Визначена раціональна динамічна характеристика автомобіля, що дозволяє розгін при мінімальних витратах енергії двигуна. Визначено закони зміни часу розгону автомобіля при реалізації граничної по зчепленню ведучих коліс з дорогою сумарної тягової сили й при реалізації запропонованого раціонального закону управління прискоренням. Проведена оцінка ефективної роботи ДВЗ при розгоні на різних передачах автомобіля

Ключові слова: динаміка розгону, раціональне управління, зниження витрат енергії, раціональна швидкість

Определена рациональная динамическая характеристика автомобиля, позволяющая разгон при минимальных затратах энергии двигателя. Определены законы изменения времени разгона автомобиля при реализации предельной по сцеплению ведущих колес с дорогой суммарной тяговой силы и при реализации предложенного рационального закона управления ускорением. Проведена оценка эффективной работы ДВС при разгоне на различных передачах автомобиля

Ключевые слова: динамика разгона, рациональное управление, снижение затрат энергии, рациональная скорость

UDC 629.3.016
DOI: 10.15587/1729-4061.2018.121568

SYNTHESIS OF ENERGY-EFFICIENT ACCELERATION CONTROL LAW OF AUTOMOBILE

M. Podrigalo
Doctor of Technical Sciences, Professor*

R. Kaidalov
PhD, Associate Professor*
E-mail: kaidalov.76@ukr.net

D. Klets
Doctor of Technical Sciences, Associate Professor
Department of Computer Technologies and Mechatronics**

N. Podrigalo
Doctor of Technical Sciences, Associate Professor
Department of Engineering and Computer Graphics**

A. Makovetskyi
PhD, Associate Professor
Department of Automobiles and Transport Infrastructure
National Aerospace University named after M. Zhukovsky Kharkiv Aviation Institute
Chkalova str., 17, Kharkiv, Ukraine, 61070

V. Hatsko
PhD
Department of machine building technology and machinery repair**

D. Abramov
PhD, Associate Professor
Department of machine building technology and machinery repair**

Yu. Tarasov
PhD, Associate Professor*

D. Lytovchenko
PhD
Department of Theory and Design of Automotive and Special Vehicles
Ivan Kozhedub Kharkiv University of Air Force
Sumskaya str., 77/79, Kharkiv, Ukraine, 61023

O. Litvinov
Adjunct*

*Scientific Center
National Academy of the National Guard of Ukraine
Zakhysnykiv Ukrainy sq., 3, Kharkiv, Ukraine, 61001
**Kharkiv National Automobile and Highway University
Yaroslava Mudroho str., 25, Kharkiv, Ukraine, 61002

1. Introduction

Energy efficiency is the property that ensures high dynamics of a vehicle with the lowest energy consumption. The improvement of energy efficiency is required due to the growing intensity of road traffic and the requirements to reduce the harmful effect of automobile transport on the environment. Solving a given task is positively influenced by the emergence of electric cars and hybrid vehicles with a combined electromechanical drive of drive wheels. The existence of electrical part in the drive of drive wheels makes it possible to rationally control the acceleration of a vehicle, in other words, to implement such a law of change in the acceleration of a car depending on speed that would ensure the lowest consumption of energy (power consumption) by the engine.

The largest energy expenditure occurs when a vehicle accelerates on lower gears. That is why studying the influence of various factors on a given process and determining the directions aimed at reducing energy cost is an important stage in addressing the specified important scientific-technical task.

A vehicle controlled by the driver is a movable physical (energy, thermal-mechanical) system for producing mechanical energy and its immediate utilization for the implementation of a transportation process. Therefore, energy issues are particularly important for the automobile. High indicators of vehicle dynamics should be attained at minimal energy consumption by the engine. Under acceleration mode, it is expedient to control a vehicle motion making use of electric part of the drive.

Energy efficiency is one of the most important operational properties of the automobile. Maximal fuel consumption is associated with the unstable mode of operation of vehicle's internal combustion engine. The appearance of cars with a hybrid electromechanical drive of drive wheels makes it possible to reduce the additional consumption of energy under unstable mode of ICE operation.

2. Literature review and problem statement

Papers [1, 2] consider choosing rational motion speed under condition of high dynamic indicators of vehicle. Authors of [3] established the relationship between the power of the engine used to accelerate the vehicle, motion speed, and recommended acceleration

$$m_a \cdot V_a \cdot \dot{V}_a = \frac{\Delta N_e \cdot \eta_{tr}}{\delta_{rm}}, \tag{1}$$

where m_a is the mass of the vehicle; V_a, $\dot{V}_a$ are the linear speed and vehicle acceleration; ΔN_e is the additional engine power used for the acceleration of the vehicle; η_{tr} is the transmission performance efficiency; δ_{rm} is the accounting factor for the rotating masses of engine and transmission;

$$\delta_{rm} = 1 + \tau_1 + \tau_2 \cdot U_{gr}^2; \tag{2}$$

U_{gr} is the gear ratio of the gearbox; τ_1; τ_2 are the coefficients that take into account the effect of rotating masses, which are connected to the drive wheels through the constant and variable transfer ratios, τ_1=0.03–0.05; τ_2=0.04–0.06.

Acceleration of the vehicle from speed V_a to speed $V_a+\Delta V_a$ will last before a new equilibrium is reached (the traction balance [3]). In papers [1, 2], authors derived an equation that determines the condition for establishing the following equilibrium at a higher speed of the vehicle equal to $V_a+\Delta V_a$

$$\Delta N_e \cdot \eta_{tr} = m_a \cdot g \cdot \psi \cdot (V_{a1} - V_a) + k \cdot F \cdot (V_{a1}^3 - V_a^3), \tag{3}$$

where g is the free fall acceleration, g=9.81 m/s^2; ψ is the total coefficient of road resistance; k is the coefficient of air resistance; F is the frontal drag area (midsection) of the vehicle; V_{a1} is the speed at which the vehicle reaches the new equilibrium,

$$V_{a1} = V_a + \Delta V_a. \tag{4}$$

Authors of articles [1, 2] derived from equation (3), with respect to relations (1) and (4), a cubic algebraic equation relative to parameter ΔV_a

$$(\Delta V_a)^3 + 3V_a(\Delta V_a)^2 + \left(3V_a^2 + \frac{m_a g \psi}{kF}\right)\Delta V_a - \frac{m_a \delta_{rm}}{3kF} V_a \dot{V}_a = 0. \tag{5}$$

The task solved in papers [1, 2] implied determining a function $\Delta V_a(V_a)$ to be followed by obtaining rational speed (V_a), which ensures $(\Delta V_a)_{\max}$. The authors of scientific studies [1, 2], rather than solving equation (5) using the Cardano's expression, applied approximate and numerical solutions. Fig. 1 shows dependences $\Delta V_a(V_a)$, constructed in papers [1, 2] employing the approximate and numerical methods without an accurate analytical solution to equation (5).

