Представлена математична модель комбінованого вилучення органічних забруднень (ОЗ) в аеротенках-змішувачах завислим та закріпленим біоценозом. Наведені чисельні розрахунки при різних значеннях впливових параметрів. Використання рівняння Моно для опису швидкості окислення дозволяє моделювати процес біологічної очистки в широкому діапазоні концентрацій ОЗ та оцінити ефективність роботи аеротенків-змішувачів з біоплівкою на додатковому завантаженні

Ключові слова: органічні забруднення, аеротенк-змішувач, активний мул, біоплівка, кінетика реакцій, рівняння Моно, додаткове завантаження

Представлена математическая модель совместного извлечения органических загрязнений (ОЗ) в аэротенках-смесителях взвешенным и прикрепленным биоценозом. Выполнены численные расчеты при разных значениях влияемых параметров. Использование уравнения Моно для описания скорости окисления позволяет моделировать процесс биологической очистки в широком диапазоне концентраций ОЗ и оценить эффективность работы аэротенков-смесителей с биопленкой на дополнительной загрузке

Ключевые слова: органические загрязнения, аэротенк-смеситель, активный ил, биопленка, кинетика реакций, уравнение Моно, дополнительная загрузка

UDC 628.35
DOI: 10.15587/1729-4061.2017.118999

THE USE OF NONLINEAR MONO KINETICS IN MODELING A MIXING AERATION TANK WITH A BIOFILM ON ADDITIONAL LOADING

A. Oleynik
Doctor of Technical Sciences, Professor,
Corresponding Member of the NAS of Ukraine*
E-mail: kurganska@ukr.net
Yu. Kalugin
PhD, Senior Researcher*
E-mail: forkalugin@ukr.net
T. Airapetian
PhD, Associate Professor
Department of Water supply,
sewerage and purification of waters
O. M. Beketov National University of
Urban Economy in Kharkiv
Marshal Bazhanov str., 17, Kharkiv, Ukraine, 61002
E-mail: tamara78kh2008@rambler.ru
*Department of applied hydrodynamics
Institute of Hydromechanics of the NAS of Ukraine
Zhelyabova str., 8/4, Kyiv, Ukraine, 03680

1. Introduction

In practice, to clean wastewater from organic pollutants, it has been common to use biological methods of aerobic purification in aeration bioreactors, mainly in mixing aeration tanks – aerotanks-mixers [1–6]. It is known that in aerotanks-mixers, incoming wastewater, active sludge (suspended biocenosis in water) and oxygen are almost instantaneously mixed; therefore, the concentrations of microorganisms, pollutants, and dissolved oxygen are assumed to be identical throughout the volume of the reactor.

However, at most sewage treatment plants, traditional technologies of biological treatment in modern conditions do not provide efficient and reliable treatment of sewage, both from organic pollutants and from compounds of nitrogen and phosphorus, the standards of residual concentration of which in purified water have considerably increased [7–9]. Therefore, studies that are aimed at increasing the efficiency of aeration facilities and obtaining a mathematical description of the processes of biological wastewater treatment are essentially important.

The efficiency of biological treatment of wastewater from organic pollutants (OPs) in aerotanks and ensuring a high level of purification can be improved due to installing additional loading with a fixed biocenosis in the areas of aerotanks [1, 2, 10].

The existing theoretical developments are based on implementing simplified approaches (models) that do not reflect a number of important components of purification [11, 12]. The parameters of the biofilm, the hydrodynamic peculiarities of the intake of organic pollutants and oxygen in the biofilm, the speed of kinetic reactions with the joint removal of organic pollutants with suspended and fixed biocenoses are not taken into account sufficiently. The failure to consider these factors significantly affects the obtaining of reliable calculation results.

Only on the basis of more complete and improved mathematical models, it is possible to obtain reliable engineering methods for calculating the structural and technological parameters of bioreactors. The development of such ideal mathematical models requires a more complete study of the mechanisms and peculiarities of the effects of the processes that occur in the biofilm and in the volume of aerotanks with a mixed biocenosis in the treatment of sewage.

2. Literature review and problem statement

The analysis of literature review and the performed tests have shown the feasibility and efficiency of placing an additional loading (grids, nozzles, etc.) in the aerotank volume along with the suspended biocenosis (active sludge). The use of a fixed biocenosis due to considerably higher concentrations of microorganisms and a lower value of the half-saturation constant helps improve the quality of OP extraction in aerotanks [13, 14].

However, there are objective difficulties that relate to the reliability of simulating these processes. Thus, in [15], there is a substantiation of mathematical models based on which the methods of calculating the extraction of OPs in aerotanks-mixers are validated for the estimation and analysis of the joint extraction of OPs by suspended and fixed biocenoses. In the indicated models, when the OPs, with both suspended and fixed biocenoses, had been removed, the linear kinetics of the reactions of the first and the zero orders were adopted. However, their use is significantly limited, in particular, by the values of the wastewater concentrations that enter purification [14, 15]. Using the oxidation rate values of the zero and first order dependencies allows obtaining accurate analytical solutions, but in the range of either large values of the OP concentrations or small ones.

