

Розроблена методика математично-го опису дросельних матриць у вигляді графів. Побудована узагальнена математична модель дросельної матриці, яка є основою для аналізу функціональних можливостей схем вимірювальних перетворювачів. Розглянуті принципи синтезу схем багатофункціональних гідродинамічних вимірювальних перетворювачів параметрів плинних середовищ на основі дросельних матриць. Наведений приклад реалізації багатофункціонального вимірювального перетворювача реологічних параметрів неньютонівської рідини

Ключові слова: дросельна матриця, теорія графів, вимірювальна схема, гідродинамічний вимірювальний перетворювач

Разработана методика математического описания дросельных матриц в виде графов. Построена обобщенная математическая модель дросельной матрицы, которая является основой для анализа функциональных возможностей схем измерительных преобразователей. Рассмотрены принципы синтеза схем многофункциональных гидродинамических измерительных преобразователей параметров текучих сред на основе дросельных матриц. Приведен пример реализации многофункционального измерительного преобразователя реологических параметров неньютоновской жидкости

Ключевые слова: дросельная матрица, теория графов, измерительная схема, гидродинамический измерительный преобразователь

SYNTHESIZING THE SCHEMES OF MULTIFUNCTIONAL MEASURING TRANSDUCERS OF THE FLUID PARAMETERS

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1. Introduction

Measurement of parameters and characteristics of the fluid flow is important for many practical applications. Dynamic and kinematic viscosity, thermal conductivity and heat capacity, density, etc. are the main parameters for Newtonian fluids while rheological parameters are the main parameters for non-Newtonian fluids [1]. Pressure, velocity, flow, temperature, etc. are main flow characteristics and their measurement enables comprehensive description of the fluids [2]. These properties can be measured directly with the use of certain devices or they can be determined proceeding from several basic measurements [3].

Modern trends in the measuring technology include creation of multifunctional transducers which enable continuous and simultaneous measurement of several parameters and characteristics of fluids. Such systems are necessary for an effective conduction of technological processes and observance of qualitative indicators of end products. Therefore, development of multiparameter transducers of the fluid flow parameters for automatic control and regulation systems is an urgent problem in many industries.

Various methods can be used to measure each of these parameters [3]. However, multifunctional transducers deserve a greater attention as they are built on a single measurement method, are simpler in their technical implementation and therefore cheaper. That is why development of such multifunctional transducers is a question of expediency.

2. Literature review and problem statement

Various methods are used for measuring of fluid parameters and flow characteristics. They include vibrational, hydrodynamic, rotary, thermal and other methods. The studies conducted by many authors are aimed at improving each of them and enhancement of their functionality. For example, authors of [4] propose a system for analyzing composition of two- or three-component gas mixtures based on a measurement of the medium physical properties. The system consists of a standard Coriolis flowmeter which being combined with the sensors of density, pressure and heat flow measures the medium viscosity, density, heat capacity and flow. The mixture composition is determined continuously from the signals sent by sensors.

In [5], the Newtonian medium viscosity and density are measured by a vibrational method using an electromechanical resonator. The developed system can be applied to the quality control of motor oils and diesel fuel under production conditions. An electromechanically excited vibrational membrane rheometer [6] is suitable for analysis of rheologically complex liquids since operating frequency of the sensing element is low and the vibration amplitude is just several micrometers. Such devices require individual calibration.

The problem of continuous measurement of several fluid parameters and flow characteristics using a single transducer is solved in [7]. Based on measuring of the pressure differential in the Coriolis tube, a possibility of determining not only the fluid flow and density but also dynamic viscosity was substantiated.

The hydrogasdynamic method is promising for a continuous measurement of various parameters. Hydrogasdynamic measuring systems use throttles as the elements that create resistance in hydraulic and pneumatic diagrams. In such systems, throttle elements of various types are connected in parallel and in series to form bridge and differential measuring diagrams. This makes it possible to carry out practically all operations including arithmetic operations and form the basis for synthesis of hydrogasdynamic transducers for measuring physical and mechanical parameters of fluids. For example, an availability of constructing transducers for measuring composition of binary gas mixtures is considered in [8] based on a gas-dynamic bridge diagram. It was shown that characteristics and functionality of the transducers depend on various patterns of integration of laminar and turbulent throttles in the bridge diagram. Paper [9] investigated a gas-dynamic flow transducer in which capillary tubes included in the shoulders of the bridge measuring diagram and serve as sensing elements. The transducer does not require individual calibration and is designed to measure small and micro flows of gases. Such a flowmeter was implemented in an automated control system of the process of making blanks for fiber-optic products.

The hydrodynamic method was successfully applied in the multi-parametric system for measuring density and kinematic and dynamic viscosity of petroleum products [10, 11]. Based on it, a bridge transducer with various types of throttle elements was synthesized. The measurement principle consists in a continuous pumping of the controlled oil product through a throttle bridge transducer and an automatic astatic balancing of the bridge by varying volume flow of the liquid. Kinematic viscosity and density of the oil product are determined proceeding from the values of the volume flow and the total pressure differential on the bridge at the moment of its equilibrium. Based on these parameters, one can also calculate dynamic viscosity and cetane number.

Theoretical studies of synthesis of hydrogasdynamic measuring transducers became the basis for creation of an automated system for designing throttle diagrams [12]. A set-theoretical concept was used to describe structures of the measuring diagrams of direct transduction. Such schematic diagrams can be designed using one, two or composite throttles connected in series, in parallel or in a bridge throttle diagram. Studies in construction of measuring transducers on the basis of feedback schemes with application of the theory of sets were continued in [13]. New dependences were obtained in this work for finding the number of the feedback schemes constructed on the basis of a simple tuple

and a simple row. New procedures for determining the number of feedback schemes realized on a composite tuple and a composite row were developed. A new procedure for determining the number of feedback schemes constructed based on random-order schemes was also developed. Examples of the synthesis of such schemes and realization on their basis of transducers for measuring dynamic viscosity of a Newtonian fluid and rheological parameters of a pseudo plastic fluid were given.

