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SYNTHESIS OF ROBUST CONTROLLERS FOR THE CONTROL SYSTEMS OF TECHNOLOGICAL UNITS AT IRON ORE PROCESSING PLANTS

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Для керування процесами збагачувального виробництва в умовах зміни параметрів технологічних агрегатів, як об'єктів керування, досліджено можливість застосування робастних регуляторів. Встановлено, що за показниками номінальної і робастної якості керування доцільним є застосування робастного μ-регулятора, для зниження порядку якого виконано апроксимація з застосуванням Ганкелевої норми

Ключові слова: робастний регулятор, автоматизоване керування, збагачення руди, субоптимальний регулятор, частотні характеристики

Для управления процессами обогатительного производства в условиях изменения параметров технологических агрегатов, как объектов управления, исследована возможность применения робастных регуляторов. Установлено, что по показателям номинального и робастного качества управления целесообразно применение робастного μ-регулятора для понижения порядка которого выполнен аппроксимация с применением Ганкелевой нормы

Ключевые слова: робастный регулятор, автоматизированное управление, обогащения руды, субоптимальный регулятор, частотные характеристики

1. Introduction

Mining-metallurgical complex of Ukraine is one iron ore processing plant of the most important sectors of national industrial production. Specifically, export of ferrous metals, as well as the articles made of them, accounts for almost a quarter of the total value of the exported products. However, over the past decades, changes in the quality of iron ore raw materials have demonstrated a negative trend, which negatively affects product competitiveness of Ukrainian enterprises in the international market. At the same time, still unresolved is the problem on improving energy efficiency of technological processes at mining enterprises.

Numerous studies have proven that the most promising approach to increasing the efficiency of technological processes of the enrichment of iron ore raw materials is the comprehensive automation of control processes. That is, automation systems must engage both separate units, stages, cycles, and the entire production line or an enterprise [1, 2].

When automating control processes of the enrichment of iron ore raw materials, represented by various mineralogical-technological varieties, it is necessary to solve a task on the operational tracking of changes in the condition of control objects and to promptly bring the system to the state of dynamic equilibrium. The biggest changes in the process of enriching the iron ore raw materials happen to granulometric composition and distribution of the useful component by the classes of size of the crushed ore. A deviation of the values of these indicators from technologically substantiated values leads to the loss of the useful component, reduces productivity of technological line and increases specific electricity consumption.

At the same time, due to the existence of cyclic relations, units of the iron ore processing technological line operate mainly under transient modes, which are characterized by significant deviations of the regulated variables from preset values. For example, the optimal operation modes of the mill that works in a closed cycle with a classifier or hydrocyclone are close to the critical region. Such a region is characterized by the loss of technological stability of the process of crushing-classification of ore and a sharp increase in the variability of parameters of the original product of the cycle, associated with this. Under such conditions, it is appropriate to apply methods for constructing local systems of robust control over certain technological parameters. The task on the synthesis of such type of systems implies searching for the law of control that would maintain original variables of the system and signals of error within the assigned permissible limits, despite the existence of uncertainties in the control circuit.

At present, there is a significant number of proposed approaches to solving a task on the automation of technological processes that employ optimal, adaptive, and robust control over hierarchical structures of the enrichment production. However, one of the main factors that impede the implementation of such approaches is the non-stationarity of parameters and operating modes at numerous external and internal disturbances, which are of both systematic and random character.

2. Literature review and problem statement

Studies [1, 3, 4], with a sufficient degree of accuracy, represent technological units, as the objects of control, of the iron ore processing plant by transfer functions of the first and second order with a time delay, specifically:

– transfer function of the mill along the channel "performance – output of the solid in discharge":

$$W^{(m)}_{Q_{tv}}(p)=\frac{1}{T_{Q-\gamma_{tv}}+1}, \qquad (1)$$

where $T_{Q-\gamma_{tv}}$ is the time constant of the mill that depends on the hydraulic processes in the drum of the mill;

– transfer function of the mill along the channel "performance – output of large classes in discharge":

$$W^{(m)}_{Q-\gamma_{kr}}(p)=\frac{k_{Q-\gamma_{kr}}}{T_{Q-\gamma_{kr}}+1}, \qquad (2)$$

where

$$k_{Q-\gamma_{kr}}=\left(\alpha_{kr}k_g\right)/\left(k_{kr}+k_g\right)$$

is the transfer coefficient;

$$T_{Q-\gamma_{kr}}=1/\left(k_{kr}+k_g\right)$$

is the time constant that depends on the hydraulic processes in the drum of the mill; α_{kr} is the relative content of class size in the original ore; k_{kr} is the relative speed of the grinding of class; k_g is the coefficient, predetermined by the hydraulic processes in the drum of the mill;

– transfer function of the classifier along the channel "water flow rate in the classifier – volumetric consumption of discharge":

$$W^{(kl)}_{Q_{vh.v.}-Q_{sl}}(p)=\frac{1}{T_{Q_{vh.v.}-Q_{sl}}+1}, \qquad (3)$$

where $T_{Q_{vh.v.}-Q_{sl}}$ is the time constant of the classifier that depends on the hydraulic properties of the classifier;

– transfer function of the hydrocyclone along the channel "iron content at the input – iron content in the product":

$$W^{(hz)}_{\beta'-\beta}(p)=\frac{1}{T_{\beta'-\beta}+1}, \qquad (4)$$

where $T_{\beta'-\beta}$ is the time constant of the hydrocyclone that depends on its hydraulic properties;

– transfer function of the slugcatcher along the channel "water flow rate to the slugcatcher – output of class – 0.074 mm":

$$W^{(ds)}_{Q_v-\gamma}(p)=\frac{k_{Q_v-\gamma}}{T_{Q_v-\gamma}+1}, \qquad (5)$$

where $k_{Q_v-\gamma}$ is the transfer coefficient; $T_{Q_v-\gamma}$ is the time constant of the slugcatcher that depends on its hydraulic properties;

– transfer function of the magnetic separator along the channel "iron content in the ore – iron content in the concentrate":

$$W^{(ms)}_{\alpha-\beta}(p)=\frac{k_{\alpha-\beta}}{T_{\alpha-\beta}+1}, \qquad (6)$$

where $T_{\alpha-\beta}=V/Q$ is the time constant; $k_{\alpha-\beta}=\beta_u/\alpha$ is the transfer coefficient; V is the volume of the working area, m^3; Q is the volumetric consumption of pulp, m^3/min.;

– transfer function of the closed cycle of shredding along the channel "performance – output of the solid to discharge" is described by expression:

$$W^{(zc)}(p)=\frac{W^{(m)}_{Q-\gamma_{kr}}(p)W^{(kl)}_1(p)e^{-p\tau_1}}{1-W^{(m)}_{Q-\gamma_{kr}}(p)W^{(kl)}_2(p)e^{-p(\tau_1+\tau_2)}}, \qquad (7)$$

where $W^{(kl)}_1(p)$ is the transfer function of the classifier along the channel "consumption of the solid to the classifier – volumetric consumption of the solid with discharge"; $W^{(kl)}_2(p)$ is the transfer function of the classifier along the channel "consumption of the solid to the classifier – volumetric consumption of the solid with sand"; τ_1 and τ_2 is the time of transportation of the crushed product to the classifier and sands to the mill, respectively.

