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*Запропоновано і детально описано оригінальну конструкцію динамічного гірокомпасу на оптичних гіроскопах та метод його використання. Розроблено алгоритм застосування такого гірокомпасування у випадку, коли вібропідставка лазерного гіроскопу відсутня. Доведено високу точність роботи такого гірокомпасу. В умовах його використання можна знизити рівень шуму лазерного гіроскопа. Завдяки обертанню значно компенсується вплив повільно мінливого дрейфу та магнітна складова дрейфу*

*Ключові слова: гірокомпасування, лазерний гіроскоп, акселерометр, дрейф, кут курсу, навігаційна система, керування рухом*

*Предложена и детально описана оригинальная конструкция динамического гироскопа на оптических гироскопах и метод его использования. Разработан алгоритм применения такого гироскопирования в случае, когда виброподставка лазерного гироскопа отсутствует. Доказана высокая точность работы такого гироскопа. При условиях его использования можно снизить уровень шума лазерного гироскопа. Благодаря вращению значительно компенсируется влияние переменного дрейфа и магнитная составляющая дрейфа*

*Ключевые слова: гироскопирование, лазерный гироскоп, акселерометр, дрейф, угол курса, навигационная система, управление движением*

## 1. Introduction

One of the urgent tasks of creating and improving motion control systems for modern aerospace objects is improving the accuracy of their information subsystems in

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# DEVELOPMENT OF METHOD AND ALGORITHM OF DYNAMIC GYROCOMPASSING FOR HIGH-SPEED SYSTEMS OF NAVIGATION AND CONTROL OF MOVEMENT

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general and navigation equipment, in particular. The main direction of solving this problem is the use of redundant information coming from inertial sensors and a receiver of satellite navigation signals. This explains why the issues of rational combination of information in such systems and the

optimization of algorithms for its processing on the on-board computer are given considerable attention today. But not all tasks of increasing the accuracy of navigational definitions can be solved with the help of satellite information. Among these tasks is, in particular, the initial exhibition of the navigation system for azimuth. By the way, it is known that the accuracy of further inertial navigation essentially depends on the error in determining the initial azimuth. Thus, the listed factors, namely, the impossibility of solving the problem of the initial exhibition of the navigation system using satellite information, makes research in the field of the new gyrocompass design relevant.

## 2. Literature review and problem statement

As the analysis shows, the results obtained in the direction of increasing the accuracy of obtaining information associated with various aspects of improving navigation systems [1]. So, it should be recalled a new method of correcting the angular orientation of the aircraft, it quickly rotates around the saddle axis [2]. The concept of an inertial system rotating to compensate gyrocompass errors is considered in [3], where it is stated that in this way it is possible to compensate for the random drift of gyroscopes.

At the same time, one of the most powerful ways to improve the accuracy of the strap down inertial navigation systems (SINS) at the factory settings stage is the method of calibrating the gyro module under thermal conditions [4]. This method is developed on the basis of processing the results of experimental studies and computer modeling of thermal processes.

One of the problems of ensuring high accuracy of navigation, even in conditions of integration with satellite information, is obtaining the exact value of the initial course. Issues of the initial exhibition of inertial systems for various purposes and equipment are given considerable attention. So in [5] the problem of determining the initial angles of course, pitch and roll for a moving object is considered.

There is a whole class of navigation equipment (the so-called gyroscopes) that uses angular velocity sensors to determine the geographic azimuth or heading angle. Their work is based on the method of double gyrocompass (DGC) [6]. To further illuminate the features of the method, let's briefly outline the principle of the DGC method.

The device is a platform, mounted almost horizontally, it is possible to rotate about an axis perpendicular to the plane of the platform. On the platform are two laser gyros (LG) and two accelerometers (AA), coaxial with LG. The sensitivity axes (SA) of LG are mutually orthogonal.

The device is set so that the axis of sensitivity of the conditionally first gyro is directed at the given object, or forms a known angle with a given direction [7].

In this position, the first accumulation of data is carried out during the time  $T_1$ , during which the average values of the angular velocity projections of the Earth measured by gyroscopes are calculated. Then, the platform rotates about the vertical axis by an angle close to  $180^\circ$ , and after fixing the position in the new installation, a second accumulation of data takes place during the time  $T_2$ . The third accumulation of data during the time  $T_3$  is carried out again in the initial position and is combined with the results of the first accumulation.

Obviously, the sum of  $T_1, T_2, T_3$  and the double  $180^\circ$  turn time is the total duration of the gyrocompass session  $T$ . In order to achieve the same averaging of the measurement noise in both angular positions, it is advisable to ensure the equality  $T_1+T_3=T_2$ .

Let's consider the measurement model and the algorithm for data processing.

Let  $\psi$  be the angle of a course determined;  $\alpha_1^{(1)}, \alpha_1^{(2)}$  – the SA inclination angles of the first PH for the horizontal plane in the first (initial) and second (reverse) positions, measured by the first AA;  $\alpha_2^{(1)}, \alpha_2^{(2)}$  the SA inclination angles of the second LG to the horizontal plane in the first (initial) and second (reverse) positions, measured by the second AA;  $\Delta$  – angle of rotation of the platform in the reverse position;  $\delta\Omega_i^{(j)}$  – additive error of measurement of the  $i$ -th LG ( $i=1, 2$ ) in the  $j$ -th position ( $j=1, 2$ ) [8].