Curve 3 in Fig. 1 is built based on the numerical solution to equation (5). Curve 1 in Fig. 1 – for the case when terms $(\Delta V_a)^3$ and $(\Delta V_a)^2$ are removed from equation (5); and curve 2 – at $(\Delta V_a)^3$=0. The authors of papers [1, 2] were interested in the points of maximum for curves 1, 2, 3. An analysis of these curves allowed them to conclude that the points of maximum of the indicated curves coincide. They are determined from dependence

$$(V_a)_{\text{rat}} = \sqrt{\frac{m_a g \psi}{3kF}}. \tag{6}$$

However, papers [1, 2], to determine $(\Delta V_a)_{\max}$, proposed an approximate analytical expression since solving a cubic algebraic equation using the Cardano's expression was time consuming. In order to obtain the exact solution, it is necessary to solve equation (5).

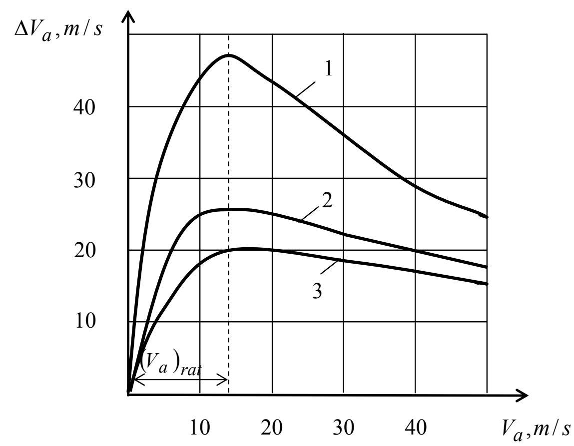

Fig. 1. Dependence chart of $\Delta V_a(V_a)$ at ψ=0.017; kF/m_a=0.0003 m^{-1}; δ_{rm}=1.176 [1, 2]

Acceleration time to the assigned speed is the indicator of traction-speed properties of the vehicle. That is why it is required to determine the acceleration time of a vehicle at $\dot{V}_a \le \dot{V}_{a\,\text{bound}}$. In addition, it is necessary to compare the functions of change in the acceleration time of a vehicle on speed, implemented at the existing rational techniques of control over acceleration and at $\dot{V}_\text{a} = \dot{V}_\text{a bound}$.

Targeted measures that affect technologies of vehicles increasingly become a tool for the developers of energy policy in the EU as a means of achieving the objectives in the field of energy efficiency, renewable energy, climate change, and ensuring energy security. There are known papers whose authors explore energy costs of vehicles from certain fleets and predict their growth: article [4] proposed a model that estimates current energy consumption by the automobile fleet in Andorra, [5] in the Republic of Ireland, and paper [6] – the city of Chang-Zhu-Tan, People's Republic of China. Authors of article [7] suggest a method for calculating energy efficiency of car fleet on the roads in terms of "traffic factor". However, these studies do not solve the problem on determining the relationship between the maximum increase in speed and the minimal increase in the vehicle engine power.

Energy efficiency of the vehicle is addressed in papers [8, 9]. To model the external speed characteristic of the internal combustion engine, the following empirical dependence is applied

$$N_e = N_{e\max}\left(a\lambda + b\lambda^2 - c\lambda^3\right), \tag{7}$$

where λ is the ratio of the current angular velocity ω_e to the angular velocity ω_N at maximum power,

$$\lambda = \frac{\omega_e}{\omega_N}; \tag{8}$$

a, *b*, *c* are the empirical coefficients of equation (7) (Table 1); $N_{e\max}$ is the maximum engine power.

Table 1

Values of coefficients for building the power using a method by S. R. Leyderman

Coefficients	Values for engines			
	carburetor	Diesel		
		Uniflow	pre-chamber	eddy-chamber
a	1	0.5	0.7	0.6
b	1	1.5	1.3	1.4
c	1	1	1.0	1.0
λ	$k=\left(a\lambda+b\lambda^2-c\lambda^3\right)$			
0.2	0.232	0.152	0.184	0.168
0.3	0.363	0.258	0.300	0.279
0.4	0.496	0.376	0.424	0.400
0.5	0.625	0.500	0.550	0.525
0.6	0.744	0.624	0.672	0.646
0.7	0.847	0.724	0.784	0.763
0.8	0.928	0.848	0.880	0.864
0.9	0.981	0.936	0.954	0.945
1.0	1.000	1.000	1.000	1.000
1.1	0.980	–	–	–

Papers [10, 11] report the study into stability of the vehicle during acceleration. However, their authors failed to assess the effective work performed by ICE when accelerating a crankshaft from angular velocity $\omega_{\min}$ (minimally stable rotation speed) to maximal angular velocity $\omega_{\max}$.

Thus, the analysis of known studies revealed the following:

– there is no analysis of the engine energy used for the acceleration of the vehicle when shifting different gears;

– there is no analytical solution to equation (5): this makes it impossible to state the law on the rational control over acceleration when accelerating a vehicle.

3. The aim and objectives of the study

The aim of present study is to determine the relationship between the maximum increase in speed and minimal increase in engine power, which would produce a rational law to control vehicle acceleration.

To accomplish the aim, the following tasks have been set:

– to build a rational dynamic characteristic of a vehicle;

– to estimate acceleration time of the vehicle to the assigned speed under various laws of control over acceleration;

– to evaluate effectiveness of ICE performance when accelerating the vehicle at different gears.

4. Rational dynamic characteristic of the vehicle

In a classic statement, first it is required to solve equation (5) relative to ΔV_a, and then proceed to search for the maximum employing known methods [1, 2]

$$\begin{cases} \dfrac{d(\Delta V_a)}{dV_a} = 0; \\ \dfrac{d^2(\Delta V_a)}{dV_a^2} < 0 \ \text{– at optimal value of } V_a. \end{cases} \tag{9}$$

First, we propose to differentiate equation (5) for V_a. In this case, the specified equation will take the form

$$3(\Delta V_a)^2 + 3\left[(\Delta V_a)^2 + 2V_a\Delta V_a\frac{d\Delta V_a}{dV_a}\right] + 3\left(V_a^2\frac{d\Delta V_a}{dV_a} + 2V_a\Delta V_a\right) + \frac{m_a g\psi}{kF}\frac{d\Delta V_a}{dV_a} - \frac{m_a\delta_{rm}}{kF}\dot{V}_a = 0. \tag{10}$$

A maximum of function (that is, the extremum shown in Fig. 1) $\Delta V_a(V_a)$ is reached at $d\Delta V_a/dV_a=0$. At $d\Delta V_a/dV_a=0$, expression (10) takes the following form

$$3(\Delta V_a)^2 + 6V_a\Delta V_a - \frac{m_a\delta_{rm}}{kF}\dot{V}_a = 0 \tag{11}$$

or

$$(\Delta V_a)^2 + 2V_a\Delta V_a - \frac{m_a\delta_{rm}}{3kF}\dot{V}_a = 0. \tag{12}$$

The solution to equation (12) with respect to the root that has a physical meaning will be obtained in the form

$$\Delta V_a = (\Delta V_a)_{\max} = -(V_a)_{\text{rat}} + \sqrt{(V_a)_{\text{rat}}^2 + \frac{m_a\delta_{rm}}{3kF}\dot{V}_a}. \tag{13}$$

Expression (13) connects the maximum increase in the motion speed $(\Delta V_a)_{\max}$ and rational speed $(V_a)_{\text{rat}}$, at which it is possible to implement the specified increase.