In paper [2], authors considered a heuristic approach to control objects of the iron ore processing production. They proposed techniques for modeling and optimizing a control system, including the least square method and genetic algorithms. They failed, however, to fully account for the uncertainty in parameters of the control object.

When solving a problem on the synthesis of robust controller, reported in [5], an analysis of a number of modifications of the algorithms of robust control was performed. The most acceptable approach was recognized to be the formation of requirements to the system in the form of frequency restrictions on the singular numbers of transfer function of closed system.

The results of comparison of robust controllers under conditions of different types of input signals are described in paper [6]. It was established that the best one, under the input influence in the form of white noise, was H_2-controller. At the same time, under similar conditions, anisotropic controller levels off white noise with lower energy consumption for control compared to the H_∞-suboptimal controller. In general, an H_∞-suboptimal controller enables better control quality at any input signal. The disadvantage of the application of such a controller is high energy consumption for control.

It was shown in paper [7] that under conditions of control over non-linear technological objects it is possible to employ a method for H_∞ circuit formation (CF) and the method of μ-synthesis. It is noted that both controllers provide high quality of control. However, it is recommended to pay more attention to the controller of lower order, synthesized by the method of H_∞ circuit formation.

Results of research into robust control systems are reported in [8, 9]. It is shown that the techniques of H_∞-optimization can be used in solving the problems of robust stabilization under conditions of parametric uncertainty in transfer functions.

A method of robust control, which ensures a preset quality of control under conditions of uncertainty in the parameters of an object, is described in work [10]. The authors substantiated advantages of the modified method for H_∞-optimization with solving problems on the synthesis of control.

A method of the synthesis of a robust controller, which ensures specified characteristics of the suboptimum controller, is presented in paper [11]. The proposed method implies solving two algebraic Riccati equations, each of which is of the same order with the system. Thus, the synthesis of controller for the higher-order systems is complicated.

Paper [12] represents inaccuracy in the control object parameters as a structural uncertainty. To synthesize a robust controller, the authors employed analysis and synthesis of the structured singular value (μ). The designed controller has low order and can work well under conditions of different input signals. Thus, such a circuit for the synthesis of controller is promising in the sense that it can adapt to a wide range of system settings without a significant increase in complexity, which is also confirmed by research results [13–15].

Applying the intelligent systems in order to control technological processes of the ore enrichment was proposed in papers [16, 17]. Underlying a given approach is the use of neural networks to model and control technological processes. The disadvantage of this approach is the considerable time required for training neural-network structures when modifying parameters of the control object.

A study into dynamic objects of control and their models under conditions of the iron ore processing production [18] points out their nonlinear and non-stationary character. It is indicated in paper [19] that the nonstationarity is characteristic not only of the control objects, but also of the parameters of technological workflows, making the synthesis of control over the processes of ore enrichment even more difficult.

A task on the accounting for changes in the parameters of control objects at mining enterprises in the process of formation automated control is dealt with in work [20]. Complexity of solving a given problem is caused by the existence of direct and inverse relationships (recycles) between technological units of the iron ore processing plant [21, 22].

The methods of adaptive control, proposed in the studies, under conditions of the non-stationarity of parameters of separate objects, were examined in papers [23, 24], however, they do not make it possible to achieve adequate quality control over a system of interconnected objects. Thus, in this case, it is expedient to study the methods of robust control.

In paper [25], authors considered the possibility of employing a robust nonlinear predictive controller and investigated issues related to the implementation of such a controller under industrial conditions. A similar technique, based on the robust approach to control using μ-synthesis, is proposed in work [26]. The uncertainty of the main process parameters is specified as a range of values of these parameters. In paper [27], to develop a digital robust controller in the presence of parametric uncertainty, authors applied a numerical theory of feedback. The controller is synthesized using a method of delta-transformation for discrete systems. The effectiveness of these approaches is confirmed by experimental results.

Thus, in order to control specific parameters of a unit in the iron ore processing production line, the circuit with a robust controller was applied (Fig. 1). Generalized structural circuit of a given automatic control system (ACS) system with the robust controller, in addition to the control object and the controller, includes weight functions that are dependent on frequency [5].

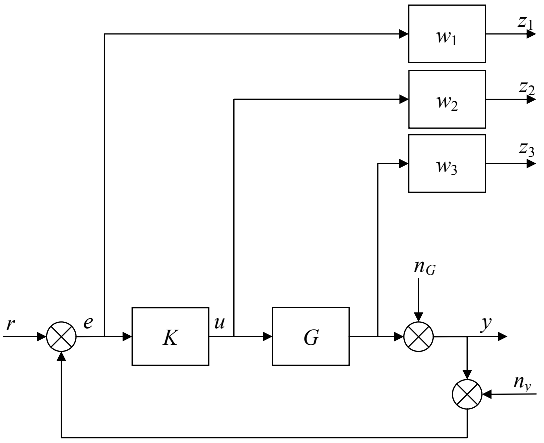


Fig. 1. Generalized structural circuit of ACS with a robust controller: G — control object, K — robust controller, r — task, n_G — disturbance that applied to the object, n_y — measurement noise, e — control error, u — controlling influence, w_1, w_2, w_3 — functions that depend on frequency

The synthesis of robust system in frequency domain allows selection of a controller at which sensitivity of the closed system would be less than a certain permissible value. At the same time, minimizing the sensitivity requires the choice of such a controller that a given indicator would equal or approach the minimum value [5]. The controller is then created to minimize the norm $\|z_1 z_2 z_3\|_\infty$ [6]. The choice of weight matrices w_1, w_2, w_3 is a nontrivial task and requires consideration of the features of the control object. Specifically, it is necessary to take into consideration information about the range of operating frequencies of the closed system, the degree of decrease in external disturbances, about the largest predictions of multiplicative disturbances.

