Одержано функціональний зв'язок температури теплопоглинаючого спаю від часу з врахуванням теплоємності комутаційних пластин і охолоджуючого об'єкта, а також теплового навантаження теплообміну на робочих спаях термоелемента з зовнішнім середовищем. Показана можливість управління інерційністю нагріву і охолодження термоелектричного пристрою шляхом зміни величини робочого струму термоелементу. Визначена ефективність такого управління у залежності від теплофізичних параметрів

Ключові слова: термоелектричний охолоджувач, нестаціонарний режим, перепад температур, управління інерційністю

Получена функциональная связь температуры теплопоглощающего спая от времени с учетом теплоемкости коммутационных пластин и охлаждающего объекта, а также тепловой нагрузки теплообмена на рабочих спаях термоэлемента с окружающей средой. Показана возможность управления инерционностью нагрева и охлаждения термоэлектрического устройства путем изменения величины рабочего тока термоэлемента. Определена эффективность такого управления в зависимости от теплофизических параметров

Ключевые слова: термоэлектрический охладитель, нестационарный режим, перепад температур, управление инерционностью

UDC 621.362.192

DOI: 10.15587/1729-4061.2017.116005

ANALYSIS OF THE POSSIBILITY TO CONTROL THE INERTIA OF THE THERMOELECTRIC COOLER

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1. Introduction

The possibilities of using thermoelectric cooling devices (TCD) in critical systems with enhanced requirements to reliability indicators are determined not only by static, but by dynamic indicators as well. Under transient modes characteristic of the operation of thermoelectric coolers in control systems, of special importance are the temporal indicators of reaching preset thermal regimes by the cooled elements of radio-electronic equipment. Under a pulsed thermal impact even a slight excess of the permissible thermal regimes of elements can lead to failure of the thermally-loaded elements. A transient characteristic depends on the heat capacity of heat load, thermal conductivity and active resistance of thermoelement branches, spatial distribution of heat flow, etc. Accounting for the effect of given components on the transitional characteristic and, accordingly, on the possibilities to control performance efficiency of TCD, is a relevant task aimed at improving failure-free operation of the thermally-loaded components of electronic equipment.

2. Literature review and problem statement

Thermoelectric cooling devices have a significantly higher performance compared with compression systems for maintaining thermal modes of radio-electronic equipment [1]. This is predetermined by the low mass of a thermoelectric cooler, solid-state principles of cold and heat generation [2], properties of the material used for thermoelements [3], and fabrication technology [4]. That is why designers of thermal mode provision systems typically confine themselves to a stationary TCD operating regime, that is, they actually neglect dynamic TCD characteristics [5–7]. However, there is a class of tasks, which are characterized by the pulse input of external heat impact, for example, laser receivers and emitters that require systems for ensuring thermal regimes [8]. For such thermally-loaded elements permissible heating temperature is limited by the values of working capacity of semiconductors, which is why it is necessary to form a cooling wave, close to the rate of rise in the heat flow of the element. This circumstance poses a problem requiring the creation of high-speed systems for ensuring thermal regimes that work in pulsed mode [9]. The issues of distribution of thermal fields in TCD are not studied enough, to say nothing of control methods. The results of research into effect of switching modes of TCD on cracking a contact between the thermoelement and the electrode [10] only partially fill this gap. This feature became the basis for the accelerated TCD test method for failure-free operation [11]. However, TCD high-speed performance issues, as well as performance efficiency control methods, were not paid due attention.

3. The aim and objectives of the study

The aim of present work is to study dynamic characteristics of thermoelements under a non-stationary operation mode of the thermoelectric cooling device.

To achieve the set aim, the following tasks must be solved:

– to design a model of the thermoelectric cooler under a non-stationary mode connecting temperature difference between the electrodes and parameters of the thermoelement, load, and working current;

– to determine a possibility to control the rate of change in the heating and cooling of the thermoelectric device.

4. Design of the model of a thermoelectric cooling device under a non-stationary mode

The operation of thermoelectric cooling devices is determined by both stationary and non-stationary cooling process. Stationary regime is characteristic of the systems for ensuring thermal regimes of thermally-loaded elements whose heat load changes insignificantly. For the non-stationary regime, important is the time for reaching the specified level of cooling time, as well as the ability to control over time a given level of temperature on the heat-absorbing layer.

In this case, of interest are the transient processes that occur when electric current of different form passes the thermoelement.

Heat conduction equation in a homogeneous medium can be represented in a general form [12]:

$$\frac{\partial t}{dt}=a^2\left(\frac{\partial^2 t}{dx^2}+\frac{\partial^2 t}{dy^2}+\frac{\partial^2 t}{dz^2}\right),$$

where $a=\sqrt{\frac{k}{c\rho}}$; k is the coefficient of thermal conductivity; c is the specific heat of the substance; ρ is the density. In this case, the initial condition that produces initial temperature distribution at t=0:

$$T|_{t=0}=f(x, y, z).$$

In the case of distribution of heat in a solid body with linear dimensions, for example in a homogeneous rod, which is located along the x axis, the equation can be written

$$\frac{\partial T}{\partial t}=a^2\frac{\partial^2 T}{\partial x^2}.$$

Given such a form of the equation, one does not take into account heat exchange between the ends and the side surface of the rod and the surrounding space.

The issues of non-stationary thermoelectric cooling were previously considered for a quite simplified thermal model of TCD [10].

