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1^2\sqrt{\omega a_2}}{\alpha_0\lambda_2a_1}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\times$$
$$\times\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1^2\omega}{\alpha_0\alpha_1a_1}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1\lambda_2\omega}{\alpha_0\alpha_2\sqrt{a_1a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\times$$
$$\times\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\alpha_0}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1). \qquad (18)$$

By substituting expressions for ΔC_{11} and ΔC_{21} into relations (10), (11), (12), we shall determine:

$$t_1(x,\omega)=\frac{t_{p0}(\omega)}{\Delta}\left(\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(x-l_1)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.$$
$$+\frac{\lambda_1\lambda_2\omega}{\alpha_1\alpha_2\sqrt{a_1a_2}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(x-l_1)\times$$
$$\times\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(x-l_1)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(x-l_1)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)-\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(x-l_1)\times$$
$$\left.\times\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)-\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(x-l_1)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)+$$
$$+\left(\mathrm{sh}\sqrt{\frac{\omega}{a_1}}x+\frac{\lambda_1}{\alpha_0}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_1}}x\right)\times$$
$$\times\frac{1}{\Delta}\left(t_{p2}(\omega)+\left(\frac{\lambda_1}{\alpha_2}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.\right.$$
$$+\frac{\lambda_1\lambda_2}{\alpha_1\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$\left.\left.+\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{a_1}\int_0^{l_1}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi-\right.$$
$$-\left(\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{\sqrt{\omega a_1}}\times$$
$$\times\int_0^{l_1}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi+\frac{\lambda_2}{\alpha_2a_2}\int_{l_1}^{l_2}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi-$$
$$\left.-\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{l_2}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi\right)+\frac{1}{\sqrt{\omega a_1}}\int_0^{x}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-x)\mathrm{d}\xi. \qquad (19)$$

$$t_2(x,\omega)=\frac{t_{p0}(\omega)}{\Delta}\times$$
$$\times\left(\frac{\lambda_1}{\alpha_2}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-x)+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-x)\right)+$$
$$+\frac{1}{\Delta}\left(\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(x-l_1)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1+\right.$$
$$+\frac{\lambda_1^2\sqrt{\omega a_2}}{\alpha_0\lambda_2a_1}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(x-l_1)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1+$$
$$+\frac{\lambda_1}{\alpha_1}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1+$$
$$+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1+\frac{\lambda_1}{\alpha_0}\sqrt{\frac{\omega}{a_1}}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\times$$
$$\left.\times\mathrm{ch}\sqrt{\frac{\omega}{a_1}}l_1+\frac{\lambda_1^2\omega}{\alpha_0\alpha_1a_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}l_1\right)\times$$
$$\times\left(t_{p2}(\omega)+\left(\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\right.\right.$$
$$+\frac{\lambda_1}{\alpha_2}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+$$
$$\left.\left.+\frac{\lambda_1\lambda_2}{\alpha_1\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\cdot\frac{1}{a_1}\times\right.$$
$$\times\int_0^{l_1}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi-$$
$$-\left(\frac{\lambda_2}{\alpha_2}\sqrt{\frac{\omega}{a_2}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(l_2-l_1)\right)\frac{1}{\sqrt{\omega a_1}}\times$$
$$\times\int_0^{l_1}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi+$$
$$+\frac{\lambda_2}{\alpha_2a_2}\int_{l_1}^{l_2}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi-$$
$$-\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{l_2}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-l_2)\mathrm{d}\xi\Bigg)+$$
$$+\left(\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{\omega}}\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(x-l_1)+\frac{\lambda_1}{\alpha_1}\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\right)\times$$
$$\times\frac{1}{a_1}\int_0^{l_1}f(\xi)\mathrm{ch}\sqrt{\frac{\omega}{a_1}}(\xi-l_1)\mathrm{d}\xi+$$
$$+\mathrm{ch}\sqrt{\frac{\omega}{a_2}}(x-l_1)\frac{1}{\sqrt{\omega a_1}}\int_0^{l_1}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-l_1)\mathrm{d}\xi+$$
$$+\frac{1}{\sqrt{\omega a_2}}\int_{l_1}^{x}f(\xi)\mathrm{sh}\sqrt{\frac{\omega}{a_2}}(\xi-x)\mathrm{d}\xi. \qquad (20)$$

