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GEOMETRICAL MODELING OF THE INERTIAL UNFOLDING OF A MULTI-LINK PENDULUM IN WEIGHTLESSNESS

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Досліджено геометричну модель розкриття каркасу орбітального об'єкта як процесу коливання багатоланкового маятника в умовах невагомості. Коливання виникають завдяки впливу імпульсу реактивного двигуна на прикінцевий вузол елементів маятника. Опис інерційного розкриття маятника виконано за допомогою рівняння Лагранжа другого роду. Результати доцільно використати при проектуванні розкриття великогабаритних конструкцій в умовах невагомості, наприклад, каркасів для сонячних дзеркал

Ключові слова: багатоланковий маятник, великомасштабна конструкція, розкриття у космосі, дзеркало у космосі, рівняння Лагранжа другого роду

Исследована геометрическая модель раскрытия каркаса орбитального объекта как процесса колебания многозвенного маятника в условиях невесомости. Колебания возникают благодаря воздействию импульса реактивного двигателя на конечный узел элементов маятника. Описание инерционного раскрытия маятника выполнено с помощью уравнения Лагранжа второго рода. Результаты целесообразно использовать при проектировании раскрытия крупногабаритных конструкций в условиях невесомости, например, каркасов для солнечных зеркал

Ключевые слова: многозвенный маятник, крупномасштабная конструкция, развертывание в космосе, зеркало в космосе, уравнение Лагранжа второго рода

1. Introduction

Development of space technologies in the leading countries of the world will contribute to the creation of large-sized structures [1] since in order to use the almost endless supplies of solar energy it is expedient to employ powerful solar power plants, solar concentrators and space-based mirrors. One of the promising directions in the development of large extraterrestrial structures is related to the creation of mirrors to illuminate regions of the Earth at night with the sunlight reflected from the orbit. Lighting of settlements at night from space will make it possible to free up electrical power used for lighting the streets and buildings, as well as utilizing it for other purposes without additional construction of new power plants.

The founder of space solar power generation pointed out [2] that the cost-effective use of space structures necessitates creation of usable areas of mirrors of the order of a few hectares. For example, in order to lighten at night one city by the reflected sun rays, it is required to have a mirror with an area exceeding 12 square kilometers. Hectares are also used to measure the area of a so-called solar sail, which is considered to be a possible engine in the future space flights. One should also take into consideration the projects for creating large-scale space structures and solar panels of different designs [3].

It is clear that large-size structures should be transported into orbit folded, to subsequently unfold in order to regain the shape planned by designers. Controlling the unfolding of large-size structures in space is a complex scientific-technical task of mechanics, which has no analogues in the ground-based equipment.

The creation of large-sized structures that are transformed in space is linked to solving several problems in engineering and mechanics predetermined by the unique objects. Their characteristic feature is the combination of conflicting requirements regarding a significant increase in the dimensions and ensuring sufficient rigidity given a rather limited mass of the framework. Such structures typically take the form of a combination of rods (frames) that is transformed, with a special fabric "stretched" onto them, which actually forms the reflective surface. Location of the rods can be absolutely different, but when it comes to the flat surface of the mirror, then the arrangements take simple geometrical shapes – triangles, quadrangles, hexagons, etc.

In 1788, Lagrange applied a variational principle for estimating mechanical structures taking into consideration kinematic connections, employing the concepts of kinetic and potential energy of a mechanical system. As a result, Lagrange derived a universal approach to describe the motion of any mechanical system in the form of equations of motion, known as the Lagrange equations of the second kind. In paper [4], authors explored the possibility of applying the Lagrange equations of the second kind under condition of weightlessness (that is, in the absence of gravity force) and, as a result, "zero" potential energy of a mechanical system. Of interest, therefore, is the issue of the implementation of this approach in practice when estimating a technology for the unfolding of cosmic structures in the form of a multilink pendulum.

Thus, it will be a relevant task to study a technique for unfolding the large-size structures in the weightlessness, the basis of which will be the rods, connected like a multilink pendulum. The set of rods is to be delivered to the orbit in the folded form (a cartridge), with subsequent operation of unfolding the rods to regain their operational design. It is proposed to perform the specified operation using an inertial technique, applying to the calculation of rod structures the Lagrangian dynamics of multilink pendulums.

2. Literature review and problem statement

In order to substantiate the choice of design parameters of the elements of an unfolding system and to confirm reliability of this process, it is required to carry out a detailed mathematical modeling employing an effective mathematical model. To build the equations of motion and the solutions, one can apply a variety of methods with some of them outlined in paper [5]. These circuits, however, utilize elements with springs, which is why they have constraints for the size of unfolding units.

Article [6] addressed applying a method of separate bodies for modeling multi-element movable structures in spacecraft designs. The method was modified for a flat system of links. Using a mechanical system for unfolding the rods of solar panels as an example, the authors constructed matrices of kinematic relations required for using the method and which determine the kinematics of relative motion of the adjacent bodies of the system. The joints that connect the adjacent rod are modeled taking into consideration elastic properties at the point of fixation. The presence of elastic elements, however, complicates the implementation of such a scheme in the case of inertial unfolding of the structure.