Under these conditions, measurements of the first LG in two positions can be written in the form

$$\Omega_1^{(1)} = \Omega_N \cos \psi \cdot \cos \alpha_1^{(1)} + \Omega_h \sin \alpha_1^{(1)} + \delta\Omega_1^{(1)},$$

$$\Omega_1^{(2)} = \Omega_N \cos(\psi + \Delta) \cdot \cos \alpha_1^{(2)} + \Omega_h \sin \alpha_1^{(2)} + \delta\Omega_1^{(2)},$$

where  $\Omega_N = \Omega \cdot \cos \phi$ ,  $\Omega_h = \Omega \cdot \sin \phi$ ,  $\Omega$  – the Earth's rotation speed,  $\phi$  – the latitude of the site.

The measurement of the second LG, which AA is rotated relative to the first by  $90^\circ$  clockwise (if "viewed" from above), have the form

$$\Omega_2^{(1)} = -\Omega_N \sin \psi \cdot \cos \alpha_2^{(1)} + \Omega_h \sin \alpha_2^{(1)} + \delta\Omega_2^{(1)},$$

$$\Omega_2^{(2)} = -\Omega_N \sin(\psi + \Delta) \cdot \cos \alpha_2^{(2)} + \Omega_h \sin \alpha_2^{(2)} + \delta\Omega_2^{(2)}.$$

Solving the above expressions for the cosine of  $C_\psi$  and the sine  $S_\psi$  of  $\psi$ , let's obtain

$$C_\psi = \frac{b_2 \cdot \cos \alpha_1^{(2)} \cdot \sin \Delta - b_1 (\cos \alpha_2^{(2)} \cdot \cos \Delta - \cos \alpha_2^{(1)})}{D \cdot \Omega_N},$$

$$S_\psi = \frac{b_1 \cdot \cos \alpha_2^{(2)} \cdot \sin \Delta + b_2 (\cos \alpha_1^{(2)} \cdot \cos \Delta - \cos \alpha_1^{(1)})}{D \cdot \Omega_N},$$

where

$$D = \cos \alpha_1^{(1)} \cos \alpha_2^{(1)} + \cos \alpha_1^{(2)} \cos \alpha_2^{(2)} - (\cos \alpha_1^{(2)} \cos \alpha_2^{(1)} + \cos \alpha_1^{(1)} \cos \alpha_2^{(2)}) \cos \Delta,$$

$$b_1 = \Omega_1^{(1)} - \Omega_1^{(2)} - \Omega_h (\sin \alpha_1^{(1)} - \sin \alpha_1^{(2)}) - (\delta\Omega_1^{(1)} - \delta\Omega_1^{(2)}),$$

$$b_2 = \Omega_2^{(1)} - \Omega_2^{(2)} - \Omega_h (\sin \alpha_2^{(1)} - \sin \alpha_2^{(2)}) - (\delta\Omega_2^{(1)} - \delta\Omega_2^{(2)}).$$

At the end, the angle  $\psi$  is determined from this, taking into account the signs of the cosine and the sine.

Under these conditions, the problem is considered, that when using LG with dither device planned for performing a measurement session, the time  $T$  may not be enough to smooth out the high-frequency component of the measurements caused by both noise and vibration. This will lead to the fact that the error in determining the course of the formulas given above will be too large. Overcoming noise in measurements when solving the problems of the exhibition as a whole and gyrocompassing, in particular, is devoted to the paper [9], which proposes the problem of determining the angles as a problem of estimating the state vector.

To solve this problem, it is proposed to abandon dither device, ensuring the normal operation of LG by forcing the optical contours of gyroscopes. Under these conditions, the level of the high-frequency component of the measurements will significantly decrease, the results of smoothing will become more effective, the accuracy of the course determination will increase.

To further fully understand the purpose of the research, let's emphasize that gyroscopes are of two types, which differ substantially in the principle of action based on mechanical and optical gyroscopes [10]. The second option is considered – gyroscopes based on optical, namely, laser gyroscopes.

The aim of gyrocompasses is the definition of a direction azimuth, established, for example, by pointing the vizier to a remote object. Given the limited functionality of the gyrocompass compared to the SINS, its instrumentation typically contains one or two angular velocity sensors (AVC) and one or two accelerometers. In this case, the turn of the measuring unit into another, reverse, angular position with high accuracy is controlled instrumentally, for example, by a mechanical lock, reduces the error in measuring the angle of rotation  $\Delta$  and the method as a whole.

The main characteristic of the gyrocompass is the accuracy of determining the angle of the course for a fixed duration of the measurement session. Under these conditions, one of the factors influencing the error in determining the heading angle is the high-frequency component of gyroscopic measurements (usually noise). To reduce this influence, either increase the duration of the session or reduce the noise level. If do not change the planned duration of the session, then in the framework of the traditional DGC scheme, which involves fixing the axis of sensitivity of gyroscopes in two anticolinear directions, no improvement can be proposed to reduce the influence of noise on the GC accuracy.