Perform expansion of the right side of characteristic equation (18) as a function of ω argument into a power series

$$\Delta(\omega)=k_2\frac{\omega^{0,5}}{a_1^{0,5}}+k_3\frac{\omega^{1,5}}{a_1^{0,5}}+..., \qquad (21)$$

where

$$k_2=\frac{\lambda_1}{\alpha_0}+\frac{\lambda_1}{\alpha_1}+\frac{\lambda_1}{\alpha_2}+l_1+\frac{\lambda_1(l_2-l_1)}{\lambda_2};$$

$$k_3=\frac{\lambda_1 l_1^2}{2\alpha_2 a_1}+\frac{\lambda_1(l_2-l_1)^2}{2\alpha_2 a_2}+\frac{\lambda_1\lambda_2(l_2-l_1)}{\alpha_1\alpha_2 a_2}+$$
$$+\frac{\lambda_1(l_2-l_1)l_1^2}{2\lambda_2 a_1}+\frac{\lambda_1(l_2-l_1)^3}{6\lambda_2 a_2}+\frac{\lambda_1 l_1^2}{2\alpha_1 a_1}+$$
$$+\frac{\lambda_1(l_2-l_1)^2}{2\alpha_1 a_2}+\frac{\lambda_2 l_1(l_2-l_1)}{\alpha_2 a_2}+\frac{l_1^3}{6a_1}+$$
$$+\frac{l_1(l_2-l_1)^2}{2a_2}+\frac{\lambda_1^2 l_1}{\alpha_0\alpha_2 a_1}+\frac{\lambda_1^2 l_1(l_2-l_1)}{\alpha_0\lambda_2 a_1}+$$
$$+\frac{\lambda_1^2 l_1}{\alpha_0\alpha_1 a_1}+\frac{\lambda_1\lambda_2(l_2-l_1)}{\alpha_0\alpha_2 a_2}+\frac{\lambda_1 l_1^2}{2\alpha_0 a_1}+\frac{\lambda_1(l_2-l_1)^2}{2\alpha_0 a_2}.$$

An analysis of characteristic equation (21) shows that it has real roots equal to zero, or imaginary roots. The roots of characteristic equation (21) were calculated employing the software MATLAB.

Expansion of the right-hand sides of expressions (19), (20) into the power series takes the form:

$$\Delta t_1(x,\omega)=k_{01}\frac{\omega^{-1,5}}{a_1^{0,5}}+k_{11}\frac{\omega^{-0,5}}{a_1^{0,5}}+..., \quad (22)$$

$$\Delta t_2(x,\omega)=k_{02}\frac{\omega^{-1,5}}{a_1^{0,5}}+k_{12}\frac{\omega^{-0,5}}{a_1^{0,5}}+..., \quad (23)$$

where

$$t_{p0}(\tau)=t_{p00}+t_{p01}\tau \text{ and } t_{p2}(\tau)=t_{p20}+t_{p21}\tau, \quad (24)$$

where $t_{p00}(\tau)$, $t_{p20}(\tau)$ is the temperature of environment outside the borders of near- surface thermal layers;

$$k_{01}=t_{p01}\left(\frac{\lambda_1}{\alpha_1}+\frac{\lambda_1}{\alpha_2}+\frac{\lambda_1(l_2-l_1)}{\lambda_2}+l_1-x\right)+$$
$$+t_{p21}\left(x+\frac{\lambda_1}{\alpha_0}\right);$$

$$k_{11}=t_{p00}\left(\frac{\lambda_1}{\alpha_1}+\frac{\lambda_1}{\alpha_2}+\frac{\lambda_1(l_2-l_1)}{\lambda_2}+l_1-x\right)+$$
$$+t_{p01}\left(\frac{\lambda_1(x-l_1)^2}{2\alpha_2 a_1}+\frac{\lambda_1(l_2-l_1)^2}{2\alpha_2 a_2}+\right.$$
$$+\frac{\lambda_1\lambda_2(l_2-l_1)}{\alpha_1\alpha_2 a_2}+\frac{\lambda_1(x-l_1)^2(l_2-l_1)}{2\lambda_2 a_1}+$$
$$+\frac{\lambda_1(l_2-l_1)^3}{6\lambda_2 a_2}+\frac{\lambda_1(x-l_1)^2}{2\alpha_1 a_1}+\frac{\lambda_1(l_2-l_1)^2}{2\alpha_1 a_2}-$$
$$\left.-\frac{\lambda_2(x-l_1)(l_2-l_1)}{\alpha_2 a_2}-\frac{(x-l_1)^3}{6a_1}-\frac{(x-l_1)(l_2-l_1)^2}{2a_2}\right)+$$
$$+t_{p20}\left(x+\frac{\lambda_1}{\alpha_0}\right)+t_{p21}\left(\frac{x^3}{6a_1}+\frac{\lambda_1 x^2}{2\alpha_0 a_1}\right).$$

$$k_{02}=t_{p01}\left(\frac{\lambda_1}{\alpha_2}+\frac{\lambda_1(l_2-x)}{\lambda_2}\right)+$$
$$+t_{p21}\left(\frac{\lambda_1}{\alpha_0}+\frac{\lambda_1}{\alpha_1}+l_1+\frac{\lambda_1(x-l_1)}{\lambda_2}\right);$$