Generally, with sufficient justification in special studies, it is suggested to remove OPs with the use of the models that are based of the kinetics of reactions which are described by the nonlinear Mono equation.

Based on the use of the obtained solutions for the reactions of the first and the zero orders, approximate methods of calculating the required parameters are proposed in accordance with the Mono equation. Thus, in articles [16, 17], to determine the concentration L in a biofilm, in particular on the external surface $L_{\delta 0}$, it is suggested to solve the problem by an iterative method. The mentioned articles provide the contents and sequence of using the proposed iteration method of calculation. In papers [18, 19], the flow of organic contamination through the biofilm in the case of the kinetics of the reaction by the Mono equation is sufficiently substantiated to be defined as the suspended average N_c that is obtained from the fluxes at zero and first order reactions. In paper [20], instead of considering N_c flows, this method is proposed to be used in calculating the concentration L_{δ_0} on the surface of the biofilm.

Using the Mono equation in a mathematical model that describes the simultaneous removal of OPs by suspended and fixed biocenoses will allow extending the scope of its application and assessing the effectiveness of aerotanks-mixers with biofilms on additional loading.

3. The aim and objectives of the study

The aim is to develop a mathematical model and calculation methods in which the kinetics of OP extraction reactions in aerotanks-mixers with suspended and fixed biocenoses are described by the nonlinear Mono equation, which is directly taken into account in solving the problem.

To achieve this aim, the following objectives need to be addressed:

– to present a general form of the basic equations and dependencies describing the process of biological purification in aerotanks-mixers with an additionally attached biomass, taking into account the peculiarities of the simultaneous removal of organic pollutants by the biofilm and the active sludge;

– to develop numerical and analytical methods for calculating the flow of pollutants withdrawn by the fixed biocenosis, as well as the input and output pollution flows to assess the possible effect of purification;

– to justify the simplifications that will help obtain analytical dependencies for the possibility of engineering calculations;

– to consider possible variants of arranging a reactor with a fixed biomass.

4. Materials and methods of researching biological purification in aerotanks-mixers with suspended and fixed biomasses

It is assumed that the process of biochemical oxidation is provided with oxygen in sufficient quantities, that is, the flow of oxygen will not limit the kinetics of biooxidation of both suspended and fixed biocenoses. Thus, in the absence of the inhibitory effect on the rate of removal response according to the nonlinear Mono equation, we describe a fixed biocenosis in the biofilm

$$R_L = \frac{\rho_m L}{K_{m_L} + L}, \tag{1}$$

$$\rho_m = \frac{\mu_m X}{Y} \tag{2}$$

and a suspended biocenosis (active sludge) in the volume of the aerotank-mixer

$$R_a = \frac{\rho_{m_a} L_a}{K_{m_a} + L_a}, \tag{3}$$

$$\rho_{m_a} = \frac{\mu_{m_a} X_a}{Y_a}. \tag{4}$$

Let us note that according to [5, 9], the value of ρ_{m_a} is taken as

$$\rho_{m_a} = \frac{\mu_{m_a} X_a}{Y_a\left(1+\phi a_i\right)}. \tag{5}$$

As is known, the general equation of the material balance of pollution in the aerotank-mixer has the form [14]

$$W_p \frac{dL_a}{dt} = Q_a\left(L_0 - L_a\right) - F_{\delta_n} N - R_{a_0} W_p. \tag{6}$$

According to [14], for practical calculations of equation (6) can be significantly simplified to the form for an ideal aerotank-mixer

$$L_0 - L_a - F_{\delta_n} T_a N - T_a\, R_{a_0} = 0, \tag{7}$$

$$F_{\delta_n} = \frac{F_{\delta_l}}{W_a} = \frac{\varepsilon F_{\delta_l}}{W_p}, \quad \varepsilon = 1 - \frac{W_\delta}{W_a} = \frac{W_p}{W_a},$$

$$T_a = \frac{W_p}{Q_a}, \quad R_{a_0} = R_a - R_{b_a}.$$

Here, the R_{b_a} is the velocity of distinguishing OPs in the process of active sludge dying; in accordance with [14, 21], it is taken as

$$R_{b_a} = b_a X_a, \tag{8}$$

where b_a is the constant of the dying rate;

$$N = K_L\left(L_a - L\big|_{z=0}\right), \tag{9}$$

$$L\big|_{z=0} = L_{\delta_0} \text{ at } \bar{z} = 0, \tag{10}$$

where N is the flow (transport) of OPs to the surface of the biofilm from the volume of the liquid (aerotank) through the liquid film (boundary layer) for further utilization by the fixed biocenosis (biofilm).