In a general case, taking account of various factors ultimately comes down to controlling the temperature of thermoelement junction in line with the assigned law.

Of greatest interest is the study of thermal processes on a heat-absorbing thermoelement junction taking into consideration heat load.

The thermal model is built under the following assumptions:

– branches of thermoelements possess identical thermophysical parameters;

– lateral surfaces of thermoelement branches are adiabatically insulated.

Then the distribution of temperature T as a function of the x coordinate, directed along the length of thermoelements from a heat-absorbing junction to the heat-generating one, and time t, is described by the equation of thermal conductivity

$$\lambda\frac{\partial^2 T}{\partial x^2}=c\frac{\partial T}{\partial t}-j^2\rho, \tag{1}$$

where λ is the thermal conductivity of thermoelement branches; c is the volumetric heat capacity of TCD; ρ is the specific electric resistance of a thermoelement branch; j is the density of supply current.

Physical parameters are related to the area of the cooled surface. At the initial point of time temperature is constant throughout the entire volume of the thermoelement, while the temperature of a heat-generating junction is constant over time due to the intensive heat exchange T_o.

At the heat-absorbing junction, one takes into account a Peltier effect, heat capacity of the switching plates and the cooled object, heat losses into the environment and a heat load.

The initial and boundary conditions for equation (1) can be written in the form:

$$T|_{t=0}=T_o; \tag{2}$$

$$\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0}=\left[\bar{e}jT+g_1\frac{\partial T}{\partial t}+\alpha_1\left(T-T_o\right)-q_l\right]\Bigg|_{x=0}; \tag{3}$$

$$T|_{x=l}=T_o,$$

where $\bar{e}$ is the thermal emf coefficient; g_1 is the heat capacity of the switching plate and the cooled object; α_1 is the heat transfer coefficient; q_l is the thermal load; l is the length of a thermoelement branch.

In a general case, power q_l consists of the power of heat sources and Joule heat power, dissipated at the heat-absorbing junction ($j^2\rho_c$, where ρ_c is the contact electric resistance of a thermoelement branch).

We shall subsequently operate with the following dimensionless parameters:

$\Theta=zT$; $\chi=x/l$; $F_0=at/l^2$; $B_{i1}=\alpha_1 l/\lambda$;

$\eta_1=g_1/(cl)$; $K_l=q_l\,(zl/\lambda)$; $\xi_n=\frac{\rho_c}{\rho l}\left(K_l=K_g\nu^2\xi_n\right)$,

where a is the coefficient of thermal diffusivity of thermoelectric material; $\bar{z}=\frac{\bar{e}^2}{\rho\lambda}$ is the thermoelectric efficiency of material.

Equation (1) with initial and boundary conditions in dimensionless units can be represented in the following form:

$$\frac{\partial^2\Theta}{\partial\chi^2}=\frac{\partial\Theta}{\partial(F_0)}-\nu^2,\ \Theta|_{F_0=0}=\Theta_0; \tag{4}$$

$$\left.\frac{\partial\Theta}{\partial\chi}\right|_{\chi=0}=\nu\Theta+\eta_1\frac{\partial\Theta}{\partial(F_0)}+B_{i1}\left(\Theta-\Theta_0\right)+K_{\mathrm{H}}; \tag{5}$$

$$\Theta|_{\chi=1}=\Theta_0.$$

Using the operational method, it is easy to find the Laplace transform for the temperature of a heat-absorbing junction:

$$\bar{\Theta}(p)\Big|_{\chi=0} = \frac{\Theta_0}{p} + \frac{\nu^2\sqrt{p}(ch\sqrt{p}-1) - p(\nu\Theta_0 - K_H)sh\sqrt{p}}{p^2\left[\sqrt{p}\,ch\sqrt{p} + (\nu + Bi + \eta_1 p)\right]sh\sqrt{p}}. \quad (6)$$

Transition to the temporal domain is carried out in line with the Riemann-Mellin formula and a deduction theorem:

$$\Theta(F_0) = \Theta_0 - \frac{\nu\Theta_0 - 0{,}5\nu^2 - K_l}{1 + \nu + B_{i1}} + \sum_{k=1}^{\infty} A_k \exp\left(-\delta_k^2 F_0\right), \quad (7)$$

where

$$A_k = 2\frac{\left[\delta_k\left(\nu\Theta_0 - K_l\right)\sin\delta_k - \nu^2\left(1 - \cos\delta_k\right)\right]}{\delta_k^2\left[\delta_k\left(2\eta_1 + 1\right)\sin\delta_k + \left(\eta_1\delta_k^2 - 1 - \nu - B_{i1}\right)\cos\delta_k\right]}.$$

The magnitudes δ_k that are included here are the positive roots of transcendental equation

$$tg\delta = \frac{\delta}{\eta_1\delta^2 - \nu - Bi_1}. \quad (8)$$

It follows from ratio (7) that temperature difference $\Delta\Theta = \Theta_0 - \Theta$ between the thermoelement junctions can be represented in the form

$$\Delta\Theta = \Theta_{stat} - f(F_0), \quad (9)$$

where

$$f(F_0) = \sum_{k=1}^{\infty} A_k \exp\left(-\delta_k^2 F_0\right). \quad (10)$$