Closed cycles of ore crushing are the main technological structures that determine the disclosure of useful component. These cycles are affected more, compared to other technological units, by the disturbance factors, which is caused by a change in the characteristics of original ore and condition of the equipment. That is why we shall consider as a model example the synthesis of a robust controller for local ACS of parameters of the original product from a hydrocyclone of the second stage of shredding along the channel "pressure at the input – the output of class 0.074 mm".

In a general form, mathematical expression of the second-order control object can be recorded in the following way [1, 3, 4]:

$$\mu\ddot{x}(t)+\theta\dot{x}(t)+\kappa x(t)=u(t). \tag{8}$$

In an actual system, control object parameters μ, θ, κ are not known precisely. Therefore, we can assume that these values are within certain intervals [7]:

$$\mu=\bar{\mu}\left(1+p_{\mu}\delta_{\mu}\right);$$

$$\theta=\bar{\theta}\left(1+p_{\theta}\delta_{\theta}\right);$$

$$\kappa=\bar{\kappa}\left(1+p_{\kappa}\delta_{\kappa}\right), \tag{9}$$

where $\bar{\mu}$, $\bar{\theta}$, $\bar{\kappa}$ are the nominal values of parameters; p_μ, p_θ, p_κ, δ_μ, δ_θ, δ_κ are the possible deviations of parameters.

3. The aim and objectives of the study

The aim of present study is to investigate and synthesize robust controllers in the local systems of control over technological iron ore processing units in order to improve efficiency and quality of control under conditions of the non-stationarity of parameters and operation modes at numerous external and internal disturbances. The stated goal of present work necessitated solving the following tasks:

– to synthesize and investigate robust stability of control systems of enrichment units by using various types of controllers;

– to explore robust quality of control, formed by applying different types of robust controllers;

– to substantiate the choice of robust controller to control technological iron ore processing units under conditions of uncertainty in parameters.

4. Materials and methods to examine quality of control created by different types of robust controllers

4. 1. Basic principles for the construction of robust control over objects of the iron ore processing production

In accordance with the approach proposed in paper [7], taking into account the uncertainty of parameters μ, θ, κ, it is advisable to represent them as two blocks, obtained with the use of fractional linear transformation and by connecting on top the respective blocks δ_μ, δ_θ, δ_κ. For this purpose, we shall perform transformation:

$$\frac{1}{\mu}=F_U\left(M_{\mu},\delta_{\mu}\right)=\frac{1}{\bar{\mu}\left(1+p_{\mu}\delta_{\mu}\right)}=\frac{1}{\bar{\mu}}-\frac{p_{\mu}}{\bar{\mu}\left(1+p_{\mu}\delta_{\mu}\right)}, \tag{10}$$

where

$$M_{\mu}=\begin{bmatrix}-p_{\mu} & \left(\bar{\mu}\right)^{-1}\\ -p_{\mu} & \left(\bar{\mu}\right)^{-1}\end{bmatrix}. \tag{11}$$

We shall represent in the same fashion:

$$\bar{\theta}=F_U\left(M_{\theta},\delta_{\theta}\right);\quad \bar{\kappa}=F_U\left(M_{\kappa},\delta_{\kappa}\right), \tag{12}$$

where

$$M_{\theta}=\begin{bmatrix}0 & \bar{\theta}\\ p_{\theta} & \bar{\theta}\end{bmatrix};\quad M_{\kappa}=\begin{bmatrix}0 & \bar{\kappa}\\ p_{\kappa} & \bar{\kappa}\end{bmatrix}. \tag{13}$$

With respect to these transformations, block diagram of the control object will take the form shown in Fig. 2.

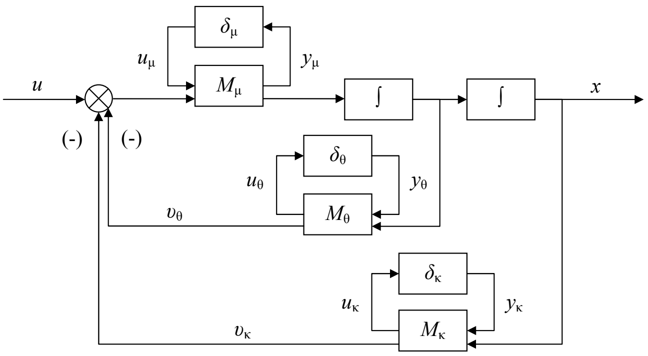


Fig. 2. Block diagram of a second-order control object with uncertain parameters

With respect to the performed transformations, the output signals of blocks of parameters with uncertainty will be recorded in the following way:

$$\begin{bmatrix}y_{\mu}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}-p_{\mu} & 1/\bar{\mu}\\ -p_{m} & 1/\bar{\mu}\end{bmatrix}\begin{bmatrix}u_{\mu}\\ u-\upsilon_{\theta}-\upsilon_{\kappa}\end{bmatrix},$$

$$\begin{bmatrix}y_{\theta}\\ \upsilon_{\theta}\end{bmatrix}=\begin{bmatrix}0 & \bar{\theta}\\ p_{\theta} & \bar{\theta}\end{bmatrix}\begin{bmatrix}u_{\theta}\\ \dot{x}\end{bmatrix},$$

$$\begin{bmatrix}y_{\kappa}\\ \upsilon_{\kappa}\end{bmatrix}=\begin{bmatrix}0 & \bar{\kappa}\\ p_{k} & \bar{\kappa}\end{bmatrix}\begin{bmatrix}u_{\kappa}\\ x\end{bmatrix},$$

$$u_{\mu}=\delta_{\mu}y_{\mu},\quad u_{\theta}=\delta_{\theta}y_{\theta},\quad u_{k}=\delta_{k}y_{k}, \tag{14}$$

substituting $x_1=x$, $x_2=x_1'$, $y=x_1$ and eliminating variables υ_θ and υ_κ, we shall obtain:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ y_{\mu}\\ y_{\theta}\\ y_{\kappa}\\ y\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0 & 0 & 0\\ -\bar{k}/\bar{\mu} & -\bar{\theta}/\bar{\mu} & -p_{m} & -p_{\theta}/\bar{\mu} & -p_{\kappa}/\bar{\mu} & 1/\bar{\mu}\\ -\bar{k}/\bar{\mu} & -\bar{\theta}/\bar{\mu} & -p_{m} & -p_{\theta}/\bar{\mu} & -p_{\kappa}/\bar{\mu} & 1/\bar{\mu}\\ 0 & \bar{\theta} & 0 & 0 & 0 & 0\\ \bar{k} & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ u_{\mu}\\ u_{\theta}\\ u_{\kappa}\\ u\end{bmatrix},$$

$$\begin{bmatrix} u_\mu \\ u_\theta \\ u_\kappa \end{bmatrix} = \begin{bmatrix} \delta_\mu & 0 & 0 \\ 0 & \delta_\theta & 0 \\ 0 & 0 & \delta_\kappa \end{bmatrix}. \tag{15}$$