Therefore, the problem of high-precision determination of the initial azimuth with the use of special equipment – gyrocompass, built on optical gyroscopes, is then considered. The idea of using a dynamic gyrocompass on fiber-optic gyroscopes is proposed in [11]. However, in the given case it is a different type of gyroscope, and the task is not to reduce the influence of the noise component on the dither device.

To obtain a positive effect under these conditions, it is possible to propose a new gyrocompass design, an original technique for performing and processing measurements, and an algorithm for calculating the azimuth that is being sought. All this series of measures ensure high efficiency of a new solution to the problem of initial course determination.

**3. The aim and objectives of research**

The aim of research is development of a more accurate method and algorithm for dynamic gyrocompassing and proving the hypothesis about the possibility of increasing the accuracy of the course determination in the gyrocompass with LG by replacing the dither device with forced regular rotation.

To achieve this aim, the following tasks are identified:

- to propose the composition and construction of the device;

- to construct the corresponding mathematical model of measurements and describe the measurement procedure, to develop an algorithm for data processing;
- to conduct an experiment and obtain results of semi-realistic modeling;
- to analyze sources of errors in the course determination and formulate recommendations for practical use.

**4. Materials and methods of research**

Let's consider an alternative scheme of gyroscopes that has two laser gyroscopes (LG) with a vibration-type stand and two accelerometers. In contrast to the circuit, it uses LG with dither device, sensor sensitivity axes are located at an angle to the vertical axis and the whole block rotates about the vertical axis so that the role of dither device in LG plays rotations with a constant speed. Under these conditions, a significant part of the high-frequency components will be absent in the LG measurements, which arises in the "normal conditions" as a result of a mechanical dither device. Thus, the LG rotation factor makes it possible to reduce the noise level in gyroscopic measurements in comparison with the traditional scheme of using similar gyroscopes, which in the end should improve the accuracy of determining the course for the same duration of the session. Taking into account the mobility of the LG unit during the measurement let's call this gyrocompass dynamic.

Let's describe the proposed construction of a dynamic gyrocompass.

The instrumental composition (Fig. 1) contains two LGs and two accelerometers (AA) located on the platform, for a measurement session, approximately horizon. The LG and AA block located on it with the help of the engine can rotate about an axis normal to the platform at a given speed.

The sensitivity axes (SA) of the LG form nominally defined angles with the platform and, respectively (positive count upwards). LG projections on the platform are mutually orthogonal. The sensitivity AA axes coincide with the SA LG.

The angle of rotation of the LG-AA block relative to the so-called "zero reference", reproducing the direction of the line of sight, is measured by a non-gyroscopic angle-of-rotation sensor (ARS).

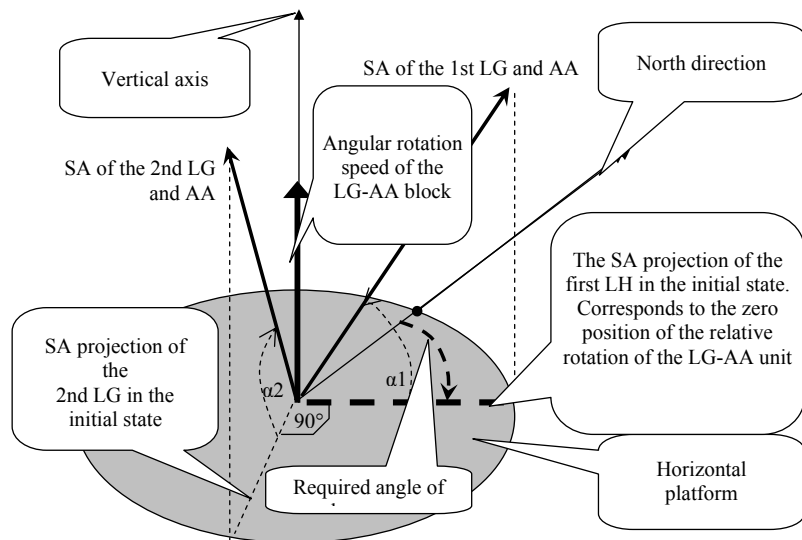


Fig. 1. Illustration for the construction of a dynamic gyrocompass

After switching on the device for a short time, one turn of the LG-AA unit takes place with a fixation of the fixed position (for 5 seconds) every 45° for accelerometers to measure the possible precession of the rotation axis of the platform. According to these data, the dependences of the deviation angles from the horizontal plane

$$\alpha_1(\theta) = \alpha_1^H + \delta\alpha_1(\theta),$$

$$\alpha_2(\theta) = \alpha_2^H + \delta\alpha_2(\theta),$$

in which  $\alpha_1^H, \alpha_2^H$  – the nominal values of the angles;  $\delta\alpha_1, \delta\alpha_2$  – small deviations of the actual values of the angles from the nominal, resulting from inaccurate platform horizon and possible precession of the rotation axis;  $\theta$  – the angle of the relative rotation of the LG-AA block around the axis of the platform, measured with the ARS help. Further, these dependences are used in the processing of gyroscopic measurements.

After making the first turn without stopping in the initial position (to exclude the acceleration section), the LG-AA unit rotates with a nominal constant speed  $v$ . Starting from the moment of passing through the “zero reference” point, processing of LG measurements is carried out at the real-time tempo. After each complete turnover, controlled by the ARS, the sum of all previous measurements is used to estimate the value of the heading angle  $\psi$  of the line of sight.