Series (10) determines the speed at which a heat-absorbing junction temperature approaches the stationary value. In this case

$$\Delta\Theta_{stat} = \frac{\nu\Theta_0\left(0{,}5 + \xi_k\right)\nu^2 - K_l}{1 + \nu + B_{i1}}. \quad (11)$$

The maximum temperature difference is reached at optimum current density ν_T^o, which can be determined from ratio

$$\nu_T^o = \frac{\left[B^2 + 2\Theta_0 B + 2K_l\left(1 + 2\xi_k\right)\right]^{1/2} - B}{1 + 2\xi_k}. \quad (12)$$

The maximum temperature difference can be determined from relation

$$\Delta\Theta_{stat}^{max} = \Theta_0 + B - \left[B^2 + 2\Theta_0 B + 2K_l\left(1 + 2\xi_k\right)\right]^{\frac{1}{2}}, \quad (13)$$

where

$$B = (1 + B_{i1})(1 + 2\zeta_k).$$

5. Analysis of the model of a thermoelectric cooler under a non-stationary mode

Calculating temperature difference Θ from formula (7) implies a knowledge of the roots of equation (8). We calculated values of the two lowest roots δ_1 and δ_2 for a wide range of change in the parameters $\nu + B_{i1}$ and η_1. Calculation results are given in Table 1.

Table 1

Values of roots δ_1 and δ_2

η_1 \ $\nu+B_{i1}$	0	0.5	1.0	2.0	3.0	5.0	10	15	20
0	1.57	1.84	2.03	2.29	2.46	2.65	2.86	2.95	3.0
	4.71	4.82	4.91	5.1	5.23	5.45	5.76	5.91	6.0
0.10	1.43	1.70	1.90	2.18	2.37	2.60	2.84	2.94	2.98
	4.31	4.40	4.49	4.67	4.85	5.14	5.60	5.82	5.94
0.25	1.27	1.52	1.72	2.02	2.24	2.51	2.81	2.92	2.98
	3.94	4.0	4.06	4.19	4.34	4.63	5.25	5.62	5.82
0.50	1.08	1.31	1.49	1.79	2.02	2.34	2.73	2.88	2.95
	3.64	3.67	3.71	3.78	3.86	4.05	4.59	5.092	5.46
1.00	0.86	1.05	1.21	1.47	1.68	2.01	2.53	2.78	2.90
	3.43	3.44	3.45	3.47	3.50	3.57	3.83	4.18	4.55

The case for $\nu + B_{i1} = 0$ and $\eta_1 = 0$ is the limiting. Since the magnitudes of δ_k are localized in the respective intervals $\delta_k \in ((k-1)\pi, k\pi)$, series (10) quickly converges at large enough values of F_0.

Data from Table 1 show that an increase in heat capacity η_1 leads to a decrease in the values of δ_k, that is, to an increase in the inertia of thermoelements. An increase in heat transfer B_{i1}, as well as in current density ν, exerts an opposite effect on the magnitude of δ_k.

Pre-exponential factors A_k are also dependent on parameters η_1, ν and B_{i1}. In addition, the magnitudes A_k are affected by parameters Θ_0 and K_n. Fig. 1–4 show dependences of coefficients A_1 and A_2, as well as δ_1^2 and δ_2^2, on the ratio of current density ν to the optimum value of ν_T^o for various combinations of parameters B_{i1} and η_1. Dependence of A_1 on current density ν has a maximum, after which A_1 monotonically decreases, passing a zero to the negative region. Coefficient A_2 monotonically increases with an increase in current ν. With an increase in heat transfer B_{i1}, coefficients of the series decrease, which corresponds to a reduction of the stationary temperature difference (Fig. 1, 2). Values δ_1^2 and δ_2^2 increase monotonically with an increase in current power. In the cases where parameters B_{i1} and η_1 accept limiting values close to zero, or sufficiently large, it is easy to determine analytically relevant values of δ_k and A_k, specifically:

– at $\eta_1 \to 0$ $\nu + B_{i1} \to 0$ $K_l = 0$

$$\delta_к = \frac{\pi}{2}(2k-1);\; A_k = \frac{8\pi(2k-1)\nu\Theta_0 + 2(-1)^{k\nu^2}}{(2k-1)^3\pi^3};$$

– at $\nu + B_{i1} >> 0$ $K_l = 0$

$$\delta_k = k\pi;\; A_k = \frac{2\nu^2\left[(-1)^k - 1\right]}{\pi^2 k^2(\nu + B_{i1})};$$

– at $\eta_1 >> 0$ $K_l = 0$

$$\delta_1 = \sqrt{\frac{1 + \nu + B_{i1}}{\eta_1}};\quad \delta_k = (k-1)\pi,\ (k = 2,3\ldots),$$

$$A_1 = \frac{\nu\Theta_0 - 0{,}5\nu^2}{1 + \nu + B_{i1}};\; A_k = \frac{2\nu^2\left[1 + (-1)^k\right]}{\pi^4(k-1)^4\eta_1}\ (k = 2,3\ldots).$$