When estimating the systems for unfolding the structures of the multilink pendulum type, the questions arise on choosing an engine, which would ensure the required resulting arrangement of its links. In practice, more common are the framed rope unfolding systems.

Paper [7] proposed an approach to describe the dynamics of solar panels in the process of their unfolding taking into consideration elastic properties of the elements. The authors describe mathematical models for unfolding mechanisms and rope synchronization. This issue is also tackled in article [8], which describes mathematical models for unfolding mechanisms, rope synchronization, braking and fixing the panels. The authors determine integral dynamic characteristics and the characteristics of loading the elements of a solar cell. The specified studies, however, do not consider an inertial technique for unfolding large-size solar panels, with preference given to the rope synchronization.

Paper [9] presents a mathematical model for the process of unfolding a multilink framework design of a solar battery with the rope system of synchronization. Fig. 1 shows a schematic of unfolding the structure, in which using the electric motors and ropes enables synchronization of change in the magnitudes of angles between adjacent links.

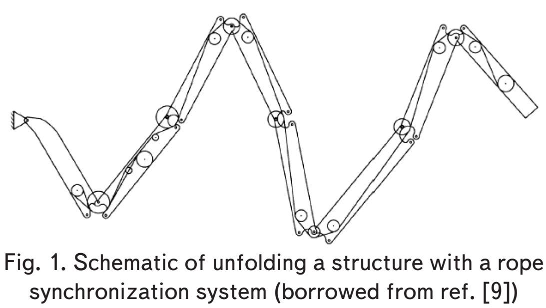

Fig. 1. Schematic of unfolding a structure with a rope synchronization system (borrowed from ref. [9])

In this case, based on an analysis of the kinematic scheme of an unfolding system, it is required to choose the size of rollers' radii and the transfer ratio of two types of gear mechanisms that enable the preset sequence of fixing the links. To study the process of unfolding a solar panel, the Lagrange equation of the second kind is employed.

In addition to the framed rope system for unfolding the structures of the multilink pendulum type, it is worthwhile considering a technique for enabling the required ultimate arrangement of its links through a limited momentum, applied to the first link. In paper [10], authors derived a governing law for a multilink pendulum on the plane in the vicinity of the assigned position of equilibrium in the form of a feedback that makes it possible to set the pendulum over a finite period to the position of equilibrium via a limited momentum, applied to the first link. The efficiency of the resulting governing law based on the Lyapunov function is demonstrated by means of computer simulation of the dynamics of a three-link pendulum. In contrast to a flat pendulum, driven by the scalar momentum, paper [11] represents control over a pendulum with two-stage hinges in the form of a two-dimensional vector. This circumstance required a modification in the algorithm for constructing the control. Control is implemented in the form of a feedback that sets the pendulum from the edge of an arbitrary equilibrium position to a given position through a limited momentum, applied to the first link. The authors substantiated applicability of the resulting governing law for a nonlinear multilink pendulum. These studies, however, are focused mainly on controlling the equilibrium of the inverse pendulum on a cart in the field of gravity.

When designing circuits for unfolding the multilink structures, it is required to construct mathematical models that adequately describe dynamic properties. The application of the models at the design stage makes it possible to estimate parameters of functioning of the structure. In order to run a numerical analysis of the unfolding process of structures that are transformed, the possibilities of modern packages for modeling the dynamics of mechanical systems are utilized. Paper [12] examines a method for calculating the large-sized unfolding structures using the MSC.Software programming complexes. Article [13] gives an example of the estimation of unfolding by using the complex of automated dynamic analysis of multicomponent mechanical systems EULER. The specified software, however, are not designed, without appropriate add-ins, to implement the inertial technique of unfolding of multilink structures. Other variants of unfolding systems are reported in the review of scientific literature [14]. It lacks, however, any information on the inertial technique of unfolding the multilink pendulums with preference given to rope unfolding systems. Paper [15] describes an actual example of a simulation of the process of unfolding of solar batteries of the satellite "Yamal-200".

Thus, the prototype of the considered technique for unfolding of the multilink rod structure is the rope system of unfolding. A review of the scientific literature that we performed revealed that the existing circuits for unfolding the rope systems are too complex to implement in the case of large size of the links (of the order of tens of meters). This conclusion is based on the necessity of synchronization and switching the motors to adjust the magnitudes of angles in the units of structures in order to provide a multilink structure with calculated geometrical shape, which is a separate task.

A review of the scientific literature allowed us to identify the issues that are not yet studied by other authors, which made it possible to formulate the following subject of research. In order to implement the idea of unfolding the multilink large-sized structures in the weightlessness, it is required to explore the inertial system in which the initiation of oscillations is driven by a pulse from a jet engine that must influence one of the nodal elements of the structure. We should also examine the issue of fixing a multilink structure when unfolded. In this case, the presence of extended links of the structure should not fundamentally affect the generality of implementation of the inertial technology of unfolding.