In order to justify the data processing algorithm, let's construct a model of LH measurements in the described conditions.

Let the SA of the first LG and its projection to the horizontal plane in the initial state corresponds to the zero reference angle of the relative rotation and forms the heading angle  $\psi$  to be determined with northward direction. When LG rotates around the axis of the platform at a given speed (positive direction is “clockwise”), the instantaneous measurements of the first LG can be represented in the form

$$\Omega_1(t) = \Omega_N \cdot \cos(\psi + vt) \cdot \cos\alpha_1(\theta) + \Omega_h \cdot \sin\alpha_1(\theta) - v \cdot \sin\alpha_1^H + \delta\Omega_1 + \delta\Omega_1^M(\psi + vt) + \xi_1(t), \quad (1)$$

where  $\Omega_N = \Omega \cdot \cos\phi$ ,  $\Omega_h = \Omega \cdot \sin\phi$  – projections of the angular velocity of the Earth's rotation on the northern and vertical axes of the local geographic coordinate system (SC);  $\delta\Omega_1$  – quasistationary component of the measurement error of the 1st LG;  $\delta\Omega_1^M$  – the magnetic component of the measurement error, depends on the current azimuth  $\psi + v \cdot t$ ;  $t$  – the current time, which is counted from the moment of the first passage of the “zero reference” point;  $\alpha_1(\theta), \alpha_2(\theta)$  – the dependence of the angles between the SA LG and the horizontal plane on the angle of relative rotation  $\theta(t) = v \cdot t$ , obtained at the initial stage of the work on the AA measurements;  $\xi_1(t)$  – measurement noise of the 1st LG.

Thus, in (1) the first term is the contribution of the northern component of the Earth's rotation speed at a variable rate, which changes as a result of unit rotation. The second term is the projection of the vertical velocity of the Earth's rotation on the SA. The third term is the contribution of the relative rotation speed of the platform. The fourth term is a constant and slowly drifting LG (the characteristic time for estimating the rate of change is the period of revolution). The fifth term is the magnetic component of the drift, the magnitude of which depends on the current SA azimuth.

For the second LG, the measurement model is similar:

$$\Omega_2(t) = -\Omega_N \cdot \sin(\psi + vt) \cdot \cos\alpha_2(\theta) + \Omega_h \cdot \sin\alpha_2(\theta) - v \cdot \sin\alpha_2^H + \delta\Omega_2 + \delta\Omega_2^M(\psi + vt) + \xi_2(t). \quad (2)$$

The meaning of the variables is the same as for (1), but they are attributed to the second LG.

The measurements are carried out and processed on a computer with a sufficiently high refresh rate, for example 1000 Hz.

Thus, the formulation of the problem of determining the course under these conditions is the following: having updated measurements (1), (2), as well as latitude, relative rotation speed  $v$  (it can be refined from the measurements by means of the ARS of the actual angle  $\theta = v \cdot t$  of relative rotation), and the angles  $\alpha_1(\theta), \alpha_2(\theta)$ , dependences are necessary calculate the angle  $\psi$ . In this case, the angle  $\psi$  corresponds to the angle between the direction to the north and the “zero reference” of the platform (Fig. 1).

Let's obtain the calculation formulas for the method of solution.

To isolate the initial phase of the useful signal, is the first term in (1), (2), algorithmically multiply the measurements  $\Omega_1(t)$  (1) by  $\cos(vt) / \cos\alpha_1(\theta)$ , and measurements  $\Omega_2(t)$  (2) by  $\sin(v \cdot t) / \cos\alpha_2(\theta)$  and subtract them from the first expression:

$$\begin{aligned} & \Omega_1 \cdot \cos(vt) / \cos\alpha_1(\theta) - \Omega_2 \cdot \sin(vt) / \cos\alpha_2(\theta) = \\ & = \Omega_N \cdot (\cos(\psi + vt) \cdot \cos(vt) + \sin(\psi + vt) \cdot \sin(vt)) + \\ & + \Omega_h \cdot (tg\alpha_1(\theta) \cdot \cos(vt) - tg\alpha_2(\theta) \cdot \sin(vt)) - \\ & - v \cdot (\sin\alpha_1^H / \cos\alpha_1(\theta)) \cdot \cos(vt) + \\ & + v \cdot (\sin\alpha_2^H / \cos\alpha_2(\theta)) \cdot \sin(vt) + \\ & + \delta\Omega_1 \cdot \cos(vt) / \cos\alpha_1(\theta) - \delta\Omega_2 \cdot \sin(vt) / \cos\alpha_2(\theta) + \\ & + \delta\Omega_1^M \cdot \cos(vt) / \cos\alpha_1(\theta) - \delta\Omega_2^M \cdot \sin(vt) / \cos\alpha_2(\theta) + \\ & + \xi_1(t) \cdot \cos(vt) / \cos\alpha_1(\theta) - \xi_2(t) \cdot \sin(vt) / \cos\alpha_2(\theta). \quad (3) \end{aligned}$$