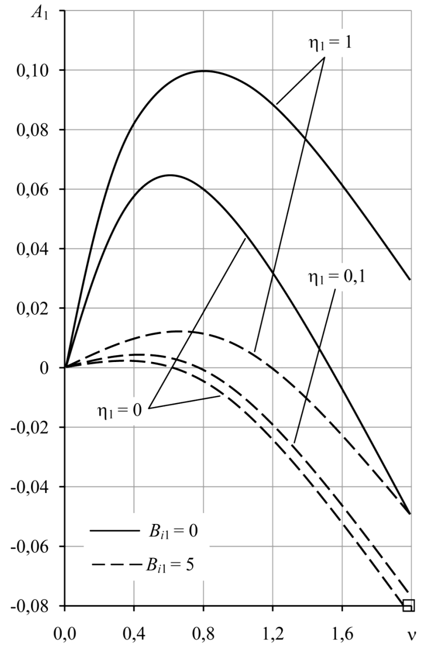

Fig. 1. Dependence of coefficient A_1 on current density ν at Θ_0=0.6 and T=300 K for various values of B_{i1} and η_1

Accordingly, a temperature change at large enough values of F_0 is determined from the following relations:

– at $\eta_1 \to 0$ $\nu+B_{i1}\to 0$ $K_l=0$

$$\Delta\Theta=\Delta\Theta_{\text{stat}}-8\frac{\left(\pi\nu\Theta_0-2\nu^2\right)}{\pi^3}\exp\left(-\frac{\pi^2}{4}F_0\right); \tag{14}$$

– at $\nu+B_{i1}>>1$ $K_l=0$

$$\Delta\Theta=\Delta\Theta_{\text{stat}}+8\frac{4\nu^2}{\pi^2\left(\nu+B_{i1}\right)}\exp\left(-\pi^2F_0\right); \tag{15}$$

– at $\eta_1>>1$

$$\Delta\Theta=\Delta\Theta_{\text{stat}}\left[1-\exp\left(-\frac{1+\nu+B_{i1}}{\eta_1}F_0\right)\right]. \tag{16}$$

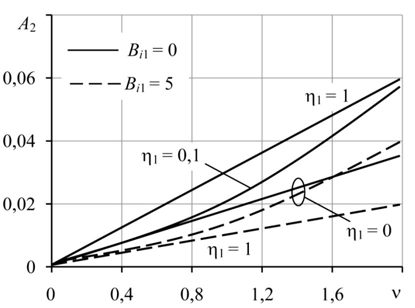

Fig. 2. Dependence of coefficient A_2 on current density ν at Θ_0=0.6 and T=300 K for various values of B_{i1} and η_1

Temperature of the heat-absorbing junction, depending on the current strength, can approach stationary from the side of larger or smaller values, that is, either monotonically or passing through a minimum. The condition that differentiates these two cases implies that coefficient A_1 is equal to zero:

$$\delta_1\left(\Theta_0-\frac{K_{\text{н}}}{\nu}\right)\sin\delta_1-\nu\left(1-\cos\delta_1\right)=0. \tag{17}$$

Determining current density that satisfies this condition (ν_0) should be carried out numerically, in combination with solving equation (8). The values of ν_0, obtained in this case, can be compared with the optimal under a stationary mode. The ν_0 calculation data for some values of parameters η_1 and B_{i1} (at Θ_0=0.6 and K_l=0) are given in Table 2.

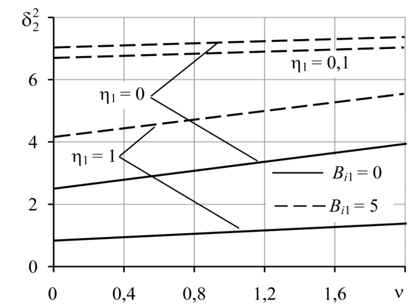

Fig. 3. Dependence of magnitude δ_1^2 on current density at Θ_0=0.6 and T=300 K for various values of B_{i1} and η_1

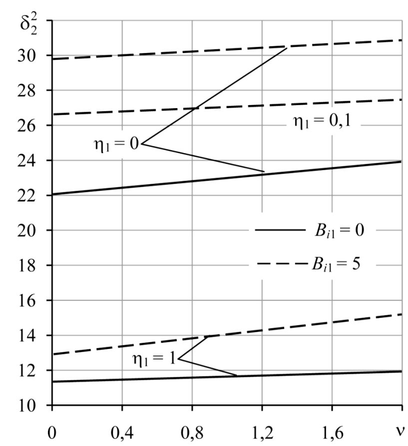

Fig. 4. Dependence of magnitude δ_2^2 on current density at Θ_0=0.6 and T=300 K for various values of B_{i1} and η_1

The value of ν_0 can be derived analytically, at $K_l \to 0$, in this case

$$\nu_0=\frac{\Theta_0\arccos\left(-\frac{\Theta_0}{1+\Theta_0}\right)}{\sqrt{1+2\Theta_0}}. \tag{18}$$

Since $\nu_T^o=\sqrt{1+2\Theta_0}-1,$ then

$$\frac{\nu_0}{\nu_T^o}=\frac{1}{2}\left(1+\frac{1}{\sqrt{1+2\Theta_0}}\right)\arccos\left(-\frac{\Theta_0}{1+\Theta_0}\right). \tag{19}$$

Ratio (19) for any Θ_0 exceeds $\pi/2$, that is, it is larger than unity. An increase in heat load η_1 contributes to the growth of current ν_0 while heat transfer B_{i1} reduces magnitude ν_0 (Table 2).