3. The aim and objectives of the study

The aim of present work is to construct a geometrical model of the process of unfolding in weightlessness of a multilink construction with the inertial system of unfolding, provided that the structure is identified as a multilink pendulum in the imaginary plane. This will make it possible to implement the unfolding of a structure using only one jet engine mounted on the final mode of the pendulum.

To achieve the set aim, the following tasks have been solved:

– to construct and solve a system of the Lagrange differential equations of the second kind in order to describe oscillations in weightlessness of a four-link (as an example) pendulum;

– to design a circuit for the initiation of oscillations through the influence of a pulse on one of the nodal elements of a pendulum (a model of pulse jet engine);

– to propose and explore a technique for fixing the elements of the structure in the unfolded state;

– to draw test examples of unfolding a four-link frame in weightlessness.

4. Geometrical model of the process of unfolding in weightlessness of a four-link structure with the inertial system of unfolding

4. 1. Description of the multilink pendulum oscillations in weightlessness using the Lagrange equations of the second kind

We shall assume under conditions of weightlessness an imaginary plane with the *Oxy* Cartesian coordinates and consider an idealized mathematical model of a multilink pendulum on it. We shall consider n to be a link pendulum consisting of n weightless inextensible rods of lengths L_i (i=1...n), interconnected by hinges between final nodal points that hold loads (to simplify – balls) of masses m_i(i=1...n). The motion of hinges should ensure movement of loads within the chosen plane only. In order to simplify, we shall assume that the friction in the nodes is absent, and the fixing point is unmovable in the coordinate system of the plane because it is connected to a spacecraft whose mass is orders of magnitude larger than the mass of loads in the nodes.

Let the beginning of the first link of the pendulum coincide with the coordinates origin. The direction of reference will be the *Oy* axis. Generalized coordinates will be angles $u_i(t)$ (i=1...n), formed by the corresponding links along the direction of the *Oy* axis on the plane (Fig. 2).

To determine mutual position in time relative to the *Oy* axis of the elements of a multilink pendulum at oscillations in weightlessness under conditions of absence of dissipative forces, we shall use the Lagrange equations of the second kind [16, 17].

Notation of a pendulum oscillation on the plane, taking into consideration the absence of potential energy, will be performed based on the Lagrange equations of the second kind

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial u_i'}L(n)\right)-\frac{\partial}{\partial u_i'}\big(L(n)\big)=0,\ \ (i=1...n), \quad (1)$$

where $L(n)$ is the Lagrangian whose expression coincides with the description of the kinetic energy of the system; $u_i(t)$ is the generalized coordinate (that is, the value in time of angle between the chosen direction of the Oy axis and the i-th link); $u_i' = \frac{d}{dt}u_i(t)$ is the derivative from the function of notation of the generalized coordinate (that is, the value of the "initial" instantaneous velocity of an increase in the i-th angle).

To calculate the Lagrangian, we have expression:

$$L(n)=$$
$$=\frac{1}{2}\sum_{k=1}^{n} m_k\left[\left(-\sum_{i=1}^{k-1} L_i\cos(u_i(t))\frac{du_i(t)}{dt}-L_k\cos(u_k(t))\frac{du_k(t)}{dt}\right)^2+\right.$$
$$\left.+\left(\sum_{i=1}^{k-1}-L_i\sin(u_i(t))\frac{du_i(t)}{dt}-L_k\sin(u_k(t))\frac{du_k(t)}{dt}\right)^2\right]. \quad (2)$$

As a result, after substituting formula (2) in the expressions of the Lagrange equations of the second kind (1), a notation of the motion of an n-link pendulum will be obtained in the form of a system of n differential equations relative to angles $u_i(t)$ (i=1...n).

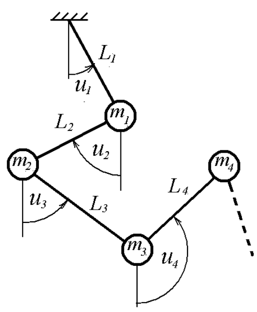

Fig. 2. Schematic of n-link pendulum

In order to estimate the arrangement of pendulum elements over time, we developed software in the maple programming environment [18, 19]. We considered a four-link pendulum (n=4) as an example. By using the generalized coordinates, we compute coordinates of the nodes of the pendulum:

$$x_1(t)=L_1\sin(u_1(t));\quad y_1(t)=L_1\cos(u_1(t));$$
$$x_2(t)=x_1(t)+L_2\sin(u_2(t));$$
$$y_2(t)=y_1(t)+L_2\cos(u_2(t)); \quad (3)$$
$$x_3(t)=x_2(t)+L_3\sin(u_3(t));\quad y_3(t)=y_2(t)+L_3\cos(u_3(t));$$
$$x_4(t)=x_3(t)+L_4\sin(u_4(t));\quad y_4(t)=y_3(t)+L_4\cos(u_4(t))$$

and determine the Lagrangian, which is identified with the kinetic energy of the pendulum:

$$L=0,5\left[m_1\left(\dot{x}_1^2+\dot{y}_1^2\right)+m_2\left(\dot{x}_2^2+\dot{y}_2^2\right)+\right.$$
$$\left.+m_3\left(\dot{x}_3^2+\dot{y}_3^2\right)+m_4\left(\dot{x}_4^2+\dot{y}_4^2\right)\right]. \quad (4)$$