In accordance with the existing norms [5], it is proposed to determine the aeration duration T_a in aerotanks at known (given) concentrations of sewage entering the aerotank L_o and purified sewage L_a by the formula

$$T_a = \frac{L_0 - L_a}{F_{\delta_n} N + R_{a_0}}. \tag{11}$$

The value of the OP flux on the surface of the biofilm N, as well as the changes in the concentration $L(z)$ in the thickness of the biofilm, including the concentrations $L_{\delta 0}$ and $L_{\delta 1}$, are the result of solving the equation that characterizes the degree of removing the OPs with the biofilm. In this case, the biofilm is formed on the surface of the loading material, and the concentration X is saturated with microorganisms. Generally, this equation for the biofilm has the form

$$\frac{\partial L}{\partial t} = D_L \frac{\partial^2 L}{\partial z^2} - R_L. \tag{12}$$

In the conditions of cylindrical loading elements on which the biofilm is formed, the equation for the biofilm will have the form

$$\frac{\partial L}{\partial t} = D_L \left(\frac{\partial^2 L}{\partial z^2} + \frac{1}{r}\frac{\partial L}{\partial r} \right) - R_L. \tag{13}$$

5. Results of researching the biological purification process in aerotanks-mixers with a fixed biomass

The solution of equations (12) and (13) allows us to determine the concentration of OPs in the thickness of the biofilm L and, most importantly, for further calculations, the concentration of OPs on the external and internal surfaces of the biofilm – L_{δ_0} and L_{δ_1}.

According to the analysis that was carried out for practical calculations, the solution of equations (12) and (13) is sufficiently performed in stationary conditions that occur fairly quickly, that is, at

$$\frac{\partial L}{\partial t} = 0.$$

In this case, equations (12) and (13) are somewhat simplified. In particular, the solution of these equations for reactions of the first and the zero orders is obtained in [14, 15]. Let us consider the solution of equation (12) under the conditions of the nonlinear reaction R_L according to the Mono equation (1), which in a dimensionless form and under the accepted boundary conditions will have the form

$$\frac{d^2\bar{L}}{d\bar{z}^2} - \alpha_L \frac{\bar{L}}{\bar{K}_{m_L} + \bar{L}} = 0, \tag{14}$$

$$\text{– at } \bar{z} = 0 \quad -\frac{d\bar{L}}{d\bar{z}} = \beta_L\left(1 - \bar{L}\right),$$

$$\text{– at } \bar{z} = 1 \quad \frac{d\bar{L}}{d\bar{z}} = 0. \tag{15}$$

$$\bar{z} = \frac{z}{\delta},$$

$$\bar{L} = \frac{L}{L_a},$$

$$\bar{K}_{m_L} = \frac{K_{m_L}}{L_a},$$

$$\alpha_L = \frac{\mu_m X \delta^2}{Y D_L L_a},$$

$$\beta_L = \frac{K_L \delta}{D_L}.$$

As a result of solving equation (14), numerical methods based on the developed program have helped construct calculation graphs for determining the concentrations on the outer surface of the biofilm $L_{z=0} = L_{\delta_0}$ $(z = 0)$ and on the inner surface $L_{z=\delta} = L_{\delta_1}$ $(z = \delta)$ (Fig. 1). These graphs are constructed for different values of the coefficient of the OP transfer in the liquid film K_L, which largely depends on its thickness δ_p, and different saturation (half-saturation) constants $\bar{K}_{m_L}$.

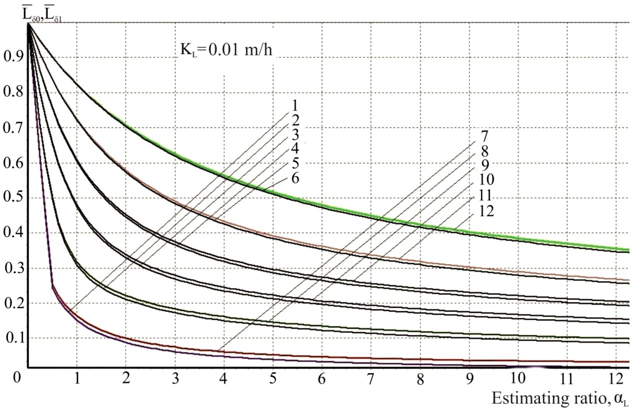

Fig. 1. Graphs for determining the OP concentrations on the outer $\bar{L}_{\delta_0} = \frac{L_{\delta_0}}{L_a}$ and inner $\bar{L}_{\delta_1} = \frac{L_{\delta_1}}{L_a}$ surfaces of the biofilm for different values of $\bar{K}_{m_L} = \frac{K_{m_L}}{L_a}$ for the concentration $\bar{L}_{\delta_1}$ (at $\bar{z} = \frac{z}{\delta} = 1$), mg/l: 1 – 0; 2 – 0.2; 3 – 0.7; 4 – 1.5; 5 – 3.0; 6 – 6.0; for the concentration $\bar{L}_{\delta_0}$ (at $\bar{z} = 0$): 7 – 0; 8 – 0.2; 9 – 0.7; 10 – 1.5; 11 – 3.0; 12 – 6.0 at $K_L = 0.01$ m/h