In a general case, ratio $\frac{\nu_0}{\nu_T^o}$ can be both larger and lower than unity. Thus, an extremum at the dependence curve $\Theta(F_0)$, occurring at $\nu=\nu_0$, may emerge at the current values that are less than optimal (for sufficiently large values of B_{i1}) and which exceed the optimal under a stationary mode. Because $\delta_2>\delta_1$, the mode, which is transitional from monotonic fall in temperature over time to a change in temperature with an extremum, is the "fastest". Current density ν_0 corresponds to the least time of the transition process, while mode $\nu=\nu_0$

appears appropriate under condition that current ν_0 ensures sufficient reduction in temperature under a stationary state.

Table 2

Ratio between ν_0 and ν_T^o at different thermal load on a heat-absorbing junction

η_1	0	0.1	0.5	1.0	∞	0	0	0
B_{i1}	0	0	0	0	0	0	1.0	5.0
ν_T^o	0.483	0.483	0.483	0.483	0.483	0.483	0.53	0.573
ν_0	0.79	0.83	0.97	1.05	1.20	0.79	0.66	0.37
$\bar{\nu}=\nu_0/\nu_T^o$	1.64	1.74	2.0	2.17	2.48	1.64	1.25	0.65
η_1	0	0	0	0.1	0.5	1.0	0	–
B_{i1}	10	∞	5.0	5.0	5.0	5.0	5.0	–
ν_T^o	0.584	0.60	0.573	0.573	0.573	0.573	0.573	–
ν_0	0.24	0	0.37	0.41	0.55	0.72	1.20	–
$\bar{\nu}=\nu_0/\nu_T^o$	0.41	0	0.65	0.71	0.97	1.25	2.09	–

When solving heat equation (4) in the form (7), we observe a fast convergence for sufficiently large values of time F_0. To build a transition process for small values of time F_0, it is necessary to apply a solution to equation (4) as a series for powers of F_0. It is possible to obtain a decomposition for powers $p^{-1/2}$ from formula (6) $\Theta(p)$:

$$\Theta(p)=\frac{\Theta_0}{p}-\frac{\nu\Theta_0-K_l}{\eta_1}\frac{1}{p^2}+\frac{\eta_1\nu^2+\nu\Theta_0-K_l}{\eta_1^2 p^{5/2}}-$$
$$-\frac{(\nu\Theta_0-K_l)\left[1-\eta_1(\nu+B_{i1})\right]+\eta_1\nu^2}{\eta_1^3 p^3}+$$
$$+\frac{(\nu\Theta_0-K_l)\left[1-2\eta_1(\nu+B_{i1})\right]+\eta_1\nu^2\left[1-2\eta_1(\nu+B_{i1})\right]}{\eta_1^4 p^{7/2}}\ldots,$$

which corresponds to the following series in a temporal domain

$$\Theta(F_0)=\Theta_0-\frac{\nu\Theta_0-K_l}{\eta_1}F_0+\frac{4}{3\sqrt{\pi}}\frac{\nu\Theta_0-K_l+\eta_1\nu^2}{\eta_1^2}F_0^{3/2}-$$
$$-\frac{(\nu\Theta_0-K_l)\left[1-\eta_1(\nu+B_{i1})\right]+\eta_1\nu^2}{2\eta_1^3}F_0^2+$$
$$+\frac{8}{15\sqrt{\pi}}\frac{(\nu\Theta_0-K_l)\left[1-2\eta_1(\nu+B_{i1})\right]+\eta_1\nu^2\left[1-\eta_1(\nu+B_{i1})\right]}{\eta_1^4}F_0^{5/2}+\ldots \quad (20)$$

Similarly, it is possible to find decomposition $\Theta(F_0)$ for powers F_0 for the case of negligible heat capacity η_1:

$$\Theta(F_0)=\Theta_0-\frac{2}{\sqrt{\pi}}(\nu\Theta_0-K_l)\sqrt{F_0}+$$
$$+\left[(\nu\Theta_0-K_l)(\nu+B_{i1})+\nu^2\right]F_0-$$
$$-\frac{4}{3\sqrt{\pi}}\left[(\nu\Theta_0-K_l)(\nu+B_{i1})+\nu^2\right](\nu+B_{i1})F_0^{3/2}+$$
$$+\frac{1}{2}\left[(\nu\Theta_0-K_l)(\nu+B_{i1})+\nu^2\right](\nu+B_{i1})^2F_0^2-\ldots \quad (21)$$

Comparing formulae (20) and (21) reveals that the presence of heat capacity on a heat-absorbing junction leads to a decrease in the cooling rate during initial period. The first two terms of decomposition (21) coincide with the corresponding decomposition terms for the case of cooling the plates from a source of constant cooling capacity equal to $\nu\Theta_0-K_l$.

In order to estimate an error in temperature calculations by analytical formulae using the first terms of the corresponding series, it is possible to apply data from the numerical solution to equation (4). Fig. 5 shows temperature difference dependences on time for various values of ratio $\bar{\nu}=\nu_0/\nu_T^o$ and parameters η_1 and B_{i1} (Θ_0=0.6; K_l=0).