Upon substituting formula (4) in the expressions of equations (1), we shall obtain a system of four Lagrange differential equations of the second kind relative to functions $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ (not given here because of the bulkiness).

4. 2. Solving a system of Lagrange equations of the second kind

When solving the system of Lagrange equations of the second kind, it is required to take into consideration the following parameters (all values of parameters are given in conditional magnitudes):

– vector of lengths of pendulum links: $\mathbf{L}=\{L_1, L_2, L_3, L_4\}$;

– vector of values of the masses of balls: $\mathbf{m}=\{m_1, m_2, m_3, m_4\}$.

In addition, when solving a system of equations, one should take into consideration initial conditions:

– vector of values of the initial angles of deviation: $\theta=\{u_1(0), u_2(0), u_3(0), u_4(0)\}$;

– vector of values of the initial velocities assigned to the angles of deviations:

$$\theta'=\{u_1'(0), u_2'(0), u_3'(0), u_4'(0)\}.$$

Taking into consideration the corresponding initial conditions, the system of Lagrange equations of the second kind is approximately solved by the Runge-Kutta method in the environment of the mathematical software package maple; the obtained solutions are denoted by symbols $U_1(t)$, $U_2(t)$, $U_3(t)$ and $U_4(t)$.

In the Oxy coordinate system selected on the plane, employing the obtained solutions, we determine coordinates of the nodal points in time t. For this purpose, we apply expressions (3) to calculate coordinates of the pendulum nodes using the generalized coordinates, replacing small letters u with large U. By using the developed software maple, in addition to the displacement of nodal points, it is possible to determine velocities, which makes it possible to construct appropriate phase trajectories of displacement.

4. 3. Explanation of the idea of initiating oscillations in weightlessness

We shall explain the idea of initiating in weightlessness of a multilink pendulum using its four-link variant as an example.

A multilink frame structure is delivered folded to the orbit (visually, it resembles a household meter in a folded state). In other words, the initial position of the set of pendulum links takes a "folded" form with the vector of the values of the initial angles of deviations always accepting coordinates $\theta=\{\pi/2, -\pi/2, \pi/2, -\pi/2\}$.

Initiation of the pendulum oscillations will be carried out by choosing the coordinates of the vector of initial velocities, assigned to one of the angles of deviations. For example, $\theta'=\{0, 0, 0, u_4'(0)\}$ means that only ball No. 4 of mass m_4 is given a pulse of magnitude $m_4u_4'(0)$ (or, the unfolding angle $u_4(0)$ is assigned with the initial velocity $u_4'(0)$). Vector of direction $\mathbf{R}$ of setting the velocity is perpendicular to the fourth link L_4 of the pendulum in a final point (Fig. 3). It determines the rate of change in the unfolding angle $u_4(0)$ of the fourth link.

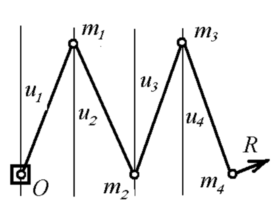

Fig. 3. Four-link variant of the pendulum

In other words, the specified oscillation initiation can be assigned by $\theta'=\{0, 0, 0, u_4'(0)\}$. The selected initiation is the modeling of the action of a pulse jet engine. Taking into consideration velocity $u_4'(0)$ given by the jet engine, the pendulum system must unfold by inertia. This explains the term "inertial system of unfolding".

One should note that in the folded position, the starting position of a relatively massive jet engine is in the region where the pendulum is attached to a spacecraft. This simplifies the assembly of a transportation device, as well as makes it possible to compensate for the effect of the pulse on the system by placing a similar multilink pendulum symmetrically to the apparatus.

5. Computer simulation of unfolding a four-link frame

5. 1. Test example of geometrical modeling of the unfolding of a four-link frame

To test the example of solving a system of the Lagrange equations of the second kind, we selected the following parameters (all are given in conditional magnitudes):

– lengths of the pendulum links: $\mathbf{L}=\{2, 2, 2, 2\}$;

– values of the masses of balls: $\mathbf{m}=\{0.01, 0.01, 0.01, 9.85\}$;

– values of the initial angles of deviations: $\theta=\{\pi/2, -\pi/2, \pi/2, -\pi/2\}$;

– values of the initial velocities of the angles of deviations: $\theta'=\{0, 0, 0, 1\}$.