In this case, the general dependence (9) for determining the flow into the biofilm N_L will look as follows

$$N_L = K_L L_a (1 - A), \tag{16}$$

$$A = \frac{L_{\delta_0}}{L_a}. \tag{17}$$

For the convenience of further calculations, using the dimensionless units and parameters in the model, dependence (16) can be represented as

$$N_L = \bar{N}_L \lambda_L, \tag{18}$$

$$\bar{N}_L = \beta_L (1 - A), \tag{19}$$

where

$$\lambda_L = \frac{D_L}{\delta} L_a, \quad \beta_L = \frac{K_L \delta}{D_L}.$$

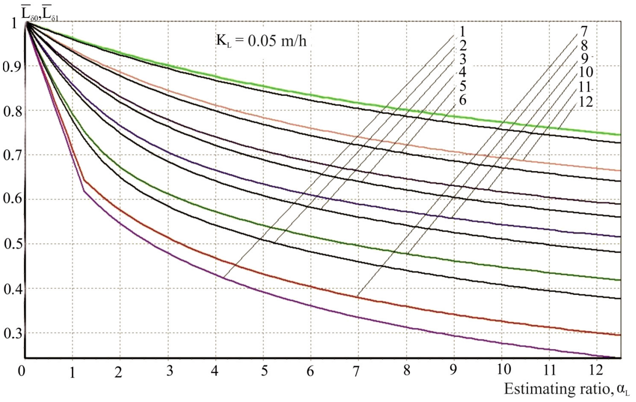

Fig. 2. Graphs for determining the OP concentrations on the outer $\bar{L}_{\delta_0} = \frac{L_{\delta_0}}{L_a}$ and inner $\bar{L}_{\delta_1} = \frac{L_{\delta_1}}{L_a}$ surfaces of the biofilm for different values of $\bar{K}_{m_L} = \frac{K_{m_L}}{L_a}$ for the concentration $\bar{L}_{\delta_1}$ (at $\bar{z} = \frac{z}{\delta} = 1$), mg/l: 1 – 0; 2 – 0.2; 3 – 0.7; 4 – 1.5; 5 – 3.0; 6 – 6.0; for the concentration $\bar{L}_{\delta_0}$ (at $\bar{z} = 0$): 7 – 0; 8 – 0.2; 9 – 0.7; 10 – 1.5; 11 – 3.0; 12 – 6.0 at $K_L = 0.05$ m/h

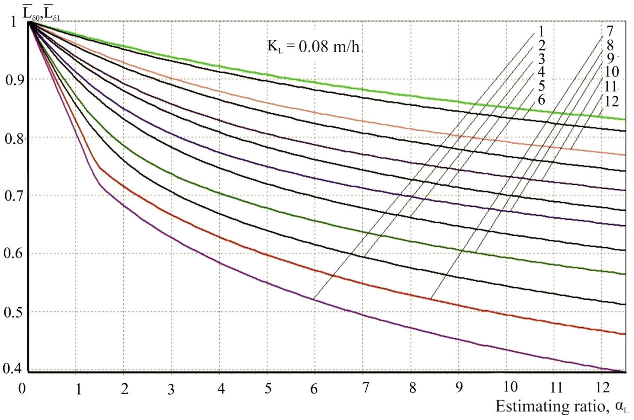

Fig. 3. Graphs for determining the OP concentrations on the outer $\bar{L}_{\delta_0} = \frac{L_{\delta_0}}{L_a}$ and inner $\bar{L}_{\delta_1} = \frac{L_{\delta_1}}{L_a}$ surfaces of the biofilm for different values of $\bar{K}_{m_L} = \frac{K_{m_L}}{L_a}$ for the concentration $\bar{L}_{\delta_1}$ (at $\bar{z} = \frac{z}{\delta} = 1$), mg/l: 1 – 0; 2 – 0.2; 3 – 0.7; 4 – 1.5; 5 – 3.0; 6 – 6.0; for the concentration $\bar{L}_{\delta_0}$ (at $\bar{z} = 0$): 7 – 0; 8 – 0.2; 9 – 0.7; 10 – 1.5; 11 – 3.0; 12 – 6.0 at $K_L = 0.08$ m/h

As was noted in the published studies [14, 20, 22, 23], at relatively low concentrations of OPs, $K_{m_L} >> L$, and at significant concentrations, $K_{m_L} << L$, so in these cases the kinetics of reactions in the biofilm R_L can be taken as the first and the zero orders, respectively. Some approximate estimates of using the practical calculations of these reactions are given, in particular, in papers [17, 20]. The obtained results of calculating OP extraction in a biofilm, in particular the concentration on the biofilm surface L_{δ_0} on the basis of the nonlinear kinetics of the Mono reaction, help obtain more reliable evaluation recommendations. To do this, a comparative analysis was carried out for the values of the concentrations L_{δ_0} and L_{δ_1}, numerically determined by the methods for the nonlinear Mono reaction, with certain analytical methods for linear reactions in works [14, 15]. In particular, let us remind that the concentration L_{δ_0} is determined in this case as follows

$$L_{\delta_{00}} = L_a \left(1 - \frac{\rho_m \delta}{K_L}\right) = L_a \left(1 - \frac{\alpha_L}{\beta_L}\right), \tag{20}$$

$$L_{\delta_{01}} = A_L L_a = L_a \frac{ch\left(\sqrt{\alpha_1}\right)}{ch\left(\sqrt{\alpha_1}\right) + \lambda_L sh\left(\sqrt{\alpha_1}\right)}, \tag{21}$$

where

$$\rho_m = \frac{\mu_m X}{Y}, \quad \alpha_1 = \frac{\alpha_L}{\bar{K}_{m_L}}, \quad \lambda_L = \frac{\sqrt{\alpha_1}}{\beta_L}, \quad \bar{K}_{m_L} = \frac{K_{m_L}}{L_a}.$$

In order to determine the parameter A_L, an estimating graph is constructed in [14, 15].