Thus, transducers based on a concrete method and on a concrete measuring scheme were used in the above-mentioned studies to measure fluid parameters. The transform functions and metrological characteristics of such transducers are being obtained in theoretical and experimental studies. Creation of transducers with other transform functions or characteristics brings about problems associated with conducting new studies with other measuring diagrams. In particular, various throttle diagrams are used in the transducers constructed based on a hydrodynamic measurement method. Usually, one measurement channel with one output signal (pressure, pressure differential or flow) is realized in a hydrodynamic throttle diagram to measure a certain parameter. Depending on the locations where the measuring channels are designed (inlets and outlets of the throttle elements), the diagram has various characteristics and transform functions. Among all throttle diagrams, a measuring diagram of a throttle matrix type in which there is a simultaneous serial and parallel connection of throttle elements deserves a particular attention. The presence of common inlets and outlets of throttle elements forms the basis for creation of hydrodynamic bridge measuring diagrams. Such features of the throttle matrix make it possible to realize many measuring channels in various locations of the schematic diagram. This predetermines simultaneous measurement of two, three or more physical and mechanical parameters and hydrodynamic characteristics of fluids in a single throttle matrix. However, the insufficiently developed mathematical description of the throttle matrix makes analysis of its functional properties impossible.

Therefore, the further development of the hydrodynamic method for a simultaneous measurement of several physical and mechanical parameters of non-Newtonian fluids with a scientifically grounded analysis of functional capabilities of the measuring diagrams based on throttle matrices is a promising idea.

3. The aim and objectives of the study

This study objective was to develop a mathematical description of the measuring schemes based on throttle matrices and its application for constructing mathematical models of multifunctional hydrodynamic measuring transducers of physical and mechanical parameters and characteristics of fluids.

To achieve this objective, the following tasks have to be addressed:

- develop a procedure of mathematical description of throttle matrices with the use of tools of the theory of graphs;
- obtain dependences for finding the number of measuring channels in a schematic diagram of a throttle matrix type and the number of possible variants of the measuring transducers constructed on the basis of such a diagram;

- construct a generalized mathematical model of a throttle matrix for analysis of functional capabilities with the use of transform functions for each of the measuring channels;
- elaborate the principles of synthesis of schemes of multifunctional measuring transducers based on a throttle matrix and apply them to construction of a multifunctional transducer for measuring rheological parameters of the non-Newtonian power law liquid.

4. Analyzing the graph theory tools for description of the measuring schematic diagrams

The graph theory is an effective tool for formalizing present-day scientific and applied problems. Its apparatus is sufficiently developed and convenient. Today, the graph theory is widely used in the practice of solving problems of automation, electronics, physics, chemistry, and the like. Layouts of roads, gas pipelines, heat and electricity supply nets are represented in a form of graphs. Let us apply the graph theory to modeling schemes of measuring transducers on the basis of throttle matrices using basic notions and symbols [14, 15].

Graphs are some sets of points in a plane connected by lines. The points are called vertices of the graph and the lines are its edges [14, 15]. Let us denote the set of vertices of the graph by symbol V and the set of edges by symbol E :

$$V = \{1, 2, \dots, n\},$$

$$E = \{(1,1), (1,2), \dots, (1,n), \dots, (i,j), \dots, (n,n)\}.$$

The number n of vertices in the graph is called the order of the graph and the number m of its edges is the size of the graph. For example, in Fig. 1, there is a graph $G(V,E)$ with four vertices and four edges for which $V = \{1, 2, 3, 4\}$, $E = \{(1,2), (2,3), (3,4), (1,4)\}$.

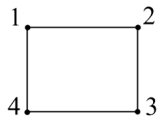


Fig. 1. Graph of the fourth order

The vertices connected by the edge are called adjacent vertices. The edge that connects the graph vertices is incidental to these vertices and the vertices themselves are incidental to the edges. A vertex incidental to only one edge is called a hung vertex and a vertex not incidental to any edge is an isolated vertex.

If the graph has many vertices and edges, then it loses visibility and it is difficult to work with. Then it is represented in a form of matrices (vertex matrices, edge matrices, incidence matrices) [14]. In particular, the vertex matrix M of the graph is a square matrix in which rows and columns correspond to the vertices of the graph and the order of the matrix coincides with the number of vertices. When filling the matrix, 1 is put at the intersection of the row and the column that correspond to the adjacent vertices and the rest of the cells are filled with zeros. For example, the vertex matrix M for the graph shown in Fig. 1 takes the form:

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}. \tag{1}$$

If all edges of the graph have no directions, that is, the edges $(i,j) = (j,i)$ are an unordered pair of vertices, then the graph is called non-oriented. A non-oriented graph is called simple if it has no loops and any pair of vertices is connected by only one edge. If a graph has no edges, it is called an empty graph and denoted by O_n . A simple graph is called complete if each pair of vertices is connected by an edge. A complete graph has C_n^2 edges [14]:

$$m = C_n^2 = \frac{n!}{2!(n-2)!}. \tag{2}$$

A graph with its edges being a subset of a complete graph and with a set of vertices coinciding with the set of vertices of the complete graph is called a partial graph. This means that not all pairs of vertices in a partial graph are connected by edges. The total number C of partial subgraphs of the graph of an m size is determined by the formula

$$C = \sum_{k=0}^m C_m^k = C_m^0 + C_m^1 + \dots + C_m^m = 2^m, \tag{3}$$

where C_m^k is the number of different partial graphs each having k ribs.

An important notion in the graph theory is a loaded graph in which each edge is put in correspondence with a certain number or a function called the edge weight.