Thus, a dynamic model of the hydrocyclone along the above control channel, taking into account the uncertainty of parameters, will have four input variables (u_μ, u_θ, u_κ, u), four output variables (y_μ, y_θ, y_κ, y) and two variables of state (x_1, x_2). In the space of states, a hydrocyclone model G_1 will take the form:

$$G_1 = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}, \tag{16}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\bar{k}/\bar{\mu} & -\bar{\theta}/\bar{\mu} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ -p_\mu & -p_\theta/\bar{\mu} & -p_\kappa/\bar{\mu} \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 1/\bar{\mu} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -\bar{k}/\bar{\mu} & -\bar{\theta}/\bar{\mu} \\ 0 & \bar{\theta} \\ \bar{k} & 0 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} -p_\mu & -p_\theta/\bar{\mu} & -p_\kappa/\bar{\mu} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 1/\bar{\mu} \\ 0 \\ 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad D_{22} = 0.$$

We shall represent uncertainty of the hydrocyclone model by two blocks, obtained with the use of fractional linear transformation and by connecting at the top a diagonal block (δ_μ, δ_θ, δ_κ) [7].

The hydrocyclone model in the form of a one-dimensional object with uncertain parameters will be recorded in the following way:

$$y = F_U(G_1, \Delta)u, \tag{17}$$

where Δ is the diagonal matrix of uncertainties (δ_μ, δ_θ, δ_κ). A family of Bode diagrams for different values of uncertain parameters (δ_μ, δ_θ, δ_κ) is shown in Fig. 3.

Representation of iron ore processing units as control object can be obtained by using a mathematical model with uncertain parameters.

We employed software package Grinding Circuit Toolbox v.2001 for computer simulation of the iron ore raw material enrichment process. The developed model is applied in the comparative analysis of the synthesized controllers for a local system of robust control over a hydrocyclone of the second stage of shredding that operates in closed cycle with a ball mill along the channel "pressure at the input – output of class 0.074 mm".

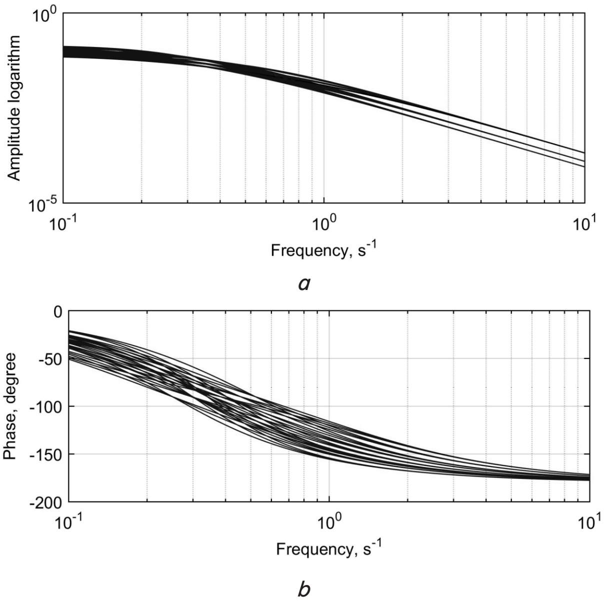


Fig. 3. Bode diagram of the open-loop control system over a hydrocyclone under conditions of variable characteristics of the ore and state of technological equipment: *a* — logarithmic amplitude-frequency characteristic; *b* — logarithmic phase-frequency characteristic

The next step is the substantiation of criterion and the synthesis of robust control over objects of iron ore processing production under conditions of non-stationarity.

4. 2. Synthesis of robust control applying the principles of H_∞-optimization of mixed sensitivity

An analysis of results of research into robust control systems [6–8] allows us to draw a conclusion about the appropriateness of using in the synthesis of control a combination of several cost functions. In order to qualitatively track a signal of the task, as well as to reduce it in the process of implementing control under conditions of changing characteristics and mass ratio of mineralogical-technological types of ore, we present a generalized circuit of the system of robust control using the principles of H_∞-optimization of mixed sensitivity.

The task with mixed sensitivity takes the following form:

$$\min_{K_{st}} \left\| \begin{matrix} (I+GK)^{-1} \\ K(I+GK)^{-1} \end{matrix} \right\|_\infty, \tag{18}$$

where G is the nominal control object; K is the controller.

To solve the task of synthesis of the concentrated local control over individual units of the technological line of iron ore processing, cost function (18) must be rewritten according to the standard procedure of H_∞-optimization (Fig. 4). For this purpose, we performed fractional-linear transformation [7] and grouping of signals in sets.

The task is to find a stabilizing controller K, which minimizes the output energy z, provided that energy w is less than, or is equal to, 1, that is, in the minimization of H_∞-norm of transfer function along the channel $w \to z$. With respect to decomposition of the generalized object P:

$$P(s)=\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}, \tag{19}$$

we shall obtain:

$$z=\left[P_{11}+P_{12}K\left(I-P_{22}K\right)^{-1}P_{21}\right]w=:F_l(P,K)w, \tag{20}$$

where $F_l(P, K)$ is the lower fractional-linear transformation of P and K.

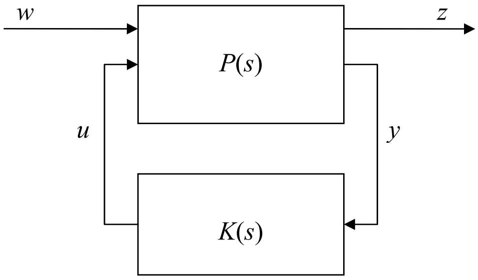


Fig. 4. Standard circuit of robust system based on H_∞-optimization

Thus, the task of synthesis of local control over technological units in the line of iron ore raw material enrichment is stated as the task on H_∞-optimization and takes the form:

$$\min_{K_{st}}\left\|F_l(P,K)\right\|_\infty, \tag{21}$$

where P is the generalized control object; K is the controller.

The solution to a given optimization task (21) for the scalar case is described in papers [9, 10]. Because of the lack of analytical formulae to solve it, it will suffice to find such stabilizing controller K, at which H_∞-norm of the transfer function of closed loop will be less than the assigned number:

$$\left\|F_l(P,K)\right\|_\infty<\gamma, \tag{22}$$

where γ is the assigned positive integer.