The mass of the fourth ball is larger due to the location of a jet engine in it.

After the execution of the program, we shall receive a sequence of N frames of animated images depending on the time of unfolding the structure. At the same time, we acquire approximated values of the current magnitudes of angles $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ for the selected time moment t. Fig. 4 shows axonometric diagrams of the initial phases of pendulum links in the process of unfolding. Jet engine is located on the site of the fourth load (shown in red). A cube denotes an unmovable node where the pendulum is attached.

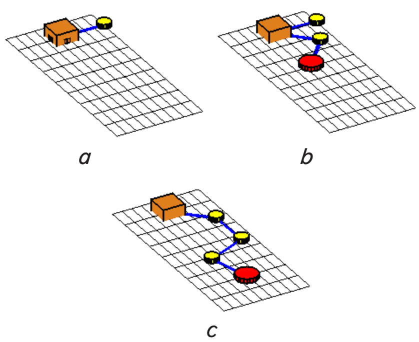

Fig. 4. Unfolding of pendulum links during initial phases of oscillations: a – t=0; u_1=1.571; u_2=–1.571; u_3=1.571; u_4=–1.571; b – t=1.43; u_1=1.572; u_2=–1.612; u_3=0.7684; u_4=–0.7224; c – t=3.07; u_1=0.7810; u_2=0.03844; u_3=–1.175; u_4=0.2586

When analyzing the animated images of oscillations during final phase of "straightening" the structure, we observed the effect of transverse oscillations (tremor) of the pendulum nodes (Fig. 5). This effect can be used to give an order to fix the elements of pendulum design in the "straightened" state. For example, in the case of a pipe variant of fabricating the structure's links, a fixing lock may take the form of an electromagnetic insert into the pipe.

This effect can be explained by using a function of the sum of modules of the obtained solutions:

$$W(t)=\sum_{k=1}^{4}\left|U_k(t)\right|, \quad (5)$$

where $U_k(t)$ are the approximated solutions to the system of Lagrange equations of the second kind; $0\le t\le T$ is the time for system integration. To calculate the test example, we selected: T=4,2; N=300. Fig. 6 shows a chart of function $W(t)$. Function W at the moment of full unfolding of a four-link frame t=4.02 has the absolute minimum with value $W(4.02)$=0.1559. The saw-tooth graph of function $W(t)$ in the vicinity of an absolute extremum is explained by the presence of transverse oscillations of pendulum nodes.

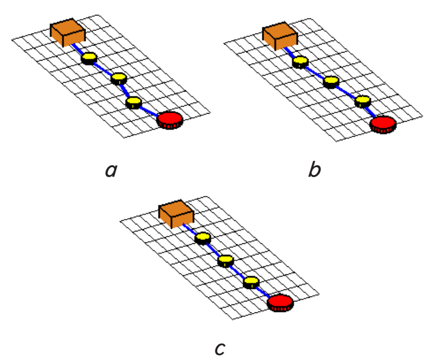

Fig. 5. Unfolding of pendulum links during final phases of oscillations: a – t=3.95; u_1=–0.1028; u_2=0.1619; u_3=–0.1965; u_4=0.2363; b – t=3.99; u_1=0.01216; u_2=0.006937; u_3=0.1984; u_4=–0.1183; c – t=4.02; u_1=–0.04290; u_2=0.1047; u_3=0.08769; u_4=–0.05071

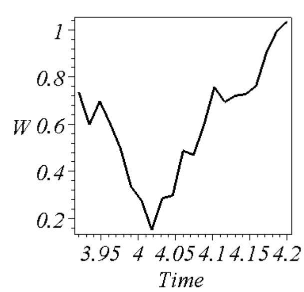

Fig. 6. Graph of function $W(t)$

To reduce the effect of the momentum of rotation around the point of attachment of a pendulum, we propose, symmetrically relative to the fixed node of pendulum attachment, to attach another similar multilink pendulum, which must oscillate in antiphase to a given one. Its oscillations would take the mirror-reflected shape of those shown in Fig. 4, 5. Fig. 7 shows some phases of compatible oscillations of the pendulums.

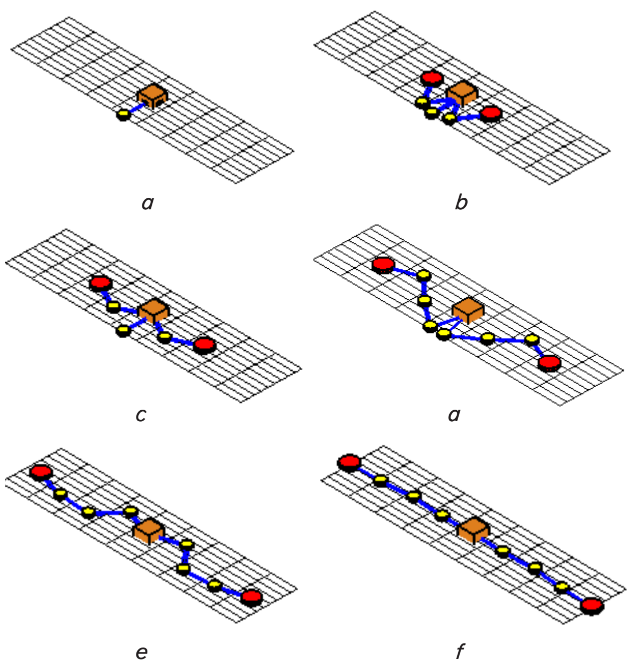

Fig. 7. Diagrams of pendulums in the process of "symmetric" oscillations depending on time t: a – t=0; b – t=1; c – t=1.77; d – t=2.7; e – t=3.5; f – t=4.02