5. Describing the measuring diagrams using the graph theory

Let us apply the graph theory to modeling measuring diagrams such as throttle matrices. The throttle matrix is a schematic diagram that contains throttle elements connected in parallel lines with the same number of the in-series connected throttle elements in each line. When applying the matrix terminology, such a diagram has the same number r of elements in each row and, accordingly, the same number p of elements in each column.

Fig. 2 shows a schematic diagram of a throttle matrix of a $p \times r$ size in which the connection points (diagram nodes) of the throttle elements are used to form measuring channels. For a throttle matrix which operates in a constant flow mode, the output signals are pressure differentials between the diagram nodes. Such nodes of the diagram numbered in a certain way are the graph vertices and the measuring channels are the graph edges. Since the output signal of the measuring channel is the pressure differential between various nodes of the diagram, we can describe the measuring channel by a straight edge connecting two graph vertices. In this case, we have a non-oriented simple graph. Consequently, graphs enable mathematical description of a set of nodes of the throttle matrix and a set of measuring channels. Taking into account functions of converting each measuring channel corresponding to the graph edges, we obtain a loaded graph which can serve as a basis for construction of mathematical models of measuring transducers.

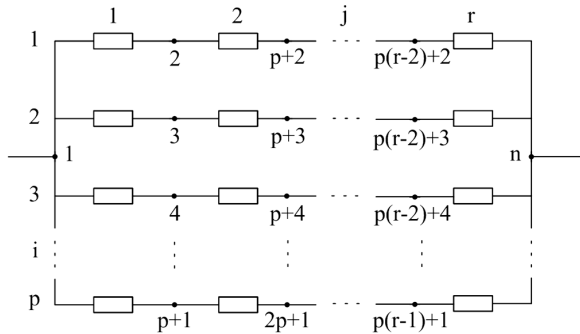


Fig. 2. The schematic diagram of a throttle matrix of a $p \times r$ size

Considering that the matrix size determines the number $p \times r$ of throttle elements in the diagram, the authors have proposed a formula for finding the number n of nodes in the diagram of the throttle matrix of a $p \times r$ size:

$$n = p \cdot (r - 1) + 2. \tag{4}$$

It is seen from formula (4) that throttle matrices of different sizes have different numbers of nodes. Fig. 3 shows schematic diagrams of the throttle matrices of various sizes with marked nodes.

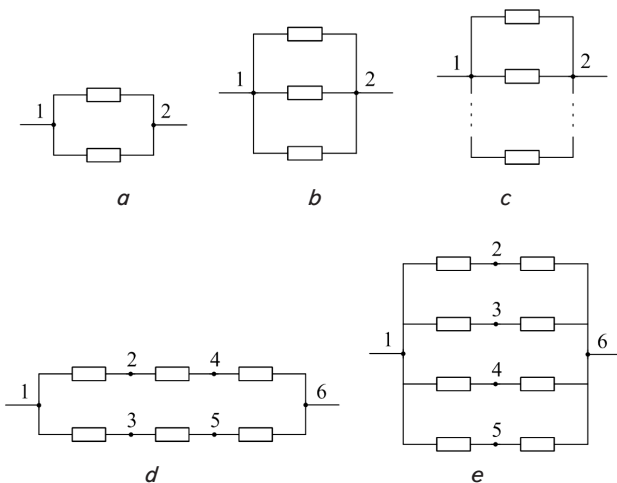


Fig. 3. Schematic diagrams of throttle matrices: 2×1 size (a); 3×1 size (b); $p \times 1$ size (c); 2×3 size (d); 4×2 size (e)

At first glance, the number of nodes in the diagram increases with an increase in the number of throttle elements. For example, the diagram in Fig. 3, b which contains 3 throttles has 2 nodes and the diagram in Fig. 3, d with 6 throttles has 6 nodes. At the same time, the number of nodes in the throttle matrices of 2×1 , 3×1 , ..., $p \times 1$ sizes which are actually a parallel connection of the elements is equal to 2 regardless of the number of throttle elements (Fig. 3, a–c). The matrices shown in Fig. 3, d, e are the throttling matrices of different sizes 2×3 and 4×2 with correspondingly different numbers (6 and 8) of throttling elements have the same number of nodes $n = 6$. Proceeding from formula (4), we can conclude that the number of nodes in the throttle matrices with the same number of elements will be larger for the diagrams with a larger number r of columns.

Next, to analyze the throttle matrix, let us apply such concepts of the graph theory as empty and complete graphs.

An empty graph describes a throttle matrix with no measuring channels and a complete graph completely covers all possible measuring channels that can be constructed based on a given schematic diagram. Therefore, the quantity m of all measuring channels that can be formed on a throttle matrix of a given size, taking into account (2), is determined by the formula:

$$m = \frac{(n-1) \cdot n}{2}. \tag{5}$$

Usually, when measuring physical and mechanical parameters of fluid using a throttle matrix, all measuring channels are not involved simultaneously. Such separate variants of construction of measuring transducers with one, two or more measuring channels should be described by partial graphs. The set of all partial graphs of a complete graph covers all possible variants of construction of the measuring transducers based on a throttle matrix of a certain size and differing by patterns of their measuring channels.

A schematic diagram with no measuring channels is described by an empty graph, that is, the number of edges $k = 0$. With this in mind, based on formula (3), we obtain a dependence for finding the number n_{VP} of all variants of construction of measuring transducers on a throttle matrix of a certain size with various measuring channels:

$$n_{VP} = 2^m - 1. \tag{6}$$

Let us consider some diagrams of throttle matrices of different sizes and their equivalent graphs shown in Fig. 4. For example, in accordance with formulas (4) and (5), the throttle matrix of a 2×2 size (Fig. 4, a) has 4 nodes in which 6 different measuring channels can be formed (the graph shown in Fig. 4, b). The throttle matrix of a 3×2 size (Fig. 4, c) has 5 nodes in which 10 different measuring channels can be realized (the graph shown in Fig. 4, d). The throttle matrix of a 2×3 size (Fig. 4, e) has 6 nodes and 15 different measuring channels (the graph shown in Fig. 4, f). The throttle matrix of a 2×4 size (Fig. 4, g) has 8 nodes and 28 different measuring channels (the graph shown in Fig. 4, h).