It is proposed to solve equation (5) relative to V_a. In this case, the indicated equation transforms to the form

$$V_a^2+\left(\Delta V_a-\frac{m_a\dot{V}_a\delta_{rm}}{3kF\Delta V_a}\right)V_a+\frac{m_a g\psi}{3kF}=0. \qquad (14)$$

Solution to quadratic solution (14)

$$V_{a1,2}=\frac{m_a\dot{V}_a\delta_{rm}}{6kF\Delta V_a}-\frac{\Delta V_a}{2}\pm$$
$$\pm\sqrt{\left(\frac{m_a\dot{V}_a\delta_{rm}}{6kF\Delta V_a}-\frac{\Delta V_a}{2}\right)^2-\frac{m_a g\psi}{3kF}}. \qquad (15)$$

Dependence chart of $V_a(V_a)$ will represent a chart turned at 90° shown in Fig. 1. The chart shown in Fig. 2 demonstrates that $(V_a)_{max}$ is implemented at $V_a=(V_a)_{rat}$ and at $V_{a1}=V_{a2}$. Condition $V_{a1}=V_{a2}$ is satisfied when a discriminant of the quadratic equation equals zero, that is, at

$$\left(\frac{m_a\dot{V}_a\delta_{rm}}{6kF\Delta V_a}-\frac{\Delta V_a}{2}\right)^2-\frac{m_a g\psi}{3kF}=0. \qquad (16)$$

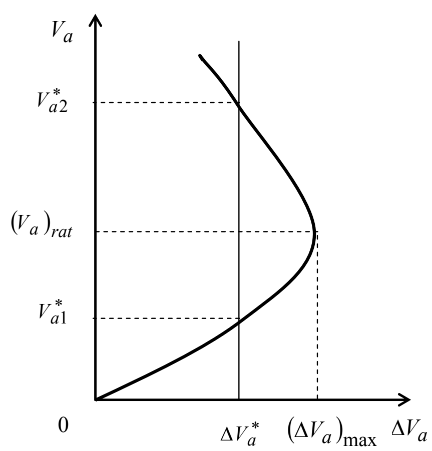

Fig. 2. Dependence chart of $\Delta V_a(V_a)$

We shall derive from equation (16)

$$\Delta V_a=(\Delta V_a)_{max}=$$
$$=\sqrt{\frac{m_a g\psi}{3kF}\left[\frac{\dot{V}_a\delta_{rm}}{g\psi}+2\left(1-\sqrt{1+\frac{\dot{V}_a\delta_{rm}}{g\psi}}\right)\right]}. \qquad (17)$$

Thus, considering expression (15)

$$V_a=(V_a)_{rat}=$$
$$=\sqrt{\frac{m_a g\psi}{3kF}}\frac{\sqrt{1+\frac{\dot{V}_a\delta_{rm}}{g\psi}}-1}{\sqrt{\frac{\dot{V}_a\delta_{rm}}{g\psi}+2\left(\sqrt{1+\frac{\dot{V}_a\delta_{rm}}{g\psi}}-1\right)}}. \qquad (18)$$

By comparing expressions (6) and (18), we can conclude that the analytical solution to equation (5) allowed us to refine the approximated result that was obtained earlier (6). Correction factor

$$K_{cor}=\frac{\sqrt{1+\frac{\dot{V}_a\delta_{rm}}{g\psi}}-1}{\sqrt{\frac{\dot{V}_a\delta_{rm}}{g\psi}+2\left(\sqrt{1+\frac{\dot{V}_a\delta_{rm}}{g\psi}}-1\right)}}. \qquad (19)$$

Expressions (17) and (18) allow us not only to determine parameters of $((\Delta V_a)_{max}$ and $(V_a)_{rat}$, but also to obtain a rational dynamic characteristic of the vehicle.

Equation (18) will be solved relative to acceleration $\dot{V}_a$. We shall obtain as a result

$$\dot{V}_a=\frac{8g\psi V_a^2}{\delta_{rm}}\frac{\frac{m_a g\psi}{3kF}+V_a^2}{\left(\frac{m_a g\psi}{3kF}-V_a^2\right)^2}. \qquad (20)$$

An analysis of dependence (20) shows that when equality (6) is fulfilled, the magnitude $\dot{V}_a\to\infty$.

Equation (20) describes the law of acceleration control, which ensures acceleration dynamic of the vehicle at the lowest energy consumption. We shall obtain a differential equation of vehicle acceleration from expression (20)

$$\frac{dV_a}{dt}=\frac{8g\psi V_a^2}{\delta_{rm}}\frac{\frac{m_a g\psi}{3kF}+V_a^2}{\left(\frac{m_a g\psi}{3kF}-V_a^2\right)^2}. \qquad (21)$$

Equation (21) is a differential equation with variables that are divided

$$\int_{V_a}^{V_{a1}}\frac{\left(\frac{m_a g\psi}{3kF}-V_a^2\right)^2}{\left(\frac{m_a g\psi}{3kF}+V_a^2\right)V_a^2}dV_a=\frac{8g\psi}{\delta_{rm}}\int_0^{t_p}\mathrm{d}t. \qquad (22)$$

As a result of solving differential equation (22), we find acceleration time t_p from speed V_a to speed $V_{a1}=V_a+(\Delta V_a)_{max}$ at rational control over vehicle acceleration

$$t_p=\frac{\delta_{rm}}{8g\psi}\left\{(V_{a1}-V_a)\left(\begin{array}{l}1+\frac{m_a g\psi}{3kF\cdot V_a\cdot V_{a1}}-4\sqrt{\frac{m_a g\psi}{3kF}}\times\\ \times\left[\mathrm{arctg}\left(V_{a1}\sqrt{\frac{3kF}{m_a g\psi}}\right)-\mathrm{arctg}\left(V_a\sqrt{\frac{3kF}{m_a g\psi}}\right)\right]\end{array}\right)\right\}. \qquad (23)$$

An analysis of equation (23) shows that a decrease in δ_{rm} leads to a decrease in the acceleration time t_p of vehicle from speed V_a to speed $V_{a1}=V_a+\Delta V_a$. The magnitude of accounting factor for the rotating masses of transmission and engine is close to unity if a crankshaft is under constant speed mode with the vehicle acceleration driven by electric motors.