The second line of expression (3) contains the cosine of the difference and is reduced to the form  $\Omega_N \cdot \cos\psi$ . Let's integrate (3) for the period of relative rotation, that is, on the time interval  $T = 2\pi / v$ , for which LG-AA block completes the complete revolution:

$$\begin{aligned} & \int_0^T (\Omega_1 \cdot \cos(vt) / \cos\alpha_1(\theta) - \Omega_2 \cdot \sin(vt) / \cos\alpha_2(\theta)) dt = \\ & = \Omega_N \cdot T \cdot \cos\psi + \Omega_h \cdot \int_0^T (tg\alpha_1(\theta) \cdot \cos(vt) - tg\alpha_2(\theta) \cdot \sin(vt)) dt - \\ & - v \cdot \int_0^T (\sin\alpha_1^H \cdot \cos(vt) / \cos\alpha_1(\theta) - \sin\alpha_2^H \cdot \sin(vt) / \cos\alpha_2(\theta)) dt + \\ & + \delta\Omega_1 \cdot \int_0^T (\cos(vt) / \cos\alpha_1(\theta)) dt - \delta\Omega_2 \cdot \int_0^T (\sin(vt) / \cos\alpha_2(\theta)) dt + \\ & + \delta\Omega_1 \cdot \int_0^T (\cos(vt) / \cos\alpha_1(\theta)) dt - \delta\Omega_2 \times \\ & \times \int_0^T (\sin(vt) / \cos\alpha_2(\theta)) dt + \int_0^T (\delta\Omega_1^M \cdot \cos(vt) / \cos\alpha_1(\theta)) dt - \\ & - \int_0^T (\delta\Omega_2^M \cdot \sin(vt) / \cos\alpha_2(\theta)) dt + \int_0^T (\xi_1(t) \cdot \cos(vt) / \cos\alpha_1(\theta)) dt - \\ & - \int_0^T (\xi_2(t) \cdot \sin(vt) / \cos\alpha_2(\theta)) dt. \quad (4) \end{aligned}$$



The value of the period is controlled by the actual time of completion of the full turnover, fixed by the ARS.

With (4) let's obtain the formula for definition:

$$\cos \Psi = \frac{1}{\Omega_N \cdot T} \cdot \int_0^T \left[ \begin{aligned} & \left( \Omega_1 \cdot \cos(vt) / \cos \alpha_1(\theta) - \right. \\ & - \Omega_2 \cdot \sin(vt) / \cos \alpha_2(\theta) - \\ & - \Omega_h \cdot (tg \alpha_1(\theta) \cdot \cos(vt) - \\ & - tg \alpha_2(\theta) \cdot \sin(vt)) + \\ & + v \cdot (\sin \alpha_1^H \cdot \cos(vt) / \cos \alpha_1(\theta) - \\ & \left. - \sin \alpha_2^H \cdot \sin(vt) / \cos \alpha_2(\theta)) \right) \end{aligned} \right] dt - \\ - \frac{\delta \Omega_1}{\Omega_N \cdot T} \cdot \int_0^T (\cos(vt) / \cos \alpha_1(\theta)) dt + \\ + \frac{\delta \Omega_2}{\Omega_N \cdot T} \cdot \int_0^T (\sin(vt) / \cos \alpha_2(\theta)) dt - \\ - \frac{1}{\Omega_N \cdot T} \int_0^T (\delta \Omega_1^M \cdot \cos(vt) / \cos \alpha_1(\theta)) dt + \\ + \frac{1}{\Omega_N \cdot T} \int_0^T (\delta \Omega_2^M \cdot \sin(vt) / \cos \alpha_2(\theta)) dt - \\ - \frac{1}{\Omega_N \cdot T} \int_0^T (\xi_1(t) \cdot \cos(vt) / \cos \alpha_1(\theta)) dt + \\ + \frac{1}{\Omega_N \cdot T} \int_0^T (\xi_2(t) \cdot \sin(vt) / \cos \alpha_2(\theta)) dt. \quad (5)$$

And the first integral is calculated by the current measurement, and all subsequent components are unknown, they have to be ignored and make up the error of the method.

Let's discuss the components of the error.

1. If  $\alpha_1(\theta)$ ,  $\alpha_2(\theta)$  on the reverse vary slightly, that is, they can be considered constants, then

$$\int_0^T (\cos(vt) / \cos \alpha_1) dt = \frac{1}{\cos \alpha_1} \int_0^T (\cos(vt)) dt = 0, \\ \int_0^T (\sin(vt) / \cos \alpha_2) dt = \frac{1}{\cos \alpha_2} \int_0^T (\sin(vt)) dt = 0,$$

that is, the contribution of a systematic and slowly varying drift is absent. In addition, the formula for the calculated integral is also simplified. Only the first line of the expression in square brackets remains under the integral sign.