As seen from Fig. 4, an increase in the number of nodes in the diagram results in an increase in the number of measuring channels. This is a prerequisite for forming of a large number of measuring transducers of physical and mechanical parameters of fluids based on a throttle matrix of a certain size.

For an example, let us analyze a rather simple throttle matrix of a 3×2 size (Fig. 4, c). Using formulas (4), (5), calculate the number n of nodes in the diagram and the number m of all measuring channels that can be formed in the diagram:

$$n = 3 \cdot (2 - 1) + 2 = 5;$$

$$m = \frac{(5 - 1) \cdot 5}{2} = 10.$$

Accordingly, the number n_{VP} of all variants of construction of measuring transducers with a variety of combinations of measuring channels is determined by formula (6):

$$n_{VP} = 2^{10} - 1 = 1023.$$

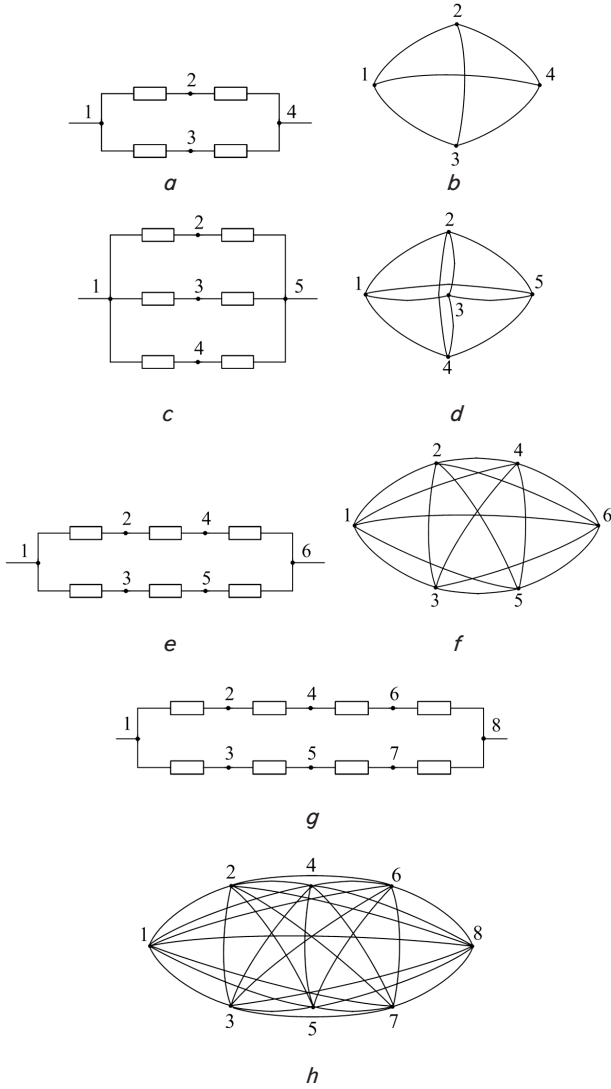


Fig. 4. Schematic diagrams of throttle matrices and their equivalent graphs: a diagram of a throttle matrix of a 2×2 size (a); a graph of the 4th order (b); a diagram of a throttle matrix of a 3×2 size (c); a graph of the 5th order (d); a diagram of a throttle matrix of a 2×3 size (e); a graph of the 6th order (f); a diagram of a throttle matrix of a 2×4 size (g); a graph of the 8th order (h)

The set of all measuring channels of the throttle matrix of Fig. 4, c is described by the graph of Fig. 4, d with a set of vertices (the diagram nodes) $V=\{1, 2, 3, 4, 5\}$ and a set of edges (measuring channels) $E=\{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$.

The vertex matrix of this graph takes the form:

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}. \quad (7)$$

By applying software realization of an edge matrix or a vertex matrix, one can obtain all possible measuring channels in the diagram and synthesize new measuring transducers based on the throttle matrices of given sizes. Thus, rep-

resentation of the measuring diagrams of a throttle-matrix type in a form of graphs is an effective means of their study.

6. Synthesizing schemes of measuring transducers based on throttle matrices with application of the graph theory

6.1. Developing the mathematical model of the throttle matrix

Throttle matrices feature their ability to perform certain mathematical operations with certain physical quantities. For example, in a schematic diagram composed of in-series connected throttle elements, the total pressure differential is equal to the sum of the pressure differentials at each element. In a diagram consisting of in-parallel connected throttle elements, the total flow of liquid through the diagram is equal to the sum of flows through each element, etc.

As noted above, the output signals of the throttle matrix are the pressure differentials arising between the diagram nodes during fluid flow through the throttle elements. The medium whose parameters must be measured is supplied to the throttle elements connected by a common inlet and therefore passes through the throttle elements to the common outlet. Let us denote the pressure differential on the throttle element of the i -th row and the j -th column of the matrix by ΔP_{ij} , and the pressure at the node in which the throttle element of the i -th row and the j -th column is connected with the next element by P_{ij} (Fig. 5)