By multiplying the left and right sides of equation (20) by the magnitude $m_aV_a\delta_{rm}/\eta_{tr}$, we shall obtain a rational law of engine power control at vehicle acceleration

$$(\Delta N_e)_{rat}=\frac{8m_a g\psi V_a^3}{\eta_{tr}}\frac{\frac{m_a g\psi}{3kF}+V_a^2}{\left(\frac{m_a g\psi}{3kF}-V_a^2\right)^2}, \qquad (24)$$

where η_{tr} is the performance efficiency of vehicle transmission, it is possible to accept that $\eta_{tr}\approx0.8$.

Fig. 3 shows dependence charts of $\dot{V}_a(V_a)$, built at different values of ψ (curves 2, 3). The same figure shows a chart (curve 1) of dependence of boundary acceleration $\left(\dot{V}_a\right)_{\text{bound}}$ on speed, which corresponds to the implementation of boundary forces for adhesion on the drive wheels of the vehicle. For an all-wheel drive vehicle, at the implementation of boundary forces for adhesion on the drive wheels

$$\dot{V}_{a\,\text{bound}}=\frac{1}{\delta_{rm}}\left(g\phi_x-\frac{kF}{m_a}V_a^2\right), \tag{25}$$

where φ_x is the longitudinal coefficient of drive wheels adhesion to the road, we accept φ_x=0.8.

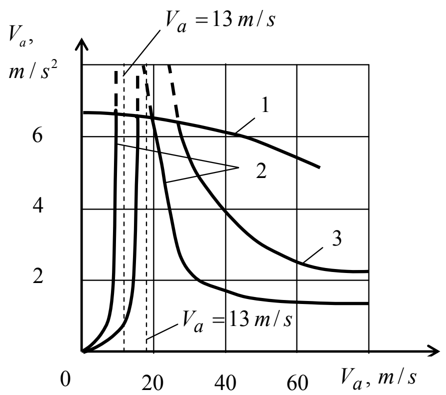

Fig. 3. Dependence $\dot{V}_a(V_a)$ for the vehicle with parameter at kF/m_a=0.0003 m^{-1}: 1 – $\dot{V}_{\text{bound}}(V_a)$; 2 – ψ=0.017; 3 – ψ=0.030; δ_{rm}=1.176 [1, 2]

Therefore, the obtained analytical expressions make it possible to implement a rational dynamic characteristic of a vehicle. It was determined for the example under consideration (Fig. 3) that the rational dynamic characteristic can be implemented over the entire range of possible speeds from zero to V_a=70 m/s (252 km/h). In the interval of speeds from 10 m/s up to 30 m/s maximal vehicle acceleration is limited by the boundary adhesive capacity of drive wheels to the road. At the point that corresponds to the fulfilment of condition (6), there is a rupture of function $\dot{V}_a(V_a)$.

5. Estimation of vehicle acceleration time to the assigned speed under various acceleration control laws

A perfect dynamic characteristic of the vehicle (an acceleration characteristic) is implemented at equality of tangential reactions on wheels to the boundary forces for adhesion. Assuming $\dot{V}_a=\dot{V}_{a\,\text{bound}}$ and upon solving differential equation (25) with the variables that are divided, we shall obtain, at vehicle acceleration from speed V_a to speed V_{a1},

$$t_p=\frac{\delta_{rm}}{2\sqrt{g\phi_x\dfrac{kF}{m_a}}}\cdot\ln\left|\frac{\sqrt{\dfrac{m_a g\phi_x}{kF}}+V_{a1}}{\sqrt{\dfrac{m_a g\phi_x}{kF}}-V_{a1}}-\frac{\sqrt{\dfrac{m_a g\phi_x}{kF}}-V_a}{\sqrt{\dfrac{m_a g\phi_x}{kF}}+V_a}\right|. \tag{26}$$

Fig. 4 shows dependence charts $t_p=t_p(V_a)$, built at different values of φ_x. A decrease in adhesion coefficient φ_x from 0.8 to 0.2 increases the minimum possible acceleration time (for example, from 0 to V_{a1}=30 m/s) by about four times (Fig. 4).

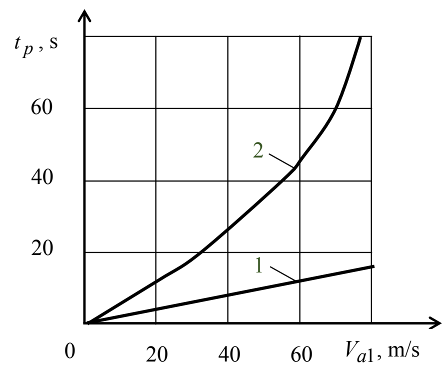

Fig. 4. Dependence of vehicle acceleration time from zero to the speed of V_{a1} when implementing a perfect dynamic characteristic kF/m_a=0.0003 m^{-1}: 1 – φ_x=0.8; 2 – φ_x=0.2

The existing technique to control vehicle acceleration implies combined control over the engine and the gearbox. Engine acceleration is associated with significant additional energy expenditures, since the moment of inertia of its moving parts, applied to the drive wheels, reaches considerable magnitudes.

To model the impact of an ICE speed characteristic, we employ dependences [4]

$$N_e=N_{e\max}\left(a\lambda+b\lambda^2-c\lambda^3\right), \tag{27}$$

where λ is the ratio of the current angular velocity ω_e to the angular velocity ω_N of maximum power $N_{e\max}$,

$$\lambda=\frac{\omega_e}{\omega_N}; \tag{28}$$

a, *b*, *c* are the empirical coefficients of equation (27) [8] (Table 1); $N_{e\,\max}$ is the maximum engine power.

To model partial speed characteristics of ICE, we shall use the following dependence:

$$N_{ep}=K_N\cdot N_{e\max}\left(a\lambda+b\lambda^2-c\lambda^3\right), \tag{29}$$

where K_N is the coefficient of engine power utilization, equal to the ratio of maximum power at a speed characteristic that is considered relative to the maximum power of the engine

$$K_N=\frac{N_{e\max p}}{N_{e\max}}. \tag{30}$$

The assumption, made in [3] for expression (29), implies that ω_N=const does not change when shifting from one speed characteristic to the other.

Effective torque at the engine shaft can be determined from:

$$M_e=\frac{N_e}{\omega_e}=N_{e\max}\left(a\frac{\lambda}{\omega_e}+b\frac{\lambda^2}{\omega_e}-c\frac{\lambda^3}{\omega_e}\right). \tag{31}$$

With respect to ratio (28)

$$M_e = N_{e\max}\left(\frac{a}{\omega_N} + b\frac{\omega_e}{\omega_N^2} - c\frac{\omega_e^2}{\omega_N^3}\right). \qquad (32)$$

Equation of vehicle dynamics can be written in the form

$$m_a \delta_{rm} \cdot \dot{V}_a = P_k - m_a g\psi - kFV_a^2, \qquad (33)$$

where P_k is the traction force on the drive wheels

$$P_k = \frac{M_e \cdot U_o \cdot U_{gr} \cdot \eta_{tr}}{r_d}, \qquad (34)$$

U_0, U_{gr} are the transfer ratios of the main transmission and the gearbox at the *i*-th gear; r_d is the dynamic radius of the drive wheels; η_{tr} is the transmission performance efficiency.