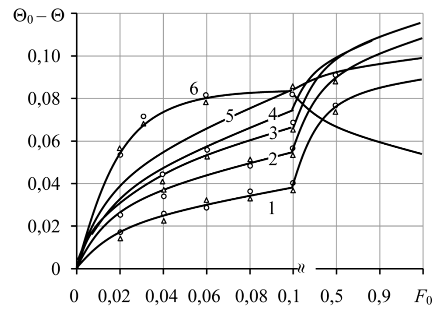

Fig. 5. Temperature difference dependence at the thermoelement on time F_0 at Θ_0=0.6; T=300 K; η_1=0 and B_{i1}=0 for different values of current density $\bar{\nu}$: 1 – 0.5; 2 – 0.75; 3 – 1.0; 4 – 1.25; 5 – 1.5; 6 – 2.0

For the case η_1=0; B_{i1}=0 (Fig. 5), high current values are matched by curves with a larger lifting angle at the coordinate origin. The curves for currents that are smaller than ν_0 (for the examined case ν_0=1.64ν_T^o) are monotonic in character and do not intersect at $\nu_0<\nu_T^o$, while the curves that correspond to currents exceeding values ν_T^o, start to intersect with the preceding ones. When current exceeds magnitude ν_0, the temperature course has an extremum. The highest speed of establishing a stationary differential for the represented dependences is demonstrated by curve (5) ($\nu=1{,}5\nu_T^o$ and close to value ν_0).

Graphic dependences, shown in Fig. 6 (η_1=1; B_{i1}=0), allow us to observe the effect of heat capacity of the load on the cooling progress. An extremum on curve $\Delta\Theta_0=f(F_0)$ is not observed even at $\bar{\nu}=2$ (in this case, $\nu_0=2{,}17\nu_T^o$). The highest rate of establishing a stationary state corresponds to $\bar{\nu}=2$.

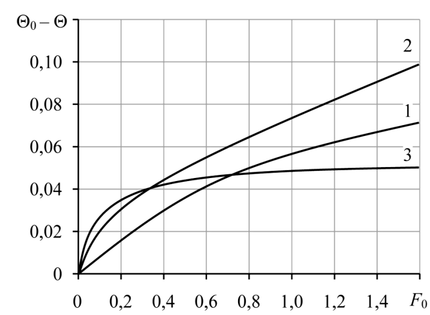

Fig. 6. Temperature difference dependence at the thermoelement on time F_0 at Θ_0=0.6; T=300 K; η_1=0 and B_{i1}=0 for different values of current density $\bar{\nu}$: 1 – 0.5; 2 – 1.0; 3 – 1.5

For large values of heat transfer coefficient (Fig. 7) (η_1=0; B_{i1}=5), the value of $\bar{\nu}$, at which an extremum of temperature progress appears, is less than unity, while curve $\Delta\Theta(F_0)$ for $\nu=\nu_T^o$ has an extremum. The magnitude of temperature difference at the extremum point of curve $\Delta\Theta=f(F_0)$ is larger than the stationary. At $B_{i1}=5$; $\eta_1=1$ (Fig. 8) is characterized by a mutual compensation of the effect of temperature and heat capacity of the load, resulting in an extremum occurring at $\bar{\nu}=1{,}25$.

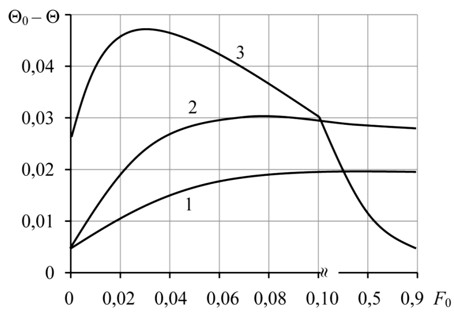

Fig. 7. Temperature difference dependence at the thermoelement on time F_0 at Θ_0=0.6; T=300 K; η_1=0 and B_{i1}=5 for different values of current density $\bar{\nu}$: 1 – 0.5; 2 – 1.0; 3 – 1.5

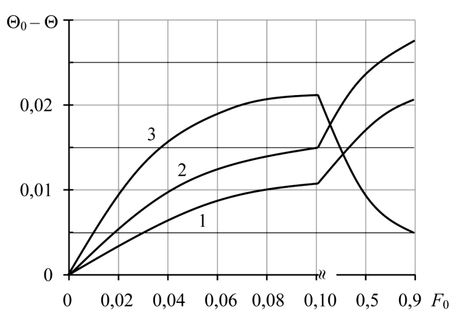

Fig. 8. Temperature difference dependence at the thermoelement on time F_0 at Θ_0=0.6; T=300 K; η_1=0 and B_{i1}=5 for different values of current density $\bar{\nu}$: 1 – 0.5; 2 – 1.0; 3 – 2.0

In Fig. 5–7, circles denote data on the estimation of function $\Delta\Theta(F_0)$ according to formula (7); triangles – using formulae (20) and (21). Comparison with the curves, built based on the results of calculating function $\Delta\Theta(F_0)$ by numerical methods, shows that at $F_0\in(0;0{,}05)$ a good approximation accuracy is ensured by only three terms of series (11) or (12). At $F_0>0.04$, sufficient accuracy is achieved when considering two terms of series (7).

The obtained ratios for calculating the cooling process could be used for a heating mode (in the case when the temperature at a heat-absorbing junction is maintained constant), after replacing magnitude ν in the relevant formulae with $-\nu$. Parameters δ_k will correspond to the roots of equation

$$\operatorname{tg}\delta=\frac{\delta}{\eta_1\delta^2+\nu-B_{i1}}.$$

The values of δ_k for the case of a heating mode are less than during cooling at identical values of ν, η_1 and B_{i1}. To determine the magnitudes of δ_1 and δ_2, it is possible to employ data from Table 1; in this case, the input parameter should be the difference $B_{i1}-\nu$.