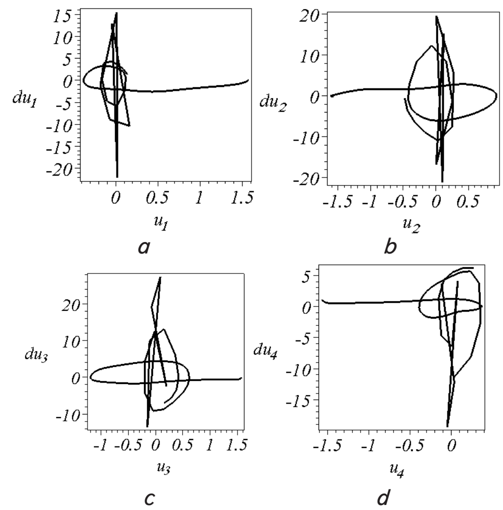

Fig. 8. Phase trajectories of the generalized variables for variant 1: a – $u_1(t)$; b – $u_2(t)$; c – $u_3(t)$; d – $u_4(t)$

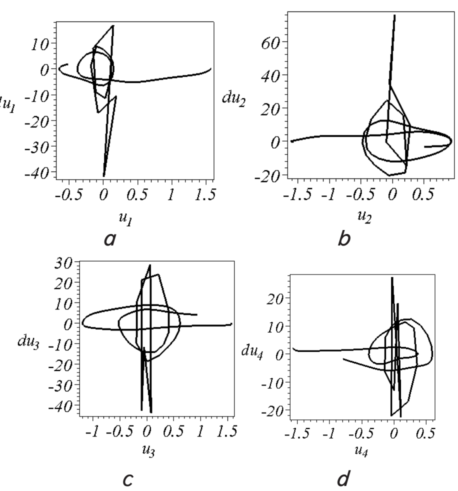

Fig. 9. Phase trajectories of the generalized variables for variant 2: a – $u_1(t)$; b – $u_2(t)$; c – $u_3(t)$; d – $u_4(t)$

5. 2. Substantiation of technique for fixing the elements of a structure

In the previous chapter, we explained on the qualitative level the effect of transverse oscillations of pendulum nodes. Next, we shall confirm these provisions numerically. For this purpose, we shall construct phase trajectories of oscillations of the nodal elements. We chose the following input parameters for computation: lengths of the pendulum links: **L**={2, 2, 2, 2}; value of the masses of balls: **m**={0.01, 0.01, 0.01, 9.85}; values of the initial angles of deviations: θ={π/2, –π/2, π/2, –π/2}, as well as the values of the initial velocities of the angles of deviations: θ′={0, 0, 0, V}. The variants of computation will differ by the value of parameter V.

Variant 1. V=1. The minimum of function W=0.2492 is reached at time t=4.02. Coordinate functions at the moment of "straightening" accept values $u_1(4.02)=0.0208$; $u_2(4.02)=0.1175$; $u_3(4.02)=-0.07524$; $u_4(4.02)=0.03556$.

Fig. 8 shows phase trajectories of the motion of each of the nodal points for variant 1 from which it follows that for the generalized coordinates the intervals of change in the maximum velocity at the moment of straightening the structure will take the following values (in conditional units): for $u_1(t)$ – [–22; 15]; for $u_2(t)$ – [–20; 20]; for $u_3(t)$ – [–15; 25]; for $u_4(t)$ – [–20; 7].

Variant 2. V=2. The minimum of function W=0.2053 is achieved at moment t=2.012. Coordinate functions at the moment of "straightening" accept values $u_1(2.012)=0.006082$; $u_2(2.012)=0.03725$; $u_3(2.012)=-0.05320$; $u_4(2.012)=0.1087$.

Fig. 9 shows phase trajectories of the motion of each of the nodal points for variant 2 from which it follows that for the generalized coordinates the intervals of change in the maximum velocity at the moment of straightening the structure will take the following values (in conditional units): for u_1(t) – [–40; 15]; for u_2(t) – [–20; 70]; for $u_3(t)$ – [–40; 30]; for $u_4(t)$ – [–20; 30].