$$k_{12}=t_{p00}\left(\frac{\lambda_1}{\alpha_2}+\frac{\lambda_1(l_2-x)}{\lambda_2}\right)+$$
$$+t_{p20}\left(\frac{\lambda_1}{\alpha_0}+\frac{\lambda_1}{\alpha_1}+l_1+\frac{\lambda_1(x-l_1)}{\lambda_2}\right)+$$
$$+t_{p01}\left(\frac{\lambda_1(l_2-x)^2}{2\alpha_2 a_2}+\frac{\lambda_1(l_2-x)^3}{6\lambda_2 a_2}\right)+$$
$$+t_{p21}\left(\frac{\lambda_1(x-l_1)^3}{6\lambda_2 a_2}+\frac{\lambda_1 l_1^2(x-l_1)}{2\lambda_2 a_1}+\frac{\lambda_1^2(x-l_1)l_1}{\alpha_0\lambda_2 a_1}+\right.$$
$$+\frac{\lambda_1(x-l_1)^2}{2\alpha_1 a_2}+\frac{\lambda_1 l_1^2}{2\alpha_1 a_1}+\frac{l_1(x-l_1)^2}{2a_2}+$$
$$\left.+\frac{l_1^3}{6a_1}+\frac{\lambda_1(x-l_1)^2}{2\alpha_0 a_2}+\frac{\lambda_1 l_1^2}{2\alpha_0 a_1}+\frac{\lambda_1^2 l_1}{\alpha_0\alpha_1 a_1}\right).$$

An analysis of characteristic equations (22), (23) reveals that they have a valid multiple root equal to zero. After performing the inverse integral Laplace transform, we shall obtain the following expressions:

$$t_{10}(x,\tau)=\frac{k_{01}}{k_2}\tau+\frac{k_{11}}{k_2}-\frac{k_{01}k_3}{k_2^2},$$
$$t_{20}(x,\tau)=\frac{k_{02}}{k_2}\tau+\frac{k_{12}}{k_2}-\frac{k_{02}k_3}{k_2^2}.$$

The presence of imaginary roots of characteristic equations (22), (23) indicate the existence of the following terms:

$$t_{11}(x,\tau)=$$
$$=\sum_{i=1}^{\infty}\frac{e^{-\gamma_i^2\tau}}{\Delta'(\gamma_i)}\left(\left(\frac{t_{p01}}{\gamma_i^4}-\frac{t_{p00}}{\gamma_i^2}\right)\left(\frac{\lambda_1\gamma_i}{\alpha_2\sqrt{a_1}}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_1}}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}-\right.\right.$$
$$-\frac{\lambda_1\lambda_2\gamma_i^2}{\alpha_1\alpha_2\sqrt{a_1 a_2}}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_1}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+$$
$$+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_1}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+$$
$$+\frac{\lambda_1\gamma_i}{\alpha_1\sqrt{a_1}}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_1}}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+$$
$$\left.+\frac{\lambda_2\gamma_i}{\alpha_2\sqrt{a_2}}\sin\frac{\gamma_i(x-l_1)}{\sqrt{a_1}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}-\sin\frac{\gamma_i(x-l_1)}{\sqrt{a_1}}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}\right)+$$
$$+\left(\sin\frac{\gamma_i x}{\sqrt{a_1}}+\frac{\lambda_1\gamma_i}{\alpha_0\sqrt{a_1}}\cos\frac{\gamma_i x}{\sqrt{a_1}}\right)\left(\frac{t_{p21}}{\gamma_i^4}-\frac{t_{p20}}{\gamma_i^2}+\right.$$
$$+\left(\frac{\lambda_1}{\alpha_2}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1\lambda_2\gamma_i}{\alpha_1\alpha_2\sqrt{a_2}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+\right.$$
$$\left.\left.+\frac{\lambda_1\sqrt{a_2}}{\lambda_2\gamma_i}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1}{\alpha_1}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}\right)\times\right.$$
$$\times\frac{1}{a_1}\int_0^{l_1}f(\xi)\cos\frac{\gamma_i(\xi-l_1)}{\sqrt{a_1}}d\xi+\left(\frac{\lambda_2\gamma_i}{\alpha_2\sqrt{a_2}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}-\right.$$
$$-\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}\Big)\frac{1}{\gamma_i\sqrt{a_1}}\int_0^{l_1}f(\xi)\sin\frac{\gamma_i(\xi-l_1)}{\sqrt{a_1}}d\xi+$$
$$\left.+\frac{\lambda_2}{\alpha_2 a_2}\int_{l_1}^{l_2}f(\xi)\cos\frac{\gamma_i(\xi-l_2)}{\sqrt{a_2}}d\xi-\frac{1}{\gamma_i\sqrt{a_2}}\int_{l_1}^{l_2}f(\xi)\sin\frac{\gamma_i(\xi-l_2)}{\sqrt{a_2}}d\xi\right)\Big);$$