Expression for a non-stationary temperature progress at stepwise current form can generally be represented only in the form of a series. This circumstance greatly complicates its analysis.

To obtain expression $\Delta\Theta$ in a time function, it is convenient to use a model of the thermoelement with an infinite length of branches. In this case, it should be taken into consideration that at a sufficiently short duration of the cooling process, after switching on current, the temperature field at a heat-generating junction does not affect temperature of the heat-absorbing junction. While a Fourier criterion $F_0=at/l^2$ is much smaller than unity, the progress of temperature at a heat-absorbing junction does not depend on the length of thermoelement branches, which is why it can be considered infinite.

Boundary conditions for heat equation (1)–(8) in this case are:

– at $x=0$

$$\lambda\frac{\partial T}{\partial x}=ejT-\alpha_1(T_0-T)+g_1\frac{\partial T}{\partial t}-\gamma^2\rho_k-q_l; \qquad (22)$$

– at $x\to\infty$

$$\frac{\partial T}{\partial x}=0. \qquad (23)$$

The heat-absorbing junction boundary condition takes into consideration the Peltier heat, heat transfer into the surrounding environment, heat capacity of the cooled object, Joule heat released on contacts, and the power of permanent sources of heat.

We shall consider the case when a heat-absorbing junction is insulated from the surrounding environment; then at $x=0$

$$\lambda\frac{\partial T}{\partial x}=ejT.$$

We shall reduce equation (1) to the dimensionless form; in this case, we shall employ ratio $l_e=\lambda/(ej)$ as the equivalent length. In this case

$$\frac{\partial^2\Theta}{\partial\chi^2}+1=\frac{\partial\Theta}{\partial(F_{0e})}; \qquad (24)$$

$$\Theta\big|_{F_{0e}}=\Theta_0; \qquad (25)$$

$$\begin{cases}\dfrac{\partial\Theta}{\partial\chi}=\Theta \text{ in } \chi=0;\\[2mm] \dfrac{\partial\Theta}{\partial\chi}=0 \text{ in } \chi\to\infty,\end{cases} \qquad (26)$$

where

$$\chi=\frac{x}{l_э}=\frac{ejx}{\lambda};\quad F_{0e}=\frac{dt}{l_e^2}=\frac{e^2\gamma^2at}{\lambda^2}.$$

Equation (24) is easily solved using an operational method, which is why a heat-absorbing junction temperature change over time can be written in the form

$$\Delta\Theta(F_0)=(1+\Theta_0)\left(1-\exp F_{0e}ezfc\sqrt{F_{0e}}\right)-2\sqrt{\frac{F_{0e}}{\pi}}. \qquad (27)$$

Thus, a heat-absorbing junction temperature change depends only on parameter F_{0e}, that is, on product j^2t. Expression (27) has a maximum at a specific value of argument $F_{0e}=F_{0e}^*$, derived from equation

$$\sqrt{\pi F_{0e}^*}\exp F_{0e}^*\cdot ezfc\sqrt{F_{0e}^*}=\frac{\Theta_0}{1+\Theta_0}. \qquad (28)$$

Thus, at Θ_0=0.6 $F_{0e}^*=0{,}081$, $\Delta\Theta_{\max}$=0.0895 and an increase in current j causes a corresponding decrease in the time of reaching a maximum, and vice versa. The magnitude of maximum $\Delta\Theta$ does not depend on current. The largest temperature difference at the thermoelement is achieved under a stationary mode:

$$\Delta\Theta'_{\max}=1+\Theta_0-\sqrt{1+2\Theta_0} \text{ at } \Theta_0=0.6,\ \Delta\Theta'_{\max}=0{,}117.$$

The largest temperature difference under a non-stationary mode is less than the maximum temperature difference under a stationary mode. Thus, at Θ_0=0.6, the ratio between these magnitudes is equal to 0.766. These data are also correct for a thermoelement with ultimate branch length l if the Fourier criterion at/l^2<<1.

The maximum drop of temperature at a heat-absorbing junction for a semi-infinite thermoelement occurs at

$$F_{0e}=F_{0e}^*=\frac{e^2j^2at^*}{\lambda^2},$$

that is, at

$$\frac{at^*}{l^2}=\left(\frac{\lambda}{ejl}\right)^2F_{0e}^*.$$

Thus, condition at/l^2<<1 is reduced to inequality $\nu>>\sqrt{F_{0e}^*}$. As demonstrated earlier (Table 2, formula (18)), the minimum value of current ν_0, at which there appears an extremum, is 0.791, and $\sqrt{F_0^*}=0{,}28$.

Starting from value $\nu=\nu_0$, a temperature difference magnitude at the point of extremum, calculated for models with branches of finite and semi-infinite length, is almost identical. Fig. 9 shows dependence of temperature difference $\Delta\Theta$ on ν (Curve 1) and value of $\Delta\Theta$ at the point of extremum of function $\Delta\Theta(F_0)$ (Curve 2) for the thermoelement with a finite length of branches. At $\nu=\nu_0$ the maximum occurs when $t\to\infty$, with an increase in ν, the value of $\sqrt{F_0^*}$ is reduced and at $\nu\to\infty$ $\sqrt{F_0^*}\to 0$.