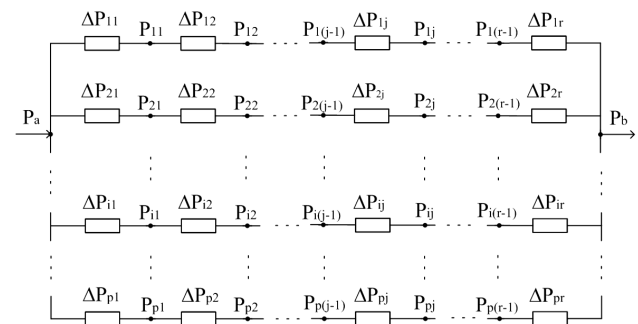


Fig. 5. Denotation of the pressure differentials at the throttle elements and of the pressures at the nodes of a throttle matrix of a $p \times r$ size

Applying the laws of fluid motion in branched hydraulic diagrams, the following can be written for the throttle matrix:

$$P_{ij} = P_a - \sum_{l=1}^j \Delta P_{il}; \quad (8)$$

$$P_{ij} = P_b + \sum_{l=j+1}^r \Delta P_{il}, \quad (9)$$

where, in addition to the known denotations, P_a is the pressure at the inlet to the throttle matrix (at the node which is a common inlet of the throttle elements, i.e. the inlet node of the throttle matrix); P_b is the pressure at the outlet of the throttle matrix (in the node connecting the outlets of the throttle elements, i.e. the node of the throttle matrix outlet); $i=1, 2, \dots, p$ is the number of rows; $j=1, 2, \dots, r$ is the number of throttle elements in a row.

Equation (8) binds pressures P_{ij} with the inlet pressure P_a and pressure differentials at the throttle elements in front of the corresponding node of the diagram in the direction of movement of the measured medium. Equation (9) binds pressures P_{ij} with the outlet pressure P_b and the pressure differentials at the throttle elements located after the corresponding diagram node in a direction of movement of the measured medium.

The following is found from equations (8) and (9):

$$P_{ij} = \frac{P_a + P_b}{2} - \frac{1}{2} \sum_{l=1}^j \Delta P_{il} + \frac{1}{2} \sum_{l=j+1}^r \Delta P_{il}. \tag{10}$$

By analogy, we have the following for the k -th row and the s -th column of the throttle matrix:

$$P_{ks} = \frac{P_a + P_b}{2} - \frac{1}{2} \sum_{l=1}^s \Delta P_{kl} + \frac{1}{2} \sum_{l=s+1}^r \Delta P_{kl}, \tag{11}$$

where $k=1, 2, \dots, p$ is the number of rows; $s=1, 2, \dots, r$ is the number of throttle elements in a row.

Pressure differentials between any two nodes except the nodes of the inlet and outlet of the throttle matrix are found from equations (10) and (11):

$$\begin{aligned} \Delta P_{ijks} &= P_{ij} - P_{ks} = \\ &= \frac{1}{2} \left(\sum_{l=1}^s \Delta P_{kl} - \sum_{l=s+1}^r \Delta P_{kl} - \sum_{l=1}^j \Delta P_{il} + \sum_{l=j+1}^r \Delta P_{il} \right). \end{aligned} \tag{12}$$

Pressure differential between the node of the throttle matrix inlet and any node except the outlet node is found using equation (8):

$$P_{aj} = P_a - P_{ij} = \sum_{l=1}^j \Delta P_{il}, \tag{13}$$

and the pressure differential between any node of the throttle matrix (except the inlet node) and the outlet node is found using equation (9):

$$P_{ib} = P_{ij} - P_b = \sum_{l=j+1}^r \Delta P_{il}. \tag{14}$$

Pressure differential between the inlet and outlet nodes of the throttle matrix is found by the formula

$$P_{ab} = P_a - P_b. \tag{15}$$

The obtained equations (12)–(15) constitute a generalized mathematical model of the throttle matrix which enables determination of a transform function for each of the measuring channels.

6. 2. Constructing a measuring transducer of rheological parameters of a non-Newtonian power law fluid on a basis of the throttle matrix

For an example, let us analyze functionality of a throttle matrix of a 2×4 size (Fig. 4, g) for constructing a multi-functional measuring transducer of rheological parameters of the non-Newtonian power law fluid. Figure 6a shows scheme of such a transducer in which sensing elements are cylindrical tubes with two different diameters. The tubes of each diameter have two different lengths.

All four types of throttle elements are placed in each line differing only in the order of connection. Tubes of diameter D_1 and length $L_{11}=L_{24}$ are placed between nodes 1-2 and 7-8 and tubes of diameter D_1 and length $L_{13}=L_{21}$ are placed between nodes 1-3 and 4-6. Tubes of diameter D_2 and length $L_{12}=L_{23}$ are placed between nodes 2-4 and 5-7 and tubes of diameter D_2 and length $L_{14}=L_{22}$ are placed between nodes 6-8 and 3-5. The measuring transducer operates in a constant flow mode. Since each line of the scheme includes two tubes of each diameter and the connection is structurally the same, hydraulic resistance of each line will be the same. Consequently, flow of the controlled fluid through each throttle element will be the same. The output signals of the transducer are pressure differentials ΔP_{1121} , ΔP_{1122} , ΔP_{1222} and ΔP_{1323} between the corresponding the scheme nodes 2-3, 2-5, 4-5 and 6-7. The output signals are represented by an equivalent partial graph shown in Fig. 6, b. The graph contains 8 vertices of which vertices 1 and 8 are isolated and four loaded edges incidental to vertices 2...7 with a corresponding weight, namely with the transform function for each measuring channel:

$$\Delta P_{ijks} = f(X_p, X_k, F_0), \tag{16}$$

where X_p is the set of rheological parameters of the fluid; X_k is a set of design characteristics of the throttle elements; F_0 is the total flow of the fluid supplied to the throttle scheme.

Pressure differentials in the selected nodes of the scheme are measured by differential pressure transducers 1 and processed in a computing device 2.