$$t_{21}(x,\tau)=$$

$$=\sum_{i=1}^{\infty}\frac{e^{-\gamma_i^2\tau}}{\Delta'(\gamma_i)}\left(\left(\frac{t_{p01}}{\gamma_i^4}-\frac{t_{p00}}{\gamma_i^2}\right)\left(\frac{\lambda_1\gamma_i}{\alpha_2\sqrt{a_1}}\cos\frac{\gamma_i(l_2-x)}{\sqrt{a_2}}+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\sin\frac{\gamma_i(l_2-x)}{\sqrt{a_2}}\right)+\right.$$

$$+\left(\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\sin\frac{\gamma_i(x-l_1)}{\sqrt{a_2}}\cos\frac{\gamma_i l_1}{\sqrt{a_1}}-\frac{\lambda_1^2\sqrt{a_2}\gamma_i}{\alpha_0\lambda_2 a_1}\sin\frac{\gamma_i(x-l_1)}{\sqrt{a_2}}\sin\frac{\gamma_i l_1}{\sqrt{a_1}}+\right.$$

$$+\frac{\lambda_1\gamma_i}{\alpha_1\sqrt{a_1}}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_2}}\cos\frac{\gamma_i l_1}{\sqrt{a_1}}+$$

$$+\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_2}}\sin\frac{\gamma_i l_1}{\sqrt{a_1}}+\frac{\lambda_1\gamma_i}{\alpha_0\sqrt{a_1}}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_2}}\cos\frac{\gamma_i l_1}{\sqrt{a_1}}-$$

$$\left.-\frac{\lambda_1^2\gamma_i^2}{\alpha_0\alpha_1 a_1}\cos\frac{\gamma_i(x-l_1)}{\sqrt{a_2}}\sin\frac{\gamma_i l_1}{\sqrt{a_1}}\right)\times$$

$$\times\left(\frac{t_{p21}}{\gamma_i^4}-\frac{t_{p20}}{\gamma_i^2}+\left(\frac{\lambda_1}{\alpha_1}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1}{\alpha_2}\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}+\right.\right.$$

$$\left.+\frac{\lambda_1\sqrt{a_2}}{\lambda_2\gamma_i}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1\lambda_2\gamma_i}{\alpha_1\alpha_2\sqrt{a_2}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}\right)\times$$

$$\times\frac{1}{a_1}\int_0^{l_1}f(\xi)\cos\frac{\gamma_i(\xi-l_1)}{\sqrt{a_1}}d\xi+$$

$$+\left(\frac{\lambda_2\gamma_i}{\alpha_2\sqrt{a_2}}\sin\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}-\cos\frac{\gamma_i(l_2-l_1)}{\sqrt{a_2}}\right)\times$$

$$\times\frac{1}{\gamma_i\sqrt{a_1}}\int_0^{l_1}f(\xi)\sin\frac{\gamma_i(\xi-l_1)}{\sqrt{a_1}}d\xi+$$

$$+\frac{\lambda_2}{\alpha_2 a_2}\int_{l_1}^{l_2}f(\xi)\cos\frac{\gamma_i(\xi-l_2)}{\sqrt{a_2}}d\xi-$$

$$\left.-\frac{1}{\gamma_i\sqrt{a_2}}\int_{l_1}^{l_2}f(\xi)\sin\frac{\gamma_i(\xi-l_2)}{\sqrt{a_2}}d\xi\right)\Bigg);$$

$$\Delta(\gamma)=\frac{\lambda_1\gamma}{\alpha_2\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{\lambda_1\lambda_2\gamma^2}{\alpha_1\alpha_2\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1}{\lambda_2}\sqrt{\frac{a_2}{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1\gamma}{\alpha_1\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{\lambda_2\gamma}{\alpha_2\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1^2\gamma^2}{\alpha_0\alpha_2 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1^2\lambda_2\gamma^3}{\alpha_0\alpha_1\alpha_2 a_1\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{\lambda_1^2\gamma\sqrt{a_2}}{\alpha_0\lambda_2 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1^2\gamma^2}{\alpha_0\alpha_1 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\times$$

$$\times\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1\lambda_2\gamma^2}{\alpha_0\alpha_2\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1\gamma}{\alpha_0\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}},$$

where

$$\Delta'(\gamma)=\frac{\lambda_1 l_1}{2\alpha_2 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{0{,}5\lambda_1}{\alpha_2\gamma\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1(l_2-l_1)}{2\alpha_2\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1\lambda_2}{\alpha_1\alpha_2\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{\lambda_1\lambda_2\gamma l_1}{2\alpha_1\alpha_2 a_1\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1\lambda_2\gamma(l_2-l_1)}{2\alpha_1\alpha_2\sqrt{a_1}a_2}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1\sqrt{a_2}l_1}{2\lambda_2\gamma a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1(l_2-l_1)}{2\lambda_2\gamma\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{0{,}5\lambda_1}{\alpha_1\gamma\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1 l_1}{2\alpha_1 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1(l_2-l_1)}{2\alpha_1\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\times$$