Through a consistent reduction of the value of γ, starting with a relatively large number, one finds a suboptimum solution. The required and sufficient conditions for the existence of H_∞-suboptimum solution, as well as the necessary constraints, are described in paper [11]. In this case, controllers $K(s)$ that satisfy $\left\|F_l(P,K)\right\|_\infty<\gamma$ are found from expression:

$$K(s)=F_l(M,\Phi), \tag{23}$$

where M is the interconnected transfer matrix; $\Phi(s)\in H_\infty$ and the conditions are satisfied:

$$\left\|\Phi(s)\right\|_\infty<\gamma. \tag{24}$$

A formula of central ($\Phi(s)=0$) suboptimum controller of the local system of control over the process of enriching the ore takes the following form in a space of states:

$$K_o(s)=\begin{bmatrix} \hat{A} & \hat{B}_1 \\ \hat{C}_1 & \hat{D}_{11} \end{bmatrix}=$$

$$=\begin{bmatrix} A+BF+\hat{B}_1\hat{D}_{21}^{-1}\hat{C}_2 & -ZL_2+\hat{B}_2\hat{D}_{12}^{-1}\hat{D}_{11} \\ F_2+\hat{D}_{11}\hat{D}_{21}^{-1}\hat{C}_2 & \hat{D}_{11} \end{bmatrix}, \tag{25}$$

where F is the matrix of feedback; L is the gain matrix of the observer; $\hat{D}_{11}$, $\hat{D}_{12}$, $\hat{D}_{21}$ are the matrices obtained by using the expansion of Kholetsky, which, respectively, are derived from expressions:

$$F:=-R_n^{-1}\left(D_{1*}{}^{T}C_1+B^TX\right)=:\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}=:\begin{bmatrix} F_{11} & F_{12} & F_2 \end{bmatrix}^T, \tag{26}$$

$$L:=-\left(B_1D_{1*}{}^{T}+YC^T\right)R_n^{-1}=:\left[L_1 \quad L_2=:\left[L_{11} \quad L_{12} \quad L_2\right]\right], \tag{27}$$

$$\hat{B}_2=Z\left(B_2+L_{12}\right)\hat{D}_{12},\quad \hat{C}_2=-\hat{D}_{21}\left(C_2+F_{12}\right),$$

$$\hat{B}_1=-ZL_2+\hat{B}_2\hat{D}_{12}^{-1}\hat{D}_{11},$$

$$\hat{C}_1=F_2+\hat{D}_{11}\hat{D}_{21}^{-1}\hat{C}_2,\ A=A+BF+\hat{B}_1\hat{D}_{21}^{-1}\hat{C}_2,$$

$$Z=\left(I-\gamma^{-2}YX\right)^{-1}. \tag{28}$$

Under these circumstances, according to the above assumption [11], system $P(s)$ is normalized.

Suboptimal controller minimizes H_∞-norm of matrix transfer function $F_L(P, K)$ of the nominal closed system along the channel "disturbance – error" ("*dist* – $[e_p\ e_u]^{\mathrm{T}}$") on the set of stabilizing K controllers. To this end, it is necessary to obtain from the model a corresponding matrix transfer function P.

An interval of change in γ is chosen between 0.1 and 10 with a tolerance of 0.001. We obtained a matrix transfer function of the closed system along the channel "disturbance – error." Following 15 iterations, we obtained controller of the sixth order.

Test of robust stability and robust quality was performed using μ-analysis, which made it possible to determine the upper and lower bounds of singular value. An analysis of frequency characteristics of the upper and lower μ-bounds (Fig. 5) leads to the conclusion that the closed system with a calculated suboptimal controller is a robust stable system. The maximum value of singular number μ is 0.4057, which indicates permissibility in the system of structured disturbances with a norm not exceeding 1/0.4057=2.4649, that is, the system maintains stability for all such Δ as $\|\Delta\|_\infty<2.4649$. It should be noted that, based on the bound of the H_∞-norm, which exceeds the value of 1 in the interval of frequencies of the order of [0.05;10] s^{-1}, robust stability is not kept for the case of unstructured uncertainty.

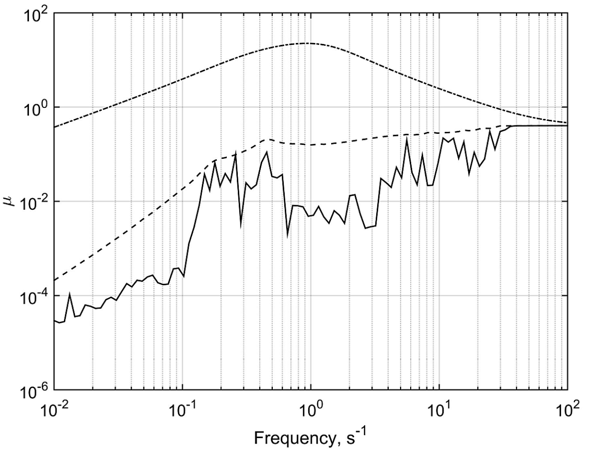


Fig. 5. Robust stability of the local system of control over a hydrocyclone of the second stage of shredding: "———" – upper bound; "- - - -" – lower bound; "-----" – bound of H_∞-norm

Let us consider results of the assessment of robust quality of control in a closed system with a suboptimal controller. Out of four inputs and five outputs in a closed system, first three inputs-outputs correspond to the three channels of uncertainty Δ. Consequently, in order to perform μ-analysis of robust quality, the circuit of the analyzed system must include a block of uncertainties Δ (3×3) and a block of quality (1×2) [7]:

$$\Delta_p := \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_F \end{bmatrix} : \Delta \in R^{3\times 3}, \Delta_F \in C^{1\times 2} \right\}, \tag{29}$$

where Δ is the block of uncertainties, a dimension of 3×3; Δ_F is the block of quality, a dimension of 1×2.

A sign of accomplishing robust quality is the fulfilment of strict condition for $\mu(\bullet)$ not to exceed the value of unity over the whole frequency range.

According to the obtained results (Fig. 6), the system of control with a developed controller does not satisfy the criterion of robust quality, since the maximum value of μ-function is equal to 1.711.