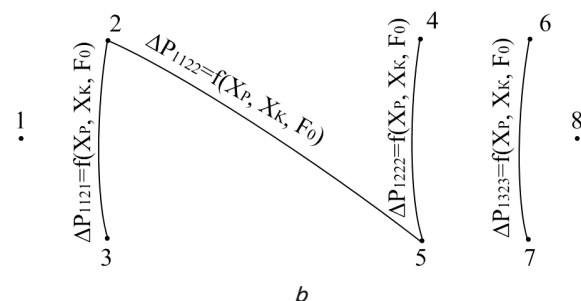
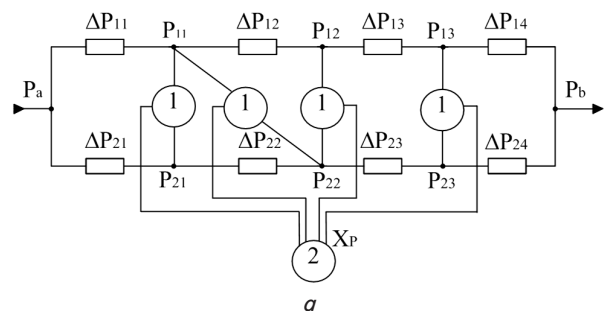


Fig. 6. The measuring transducer of rheological parameters of the non-Newtonian power law liquid: scheme of the measuring transducer constructed on a throttle matrix of a 2×4 size (a); the equivalent graph of the 8th order and the 4th size (b)

Taking into account the chosen structural characteristics of the sensing elements and connections in the scheme, the pressure differentials at the throttle elements of the same type (Fig. 6, a) will be equal to:

$$\begin{aligned}\Delta P_{11} &= \Delta P_{24}; \quad \Delta P_{12} = \Delta P_{23}; \\ \Delta P_{13} &= \Delta P_{21}; \quad \Delta P_{14} = \Delta P_{22}.\end{aligned}\quad (17)$$

The pressure differential arising during the laminar motion of the power law fluid in each tube is determined by equation [16]

$$\Delta P_{ij} = K \left(\frac{3n+1}{4n} \right)^n \left(\frac{32F}{\pi D_{ij}^3} \right)^n \frac{4L_{ij}}{D} + \Delta P_{kij}, \quad (18)$$

where $i=1, 2$ is the row number; $j=1, 2, \dots, 4$ is the column number; K is consistency coefficient; n is flow index; D_{ij} , L_{ij} are internal diameter and length of the tube; $F=F_0/2$ if fluid flow in the tube; ΔP_{kij} is the pressure differential due to the inlet effects of the tube.

Since the flow rate F of the fluid in the tubes is the same, the pressure differentials ΔP_{kij} resulting from the inlet effects of the tubes of the same internal diameter will also be the same [17]:

$$\begin{aligned}\Delta P_{k11} &= \Delta P_{k13} = \Delta P_{k21} = \Delta P_{k24}, \\ \Delta P_{k12} &= \Delta P_{k14} = \Delta P_{k22} = \Delta P_{k23}.\end{aligned}$$

Let us write the pressure differentials measured by the differential pressure meters connected as shown in the diagram of the measuring transducer (Fig. 6, *a*). Using equation (12) and taking into account equations (17), determine pressure differentials between nodes 2 and 3:

$$\Delta P_{1121} = P_{11} - P_{21} = \Delta P_{21} - \Delta P_{24}, \quad (19)$$

between nodes 2 and 5:

$$\Delta P_{1122} = P_{11} - P_{22} = \Delta P_{21} - \Delta P_{24} + \Delta P_{22}, \quad (20)$$

between nodes 4 and 5:

$$\Delta P_{1222} = P_{12} - P_{22} = \Delta P_{21} - \Delta P_{24} + \Delta P_{22} - \Delta P_{23}, \quad (21)$$

between nodes 6 and 7:

$$\Delta P_{1323} = P_{13} - P_{23} = \Delta P_{22} - \Delta P_{24}. \quad (22)$$

Substitution of the corresponding pressure differentials ΔP_{ij} from equation (18) into equations (19)–(22) produces:

$$\Delta P_{1121} = A \cdot \frac{(L_{21} - L_{24})}{D_1^{3n+1}}, \quad (23)$$

$$\Delta P_{1122} = A \cdot \left(\frac{L_{21} - L_{24}}{D_1^{3n+1}} + \frac{L_{22}}{D_2^{3n+1}} \right) + \Delta P_{k2}, \quad (24)$$

$$\Delta P_{1222} = A \cdot \left(\frac{L_{21} - L_{24}}{D_1^{3n+1}} + \frac{L_{22} - L_{23}}{D_2^{3n+1}} \right), \quad (25)$$

$$\Delta P_{1323} = A \cdot \left(\frac{L_{22}}{D_2^{3n+1}} - \frac{L_{24}}{D_1^{3n+1}} \right) + \Delta P_{k2} - \Delta P_{k1}, \quad (26)$$

where

$$A = 4K \left(\frac{3n+1}{4n} \right)^n \left(\frac{32F}{\pi} \right)^n.$$

Analysis of equations (23)–(26) shows that all pressure differentials from the chosen nodes depend on rheological parameters K and n of the controlled liquid and design characteristics of the tubes. The pressure differential ΔP_{1122} contains uncompensated pressure losses ΔP_{k2} from the inlet effects of the tube of diameter D_2 and the pressure differential ΔP_{1323} contains difference between pressure losses ΔP_{k2} and ΔP_{k1} from the inlet effects of the tubes of diameters D_2 and D_1 .

Directly from the pressure differential ΔP_{1121} , it is easy to determine apparent viscosity at an apparent shear rate $\Gamma_1 = 32F / (\pi D_1^3)$ using equation (23):

$$\mu_{a1} = \frac{\tau_{w1}}{\Gamma_1} = K \left(\frac{3n+1}{4n} \right)^n \Gamma_1^{n-1}, \quad (27)$$

where $\tau_{w1} = \Delta P_{1121} \cdot D_1 / (4 \cdot (L_{21} - L_{24}))$ is the tangential shear stress on the wall of the tubes of diameter D_1 .