According to the performed analysis, it can be assumed that the kinetics of the reactions of the first and the zero orders in practical calculations can correspond to the ratios

$$\bar{K}_{m_L} = \frac{K_{m_L}}{L_a} > 2 - 3 \text{ and } \bar{K}_{m_L} < 0.15 - 0.2.$$

That is, within the limits $0.15 < \bar{K}_{m_L} < 3$ of calculating the kinetics of the reactions R_L, it is necessary to accept the Mono equation. To use it, it is also possible to apply the obtained analytical solution of the general equation (14) which, under the boundary condition $\frac{d\bar{L}}{d\bar{z}} = 0$ at $\bar{z} = 1$ will have the form

$$\frac{d\bar{L}}{d\bar{z}} = \sqrt{2\alpha_L} \sqrt{\bar{L} - \bar{L}_{\delta_1} - \bar{K}_{m_L} \ln\left(\frac{\bar{L} + \bar{K}_{m_L}}{\bar{L}_{\delta_1} + \bar{K}_{m_L}}\right)}. \tag{22}$$

Thus, depending on the concentration $\bar{L}$, the relative flow in the biofilm $\bar{N}_L$ will be determined as follows

$$\bar{N}_L = -\frac{d\bar{L}}{d\bar{z}} = -2\sqrt{\alpha_L} \sqrt{\bar{L} - \bar{L}_{\delta_1} - \bar{K}_{m_L} \ln\left(\frac{\bar{L} + \bar{K}_{m_L}}{\bar{L}_{\delta_1} + \bar{K}_{m_L}}\right)}. \tag{23}$$

Taking in the equation that $\bar{L} = \bar{L}_{\delta_0}$ and comparing it with equation (19), we obtain the following equation that binds the concentrations $\bar{L}_{\delta_0}$ and $\bar{L}_{\delta_1}$ both on the outer and inner surfaces of the biofilm:

$$\bar{L}_{\delta_0} = 1 + \frac{\sqrt{\alpha_L}}{\beta_L} \sqrt{\bar{L}_{\delta_0} - \bar{L}_{\delta_1} - \bar{K}_{m_L} \ln\left(\frac{\bar{L}_{\delta_0} + \bar{K}_{m_L}}{\bar{L}_{\delta_1} + \bar{K}_{m_L}}\right)}. \tag{24}$$

Let us represent dependence (20) for more convenient use in calculations in the form

$$\bar{L}_{\delta_0} = \bar{L}_{\delta_1} - \bar{K}_{m_L} \ln\left|\frac{\bar{L}_{\delta_0} + \bar{K}_{m_L}}{\bar{L}_{\delta_1} + \bar{K}_{m_L}}\right| + \frac{\beta_L^2}{2\alpha_L}\left(\bar{L}_{\delta_0} - 1\right). \quad (25)$$

At $\bar{L}_{\delta_0} >> \bar{L}_{\delta_1}$, it is possible to neglect the concentration $\bar{L}_{\delta_1}$ in dependence (25). This helps, in practical calculations, determine the concentration L_{δ_0} and the flow *N*, at which there is a transition from a partially penetrated to a fully permeable biofilm. In this case, the initial parameters, reactions and thickness of the biofilm are given.

Calculations using the graphs of Fig. 1 and the above dependencies allow us to estimate the work of the biofilm of the given thickness δ as to the nature of the penetration of organic pollutants into it. In particular, this allows us to establish the expediency and even the need for the flow recirculation in the adopted technological scheme of an aerotank with suspended and fixed biocenoses.

As was already noted in [14, 15], in determining the optimal parameters of aerotanks, various arrangements of the loading elements in the aerotank volume are possible, and the required surface (biofilm) area is taken as F_{δ_l}. In this case, the loading elements (nozzles, grids, etc.) may be located throughout the aerotank volume or denser and more compactly only in its individual sections. Depending on the layout of the loading elements on the length and the responses taken in the biofilm and the aerotank according to the nonlinear Mono equation, the general equations (1) and (2) can be greatly simplified.

Next, let us consider the most practically feasible technological schemes.

The load elements are not sufficiently evenly distributed throughout the length of the aerotank (Fig. 4). In this case, the area F_{δ_l} is the total loading area in the aerotank of the length *l*, and $F_{\delta_n} = \frac{F_{\delta_l}}{W_a}$ is the specific loading area.