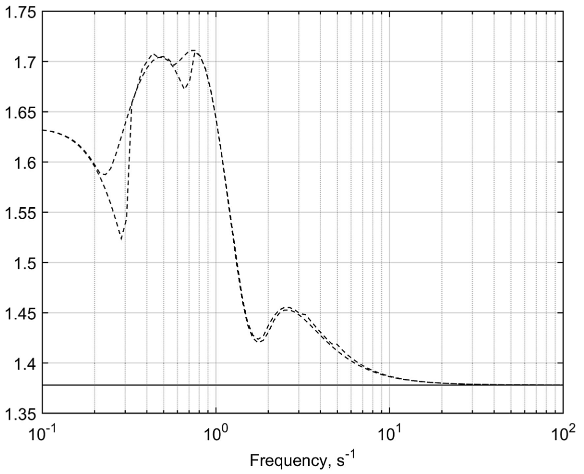


Fig. 6. Robust quality of control system with a controller: “————” — nominal quality; “– – – –” — robust quality

Thus, it is required to limit the norm $\|\Delta\|_\infty \leq (1/1.711)$ so that the quality function satisfies constraint:

$$\left\| \begin{matrix} W_p\left(I + F_U\left(G_1, \Delta_G\right)K\right)^{-1} \\ W_u K\left(I + F_U\left(G_1, \Delta_G\right)K\right)^{-1} \end{matrix} \right\|_\infty \leq 1, \tag{30}$$

where W_p, W_u are the weight functions; $F_U\left(G_1, \Delta_G\right)$ is the model of control object with respect to uncertainty; G_1 is the nominal control object; Δ_G is the diagonal matrix that takes into account the uncertainty of control object.

Thus, in order to solve the problem on the synthesis of robust control over iron ore processing units, we must investigate alternative methods.

4. 3. Synthesis of robust control by creating a control circuit using an H_∞ method

Consider the process of control circuit formation (CF) using an H_∞-method [7]. According to a given method, the circuit of the open-loop control system includes special weight blocks (Fig. 7) that ensure the quality of the “weighted” closed control system. Following this, the synthesis of a robust controller is performed, which ensures stability of the system.

By introducing to the control circuit pre-compensator W_1 and post-compensator W_2, the singular value of nominal control object G changes to the desired form.

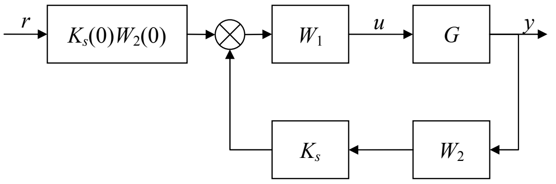


Fig. 7. Control circuit of a one-dimensional system with one degree of freedom

The smallest singular value of the weighted system should be typically greater in the low-frequency range in order to achieve high quality of control. At the same time, the largest singular value in the high frequency range should be small in the range of high frequencies. In addition, the slope of singular values near the bandwidth must be not too steep. As a result, the model of weighted system G_s does not contain hidden unstable regimes and takes the form:

$$G_s = W_2 G W_1, \tag{31}$$

where W_1 and W_2 are the transfer functions of pre-compensator and post-compensator, respectively.

Feedback controller K_s is synthesized to ensure robust stability by the normalized left mutually simple factorization of matrix G_s with a stability margin ε. The resulting notation of the feedback controller K_f, obtained from the H_∞-controller K_s with weight functions W_1 and W_2 takes the form:

$$K_f = W_1 K_s W_2, \tag{32}$$

where K_s is the H_∞-controller of feedback.

As the signal of the task enters the circuit of control system between blocks K_s and W_1, the circuit should include proportionate link $K_s(0)W_2(0)$, derived from expression:

$$K_s(0)W_2(0) = \lim_{s\to 0} K_s(s)W_2(s). \tag{33}$$

A transfer function of the closed control system of hydrocyclone of the second stage of shredding along the channel “task – output (r–y) takes the form:

$$Y = \left[I - G(s)K_f(s)\right]^{-1} I - G(0)W_2(0)R(s), \tag{34}$$

where K_f is the feedback controller.

The result of the calculations is the following matrix of controller K_{lsh}, of the sixth order, which ensures stability reserve $\epsilon_{\max}=0.4953$.

The functions of robust quality of the closed control system with controller K_{lsh} and frequency characteristics of the upper and lower μ-bounds are shown in Fig. 8.

Based on the results obtained, we can conclude that the closed system with a calculated suboptimal controller is a robust stable system. The maximum value of μ is 0.4176, which indicates permissibility in the system of structured disturbances with a norm not exceeding 1/0.4176=2.3946, that is, the system maintains stability for all such Δ that $\|\Delta\|_\infty<2.3946$.

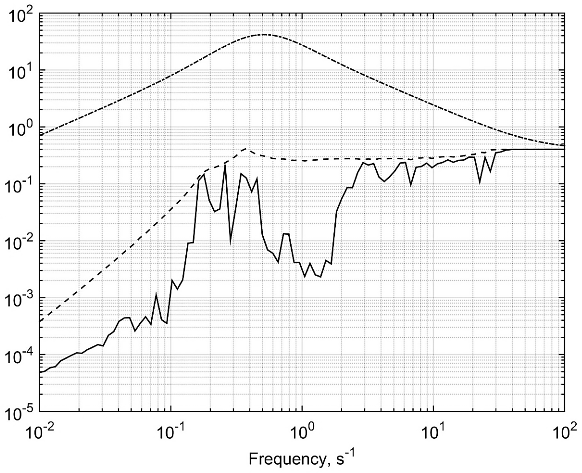


Fig. 8. Robust quality of control system over hydrocyclone of the second stage shredding: "———" — upper bound; " - - - -" — lower bound; "-–-–-" — H_∞-norm bound

4. 4. Synthesis of robust control by using μ-synthesis based on the algorithm of *D–K* iterations

While investigating μ-synthesis based on the algorithm of *D–K* iterations for the construction of control over key technological units of the iron ore processing line, we assumed that block Δ_P of the matrix transfer function of the open-loop system $P(s)$ takes the form [7]:

$$\Delta_p := \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_P \end{bmatrix} : \Delta \in R^{3\times 3}, \Delta_P \in C^{1\times 2} \right\}, \qquad (35)$$

where Δ, Δ_P are the blocks of uncertainty.

The first block of uncertainty Δ of the structured matrix (35) is diagonal and corresponds to the uncertainty that is used in the simulation of technological iron ore processing equipment. The second block Δ_P is a block of uncertainty, introduced to define requirements to the quality of control within the framework of the μ-approach [7, 12, 13].

The next optimization task is formed to minimize the upper bound of values of μ, which in turn reduces the maximum value of μ [7]:

$$\min_{K} \min_{D_\ell(s) D_r(s)} \left\| D_\ell(s) F_L(P,K) D_r^{-1}(s) \right\|_\infty , \qquad (36)$$

where

$$D_\ell(s) = \begin{bmatrix} d_1(s) & 0 & 0 & 0 \\ 0 & d_2(s) & 0 & 0 \\ 0 & 0 & d_3(s) & 0 \\ 0 & 0 & 0 & d_4(s) I_2 \end{bmatrix},$$

$$D_r(s) = \begin{bmatrix} d_1(s) & 0 & 0 & 0 \\ 0 & d_2(s) & 0 & 0 \\ 0 & 0 & d_3(s) & 0 \\ 0 & 0 & 0 & d_4(s) \end{bmatrix},$$

where $d_1(s)$, $d_2(s)$, $d_3(s)$, $d_4(s)$ are the scaling transfer function.