The apparent fluid viscosity in the tubes of diameter D_2 is determined from equations (23) and (25) by the formula

$$\mu_{a2} = \frac{\tau_{w2}}{\Gamma_2} = K \left(\frac{3n+1}{4n} \right)^n \Gamma_2^{n-1}, \quad (28)$$

where $\tau_{w2} = (\Delta P_{1222} - \Delta P_{1121}) / (4 \cdot (L_{22} - L_{23}))$ is the tangential shear stress on the wall of the tubes of diameter D_2 ; $\Gamma_2 = 32F / (\pi D_2^3)$ is the apparent rate of shear in the tubes of diameter D_2 .

Using the pressure differentials described by equations (23) and (25), the flow index n is determined as

$$n = \frac{1}{3} \ln \left(\frac{\Delta P_{1121}}{\Delta P_{1222} - \Delta P_{1121}} \cdot \frac{L_{22} - L_{23}}{L_{21} - L_{24}} \cdot \frac{D_1}{D_2} \right) / \ln \left(\frac{D_2}{D_1} \right) \quad (29)$$

and the consistency coefficient K of controlled fluid

$$K = \tau_{w1} \left(\frac{4n}{3n+1} \cdot \frac{1}{\Gamma_1} \right)^n. \quad (30)$$

Given the values of parameters K and n and design characteristics of the tubes, it is easy to determine pressure losses ΔP_{k2} and ΔP_{k1} resulting from the inlet effects of tubes of diameters D_2 and D_1 using equations (24) and (26).

Note that through the use of the measured pressure differentials at the nodes of the throttle matrix, values of the effective viscosity μ_{ef1} and μ_{ef2} can also be calculated at various shear rates which are related to the values of the apparent viscosity of the power law liquid by the following dependences [18]:

$$\mu_{ef1} = \mu_{a1} \left(\frac{4n}{3n+1} \right), \quad \mu_{ef2} = \mu_{a2} \left(\frac{4n}{3n+1} \right). \quad (31)$$

Simultaneous measurement of rheological parameters μ_{a1} , μ_{a2} , μ_{ef1} , μ_{ef2} , K , n and the inlet effects of the tubes by formulas (27)–(31) is made by the computing device 2 in which the measured pressure differentials are processed (Fig. 6, *a*).

Thus, four measuring channels were used in the scheme of the considered measuring transducer of rheological parameters and hydrodynamic characteristics of the power law

fluid. The channels were chosen in such a way as to provide automatic compensation for the method error occurring in measuring rheological parameters resulting from the inlet effects of the sensing elements.

The methodology of constructing multifunctional measuring transducers using the graph theory enables solution of the problem of measuring several parameters on a single throttle matrix in different ways. So, other throttling elements, other locations and other measuring channels can be used in the same throttle matrix of a 2×4 size since such a matrix has 28 channels (Fig. 4, *h*).

7. Discussing the results obtained in the synthesis of multifunctional hydrodynamic measuring transducers based on throttle matrices

The conducted studies have shown that the throttle matrix is a universal measuring scheme for constructing multifunctional hydrodynamic measuring transducers of fluid parameters. With the help of the graph theory and combinatorics tools, the problem of determining all possible variants of constructing measuring transducers based on the diagrams of the throttle-matrix type with various measuring channels was solved. In particular, a procedure for mathematical description of throttle matrices with application of the graph theory was developed. According to this procedure, locations of connection of throttle elements in the diagram, the so-called nodes of the diagram, are described as the graph vertices and the measuring channels are described as the graph edges. The measuring channel of the scheme is described by a directionless edge which connects two different vertices of the graph. In this case, a non-oriented simple graph is obtained. The complete graph fully covers all possible measuring channels that can be built on the basis of the given scheme. Since not all measuring channels are simultaneously involved in a concrete implementation of the measuring transducer, they should be described with the help of partial graphs. Taking to consideration the transform functions of each measuring channel that correspond to the edges of the graph, we obtain a loaded graph on the basis of which mathematical models of measuring transducers can be constructed.

Applying the apparatus of combinatorics, a formula for finding the number of nodes in a throttle matrix of a certain size and a formula for calculating the number of all measuring channels that can be formed in such scheme were proposed. Also, a dependence was obtained for finding the number of all variants of construction of measuring transducers on a throttling matrix of a certain size with various measuring channels.

Finally, a procedure for describing throttle matrices with application of the graph theory allows us to obtain all possible schemes with various measuring channels based on a throttle matrix of a certain size. With such schemes in possession, it is possible to create multifunctional measurement transducers of specified parameters for a concrete manufacturing process. It is clear that the developed procedure can be applied to constructing transducers of parameters of both liquid and gaseous media.

Principles of synthesis of the schemes of multifunctional hydrodynamic measuring transducers based on throttle matrices were developed. Their essence consists in the follows:

1. According to the problem of measuring the fluid parameters, a throttle matrix of a certain size is chosen. In

doing this, the number of parameters, the fluid properties, requirements to accuracy and sensitivity of the transducer, etc. are taken into account.

2. By applying the developed procedure, the throttle matrix is represented as a loaded graph with transform functions for each of the measuring channels. Based on the obtained dependences (4)–(6), the number of nodes in the throttle matrix and the number of all measuring channels that can be formed in the chosen scheme are calculated.

3. Analyzing the transform functions of the loaded graph, measuring channels of the throttle matrix are chosen and a scheme of the multifunctional measuring transducer of the specified parameters of the fluid is synthesized.

4. Based on a generalized mathematical model (12)–(15) of the throttle matrix and the loaded graph with transform functions for individual measuring channels, a model of the measuring transducer of the specified parameters of fluid is obtained.