We shall define linear acceleration of the vehicle from expression (33) with respect to (34) and (32)

$$\dot{V}_a = \frac{1}{\delta_{rm_i}}\left[\frac{U_{gr_i} \cdot U_o \cdot \eta_{tr}}{m_a \cdot r_d} \cdot N_{e\max}\left(\frac{a}{\omega_N} + b\frac{\omega_e}{\omega_N^2} - c\frac{\omega_e^2}{\omega_N^3}\right)\right] - \\ -g\psi - \frac{kF}{m_a}V_a^2. \qquad (35)$$

Linear speed and vehicle acceleration are defined through the angular velocity and acceleration of the ICE crankshaft from the following expressions:

$$V_a = \frac{r_d}{U_0 \cdot U_{gr_i}}\omega_e; \qquad (36)$$

$$\dot{V}_a = \frac{dV_a}{dt} = \frac{r_d}{U_0 \cdot U_{gr_i}}\frac{d\omega_e}{dt}. \qquad (37)$$

Angular acceleration of the engine shaft $d\omega_e/dt$ will be derived from expression (35) with respect to (36) and (37)

$$\frac{d\omega_e}{dt} = -\omega_e^2\left(\frac{kF}{m_a}\frac{r_d}{\delta_{rm_i}} + \frac{c}{\omega_N^3}\frac{U_{gr_i}^2 \cdot U_0^2 \cdot \eta_{tr}}{m_a \cdot \delta_{rm_i} \cdot r_d^2}\right) + \\ + \frac{b}{\omega_N}\frac{U_{gr_i}^2 \cdot U_0^2 \cdot \eta_{tr}}{m_a \cdot \delta_{rm_i} \cdot r_d^2}N_{e\max} \cdot \omega + \\ + \frac{U_{gr_i} \cdot U_0}{\delta_{rm_i}}\left(\frac{a}{\omega_N}N_{e\max}\frac{U_{gr_i} \cdot U_0 \cdot \eta_{tr}}{m_a \cdot r_d^2} - \frac{g\psi}{r_d}\right) = \\ = C_i - B_i \cdot \omega_e - A_i \cdot \omega_e^2, \qquad (38)$$

where *A*, *B*, *C* are the coefficient corresponding to the *i*-th gear,

$$A_i = \frac{U_{gr_i} \cdot U_0}{\delta_{rm_i}}\left(\frac{a}{\omega_N}N_{e\max}\frac{U_{gr_i} \cdot U_0}{m_a \cdot r_d^2} - \frac{g\psi}{r_d}\right); \qquad (39)$$

$$B_i = \frac{b}{\omega_N}\frac{U_{gr_i}^2 \cdot U_0^2 \cdot \eta_{tr}}{m_a \cdot \delta_{rm_i} \cdot r_d^2}N_{e\max}; \qquad (40)$$

$$C_i = \frac{U_{gr_i} \cdot U_0}{\delta_{rm_i}}\left(\frac{a}{\omega_N}N_{e\max}\frac{U_{gr_i} \cdot U_0}{m_a \cdot r_d^2} - \frac{g\psi}{r_d}\right). \qquad (41)$$

The solution to the differential equation with variables that are divided determines vehicle acceleration time at the *i*-th gear when accelerating a crankshaft from $\omega_{\min}$ to $\omega_{\max}$

$$t_{p_i} = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega_e}{A_i + B_i\omega_e - C_i\omega_e^2} = \frac{1}{\sqrt{B_i^2 + 4A_iC_i}} \times \\ \times \ln\left|\frac{\sqrt{B_i^2 + 4A_iC_i} + 2C_i\omega_{\max} - B_i}{\sqrt{B_i^2 + 4A_iC_i} - 2C_i\omega_{\max} + B_i} \cdot \frac{\sqrt{B_i^2 + 4A_iC_i} - 2C_i\omega_{\min} + B_i}{\sqrt{B_i^2 + 4A_iC_i} + 2C_i\omega_{\min} - B_i}\right|. \qquad (42)$$

Vehicle acceleration time from V_a to $V_{a1} = V_{a\max}$ that employs all gears is determined as follows

$$t_p = \sum_{i=1}^{n} t_{p_i} + (n-1)t_{sh}, \qquad (43)$$

where *n* is the quantity of gears used in the gearbox; t_{sh} is the time of gear shifting.

When a vehicle reaches maximum speed value $V_{a\max}$ on top gear, the angular velocity of crankshaft ω_e may be less than $\omega_{\max}$. Therefore, when determining vehicle acceleration time on top gear t_{pt}, the upper bound of integration ω_t must be taken equal not to $\omega_{\max}$, but to the magnitude, which is determined from expression

$$\omega_t = \frac{V_{a\max} \cdot U_0 \cdot U_{gbt}}{r_d}, \qquad (44)$$

where U_{gbt} is the gear ratio of the gearbox on top gear.

In paper [3], authors obtained the rational dynamic characteristic of a vehicle (dependence of acceleration on car speed). Fig. 5 shows a chart of change in the rational value of $\dot{V}_a$ dependent on vehicle speed.

In the examined case, vehicle acceleration time from speed V_a to speed V_{a1} must be determined along three sections of the $0V_a$ axis. At the first section, we shall apply expression (20) within a change in V_a $[V_a; V_a^*]$

$$t_{p1} = \frac{\delta_{rm}}{8g\psi}\left\{\left(V_a^* - V_a\right)\left(\begin{aligned}&1 + \frac{m_a g\psi}{3kF \cdot V_a \cdot V^*} - 4\sqrt{\frac{m_a g\psi}{3kF}} \times \\ &\times\left[\operatorname{arctg}\left(V_a^*\sqrt{\frac{3kF}{m_a g\psi}}\right) - \operatorname{arctg}\left(V_a\sqrt{\frac{3kF}{m_a g\psi}}\right)\right]\end{aligned}\right)\right\}. \qquad (45)$$

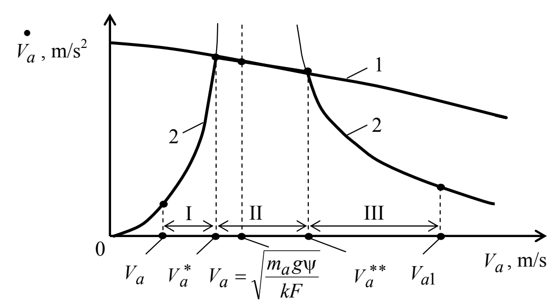

Fig. 5. Vehicle dynamic characteristics: 1 — at implementation of boundary forces for adhesion (dependence (25)); 2 — rational characteristic (dependence (12)