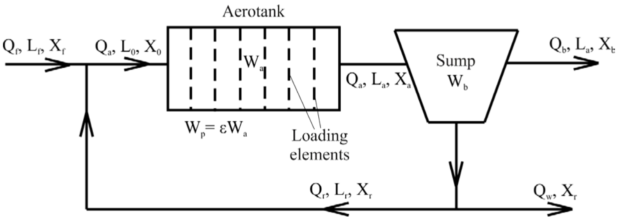

Fig. 4. A balance diagram of the aerotank-mixer with a fully secured biocenosis

It is noteworthy that in [14, 15] the solution to this problem is obtained in the case when the joint removal of OPs occurs in a zero-order reaction, and in the biofilm with a fixed biocenosis, it is the first-order reaction. In this case, in the biofilm and in the aerotank fluid volume, the OP extraction is carried out by the reaction according to the Mono equation, by formulae (1) and (2) at $R_{a_0} \approx R_a$. That is, the general equation will have the form

$$T_a = \frac{L_0 - L_a}{F_{\delta_n} N + R_a}. \quad (26)$$

In certain cases, namely, taking that *N*=0 in formula (25), we get a known dependence when the removal of OPs in aerotanks occurs only due to the suspended biocenosis (active sludge), and at $R_a = 0$, we observe a dependence when the removal of OPs in the aerotank occurs only at the expense of the fixed biocenosis (biofilm).

As it was noted above, the technological scheme of the aerotank-mixer can consist of two parts, which we will call reactors 1 and 2. In this case, in one of the reactors, OPs are extracted by the suspended biocenosis (active sludge), and in the second reactor, it is done by the fixed biocenosis (biofilm). It should be noted that in both parts of the aerotank, the reactors operate according to the reactor-mixer scheme. Such technological schemes with different arrangements of the reactors in aerotanks-mixers, in which the removal of OPs occurs due to reactions of the first and the zero orders, are considered in works [14, 15].

Let us consider the case when the removal of OPs in reactor 1 occurs due to the suspended biocenosis. Then reactor 1 operates as an ordinary aerotank-mixer, and in reactor 2 the extraction takes place at the expense of the fixed biomass (Fig. 5) on the herein installed load. Such a technological scheme of purification from a practical point of view will be expedient and more in line with the modern requirements of ensuring a high degree of purification. Then, in accordance with the general equation for determining the concentrations of OPs at the exit from reactors 1 and 2, we use the following equations:

– for reactor 1:

$$L_0 - L_{a_1} - T_{a_1} R_{a_1} = 0, \quad (27)$$

$$T_{a_1} = \frac{L_0 - L_{a_1}}{R_{a_1}}, \quad (28)$$

– for reactor 2:

$$L_{a_1} - L_{a_2} - F_{\delta_{n2}} T_{a_2} N_2 = 0, \quad (29)$$

$$T_{a_2} = \frac{L_{a_1} - L_{a_2}}{F_{\delta_{n2}} N_2}. \quad (30)$$

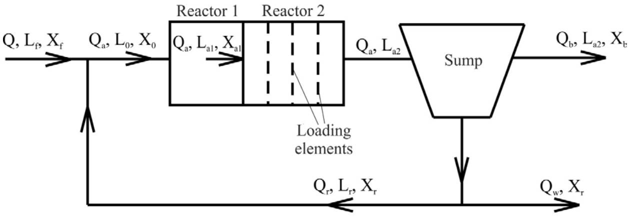

Fig. 5. A balance diagram of the aerotank-mixer with the fixed biocenosis located in reactor 2

In the given dependencies (27)–(30),

$$T_{a_1} = \frac{W_{p_1}}{Q_a},\; W_{a_1} = W_{p_1},\; T_{a_2} = \frac{W_{p_2}}{Q_a},\; W_{p_2} = W_{a_2}\varepsilon_2,$$

$$\varepsilon_2 = 1 - \frac{W_{\delta_2}}{W_{a_2}} = \frac{W_{p_2}}{W_{a_2}},\; F_{\delta_{n2}} = \frac{F_{\delta_{l2}}}{W_{p_2}},\; N_2 = K_{L_2}\left(L_{a_2} - L_{\delta_2}\right),$$

where $F_{\delta_{l2}}$ is the total area of the surface of the biofilm (loading) in reactor 2 with the length l_2 and the volume W_{a_2}. The rate of reactions of OP removal in reactors 1 and 2 by the active sludge and the biofilm corresponds to the nonlinear Mono equations, namely

$$R_{a_1} = \frac{\rho_{m_{a_1}} L_{a_1}}{K_{m_{a1}} + L_{a1}},\; \rho_{m_{a1}} = \frac{\mu_{m_{a1}} X_{a1}}{Y_{a1}} \quad (31)$$

and

$$R_{L_2}=\frac{\rho_{m_2}L_2}{K_{m_2}+L_2},\quad \rho_{m_2}=\frac{\mu_{m_2}X_2}{Y_2}. \qquad (32)$$

The parameters with index 1 refer to reactor 1; with index 2, they refer to reactor 2; and the explanation of the accepted parameters can be found in articles [14, 15].