$$\cdot\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{0{,}5\lambda_2}{\alpha_2\gamma\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_2 l_1}{2\alpha_2\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_2(l_2-l_1)}{2\alpha_2 a_2}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{l_1}{2\gamma\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{l_2-l_1}{2\gamma\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\times$$

$$\times\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1^2}{\alpha_0\alpha_2 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1^2\gamma l_1}{2\alpha_0\alpha_2 a_1\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1^2\gamma(l_2-l_1)}{2\alpha_0\alpha_2 a_1\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{3\lambda_1^2\lambda_2\gamma}{2\alpha_0\alpha_1\alpha_2 a_1\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{\lambda_1^2\lambda_2\gamma^2 l_1}{2\alpha_0\alpha_1\alpha_2 a_1\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-$$

$$-\frac{\lambda_1^2\lambda_2\gamma^2(l_2-l_1)}{2\alpha_0\alpha_1\alpha_2 a_1 a_2}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1^2\sqrt{a_2}}{2\alpha_0\lambda_2 a_1\gamma}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1^2\sqrt{a_2}l_1}{2\alpha_0\lambda_2 a_1\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1^2(l_2-l_1)}{2\alpha_0\lambda_2 a_1}\times$$

$$\times\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1^2}{\alpha_0\alpha_1 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1^2\gamma l_1}{2\alpha_0\alpha_1 a_1\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1^2\gamma(l_2-l_1)}{2\alpha_0\alpha_1 a_1\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1\lambda_2}{\alpha_0\alpha_2\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1\lambda_2\gamma l_1}{2\alpha_0\alpha_2 a_1\sqrt{a_2}}\sin\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1\lambda_2\gamma(l_2-l_1)}{2\alpha_0\alpha_2\sqrt{a_1}a_2}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}-\frac{\lambda_1}{2\alpha_0\gamma\sqrt{a_1}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+$$

$$+\frac{\lambda_1 l_1}{2\alpha_0 a_1}\sin\frac{\gamma l_1}{\sqrt{a_1}}\cos\frac{\gamma(l_2-l_1)}{\sqrt{a_2}}+\frac{\lambda_1(l_2-l_1)}{2\alpha_0\sqrt{a_1a_2}}\cos\frac{\gamma l_1}{\sqrt{a_1}}\sin\frac{\gamma(l_2-l_1)}{\sqrt{a_2}},$$

γ_i (i=1, 2), ω are the roots of characteristic equations; $t_{p01}(\tau)$, $t_{p21}(\tau)$ are the temperature changes of medium.

Thus, the solution to a boundary-value problem of heat conduction (1)–(7), taking into account expressions (24), will take the form:

$$t_1(x,\tau)=t_{10}(x,\tau)+t_{11}(x,\tau),\ \ (0\le x\le l_1);$$

$$t_2(x,\tau)=t_{20}(x,\tau)+t_{21}(x,\tau),\ \ (l_1\le x\le l_2).$$

4. 2. Determining temperature distribution in a two-layered structure "metallic wall – flame-retardant coating"

Using the developed mathematical model, we determined a temperature field distribution in a two-layered structure whose first layer is a metallic wall, the second – a protective coating for the following geometrical and thermophysical layer parameters: $a_1=7.1\cdot10^{-6}$ m^2/s, $a_2=13.8\times\times10^{-6}$ m^2/s, $l_1=0.001$ m, $l_2=0.051$ m, $l_1-l_2=0.05$ m, $\lambda_1=0.87$ W/(m·°C), $\lambda_2=55$ W/(m·°C), f(ξ)=20 °C, $\alpha_0=20$ W/(m^2·°C), $\alpha_1=240$ W/(m^2·°C), $\alpha_2=16$ W/(m^2·°C), $t_{p00}=20$ °C, $t_{p01}=1$ °C, $t_{p20}=20$ °C, $t_{p21}=0.01$ °C.

We shall calculate temperature distribution using the MATLAB software. The obtained results are given in Table 1.

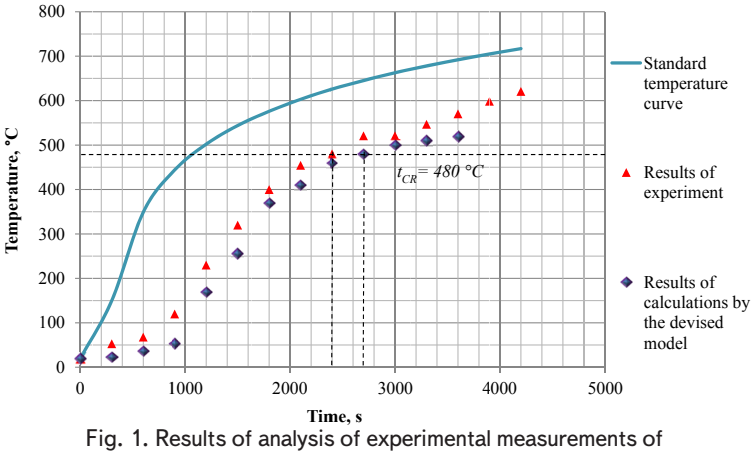

Fig. 1. Results of analysis of experimental measurements of temperature and its obtained values in line with the devised model, depending on time (coating's thickness is 0.8 mm)