At the second section, we shall apply equation (26) within a change in speed V_a $[V_a^*; V_a^{**}]$

$$t_{p2} = \frac{\delta_{rm}}{2\sqrt{g\phi_x\frac{kF}{m_a}}}\ln\left|\frac{\sqrt{\frac{m_a g\phi_x}{kF}} + V_a^{**}}{\sqrt{\frac{m_a g\phi_x}{kF}} - V_a^{**}} \cdot \frac{\sqrt{\frac{m_a g\phi_x}{kF}} - V_a^*}{\sqrt{\frac{m_a g\phi_x}{kF}} + V_a^*}\right|. \qquad (46)$$

At the third section, we shall also employ expression (20), but with the limits of change in speed V_a. In this case, $[V_a^{**}; V_{a1}]$

$$t_{p1}=\frac{\delta_{rm}}{8g\psi}\left\{\left(V_{a1}-V_a^{**}\right)\left(\begin{array}{l}1+\frac{m_a g\psi}{3kF\cdot V_{a1}\cdot V^{**}}-4\sqrt{\frac{m_a g\psi}{3kF}}\times\\ \times\left[\operatorname{arctg}\left(V_{a1}\sqrt{\frac{3kF}{m_a g\psi}}\right)-\operatorname{arctg}\left(V_a^{**}\sqrt{\frac{3kF}{m_a g\psi}}\right)\right]\end{array}\right)\right\}. \quad (47)$$

Depending on the section where speed V_{a1} is reached, a general acceleration time is determined by summing up all components of t_{p1}, t_{p2}, t_{p3}, or only parts of them.

To determine the boundaries of speed intervals V_a^* and V_a^{**}, it is required to solve a system of equations (20) and (6). A solution to the given system of equations can be obtained upon solving an algebraic equation of sixth degree

$$V_a^6+V_a^4\frac{m_a g\phi_x}{kF}\left(\frac{22}{3}\frac{\psi}{\phi_x}-1\right)+\frac{25}{9}\frac{m_a^2g^2\psi^2}{(kF)^2}V_a^2+$$

$$+\frac{2}{3}\frac{m_a^2g^2\psi\phi_x}{(kF)^2}-\frac{m_a^3g^3\psi^2\phi_x}{9(kF)^3}=0. \quad (48)$$

Two of the six roots of equation (48) will have a physical sense.

6. Estimation of the ICE operation efficiency when accelerating a vehicle at different gears

In order to model partial speed characteristics of ICE, we shall apply the following dependence:

$$N_{ep}=K_N\cdot N_{e\max}\left(a\lambda+b\lambda^2-c\lambda^3\right), \quad (49)$$

where K_N is the coefficient of engine power utilization, equal to the ratio of maximum power for a speed characteristic, which is considered relative to the maximum power of the engine

$$K_N=\frac{N_{e\max p}}{N_{e\max}}. \quad (50)$$

The assumption, made for expression (49), implies that ω_N=const does not change in the transition from one speed characteristic to the other.

For a more general representation of dependence (49), it is expedient to transform it into the following form

$$N_{rel}=K_N\left(a\lambda+b\lambda^2-c\lambda^3\right), \quad (51)$$

where N_{rel} is a relative indicator of ICE effective power;

$$N_{rel}=\frac{N_{e\,p}}{N_{e\max}}. \quad (52)$$

Dependence chart of N_{rel} is shown in Fig. 6 (with respect to the accepted assumptions).

Note that engine acceleration cannot happen in full by the external speed characteristic because a given process takes place at a gradual increase in fuel feed to the cylinders.

In this case, the engine gradually passes from one partial speed characteristic to another, until, at a certain angular speed ω_p of the crankshaft, it switches to the external ICE speed characteristic (Fig. 7).

Accelerating characteristics of automobile ICE are given in papers [1, 3]. The authors obtained boundary ICE accelerating characteristics for adhesion of the vehicle drive wheels to the road. This allowed them to estimate boundary parameters for dynamic properties of a vehicle and to determine the boundary engine power.

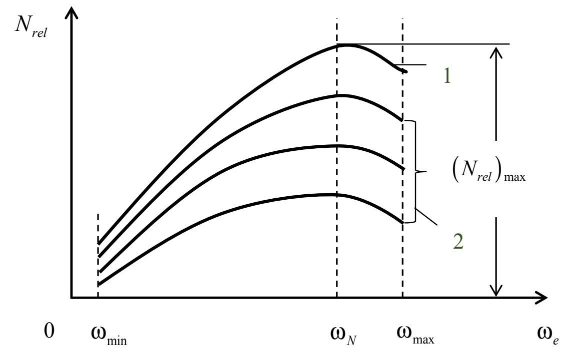

Fig. 6. Dependence of $N_{rel}(\lambda, K_N)$: 1 – external speed characteristic; 2 – partial speed characteristics

We shall denote section 2 of the engine accelerating characteristic as the characteristic's regulatory section; section 1 – the section of external speed characteristic (Fig. 7).

Determine coordinates of point P (Fig. 2) by solving the system of equations

$$\begin{cases}N_e=K_p\cdot\omega_e=K_p\cdot\omega_N\cdot\lambda; & (53)\\ N_{rel}=N_{e\max}\left(a\lambda+b\lambda^2-c\lambda^3\right), & (54)\end{cases}$$

where K_p is the angular coefficient of straight line 2 in Fig. 7.

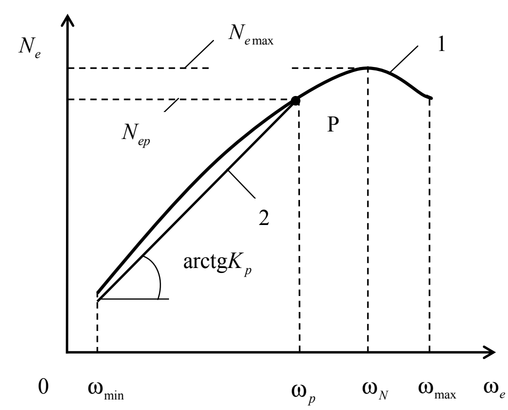

Fig. 7. ICE accelerating characteristic:
1 – working section of the external speed characteristic;
2 – section of accelerating characteristic at growing fuel feed to cylinders of the engine

Solution to the system of equations (53) and (54), with respect to the root that has a physical sense, will take the following form:

$$\lambda_p = \frac{\omega_p}{\omega_N} = \frac{b}{2c}\left[1+\sqrt{1-\frac{4c^2}{b^2}\left(\frac{K_p\cdot\omega_N}{c\cdot N_{e\max}}-\frac{a}{c}\right)}\right]. \qquad (55)$$

We shall derive from expression (54)

$$\omega_p = \frac{b\cdot\omega_N}{2c}\left[1+\sqrt{1-\frac{4c^2}{b^2}\left(\frac{K_p\cdot\omega_N}{c\cdot N_{e\max}}-\frac{a}{c}\right)}\right], \qquad (56)$$

and we shall determine from expression (53)

$$N_{ep} = K_p\cdot\omega_N\cdot\lambda_p = \frac{K_p\cdot\omega_N\cdot b}{2c}\left[1+\sqrt{1-\frac{4c^2}{b^2}\left(\frac{K_p\cdot\omega_N}{c\cdot N_{e\max}}-\frac{a}{c}\right)}\right]. \qquad (57)$$