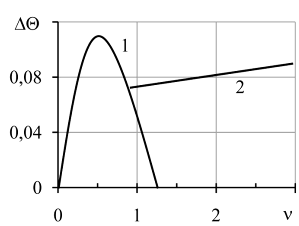

Fig. 9. Temperature difference dependence at the thermoelement with a thermally-insulated heat-absorbing junction on current density ν at Θ_0=0.6 and T=300 K under a stationary mode (1) and at the extremum point of function $\Delta\Theta(F_0)$ (2)

In a general case, when the boundary condition at a heat-absorbing junction is determined from ratio (22), it is easy to obtain an analytical expression that would yield dependence $\Delta\Theta_0$ on time. However, in this case, $\Delta\Theta_0$ is a function not only of dimensionless parameters Θ_0 and F_0, but also of dimensionless complexes:

$$B_{ie}=\frac{\alpha l_e}{\lambda}=\frac{\alpha}{ej};\ \ \eta_1=\frac{g_1}{cl_э}=\frac{g_1ej}{c\lambda};$$

$$\xi_k=\frac{\rho_k}{\rho l_e}=\frac{\rho_k ej}{\rho\lambda};\ \ K_l=\frac{q_l z l_e}{\lambda}=\frac{q_l z}{ej}.$$

The influence of thermal resistances at a junction decreases with increasing current while the effect of heat capacity and electrical resistance increases.

Fig. 10 shows, for the case B_{i1}=5 (η_1=0; ξ_k=0; K_l =0; Θ_0=0.6), dependence of $\Delta\Theta_{stat}$ (Curve 1) and $\Delta\Theta$ at the point of maximum (Curve 2) on current ν. Minimum value of current ν_0, at which there appears a maximum, is 0.37. In this case, $\nu_0<\nu_T^o$.

In the case under consideration, temperature difference at the point of maximum slightly exceeds the stationary. At $\nu\to\infty$, the magnitude of $\Delta\Theta$ coincides with a temperature difference at B_i=0. Curve 2 is built based on the results of computer calculations. Results of calculation by formula for the thermoelement with semi-infinite branches produce almost identical results, starting from $\nu\approx1$.

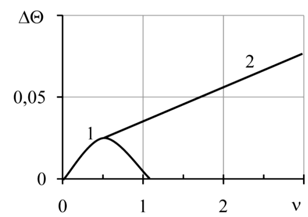

Fig. 10. Temperature difference dependence at the thermoelement on current density ν taking into consideration heat transfer at a heat-absorbing junction at T=300 K and B_{i1}=5 (η_1=0; ξ_k=0; K_l=0; Θ_0=0.6) under a stationary mode (1) and at the extremum point of function $\Delta\Theta(F_0)$ (2)

The presence of heat capacity and contact resistance results in the fact the value of $\Delta\Theta$ at the point of maximum falls with increasing current. In the case of the presence of resistance, the extremum occurs only at small enough currents at $\xi_k<\Theta_0$ or $j<\frac{\rho\lambda\Theta_0}{\rho_k l}$, when the Peltier heat magnitude exceeds, during initial point of time, the Joule heat at the contacts of a heat-absorbing junction.

6. Discussion of results of an analysis of the possibility to control the inertia of a thermoelectric cooler

The obtained ratios allow us to argue about the following:

– with a growth in heat transfer coefficient B_{i1}=5 and η_1=0 dependence $\Delta\Theta=f(\nu)$ acquires an extremum; a temperature difference at the extremum point surpasses the stationary ($\nu\geq1.0$);

– in order to obtain a simpler ratio $\Delta\Theta=f(F_0)$, it is convenient to use a model of the thermoelement with a semi-infinite branch length, at which the Fourier criterion $F_0=at/l^2$<<1. This ensures the possibility of computer implementation of the ratio in real time, as well as enables control over a temperature difference at a heat-absorbing junction in a time function;

– a growth of current leads to a decrease in the effect of thermal resistances at a heat-absorbing junction, while the effect of heat capacity and electric resistance increases;

– an increase in heat capacity at a heat-absorbing junction slows the rate of cooling.

Thus, when heat capacity of an object far exceeds the heat capacity of switching plates, additional effect of non-stationary cooling is negligible and comprises 1–2 % of the stationary.

The advantage of the proposed approach is enabling the possibility to improve response rate of the thermoelectric cooler on fluctuating thermal impact. This makes it possible to create thermoelectric systems for ensuring thermal regimes of thermally-loaded elements of enhanced energy stability. The effectiveness of such systems will be determined not only by the limitations, accepted when designing a model of the non-stationary thermoelectric cooler, but by control system as well. Input indicators of control system, the sensors applied, data processing algorithms should influence the inertia and reliability indicators of thermoelectric cooling system, which requires further research.

7. Conclusions

1. We designed a model of non-stationary working mode of the cooling thermoelement, in the framework of which we determined functional dependence of temperature of the heat-absorbing junction on time. The model takes into account heat capacity of the switching plates and the cooled object, heat load on a working junction, heat transfer to the environment.

2. We substantiated the possibility to control the rate of cooling and heating a thermoelement when changing the device's working current at different thermal loads. Thus, when current density increases by four times, the same temperature difference at the thermoelement during cooling is reached 8–10 times faster.

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