2. Under the same conditions, let's consider the effect of the magnetic component of the drift. The vector of magnetic induction can be written in the form of a sum of two components: vertical  $\vec{B}_h$  and horizontal  $\vec{B}_g$ . When SA rotation around the vertical axis (Fig. 1), the projections of the component  $\vec{B}_h$  on the SA and other axes connected with the LG do not change. So, the magnetic component  $\vec{B}_h$  of the drift from  $\vec{B}_h$  is the constant  $\delta \Omega_{10}^M = \text{const}$ . The projections of the horizontal component  $\vec{B}_g$  on the axes connected with the LG vary periodically. Therefore, formally, the effect of  $\vec{B}_g$  on the drift during SA rotation can be represented in the form

$$\delta \Omega_{11}^M \cdot \cos(\mu_0 + vt),$$

where  $\delta \Omega_{11}^M$ ,  $\mu_0$  – some amplitude and initial phase. Thus, let's write

$$\delta \Omega_1^M = \delta \Omega_{10}^M + \delta \Omega_{11}^M \cdot \cos(\mu_{01} + vt).$$

Then the integral can be taken analytically:

$$\int_0^T (\delta \Omega_1^M \cdot \cos(vt) / \cos \alpha_1) dt = \\ = \frac{1}{\cos \alpha_1} \int_0^T (\delta \Omega_{10}^M + \delta \Omega_{11}^M \cdot \cos(\mu_{01} + vt)) \cdot \cos(vt) dt = \\ = \frac{\delta \Omega_{10}^M}{\cos \alpha_1} \int_0^T \cos(vt) \cdot dt + \\ + \frac{\delta \Omega_{11}^M}{2 \cdot \cos \alpha_1} \left( \cos \mu_{01} \cdot T + \int_0^T \cos(\mu_{01} + 2vt) dt \right) = \\ = \frac{\delta \Omega_{11}^M \cdot \cos \mu_{01}}{2 \cdot \cos \alpha_1} T.$$

For the second gyroscope, carrying out similar arguments:

$$\int_0^T (\delta \Omega_1^M \cdot \sin(vt) / \cos \alpha_2) dt = \\ = \frac{1}{\cos \alpha_2} \int_0^T (\delta \Omega_{20}^M + \delta \Omega_{21}^M \cdot \cos(\mu_{02} + vt)) \cdot \sin(vt) dt = \\ = \frac{\delta \Omega_{21}^M}{\cos \alpha_2} \int_0^T \sin(vt) \cdot dt + \\ + \frac{\delta \Omega_{21}^M}{2 \cdot \cos \alpha_2} \left( -\sin \mu_{02} \cdot T + \int_0^T \sin(\mu_{02} + 2vt) dt \right) = \\ = -\frac{\delta \Omega_{21}^M \cdot \sin \mu_{02}}{2 \cdot \cos \alpha_2} T.$$

Thus, the error of the method is affected by the amplitude of the magneto-solid drift from the horizontal projection of the magnetic induction vector. The degree of such sensitivity is determined by the quality of the LG magnetic insulation.

3. It is not possible to estimate the contribution of noise components. It can only be asserted that it can be compared with the contribution of noise in the implementation of the traditional scheme of the DGC method.

Thus, the terms discarded in (5), are a methodological error, and correspond to expression

$$- \frac{\delta \Omega_{11}^M \cdot \cos \mu_{01}}{2 \cdot \cos \alpha_1 \cdot \Omega_N} + \frac{\delta \Omega_{21}^M \cdot \sin \mu_{02}}{2 \cdot \cos \alpha_2 \cdot \Omega_N} - \\ - \frac{1}{\Omega_N \cdot T \cdot \cos \alpha_1} \int_0^T (\xi_1(t) \cdot \cos(vt)) dt + \\ + \frac{1}{\Omega_N \cdot T \cdot \cos \alpha_2} \int_0^T (\xi_2(t) \cdot \sin(vt)) dt,$$

in which the first two terms correspond to the magnetic component of the drift and are determined by the amplitude of its change from the horizontal component of the magnetic induction vector when the gyroscope rotates about the vertical axis. The last two terms characterize the contribution of noise. It can be seen that the contribution of noise is minimal at  $\alpha_1 = \alpha_2 = 0$ . However, it is impossible to realize zero angles, according to the accepted concept of replacing the LG dither device by rotation with a constant angular velocity, since there will be no effect of rotation on the LH measurement process.

Let's return to the obtained formula (5) that allows to calculate  $\cos \psi$ . To avoid ambiguity in determining the heading angle  $\psi$  from the cosine, it is possible to obtain an analogous formula for estimating  $\sin \psi$ . To do this, let's algorithmically multiply the measurements  $\cos \psi$ . (1) by  $-\sin(v)/\cos \alpha_1(\theta)$ , and measurements (2) by  $\cos(v \cdot t)/\cos \alpha_2(\theta)$  and subtract from the first expression. As a result, let's obtain an expression for the  $\sin \psi$  evaluation similar to (5):