The purpose of the μ-synthesis is to find the minimum value of the objective function and to create a controller *K*, which would make it possible to reach a level of control quality quite close to the optimal level. That is, controller *K* must be so that for each frequency $\omega \in [0, \infty]$ the structured singular value would satisfy condition:

$$\mu\Delta_p\left[F_L(P,K)(j\omega)\right] < 1, \qquad (37)$$

which ensures that condition (30) is satisfied.

As a result of solving the optimization task, we obtained controller K_μ K, of the sixteenth order. Charts of frequency characteristics of the upper and lower μ-bounds are shown in Fig. 9.

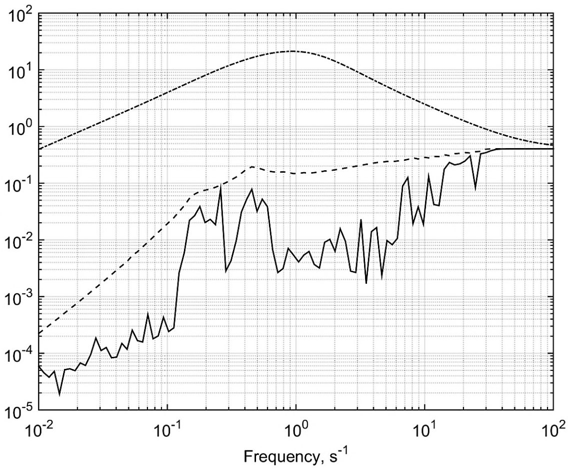


Fig. 9. Robust stability of the control system of technological unit with a controller: "———" — upper bound; "- - - -" — lower bound; "-–-–-" — H_∞-norm bound

Based on the results obtained, we can conclude that the closed system with a calculated controller is the robust stable system.

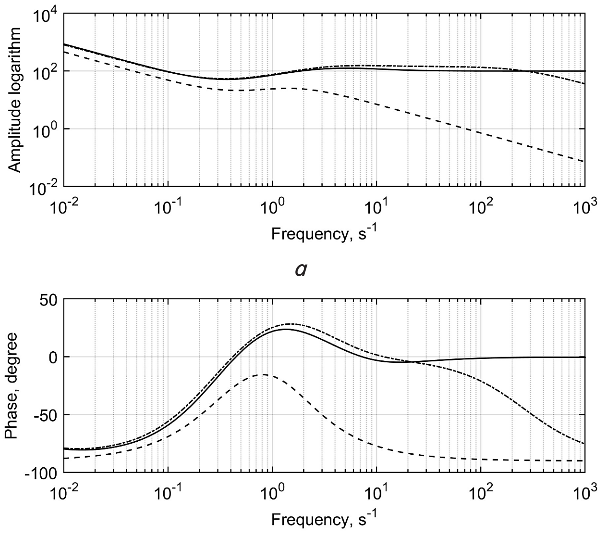
6. Discussion of results of research into methods of optimal robust control over objects of the iron ore processing production

We shall consider frequency characteristics of systems with controllers, constructed using the methods of suboptimal H_∞-optimization, circuit formation and μ-synthesis. An analysis of frequency characteristics (Fig. 10) shows that the H_∞-controller and the μ-controller are characterized by a greater gain in comparison with the CF-controller, starting at a frequency of 10 s^{-1} and higher.

Starting with a frequency of 0.2 s^{-1}, phase shift in the systems with H_∞-controller and μ-controller starts to significantly exceed the analogous value for the system with CF-controller. At the same time, starting with a frequency of 30 s^{-1}, phase shift in the system with μ-controller decreases, approaching the phase-frequency characteristic of the system with CF-controller.

Results of estimation of robust stability are shown in Fig. 11. The largest robust stability characterizes control systems with H_∞-controller and with μ-controller.

Results of estimation of the nominal quality of closed control systems are shown in Fig. 12. The best results were obtained when the control systems employed H_∞-controller and μ-controller.



b

Fig. 10. Bode diagram of controllers: “————” — H_∞-controller; “– – – –” — CF-controller; “–·–·–” — μ-controller: *a* — logarithmic amplitude-frequency characteristic; *b* — logarithmic phase frequency characteristic

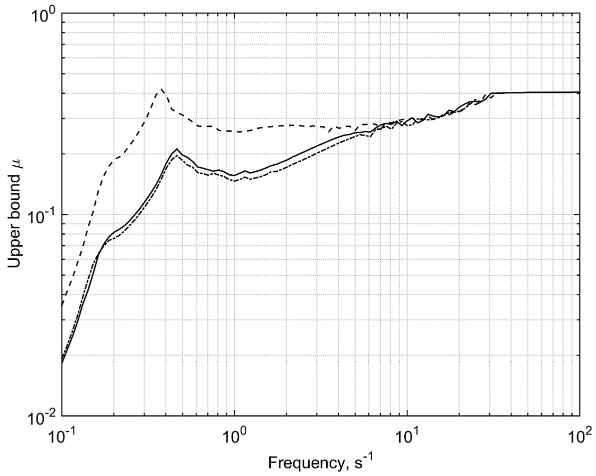


Fig. 11. Robust stability of control systems: “————” — with H_∞-controller; “– – – –” — with CF-controller; “–·–·–” — with μ-controller

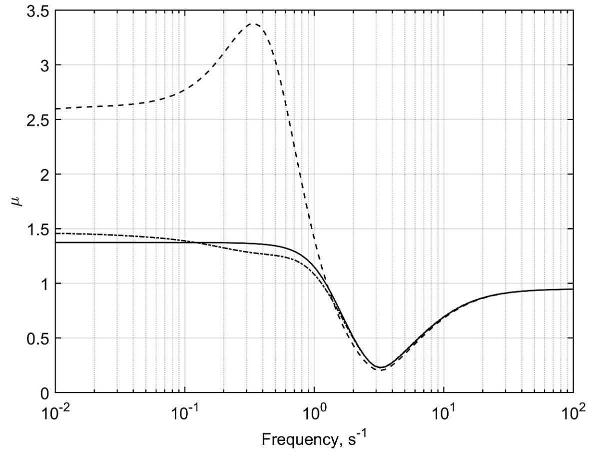


Fig. 12. Nominal quality of closed control systems: “————” — with H_∞-controller; “– – – –” — with CF-controller; “–·–·–” — with μ-controller

The charts of indicators of the robust quality of closed control systems are shown in Fig. 13. In this case, the best results were obtained when the control systems employed H_∞-controller and μ-controller.