Variant 3. V=3. The minimum of function W=0.2110 is achieved at moment t=1.34. Coordinate functions at the moment of "straightening" accept values $u_1(1.34)=0.04821$; $u_2(1.34)=-0.05611$; $u_3(1.34)=0.01042$; $u_4(1.34)=0.09566$.

Fig. 10 shows phase trajectories of the motion of each of the nodal points for variant 3 from which it follows that for the generalized coordinates the intervals of change in the maximum velocity at the moment of straightening the structure will take the following values (in conditional units): for $u_1(t)$ – [–30; 45]; for $u_2(t)$ – [–40; 70]; for $u_3(t)$ – [–120; 40]; for $u_4(t)$ – [–25; 80].

The obtained numerical estimates for the range of maximum velocities at the final phase of unfolding a four-link rod framework will help in the further calculations of dynamics and durability characteristics. Including the permissible magnitude of the pulse of oscillations initiation.

It is expedient to consider a technique for the arrangement of three pendulums with a joint attachment node (a triad), with the angles between them of 120 degrees (Fig. 11, a). Fig. 11, b–f shows some phases of unfolding the triad depending on time t.

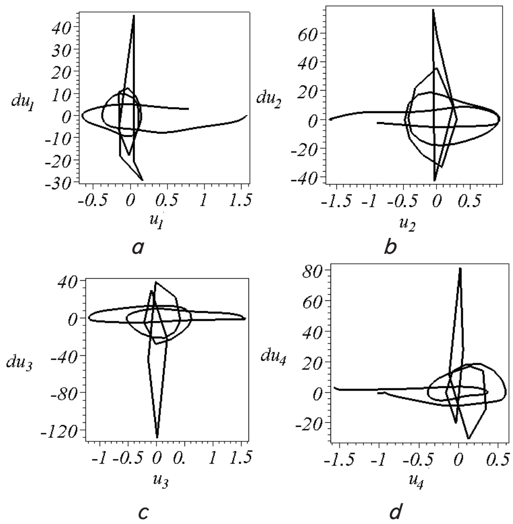

Fig. 10. Phase trajectories of the generalized variables for variant 3: *a* – $u_1(t)$; *b* – $u_2(t)$; *c* – $u_3(t)$; *d* – $u_4(t)$

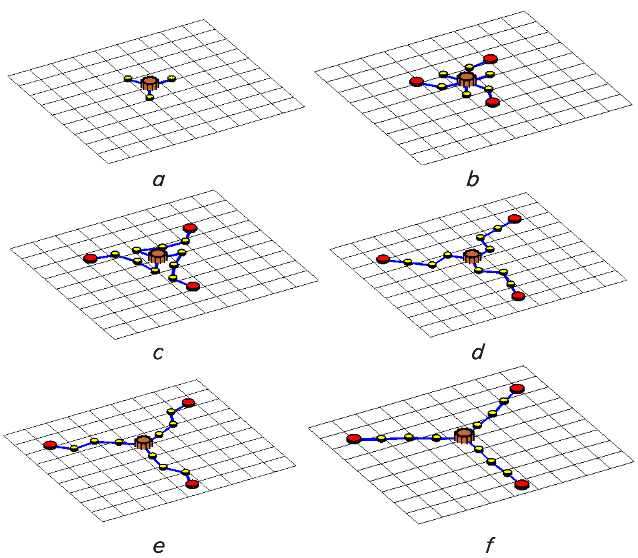

Fig. 11. Phases of unfolding the triad depending on time *t*: *a* – *t*=0; *b* – *t*=1.85; *c* – *t*=2.58; *d* – *t*=3.54; *e* – *t*=3.78; *f* – *t*=4.02

In order to estimate unfolding of the triad, we employed the Lagrangian equations of the second kind, based on variational principles. These equations allowed us to describe successive values of the magnitudes of angles between adjacent links to reach the unfolding state by a multilink pendulum. Thus, one can argue that a change in the values of magnitudes of the angles between adjacent links in a certain sense will be optimal. Any other sequence of change in the values of the magnitudes of angles will result in the increased time for unfolding a multilink pendulum.

By means of the unfolded triads one can construct within the plane large-size structures that have a cellular structure. Fig. 12 shows a cellular structure on the plane, built using the triads, with three of them highlighted in red, green and blue colors. These results could be used when designing the unfolding of large-size structures under conditions of weightlessness, for example, frames for solar mirrors.

If one continues the process of unfolding a triad after second 4.02, it is possible to watch the "collapse" of the pendulum to the new triad. Fig. 13 shows some phases of the "collapse" of a four-link pendulum to a triad depending on time *t*. Web-resource [19] gives the examples of corresponding animations.