6. Discussion of the results of studying the biological purification process in aerotanks-mixers with a fixed biomass

The implementation of the proposed models and methods of calculation allows estimating the influence of various purification factors in aerotanks-mixers in which the purification intensity increases due to additional elements with a fixed biocenosis. In the presented and implemented models, the parameters of the biofilm, the hydrodynamic peculiarities of the intake of organic pollutants, the speed of kinetic reactions with the joint removal of organic pollutants with suspended and fixed biocenoses are taken into account.

The conducted tests make it possible to substantiate the most economical and effective parameters of such structures of biological treatment as aerotanks which are used at stations of cleaning sewage. This can significantly improve the quality of using them to remove various dissolved substances, in particular of organic origin, from the wastewater.

The performed tests are a continuation of previous studies on this topic, which considered the removal of OPs in aerotanks-mixers with an additionally attached biomass if this process was adequately provided with oxygen [14, 15, 24].

Further research will be based on analyzing and evaluating the impact of such an important parameter as the thickness of the biofilm. In particular, it is essential to consider the effect produced on the formation of the active part of the biofilm in time by such important processes as growth, accumulation, decay and separation on the surface. It is also important to ensure the processes of purification (oxidation) by the required amount of oxygen, which has already been partially considered in works [25–27].

The proposed mathematical models, numerical and analytical methods of calculation with the use of kinetic reactions according to the known Mono equation, allow us to substantiate more reliably the parameters of sewage treatment against OPs in aerotanks-mixers. They also provide an opportunity to assess the significant efficiency of aerotanks through the introduction of additional loading with a fixed biocenosis (biofilm).

7. Conclusions

1. A mathematical model of the work of aerotanks-mixers is presented, taking into account simultaneous oxidation of organic pollutants (OPs) by suspended and fixed biocenoses, the velocity of which is described by the general Mono equation of kinetics. Such a model helps simulate biological purification in aerotanks-mixers by a fixed biocenosis with a wide range of OP concentrations.

2. To assess the flow of pollutants withdrawn by the fixed biocenosis, numerical calculations have been carried out at various values of the influential parameters, the results of which are presented in the form of graphs. The given graphs allow determining the concentrations of OPs on the external and internal surfaces of the biofilm.

3. The determined parameters of sewage treatment help estimate the value of pollution flows at the entrance and exit of the tank and thus assess the possible effect of cleaning.

4. A number of assumptions have been adopted to simplify the mathematical model and obtain analytical dependencies for the possibility of engineering calculations. It is assumed that in the biofilm and in the volume of aerotanks, the process of OP oxidation occurs by the reaction according to the Mono equation.

5. Dependencies have been calculated for various technological schemes of the fixed biomass location. They represent the OP concentration dependence at the outlet of the aerotank-mixer, which contains elements with a fixed biocenosis, on its parameters such as length, surface area of additional elements, sewage flow velocity, etc.

References

1. Biological Wastewater Treatment: Principles, Modelling and Design [Text] / M. Henze, M. van Loosdrecht, G. Ekama, D. Brdjanovic (Eds.). – Iwa Publishing, London, 2008. – 511 p.
2. Vasilenko, A. Reconstruction and intensification of constructions of water supply and water removal [Text]: tutorial / A. Vasilenko, P. Grabovsky, G. Larkina et. al. – Kyiv–Odessa: KNUCA-OSCA, 2007. – 307 p.
3. Zhmur, N. Technological and biochemical processes of sewage treatment in constructions with aerotanks [Text] / N. Zhmur. – Moscow: AKVAROS, 2003. – 512 p.
4. Zapolskiy, A. K. Water supply, water removal and quality of water [Text]: handbook / A. K. Zapolskiy. – Kyiv: Vyshcha shkola, 2005. – 617 p.
5. DBN V.2.5-75-2013. Sewerage. Outdoor networks and structures. The main provisions of the design [Text]. – Kyiv: Ministry of Regional Development of Ukraine, 2013. – 128 p.
6. Yakovlev, S. Water removal and waste waters treatment [Text] / S. Yakovlev, Yu. Voronov. – Moscow: ASV, 2002. – 704 p.
7. Kulikov, N. I. Theoretical bases of water treating [Text]: tutorial / N. I. Kulikov, A. Ya. Nymans, N. P. Omelchenko, V. N. Tchernyshov. – Donetsk: Izd. «Noulidzh» (Donetskoe otdelenie), 2009. – 298 p.
8. Shamsutdinova, Z. R. Analysis of the aerotanks efficiency in wastewater treatment system [Text] / Z. R. Shamsutdinova, I. I. Khafizov // Proceedings of the Voronezh State University of Engineering Technologies. – 2016. – Issue 4. – P. 245–249. doi: 10.20914/2310-1202-2016-4-245-249
9. Oleynik, A. Features of modeling wastewater treatment in the system aerotank-settle tank-regenerator [Text] / A. Oleynik, S. Zyablikov // Problems of water supply, drainage and hydraulic. – 2005. – Issue 4. – P. 46–53.