$$\begin{aligned} \sin \psi = & \frac{1}{\Omega_N \cdot T} \cdot \int_0^T \left( \begin{aligned} & -\Omega_1 \cdot \sin(vt) / \cos \alpha_1(\theta) - \\ & -\Omega_2 \cdot \cos(vt) / \cos \alpha_2(\theta) - \\ & -\Omega_h \cdot (-tg \alpha_1(\theta) \cdot \sin(vt) - \\ & -tg \alpha_2(\theta) \cdot \cos(vt)) + \\ & +v \cdot (-\sin \alpha_1^H \cdot \sin(vt) / \cos \alpha_1(\theta) - \\ & -\sin \alpha_2^H \cdot \cos(vt) / \cos \alpha_2(\theta)) \end{aligned} \right) dt + \\ & + \frac{\delta \Omega_1}{\Omega_N \cdot T} \cdot \int_0^T (\sin(vt) / \cos \alpha_1(\theta)) dt + \\ & + \frac{\delta \Omega_2}{\Omega_N \cdot T} \cdot \int_0^T (\cos(vt) / \cos \alpha_2(\theta)) dt + \\ & + \frac{1}{\Omega_N \cdot T} \cdot \int_0^T (\delta \Omega_1^M \cdot \sin(vt) / \cos \alpha_1(\theta)) dt + \\ & + \frac{1}{\Omega_N \cdot T} \cdot \int_0^T (\delta \Omega_2^M \cdot \cos(vt) / \cos \alpha_2(\theta)) dt + \\ & + \frac{1}{\Omega_N \cdot T} \cdot \int_0^T (\xi_1(t) \cdot \sin(vt) / \cos \alpha_1(\theta)) dt + \\ & + \frac{1}{\Omega_N \cdot T} \cdot \int_0^T (\xi_2(t) \cdot \cos(vt) / \cos \alpha_2(\theta)) dt. \end{aligned}$$

As for  $\cos \psi$  calculation, the first line is, in fact, an algorithm, and the following constitute a methodological error. All qualitative conclusions concerning the contribution of various components to the total error made for the formula (5) are valid here.

### 5. Results of studies of the developed method for determining the course angle

Summarizing the method for determining the course angle in a dynamic gyrocompass, let's formulate an algorithm for data processing.

The input data of the algorithm is the elementary measurements of LG and AA. Measurements are also made of the angle of relative rotation  $\theta$  and the estimate of the rotation speed  $v$ .

The algorithm is a sequence of operations:

1) the first 360° rotation takes place to determine the dependencies. For this purpose, at 8 angular positions, differing by 45°, the stationary state is realized, and the angles  $\alpha_1$ ,  $\alpha_2$  are determined from the AA measurements. Then, the sinusoidal dependence of these angles on the angle of relative rotation  $\theta$  is calculated by approximating them;

2) in the further rotation with the frequency of updating the measurements  $\Omega_1(t)$ ,  $\Omega_2(t)$  in real time, the variables

$$\begin{aligned} s_1 = & \Omega_1(t) \cdot \cos(vt) / \cos \alpha_1(\theta) - \\ & -\Omega_2(t) \cdot \sin(vt) / \cos \alpha_2(\theta) - \\ & -\Omega_h \cdot (tg \alpha_1(\theta) \cdot \cos(vt) - \\ & -tg \alpha_2(\theta) \cdot \sin(vt)) + \\ & +v \cdot (\sin \alpha_1^H \cdot \cos(vt) / \cos \alpha_1(\theta) - \\ & -\sin \alpha_2^H \cdot \sin(vt) / \cos \alpha_2(\theta)) \end{aligned}$$

and

$$\begin{aligned} s_2 = & -\Omega_1(t) \cdot \sin(vt) / \cos \alpha_1(\theta) - \Omega_2(t) \cdot \cos(vt) / \cos \alpha_2(\theta) - \\ & -\Omega_h \cdot (-tg \alpha_1(\theta) \cdot \sin(vt) - tg \alpha_2(\theta) \cdot \cos(vt)) + \\ & +v \cdot (-\sin \alpha_1^H \cdot \sin(vt) / \cos \alpha_1(\theta) - \sin \alpha_2^H \cdot \cos(vt) / \cos \alpha_2(\theta)) \end{aligned}$$

and accumulate taking into account the step of numerical integration  $\Delta t$ :

$$S_1 = S_1 + s_1 \cdot \Delta t,$$

$$S_2 = S_2 + s_2 \cdot \Delta t;$$

3) at the end of the full turnover, the variables are calculated

$$\psi_c = \frac{S_1}{\Omega_N \cdot T}, \quad \psi_s = \frac{S_2}{\Omega_N \cdot T},$$

where  $T$  – the total duration of data accumulation until the moment of fixation of the next full turn of the platform is fixed with the ARS measurements.

If  $|\psi_s| \leq 0,707$ , then

$$\psi = \begin{cases} \arcsin(\psi_s), & \text{if } \psi_c > 0, \\ \pi - \arcsin(\psi_s) \text{ or.} \end{cases}$$

If  $|\psi_s| > 0,707$ , then

$$\psi = \begin{cases} \arccos(\psi_c), & \text{if } \psi_s > 0, \\ -\arccos(\psi_c) \text{ or;} \end{cases}$$

4) the algorithm, starting from the second point, is repeated until the end of the session. The last estimate is formed at the time of the next full turnover.

Variables  $T$ ,  $S_1$ ,  $S_2$  are reset once, only at the beginning of the session.

To test the hypothesis on improving the results of gyrocompassing in the case where the LG dither device is absent, an experiment is conducted in which the LG block is forced to rotate about the vertical axis at a speed of 20 °/sec. The purpose of the experiment is collection of measurements obtained with LG dither device and without dither device. The collected measurements are processed according to the developed method.