The developed synthesis principles make it possible to construct such a variant of a multifunctional measuring transducer of the specified physical and mechanical parameters that will meet the requirements to its accuracy or sensitivity characteristics. It should also be noted that application of various types of throttle elements in the throttle diagram whose consumption characteristics depend in different ways on the fluid parameters considerably extends functionality of the hydrodynamic transducers.

In this work, an example is given concerning implementation of a multifunctional measuring transducer of rheological parameters of a power law fluid on a basis of a throttle matrix of a 2×4 size whose sensing elements are cylindrical tubes of a round cross-section with different design characteristics. Applying the developed procedure of mathematical description of the throttle matrices and the principles of synthesis of the measuring transducer schemes, a scheme (Fig. 6, *a*) and a mathematical model of such transducer were worked out. It is worth noting that rheological models of non-Newtonian fluids are quite varied and, accordingly, the models of transducers of their rheological parameters created according to the same scheme will be substantially different.

When analyzing adequacy of the mathematical model of the transducer of rheological parameters of power law fluids which was obtained in the example, we can note that experimental verification of the model is a complex practical problem. In this study, the model was constructed in two stages. First, a general model of a throttle matrix of a 2×4 size was created with the help of formulas (12)–(15) and then a mathematical model described by equations (27)–(31) was created. So, formally, both models should be checked. However, adequacy of the matrix model (12)–(15) which establishes pressure differentials between all diagram nodes can be assessed provided that the data on the types of the throttle elements in the diagram, their connections and design characteristics, properties of the investigated medium and total consumption are known. That is, adequacy of the model of hydrodynamic transducer of concrete parameters should be checked.

The model (27)–(31) of the transducer of power law fluid parameters was constructed on the basis of the equations of laminar motion of power law fluids in circular cylindrical tubes (capillaries) of a finite length which were derived and confirmed experimentally by many authors [18]. In practice, verification of adequacy of the model of the measuring

transducer of parameters of non-Newtonian media is carried out using control Newtonian fluids. The values of dynamic viscosity of these fluids should cover the range of variation of the consistency coefficient. Such a procedure makes it possible to estimate adequacy of the equations of the model only if the flow index $n=1$. Another way is to parametrically identify the rheological model of the controlled fluid in a required range of shear rate with the use of high-precision laboratory rheometers. Both methods for estimating adequacy of the transducer model require additional studies with multiple experiments.

Working out the mathematical description of throttle matrices is a continuation of the work on modeling and designing schemes of hydrogasdynamic measuring transducers of the fluid parameters having expanded functionality and improved metrological characteristics.

The obtained results will enable algorithmizing of the process for synthesizing measuring transducer schemes and can be the subject of further studies. The next task consists in a creation of a system for automated design of multifunctional measuring transducers of physical and mechanical parameters and hydrogasdynamic characteristics of fluids.

8. Conclusions

1. A procedure for mathematical description of throttle matrices in a form of graphs was developed which makes it possible to get all possible schemes with various measuring channels on the basis of a throttle matrix of a certain size. Dependences were obtained for finding the number of nodes, the number of measuring channels in the throttle matrix and

the number of possible variants of the measuring transducers constructed on the basis of such a scheme.

2. Based on the laws of fluid motion in branched schemes, a generalized mathematical model of a throttle matrix of an arbitrary size was constructed. This model is a set of equations describing the pressure differentials occurring between any two nodes of the throttle matrix during fluid motion in the sensing elements. In particular, the model equations determine the pressure differentials occurring between two arbitrary nodes in the matrix, between the inlet node and the arbitrary node, between the arbitrary node and the outlet node and between the inlet and outlet nodes of the scheme. The developed mathematical model of the throttle matrix enables determination of the transform function for each of the measuring channels. Thus, it can be used to analyze functionality of the measuring transducer scheme.

3. According to the proposed principles, a throttle matrix of a certain size is chosen taking into account the number of the measured parameters, the fluid properties, requirements to accuracy or sensitivity of the transducer. The throttle matrix is represented as a loaded graph with transform functions for each of the measuring channels. On the basis of the loaded graph and the generalized mathematical model of the matrix, a scheme and a mathematical model of the multifunctional measuring transducer of specified parameters of the fluid are synthesized.

4. The model was developed in a form of a graph and the scheme and mathematical model of a multifunctional measuring transducer of rheological parameters of a power law liquid was developed based on a throttle matrix of a 2×4 size.

The obtained results enable construction of measuring transducers of one or several parameters with specified functionality and characteristics.

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RESEARCH INTO EFFECT OF ELECTROCHEMICAL ETCHING CONDITIONS ON THE MORPHOLOGY OF POROUS GALLIUM ARSENIDE

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Проведено удосконалення способу формування поруватого арсеніду галію у розчині соляної кислоти. Досліджено основні закономірності формування поруватих просторів. Показано, що морфологічні властивості пор-GaAs залежать від умов травлення. Досліджено вплив щільності струму, часу травлення та складу електроліту на поверхневу поруватість, товщину поруватого шару та діаметр пор

Ключові слова: арсенід галію, електрохімічне травлення, морфологія, поруваті напівпровідники, умови травлення

Усовершенствован способ формирования пористого арсенида галлия в растворе соляной кислоты. Исследованы основные закономерности формирования пористых пространств. Показано, что морфологические свойства пор-GaAs зависят от условий травления. Исследовано влияние плотности тока, времени травления и состава электролита на поверхностную пористость, толщину пористого слоя и диаметр пор

Ключевые слова: арсенид галлия, электрохимическое травление, морфология, пористые полупроводники, условия травления

1. Introduction

Nanostructured materials are of considerable interest to researchers due to their unusual properties compared

with volumetric or thin-film analogs [1, 2]. Many of these properties offer prospects for the application of nanostructures of materials for electronics, photonics, power industry, etc. [3, 4].