10. Von Sperling, M. Activated Sludge and Aerobic Biofilm Reactors [Text] / M. von Sperling. – London IWA Publishing, 2007. – 328 p.
11. Vavylyn, V. Matematic modeling of the processes of the biological treatment of the waste waters [Text] / V. Vavylyn, V. Vasilyev. – Moscow: Science, 1979. – 116 p.
12. Makinia, J. Mathematical Modelling and Computer Simulation of Activated Sludge Systems [Text] / J. Makinia. – Iwa Publishing, London, New York, 2010. – 408 p.
13. Sánchez, O. Environmental engineering and activated sludge processes: models, methodologies, and applications [Text] / O. Sánchez. – Oakville, ON; Waretown, NJ: Apple Academic Press, 2016. – 354 p. doi: 10.1201/b19902
14. Oleynik, A. Modelling of the waste water treatment from the organic contaminations in the bioreactors-aerotanks with suspended (free flowing) and fixed biocenosis [Text] / A. Oleynik, T. Airapetian // Reports of the National Academy of the Sciences of the Ukraine. – 2015. – Issue 5. – P. 56–61.
15. Oleynik, A. The aerobic biological purification of the wastewaters from the organic contaminants (OC) in the aerotanks with the suspended and the fixed biocenosis [Text] / A. Oleynik, T. Airapetian // MOTROL. Commission of motorization and energetics in agriculture. – 2016. – Vol. 18, Issue 10. – P. 13–24.
16. Hu, W.-S. Engineering Principles in Biotechnology [Text] / W.-S. Hu. – John Wiley & Sons Ltd, 2017. – 504 p. doi: 10.1002/9781119159056
17. Oleynik, A. Modelling the processes of the additional treatment of the urban-communal waste waters on the filters [Text] / A. Oleynik, T. Vasilenko, S. Rybachenko, Hamid Ihao Achmad // The Problems of the water supply, drainage and hydraulic. – 2006. – Issue 7. – P. 85–97.
18. Pérez, J. Modeling biofilm and floc diffusion processes based on analytical solution of reaction-diffusion equations [Text] / J. Pérez, C. Picioreanu, M. van Loosdrecht // Water Research. – 2005. – Vol. 39, Issue 7. – P. 1311–1323. doi: 10.1016/j.watres.2004.12.020
19. Wanner, O. Matematical modeling of biofilms [Text] / O. Wanner, N. I. Ebert, B. E. Rittman // Scientificand Technical report No. 18. – 2006. – 208 p.
20. Oleynik, A. Modelling and calculation of the biological sewage treatment on the trickling biofilters [Text] / A. Oleynik, O. Kolpakova // Ecological safety and make use of nature. – 2014. – Issue 16. – P. 68–86.
21. Henze, M. Wastewater Treatment: Biological and Chemical Processes [Text] / M. Henze, P. Harremoes, J. Cour, J. la, E. Arvin. – 3rd ed. – Berlin, Springer, 2002. – 430 p.
22. Nielsen, J. Bioreactors: Description and Modelling [Text] / J. Nielsen, J. Villadsen // Biotechnology Set. – 2008. – P. 77–104. doi: 10.1002/9783527620999.ch5b
23. Lee, J. Development of a model to determine mass transfer coefficient and oxygen solubility in bioreactors [Text] / J. Lee // Heliyon. – 2017. – Vol. 3, Issue 2. – P. e00248. doi: 10.1016/j.heliyon.2017.e00248
24. Airapetian, T. S. A modeling of the oxygen regime in bioreactors-aerotanks at the purification of waste waters from organic pollutants [Text] / T. S. Airapetian, S. V. Telyma, O. J. Oliynik // Reports of the National Academy of Sciences of Ukraine. – 2017. – Issue 6. – P. 21–27. doi: 10.15407/dopovidi2017.06.021
25. Oleynik, A. Oxygen regime in research on the work of plugflow aeration tanks with fixed biomasses [Text] / A. Oleynik, T. Airapetian // Eastern-European Journal of Enterprise Technologies. – 2017. – Vol. 4, Issue 10 (88). – P. 4–11. doi: 10.15587/1729-4061.2017.107127
26. Lothman, S. Comparison of Oxygen Transfer and Uptake Between an Integrated Fixed-Film Activated Sludge (IFAS) Process and a Conventional Activated Sludge Process (ASP) [Text] / S. Lothman, W. James Gellner, A. Stone, D. Howard, P. Pitt, D. Rosso // Proceedings of the Water Environment Federation. – 2011. – Vol. 2011, Issue 18. – P. 368–390. doi: 10.2175/193864711802638977
27. Oleynik, A. Modelling and computation of the oxygen regime while extracting the organic contaminants in ideal mix aeration tank with suspended and fixed biocenose [Text] / A. Oleynik, T. Airapetian // Problems of water supply, sewerage and hydraulic. – 2016. – Issue 27. – P. 269–279.