The results of the treatment are shown in Fig. 2, where the number of full revolutions of the block is plotted along the horizontal axis, and the value of the heading angle along the vertical axis is determined from the obtained measurements. It should be noted that the true value of the angle is 164.18°.

The thin line (row 1) corresponds to the case with LG dither device, thick (p 2) – without it. From the graph (Fig. 2), it can be seen that the convergence of the course

estimate in the second case is much better than for the LG dither device, which confirms the hypothesis proposed for reducing the noise level in the LH measurements when rotating the dither device by rotation.

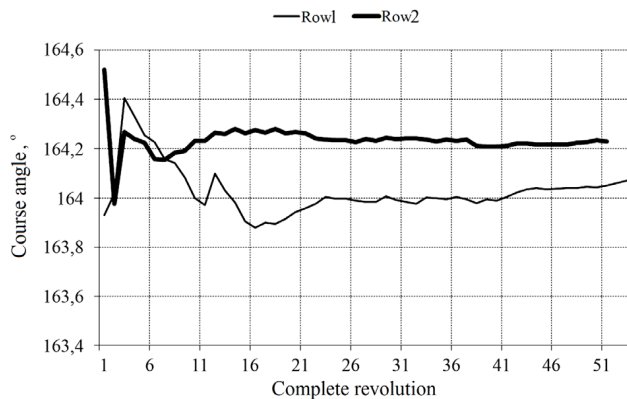


Fig. 2. The results of the method of dynamic gyrocompassing in cases with LG dither device and without it, depending on the number of full revolutions

## 6. Discussion of the gyrocompass simulation results

From the results of modeling the work of the gyrocompass of the proposed design, it is seen that the rejection of LG dither device, firstly, helps to reduce the spread of course estimates in comparison with the traditional scheme of using LG; second, it accelerates the convergence of the result; and third, I increase the accuracy of the course determination at LG the same duration of the session. The explanation of these positive effects is as follows. Reducing the spread of the result and accelerating its convergence is a consequence of a decrease in the noise level in the initial measurements in the absence of dither device. This fact contributes to a better averaging of the noise component, which is realized by numerical integration according to the formulas of point 2 of the algorithm. The main reason for the increase in the accuracy of the gyrocompass in the absence of dither device, in comparison with its case, appears in the fact that for the second case there was not enough time devoted to the accumulation and averaging of the data.

This is a great advantage of this method, so its use is quite appropriate. At the same time, the given graph (Fig. 2) also shows the presence of the last gyrocompass error, in part can be attributed to the lack of the proposed method.

For a better understanding of the problem, let's cite possible sources of error in the method and previous estimates of their significance:

- the magnetic component of drift – the significance is determined by the magnitude of the variability of the drift from the azimuth;

- noise component – its influence, as the simulation shows, does not exceed the influence of noise in the traditional method of DGC;

- precession and non-rotation of the axis of rotation – the effect is compensated to some extent by averaging over the period of rotation;

- the error in measuring the angle of relative rotation and the cut-off of the moments of time when the total turnover is realized.

But in view of the fact that now there are precision encoders that can be used as ARC and realizing in the technique a high frequency of surveying and data processing, the error from these factors can be neglected.

Further development of the study may concern the following aspects. First, formalization and quantitative analysis of the errors of this method, identifying the most influential factors and discussing the possibility of overcoming them. Secondly, in the formation of requirements for the required characteristics of accelerometers, gyroscopes, the sensor is the angle of rotation, based on the requirements for the device as a whole. Thirdly, the optimization of the gyrocompassing parameters, in particular, the rotation speed and the polling frequency of the sensors, by the criterion of the accuracy of the operation. Finally, certain elaboration is required and constructive and technological issues of proposed manufacturing gyroscopes.

## 7. Conclusions

1. A new design of a dynamic gyrocompass, based on optical gyroscopes with a rejection from dither device, is proposed. The normal LG operation with this solution is provided by forcibly rotating the optical contours of the gyroscopes.

2. The corresponding dynamic LG model for measurements in DGC is constructed, which takes into account the variable geometry of the structure; quasistationary, azimuthally dependent and stochastic components of error. With a look at the proposed design, a measurement procedure for such a gyrocompass and an algorithm for processing them with a view to obtaining an azimuth estimate has been developed. Unlike well-known algorithms used in static gyrocompasses, it performs high-frequency processing of the current measurement variables of two LGs in which the dither device is replaced by a controlled rotation of the optical circuits. This technology has made it possible to improve the autocompensation of the quasi-stationary component of the error; to reduce the sensitivity of the azimuth estimate to the effect of an external magnetic field, which ultimately led to an increase in the accuracy of determining the azimuth using a dynamic gyrocompass. Reducing the noise level due to the failure of the stand, increased the rate of convergence of the current estimate to the true value.

3. Semi-detailed computer simulation of the developed method for the use of dynamic gyroscopes show significant reduction (by an order of magnitude) of the noise level in the LG measurements when the dither device is replaced by rotation.

4. The advantages of dynamic gyrocompass are determined, namely, potentially high accuracy of operation. That is, refusing dither device, it is possible to reduce the noise level of the laser gyroscope. It is also better to compensate for the effect of a slowly changing drift (enough that drift changes are not significant during the rotation period, that is, 30–50 s). And the magnetic component of drift is better compensated, if it is.

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