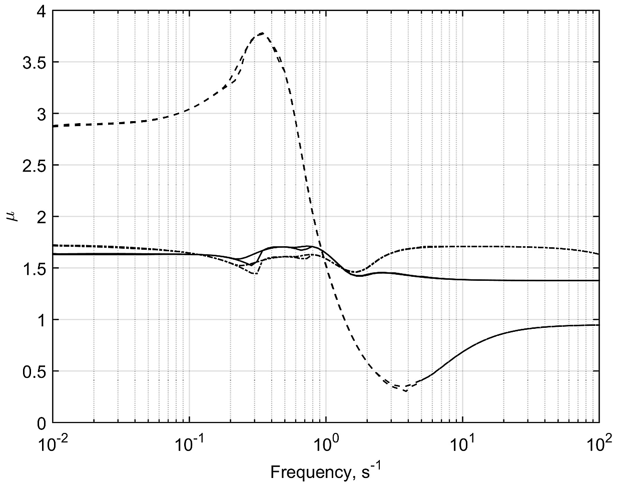


Fig. 13. Robust quality of closed control systems: “————” — with H_∞-controller; “– – – –” — with CF-controller; “–·–·–” — with μ-controller

All three examined types of controllers ensure guaranteed stability of the closed-loop system relative to parametric disturbances, which are included in the diagonal matrix of uncertainties, dimension (3×3). However, quality of control under these circumstances varies over a significant range.

Results of estimation of quality indicators for control systems in the worst scenario are shown in Fig. 14. About the same rates of deterioration in the quality of control were demonstrated by system with H_∞-controller and with μ-controller, though showing at the same time significantly better results compared to CF-controller.

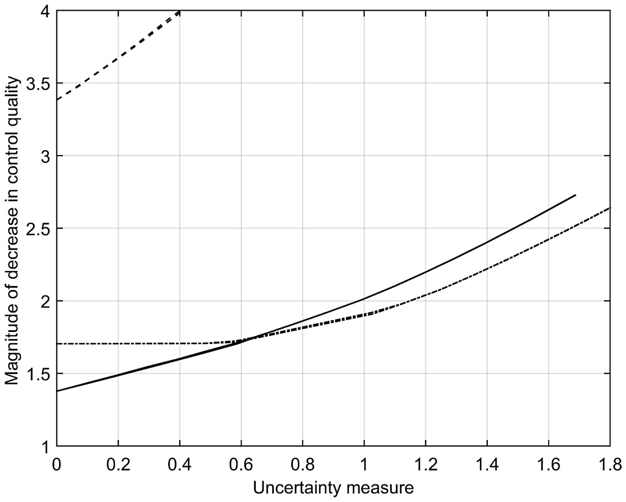


Fig. 14. Quality of the closed control systems at a growth in uncertainty: “————” — with H_∞-controller; “– – – –” — with CF-controller; “–·–·–” — with μ-controller

The best results were obtained when the control systems used μ-controller. For the actual application, the order of μ-controller, which is the controller of the sixteenth order, was reduced by using the algorithms of reduction. As a result of approximation using the Hankel norm, controller of the sixteenth order was reduced to the fourth order (Fig. 15).

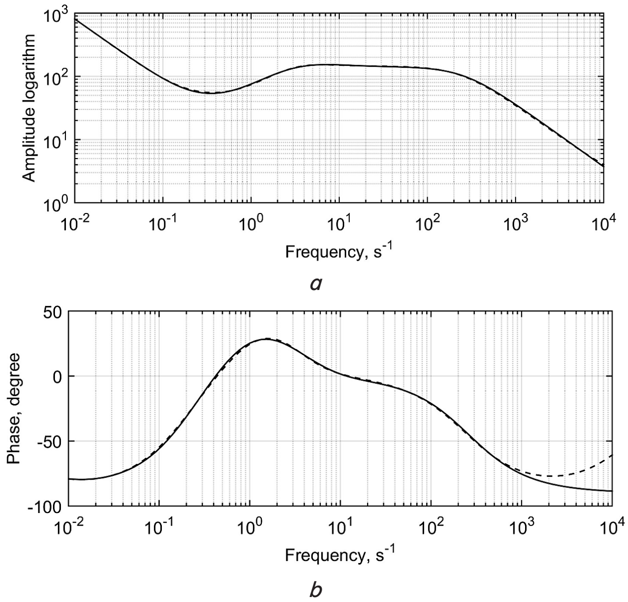


Fig. 15. Bode diagram of μ-controller: "————" — basic controller of the sixteenth order; "– – – –" — obtained controller of the fourth order: *a* — logarithmic amplitude-frequency characteristic; *b* — logarithmic phase frequency characteristic

Frequency characteristics of controller of the fourth order, shown in Fig. 15, repeat frequency characteristics of the base controller of the sixteenth order with a root-mean-square error of 0.027. This makes it possible to use the obtained controller when constructing local systems for the allocation of useful component by the granulometric characteristic of the processed ore in the process of iron ore processing.

7. Conclusions

1. We have substantiated that in order to control technological units in the iron ore processing production under conditions of uncertainty of parameters it is appropriate to use the methods of robust control with the representation of uncertainty in the form of a diagonal block connected at the top.

2. We have investigated robust stability of control systems for the iron ore processing units with H_∞-suboptimal controller. The maximum value of structural singular number μ is 0.4057, which indicates permissibility in the system of structured disturbances Δ with a norm of $\|\Delta\|_\infty<2.4649$. We have investigated robust stability of systems with the controller, synthesized using the H_∞-method of circuit formation. The maximum value of a structural singular number is $\mu=0.4176$, which indicates permissibility in the system of structured disturbances with a norm of $\|\Delta\|_\infty<2.3946$.

3. The analysis was performed of the results of study into indicators of control quality, created based on H_∞-controller, CF-controller, and μ-controller in transient modes. The best approach is to use μ-controller, which ensures a minimum overshoot value of 2 %.

4. Based on the analysis of research results of the nominal and robust quality of controls, formed on the base of H_∞-controller, CF-controller, and μ-controller, it was established that in order to control the processes of iron ore raw material enrichment, the most appropriate is to use μ-controller. To reduce its order to the fourth order, we performed approximation using the Hankel norm. Under such a condition, a root-mean-square error relative to the base controller is 0.027.

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