Table 1

Estimated results of determining the temperatures of a near-surface layer, surface of the coating and the surface that is not heated, depending on the duration of heating and temperature of the environment (coating's thickness is 0.8 mm)

Duration of heating, min.	Temperature of the environment, °C	Temperature of a near-surface layer, °C	Temperature of the applied coating, °C	Temperature of the surface that is not heated, °C
1	80	26.1	24.9	20.1
5	320	53.8	47.7	23.3
15	920	142	124	52.7
30	950	442	411	370
45	980	712	514	480
60	1080	804	627	520

To confirm the adequacy of the developed mathematical model for calculating a temperature field in the examined system, we shall will carry out a comparative analysis of the obtained numerical results with the results obtained experimentally.

5. Analysis of the obtained numerical estimations of temperature and the experimental data

Fire-retardant efficiency of the examined materials was obtained in line with the procedure by VNIIPO [14].

Composition of the flame-retardant coating [15]: binder – 30 %, K-2104 (LAC carborane siloxane varnish), fillers – 35 % Al_2O_3, 20 % ZrO_2, 15% kaolin. It was established in study [16] that the time for reaching the critical temperature is 41 min. (Fig. 1).

The influence of thickness of fire-retardant coatings on its fire-retardant efficiency was studied experimentally for a metallic plate. Corresponding dependence is shown in Fig. 2.

Experimental studies have shown that an increase in the thickness of coating exerts a positive effect on the fire-retardant efficiency. The mechanism of a fire-retardant coating is based on the creation of a heat-insulating and temperature-resistant layer at the surface of the material due to bloating, when heated, with the formation of a porous and strong structure. When heating the samples to a temperature well above 400 °C, due the gaseous products of thermal destruction of carborane siloxane, the bloating of the coating occurs and its volume increases by 8.3–12.4 times at satisfactory adhesion strength to metal with an optimum thickness of the coating of 0.8 mm.

It was established that an increase in the thickness of a coating from 0.8 to 2.0 mm results in an increase in the time needed to reach the critical temperature (t_{CR}) for metallic structures from 38 to 52 minutes. Therefore, for a maximum limit of fire resistance of a coating, effective thickness equals 0.8 mm.

Fire-retardant efficiency of coating η will be determined from ratio:

$$\eta=\tau/\tau_{max}\cdot100\ \%=$$
$$=41\ \text{min.}/52\ \text{min.}\cdot100\ \%=78.8\ \%.$$

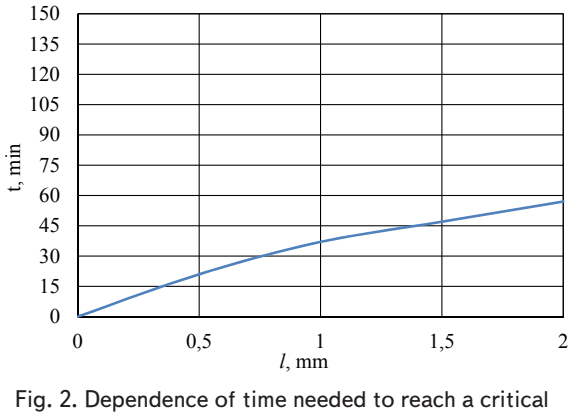

Fig. 2. Dependence of time needed to reach a critical temperature on the coating's thickness for a metallic plate

An analysis of the presented experimental data and the obtained numerical values for temperature distributions (Fig. 1) shows that the difference between them is 9.7 %.

6. Discussion of modeling and experimental results of research into a temperature field in the structure "metallic wall – flame-retardant coating"

Comparison of the results obtained based on the devised model and those received experimentally shows that error is 9.7 %. This is caused by the inaccuracy of the mathematical model (model is linear) and an error in the experiment.

Heat transfer conditions can acquire a more complicated form, in the case when complex physical-mechanical or chemical processes occur in the contact area, leading, for example, to the emergence of sources of heat (as a result of bloating of a fire-retardant layer) whose power needs to be considered for these conditions. Presence of heat sources in the contact area leads to a jump in both temperature and heat flow. Depending on the sequence of distribution of heat sources, the jump occurs both in the direction of temperature rise and temperature fall.