Thus, the effective work of ICE when accelerating the crankshaft from ω_{min} to ω_{max} can be determined from

$$A_e = \int_{\omega_{\min}}^{\omega_{\max}} N_e dt = \int_{\omega_{\min}}^{\omega_{\max}} \frac{N_e d\omega_e}{d\omega_e/dt} = K_p\int_{\omega_{\min}}^{\omega_p}\frac{\omega_e d\omega_e}{d\omega_e/dt} + N_{e\max}\int_{\omega_p}^{\omega_{\max}}\frac{a\frac{\omega_e}{\omega_N}+b\frac{\omega_e^2}{\omega_N^2}-c\frac{\omega_e^3}{\omega_N^3}}{d\omega_e/dt}d\omega_e. \qquad (58)$$

By dividing the left and right sides of equation (58) by $N_{e\,max}$, we shall obtain

$$\frac{A_e}{N_{e\max}} = \frac{K_p\cdot\omega_N}{N_{e\max}}\int_{\lambda_{\min}}^{\lambda_p}\frac{\lambda d\lambda}{d\lambda/dt} + \int_{\lambda_p}^{\lambda_{\max}}\frac{a\lambda+b\lambda^2-c\lambda^3}{d\lambda/dt}d\lambda, \qquad (59)$$

where λ_{min}, λ_{max} are the minimum and maximum value for parameter λ, corresponding to ω_{min} and ω_{max}.

Determine $d\lambda/dt$. To this end, we shall apply equation of the vehicle power balance

$$N_e\cdot\eta_{tr} = \left(m_a\cdot g\cdot\psi + \frac{C_x}{2}\cdot\rho\cdot F\cdot V_a^2 + \delta_{rm}\cdot m_a\cdot\dot{V}_a\right)V_a, \qquad (60)$$

where C_x is the coefficient of frontal drag; ρ is the air density; F is the area of the frontal drag (a midsection) of the vehicle; δ_{rm} is the accounting factor for the rotating masses of the transmission and the engine,

$$\delta_{rm} = 1{,}04 + 0{,}05U_{gb}^2, \qquad (61)$$

where U_{gb} is the gear ratio of the gearbox.

Linear speed and acceleration of the vehicle

$$V_a = \frac{\omega_e\cdot r_k}{U_o\cdot U_{gb}} = \frac{\lambda\cdot\omega_N\cdot r_k}{U_o\cdot U_{kb}}; \qquad (62)$$

$$\dot{V}_a = \frac{r_k}{U_o\cdot U_{gb}}\cdot\frac{d\omega_e}{dt} = \frac{\omega_N\cdot r_k}{U_o\cdot U_{gb}}\cdot\frac{d\lambda}{dt}. \qquad (63)$$

Equation (60) with respect to (62) and (63) takes the following form:

$$N_e\cdot\eta_{tr} = \left(\begin{array}{l} m_a\cdot g\cdot\psi + \frac{C_x}{2}\cdot\rho\cdot F\cdot\frac{\lambda^2\cdot\omega_N^2\cdot r_k^2}{U_o^2\cdot U_{gb}^2} + \\ +\delta_{rm}\cdot m_a\cdot\frac{\omega_N\cdot r_k}{U_o\cdot U_{gb}}\cdot\frac{d\lambda}{dt}\end{array}\right)\frac{\lambda\cdot\omega_N\cdot r_k}{U_o\cdot U_{gb}}. \qquad (64)$$

We derive from equation (64)

$$\frac{d\lambda}{dt} = \frac{U_o\cdot U_{gb}}{\delta_{rm}\cdot m_a\cdot\omega_N\cdot r_k}\times\left(N_e\cdot\eta_{tr}\cdot\frac{U_o\cdot U_{gb}}{\omega_N\cdot r_k} - g\cdot\psi\cdot m_a - \lambda^2\cdot\frac{C_x\cdot\rho\cdot F\cdot\omega_N^2\cdot r_k^2}{U_o^2\cdot U_{gb}^2}\right). \qquad (65)$$

Considering equation (58), we shall obtain upon transforms

$$\frac{d\lambda}{dt} = \frac{U_o\cdot U_{gb}}{\delta_{rm}\cdot m_a\cdot\omega_N\cdot r_k}\times\left[\begin{array}{l}\left(\frac{N_{e\max}\cdot\eta_{tr}\cdot U_o\cdot U_{gb}\cdot a}{\omega_N\cdot r_k} - g\cdot\psi\cdot m_a\right) + \\ + \frac{N_{e\max}\cdot\eta_{tr}\cdot U_o\cdot U_{gb}\cdot b}{\omega_N\cdot r_k}\cdot\lambda - \\ -\left(\frac{N_{e\max}\cdot\eta_{tr}\cdot U_o\cdot U_{gb}\cdot c}{\omega_N\cdot r_{kgb}} + \frac{\omega_N^2\cdot r_k^2}{U^2\cdot U^2}\cdot\frac{C_x\cdot\rho}{2}\cdot F\right)\lambda^2\end{array}\right]. \qquad (66)$$

Introduce denotations

$$K_o = N_{e\max}\cdot\eta_{tr}\times\frac{U_o^2\cdot U_{gb}^2\cdot a}{\delta_{rm}\cdot m_a\cdot\omega_N^2\cdot r_k^2} - \frac{g\cdot\psi\cdot U_o\cdot U_{gb}\cdot\omega_N}{\delta_{rm}\cdot r_k}; \qquad (67)$$

$$B_o = \frac{N_{e\max}\cdot\eta_{tr}\cdot U_o\cdot U_{gb}\cdot b}{\omega_N\cdot r_k}; \qquad (68)$$

$$E_o = \frac{N_{e\max}\cdot\eta_{tr}\cdot U_o\cdot U_{gb}\cdot c}{\omega_N\cdot r_k} + \frac{\omega_N^2\cdot r_k^2}{U_o^2\cdot U_{gb}^2}\cdot\frac{C_x}{2}\cdot\rho\cdot F. \qquad (69)$$

Expression (66) with respect to (67)–(69) will take the form

$$\frac{d\lambda}{dt} = \frac{U_o\cdot U_{gb}}{\delta_{rm}\cdot m_a\cdot\omega_N\cdot r_k}\cdot\left(K_o + B_o\cdot\lambda - E_o\cdot\lambda^2\right). \qquad (70)$$

Equation (59) considering (70) will be transformed in the following way:

$$A_e = \frac{N_{e\max}\cdot\delta_{rm}\cdot m_a\cdot\omega_N\cdot r_k}{U_o\cdot U_{gb}}\times\left(\frac{K_p\cdot\omega_N}{N_{e\max}}\int_{\lambda_{\min}}^{\lambda_p}\frac{\lambda d\lambda}{K_o+B_o\cdot\lambda-E_o\cdot\lambda^2} + \int_{\lambda_p}^{\lambda_{\max}}\frac{a\lambda+b\lambda^2-c\lambda^3}{K_o+B_o\cdot\lambda-E_o\cdot\lambda^2}d\lambda\right). \qquad (71)$$

Fig. 8 shows dependence of vehicle's ICE effective work (using KrAZ-5233 as an example) on the gear of its GB, obtained using dependence (71).