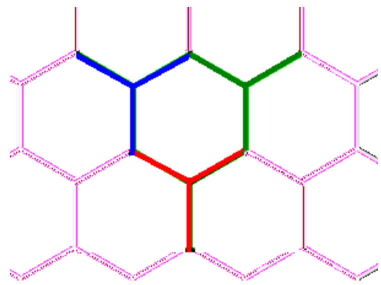

Fig. 12. A cellular structure on the plane, built using the triads (three of them are highlighted in colors)

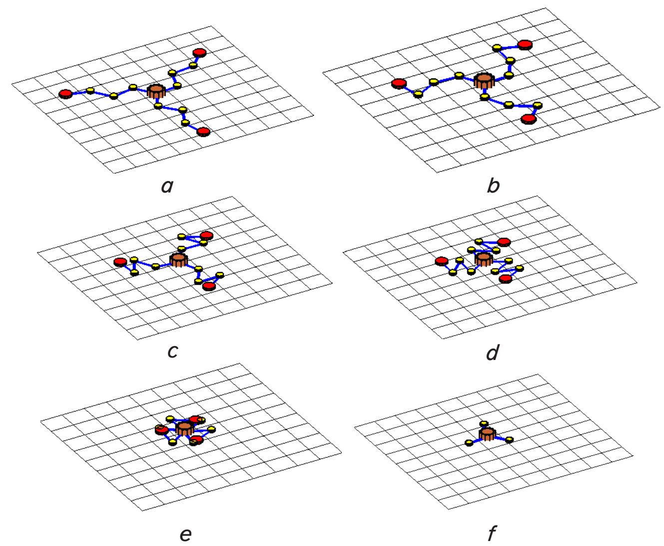

Fig. 13. Phases of "folding" a four-link pendulum to the triad depending on time *t*: *a* – *t*=4.5; *b* – *t*=5.07; *c* – *t*=5.71; *d* – *t*=6.35; *e* – *t*=7.16; *f* – *t*=8.04

In summary, it should be noted that the result of the research conducted is the constructed geometrical model of unfolding a multilink pendulum on the imaginary plane in weightlessness, which would enable creation of actual rod basis for structures in space.

6. Discussion of results of research into inertial technique for unfolding a multilink pendulum in weightlessness

The advantage of the examined inertial technique for unfolding a multilink structure in weightlessness is in the following:

– there is no need to synchronize the means of control over the magnitudes of angles in separate nodes of a multilink structure;

– technology of the inertial technique for unfolding is not critical in terms of the size of the elements of structure's parts;

– transverse oscillations of pendulum nodes (tremor) can be used to lock the fixing positions of the adjacent parts of the pendulum;

– for a four-link (and other designs with an even number of links) pendulum, the starting location of the jet engine is in the region where the pendulum is attached to a spacecraft, which is convenient when assembling a construction and taking into consideration a compensation for the pulse action;

– based on the circuit of unfolding one multilink structure, it is possible to create multi-beam circuits with many pendulums with a joint non-movable mounting node (a triad as an example);

– given the use of a multilink structure, coupled in pairs and with links oscillating in antiphase, it is possible to try to weaken (or even eliminate) the torque of the structure in general.

The results obtained can be explained by the possibility of applying the variational principle of Lagrange to the estimation of mechanical structures taking into consideration kinematic relations and using the "zero" potential energy of a mechanical system. This allowed us to employ the Lagrange equations of the second kind to describe the motion of a pendulum system in weightlessness.

The possibilities of research into the motion of a pendulum system in weightlessness that are not implemented as yet include consideration of unequal lengths of pendulum links, as well as irregular masses of nodal elements and links. The development of an illustrative geometrical model for the inertial unfolding of a multilink pendulum explains the use of conditional units for parameters in the test examples.

Further research in this direction implies employing other variants of multilink pendulums – in which the intermediate nodes of a "parent" pendulum can serve as the initial nodes of the "child" multilink pendulums. This direction is important when calculating the circuits for unfolding "star-shaped" structures (for example, space antennas). The difficulties in the development of research in this direction are related to the requirement of solving an inverse problem of assembly – that is, for a given resulting arrangement of pendulum elements, it is necessary to determine a rational set of parameters for a multilink pendulum and initial conditions for its motion, which will enable such unfolding.

The study performed would also form the basis for calculating spatial multilink pendulums whose links in the process of unfolding would not be limited by one plane. This is expedient when designing construction work in weightlessness applying the unfolding of large-size 3d-structures.

7. Conclusions

We proposed a technique of unfolding in weightlessness of a multilink structure with the inertial system of unfolding, provided that the structure is identified with a multilink pendulum on the imaginary plane, attached to a spacecraft. To unfold the structure of a multilink pendulum, it is proposed to give a momentum by a jet engine to one of the nodal elements of a pendulum (in the present work – to the final one). This allowed us to implement the circuit for inertial unfolding of a multilink pendulum by using a single engine, which does not require synchronization of the means of control over the magnitudes of angles in separate nodes of a multilink structure.

For the actuation of sensors in order to lock fixing position of the adjacent links of the pendulum in the unfolded state, it is proposed to employ the transverse oscillations of nodes (tremor) at the final phases of pendulum unfolding.

The results might be used when designing the unfolding of large-size structures under conditions of weightlessness, for example, frames for solar mirrors.

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