The devised mathematical model for determining a temperature field in a fire-retardant coating allows us to argue that the effectiveness of a fire-retardant coating, applied onto metallic wall, when reaching a critical value of temperature on the surface of the side, which is not heated (480 °C), has a value that is 80.07 %.

In future, the aim of research is to construct a non-linear mathematical model for determining temperature regimes in heat-sensitive layered structures (thermophysical parameters depend on temperature), as well as consideration in the boundary conditions of locally arranged internal sources of heat.

A direct benefit of the mathematical model for determining a temperature field is the presence of conditions for a non-ideal thermal contact on the boundary conjugating surfaces of the structure "metallic wall – protective coating". Our research results should subsequently serve to design new fire-retardant coatings. This will make it possible to effectively protect various layered systems under fire conditions.

7. Conclusions

1. By applying the Laplace transform, we obtained an analytic solution to the nonstationary heat conduction problem for a two-layered structure (metallic structure – protective coating). The mathematical model takes into account a non-ideal contact of the surfaces, a nonstationary temperature regime.

2. By employing the obtained mathematical model for a nonstationary temperature field, we determined numerical values for temperature field distribution. Calculations showed that the effectiveness of a fire-retardant coating applied onto a metallic plate, has an estimated value of 45 minutes when reaching a critical temperature on the unheated surface of 480 °C.

3. Comparisons are performed between the analytical calculations and the results of experimental research into temperature distribution throughout the thickness of a metallic structure protected by a flame-retardant coating. It was established that the fire-retardant efficiency for a given structure is 41 minutes.

References

1. Bashkirtsev, M. P. Fundamentals of fire thermal physics [Text]: textbook / M. P. Bashkirtsev, N. F. Bubir, N. A. Minaev, D. N. Onchukov. – Moscow: Stroyizdat, 1984. – 200 p.
2. Timoshenko, M. V. Numerical simulation of heat transfer in multilayer structures with generalized nonideal contact [Text] / M. V. Timoshenko // Journal of Engineering Physics and Thermophysics. – 1996. – Vol. 69, Issue 5. – P. 590–595. doi: 10.1007/bf02606174
3. Yevtushenko, A. Non-stationary temperature field of discrete sliding contact of elastic bodies [Text] / A. Yevtushenko, O. Ukhanska // Wear. – 1994. – Vol. 176, Issue 1. – P. 19–23. doi: 10.1016/0043-1648(94)90192-9
4. Brigola, R. On initial conditions, generalized functions and the Laplace transform [Text] / R. Brigola, P. Singer // Electrical Engineering. – 2009. – Vol. 91, Issue 1. – P. 9–13. doi: 10.1007/s00202-009-0110-5
5. Procyuk, B. V. Investigation of the nonstationary temperature field in a multilayer flat structure [Text] / B. V. Procyuk, M. M. Semerak, R. B. Veselivs'kij, V. M. Sinyuta // Journal fire safety. – 2012. – Issue 20. – P. 111–117.
6. Yang, B. A Distributed Transfer Function Method for Heat Conduction Problems in Multilayer Composites [Text] / B. Yang // Numerical Heat Transfer, Part B: Fundamentals. – 2008. – Vol. 54, Issue 4. – P. 314–337. doi: 10.1080/10407790802359038
7. Belyakov, N. S. Mathematical simulation of thermal friction processes under conditions of nonideal contact [Text] / N. S. Belyakov, A. P. Nosko // High Temperature. – 2009. – Vol. 47, Issue 1. – P. 123–130. doi: 10.1134/s0018151x09010167
8. Toutain, J. Numerical Inversion of Laplace Transform for Time Resolved Thermal Characterization Experiment [Text] / J. Toutain, J.-L. Battaglia, C. Pradere, J. Pailhes, A. Kusiak, W. Aregba, J.-C. Batsale // Journal of Heat Transfer. – 2011. – Vol. 133, Issue 4. – P. 044504. doi: 10.1115/1.4002777
9. Havrysh, V. I. Nonlinear Boundary-Value Problem of Heat Conduction for a Layered Plate with Inclusion [Text] / V. I. Havrysh // Materials Science. – 2015. – Vol. 51, Issue 3. – P. 331–339. doi: 10.1007/s11003-015-9846-4
10. Ovchinnikov, V. A. Modeling of the Thermal Protection of a Multilayer Material Under Fire Conditions [Text] / V. A. Ovchinnikov, A. S. Yakimov // Journal of Engineering Physics and Thermophysics. – 2016. – Vol. 89, Issue 3. – P. 569–578. doi: 10.1007/s10891-016-1413-9
11. Rodrigo, M. R. Solution of multilayer diffusion problems via the Laplace transform [Text] / M. R. Rodrigo, A. L. Worthy // Journal of Mathematical Analysis and Applications. – 2016. – Vol. 444, Issue 1. – P. 475–502. doi: 10.1016/j.jmaa.2016.06.042
12. Tatsii, R. M. General Boundary-Value Problems for the Heat Conduction Equation with Piecewise-Continuous Coefficients [Text] / R. M. Tatsii, O. Yu. Pazen // Journal of Engineering Physics and Thermophysics. – 2016. – Vol. 89, Issue 2. – P. 357–368. doi: 10.1007/s10891-016-1386-8