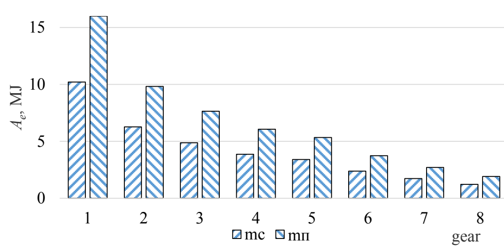

Fig. 8. Effective work of vehicle's ICE (KrAZ-5233)

The effective work of ICE of the truck KrAZ-5233, as shown in Fig. 8, decreases when the vehicle shifts to a higher gear.

An analysis of the results obtained reveals that a transition from lower to higher gears leads to a significant reduction in the engine energy consumption required to accelerate the vehicle. When accelerating, in the same range of speeds, the internal combustion engine (from ω_{min} to ω_{max}) of the vehicle KrAZ-5233, energy expenditure at gear I is A_e=16.0 MJ, and at gear VIII – A_e=2.0 MJ; in other words, eight times less. This allows us to recommend that hybrid vehicles should accelerate at lower gears employing an electric drive. It is appropriate to implement a rational law for vehicle acceleration control at all gears.

7. Discussion of results of research into a vehicle acceleration control law

The obtained rational law of acceleration control makes it possible to reach a maximum increase in the vehicle speed at minimum consumption of the engine energy. If, at each minimal interval of speed change, one would change, in a stepwise manner, engine power, then the result of acceleration to the maximal speed will be a minimal expenditure of engine energy.

The synthesis of the rational law for vehicle acceleration control became possible through the transformation of equation for vehicle acceleration dynamics when considering it over a small range of change in speed ΔV.

Similar research aimed at synthesizing rational laws for vehicle acceleration control implemented tangential reactions of the road, boundary for adhesion, on drive wheels. However, known laws demanded a multiple increase in the engine power and considerable energy consumption to control a motor-transmission unit.

The results proposed could be further developed for electric cars and hybrid vehicles, in which the implementation of the resulting rational law for acceleration control might be achieved by the simplest means with the lowest consumption of energy.

8. Conclusions

1. The analytical expressions obtained make it possible to implement such a change in the vehicle acceleration depending on its speed that ensures maximum dynamics at minimum engine power consumption, taking into consideration a nonlinear change in external resistance. The maximum acceleration, which is possible to implement using the rational dynamic characteristic, can amount to $\dot{V}_a = 7$ m/s^2.

2. Based on the dependences obtained, it is possible to determine effective work of ICE required to accelerate a vehicle at different gears. An analysis of calculation results revealed that the transition from lower to higher gears is accompanied by a sharp decrease in engine energy expenditure required to accelerate the vehicle (from A_e=16.0 MJ at gear I of the truck KrAZ-5233 to A_e=2.0 MJ at gear VIII).

3. In the case of hybrid vehicles, acceleration using the electric drive, rather than accelerating at lower gears of the mechanical drive, makes it possible to reduce energy losses by 20 % (for a four-cylinder internal combustion engine). Energy preservation is accomplished by reducing the fluctuation of traction force, as well as the possibility of a step-free change in motion speed.

References

1. A method of evaluating vehicle controllability according to the dynamic factor / Turenko A., Podrygalo M., Klets D., Hatsko V., Barun M. // Eastern-European Journal of Enterprise Technologies. 2016. Vol. 3, Issue 7 (81). P. 29–33. doi: 10.15587/1729-4061.2016.72117
2. Heaviside based optimal control for ride comfort and actuation energy optimisation in half-car suspension systems / Nkomo L. I., Dove A., Ngwako M. T., Nyandoro O. T. // IFAC-PapersOnLine. 2017. Vol. 50, Issue 2. P. 259–264. doi: 10.1016/j.ifacol.2017.12.055
3. Creation of the energy approach for estimating automobile dynamics and fuel efficiency / Podrigalo M., Klets D., Podrigalo N., Abramov D., Tarasov Y., Kaidalov R. et. al. // Eastern-European Journal of Enterprise Technologies. 2017. Vol. 5, Issue 7 (89). P. 58–64. doi: 10.15587/1729-4061.2017.110248
4. Transport energy demand in Andorra. Assessing private car futures through sensitivity and scenario analysis / Travesset-Baro O., Gallachóir B. P. Ó., Jover E., Rosas-Casals M. // Energy Policy. 2016. Vol. 96. P. 78–92. doi: 10.1016/j.enpol.2016.05.041
5. Daly H. E., Ó Gallachóir B. P. Modelling future private car energy demand in Ireland // Energy Policy. 2011. Vol. 39, Issue 12. P. 7815–7824. doi: 10.1016/j.enpol.2011.09.027
6. Yu F., Liu Z. Direct Energy Rebound Effect of Family Cars: An Analysis Based on a Survey in Chang-Zhu-Tan City Group // Energy Procedia. 2016. Vol. 104. P. 197–202. doi: 10.1016/j.egypro.2016.12.034
7. Daly H., Ó Gallachóir B. P. Modelling private car energy demand using a technological car stock model // Transportation Research Part D: Transport and Environment. 2011. Vol. 16, Issue 2. P. 93–101. doi: 10.1016/j.trd.2010.08.009
8. Matas A., Raymond J.-L., Dominguez A. Changes in fuel economy: An analysis of the Spanish car market // Transportation Research Part D: Transport and Environment. 2017. Vol. 55. P. 175–201. doi: 10.1016/j.trd.2017.06.025
9. Ohara H., Murakami T. A Stability Control by Active Angle Control of Front-Wheel in a Vehicle System // IEEE Transactions on Industrial Electronics. 2008. Vol. 55, Issue 3. P. 1277–1285. doi: 10.1109/tie.2007.909051
10. Hsu J.-Y., Chen B.-R., Hu T.-H. Vehicle stability control method and system: Pat. 20130103263 TW. USPC Class: 701 42. Class name: Vehicle subsystem or accessory control steering control feedback, transfer function or proportional and derivative (p&d) control. AB62D600FI; published: 25.04.2013. 4 p.
11. Chung T., Yi K. Design and evaluation of side slip angle-based vehicle stability control scheme on a virtual test track // IEEE Transactions on Control Systems Technology. 2006. Vol. 14, Issue 2. P. 224–234. doi: 10.1109/tcst.2005.863649