13. Lukomski, M. Fire Resistance of Unprotected Steel Beams – Comparison between Fire Tests and Calculation Models [Text] / M. Lukomski, P. Turkowski, P. Roszkowski, B. Papis // Procedia Engineering. – 2017. – Vol. 172. – P. 665–672. doi: 10.1016/j.proeng.2017.02.078
14. Determination of the heat-insulating properties of fire-retardant coatings for metal: Methodology [Text]. – Moscow: VNIIPO, 1998. – 19 p.
15. Pat. No. 39860 UA. Kompozytsiya dlia zharo- i vohnestiykoho zakhysnoho pokryttia. MPK(2009) S09K 21/00; E04V 1/94 [Text] / Loik V. B., Givlyud M. M., Guculyak Yu. V., Yuzkiv T. B., Emchenko I. V., Peredrij O. I.; zaiavnyk i patentovlasnyk Lviv. derzh. un-t bezpeky zhyttiediyalnosti. – No. u200813207; declareted: 14.11.2008; published: 10.03.2009, Bul. No. 5. – 3 p.
16. Loik, V. B. Influence of the heating temperature on mass transfer processes in the contact zone "flame retardant coating-metal" [Text] / V. B. Loik // Journal fire safety. – 2010. – Issue 16. – P. 44–49.

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A STUDY OF THE THIRD-ORDER NONLINEAR SUSCEPTIBILITY AND NONLINEAR ABSORPTION OF INAS IN THE MIDDLE INFRARED REGION

M. Musaev
Doctor of physical sciences, Professor*
E-mail: m_musaver@yahoo.com
I. Abbasov
PhD, Associate Professor*
E-mail: ibrahimabbasov179@gmail.com
A. Baxtiyarov
PhD, Associate Professor*
E-mail: aliashrafbaxtiyarov@gmail.com
*Azerbaijan State Oil and Industry University
Azadliq ave., 20, Baku, Azerbaijan, AZ 1010

Виміряні нелінійні сприйнятливості третього порядку $\chi^{(3)}$ і коефіцієнт нелінійного поглинання в InAs n-типу з різним ступенем легування. Отримані значення $\chi^{(3)}\simeq 10^{-7}$ СГСЕ істотно перевищують значення, обчислені на основі моделі, що описує нелінійну сприйнятливість електронів, які знаходяться в непараболічній зоні провідності. Показано, що спостережувана розбіжність усувається, якщо в розрахунку врахувати дисипації енергії електронів. Зростання ефективності при чотирьоххвильовій взаємодії в вузькозонних напівпровідниках обмежується нелінійним поглинанням взаємодіючих хвиль

Ключові слова: нелінійна сприйнятливість третього порядку, чотирьох хвильова взаємодія, вузькозонні напівпровідники, поріг пробою

Измерены нелинейные восприимчивости третьего порядка $\chi^{(3)}$ и коэффициент нелинейного поглощения в InAs n-типа с разной степенью легирования. Полученные значения $\chi^{(3)}\simeq 10^{-7}$ СГСЭ существенно превышают значения, вычисленные на основе модели, описывающей нелинейную восприимчивость электронов, находящихся в непараболичной зоне проводимости. Показано, что наблюдаемое расхождение устраняется, если в расчете учесть диссипацию энергии электронов. Рост эффективности при четырехволновом взаимодействии в узкозонных полупроводниках ограничивается нелинейным поглощением взаимодействующих волн

Ключевые слова: нелинейная восприимчивость третьего порядка, четырехволновое взаимодействие, узкозонные полупроводники, порог пробоя

1. Introduction

Nonlinear optical materials have numerous applications, including photodynamic therapy, nonlinear photonics, 3D optical data storage, frequency upconverted lasing, and fluorescence imaging [1–5]. One of the most important problems of applied nonlinear optics is the search for media with possibly large values of nonlinear susceptibilities. In this regard, semiconductors, as experiments have shown, are among the most promising media [6, 7]. The large nonlinearity of semiconductors basically comes from the fact that they, with their relatively small bandgap E_g, are characterized by sufficiently low internal fields, which determine the couple forces acting on optical electrons. Therefore, even not too high laser fields should already provide a large contribution to the susceptibility of nonlinear electronic polarization.

2. Literature review and problem statement

The study of cubic susceptibilities is the central problem of nonlinear spectroscopy [8]. The effects due to the cubic susceptibility are the basis of such methods of nonlinear spectroscopy as two-photon spectroscopy, saturation spectroscopy and also allow solving such an important practical problem as the correction of phase distortions by the four-