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*Пропонуються адаптивні комбіновані моделі гібридного та селективного типів для прогнозування часових рядів на основі програмного набору з адаптивних поліноміальних моделей різних порядків. Пропонуються адаптивні комбіновані моделі прогнозування часових рядів з врахуванням результатів ідентифікації подібностей в ретроспекції цих часових рядів. Оцінена ефективність прогнозування різних комбінованих моделей залежно від рівня персистентності часових рядів. Розроблені моделі дозволяють підвищити точність у випадку середньострокового прогнозування нестационарних часових рядів, зокрема фінансових показників*

*Ключові слова: прогнозування часових рядів, пошук подібностей, адаптивна комбінована модель, показник Герста*

*Предлагаются адаптивные комбинированные модели гибридного и селективного типов для прогнозирования временных рядов на основе программного набора из адаптивных полиномиальных моделей разных порядков. Предлагаются адаптивные комбинированные модели прогнозирования временных рядов с учетом результатов идентификации подобий в ретроспекции этих временных рядов. Оценена эффективность прогнозирования различных комбинированных моделей в зависимости от уровня персистентности временных рядов. Разработанные модели позволяют повысить точность в случае среднесрочного прогнозирования нестационарных временных рядов, в частности финансовых показателей*

*Ключевые слова: прогнозирование временных рядов, поиск подобий, адаптивная комбинированная модель, показатель Херста*

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# DEVELOPMENT OF ADAPTIVE COMBINED MODELS FOR PREDICTING TIME SERIES BASED ON SIMILARITY IDENTIFICATION

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## 1. Introduction

It is known that the overwhelming majority of financial, technical and physical processes for which the problem of predicting arises are characterized by nonlinearity and

instability with respect to the average level. Application of classical econometric prediction models and appropriate methods of predicting such time series that reflect these processes is rather limited. This is because of a low efficiency of these models in such conditions. Prospective directions

in development of information systems for analysis and prediction of time series and related decision-making support systems include construction of combined multi-level models and application of methods for intelligent data analysis. In addition, the pre-prediction evaluation of input data including financial indicators that can be implemented on the basis of fractal R/S analysis is relevant.

Contradictions between the requirements to planning strategies of activities of financial institutions and the tasks of scientifically grounded decision-making support can be solved by the use of mathematical models for prediction of financial time series. Known mathematical models do not ensure a necessary precision in predicting time series for solving problems of financial risk management under uncertainty. In this connection, the studies that consist in solving this contradiction by development of combined adaptive models using the results of intelligent analysis of time series, in particular, identification of similarities by the methods of the nearest neighbor and application of the apparatus of fractal analysis of data for pre-predictive analysis of time series should be considered relevant.

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## 2. Literature review and problem statement

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The problem of identifying similarities in the dynamics of time series is an important component in the study of changes in stock exchange price values, search for the goods with similar characteristics in terms and conditions of implementation. Besides, the problem of identifying similarities is used to analyze similarity of acoustic, textual, numeric or graphical information, detection of anomalies or events in signals of different nature, etc.

In [1], the concept of “approximately similar time series data” was developed which ensures understanding similarity between time series and is the basis for identification of similarities. However, this concept does not include a method for calculating similarity between time series by routine calculation of metric distances between them. The classical foundations of the problem of identifying similarities and their classification have been described in [2, 3]. In [4], basic theses of the problem of search for similarities are described by comparison with a sample, the methods of calculating metric distances, the problem of segmentation and approximation of time series are described.

Numerous papers [5–26] describing application of similarity identification in various applied problems can be mentioned. In particular, predictive power of the clustering method using similar behavior of the stock market time series as well as the use of this method for efficient prediction of share prices is investigated in [5]. The method for modeling samples for the problem of short-term prediction of time series is proposed in [6]. Work [7] considers the method of selective comparison with the sample. This method is used for the problem of constructing combined models of prediction of increment signs of time series with an unstable nature of fluctuations taking into account identification of similarities or indexation. The method for prediction of increments of time series in conditions of uncertainty using trend models of fluid averages is described in [8]. In papers [9, 10], the method of the closest neighbor is considered for this problem but not the comparison with the sample. The method of indexing time series based on identification of similarities between them is described in [11]. This method

is effective for time series of different lengths. The method of clustering for the search for similarities in multidimensional spaces represented by multidimensional vectors or time series of a fixed length is described in [12].

The use of similarity identification methodologies is also useful in education to predict potentials of development of scientific directions. The method that ensures calculation of prediction of the potential of scientific directions is described in [13]. The values of time series in this case are the ratio of estimations of the results of scientific research activities of scientists for various periods of time. The method can be used to identify prospective directions of research that are formed in the scientific medium. The method for classifying scholars by the research directions based on identification of similarities is described in [14]. The method for constructing evaluations of the results of scientific research activities of scientists based on the analysis of citations of publications is described in [15]. Paper [16] describes a parametric model for assessing and predicting quality of educational institutions which uses approaches to comparison of grades. The method enabling assessment of not only educational institutions but also structural units of higher educational institutions using transformation of a complex of developed estimations from qualitative to quantitative ones is described in paper [17]. The task of identifying similarities in this case may be useful for constructing relevant assessments taking into account other educational institutions or structural units of a similar weight.

The similarity identification task is also used to detect incomplete duplicates in text information for the purpose of preventing plagiarism and illegal dissemination of information. In particular, authors of work [18] have developed a method for detecting incomplete duplicates in tables which is based on the methods of the nearest neighbor and locally sensitive hashing. In work [19], a conceptual model of the system for finding incomplete duplicates using identification of similarities in electronic documents is described. The model enables identification of incomplete duplicates in documents containing data of various types: text information, numeric data, tables, mathematical formulas, schemes, diagrams and other graphic images.

For predicting time series, in addition to identification of similarities, conventional models can also be used. However, it should be borne in mind that traditional econometric models often show results with a significant margin of error, especially when applied to financial time series. This is explained by the fact that in the conditions of crisis phenomena, financial time series are often weakly persistent or even close to random, that is, they lose the memory of their initial conditions. Nevertheless, if incoming time series are trend-resistant, the use of traditional prediction models makes sense. In this case, the main stages in the prediction construction are [7]: predictive retrospection, construction of a model for prediction and formalization of the model, construction of the prospectus, estimation of the prediction and its verification.

Paper [20] describes approaches to intellectual decision-making methods in business using classical models of time series prediction. Adaptive models of short-term prediction of time series and methods of constructing combined methods are described in [21]. Models and methods for prediction of time series using intelligent data analysis: neural network modeling, genetic algorithms, fuzzy analysis, etc. are considered in work [22]. All these methods can use the

concept of identifying similarities and, as a result, increase their effectiveness.

Both classical prediction models and the prediction models using identification of similarities are often used as components of complex prediction, modeling and decision-making systems. Analysis of the peculiarities of designing information-analytical systems for prediction of time series with the use of expert evaluation has been carried out in work [23]. In particular, the method of fuzzy clustering which uses the identification problem of similarities is considered in [24]. The use of expert estimation to predict time series in information systems is also described in [25]. Peculiarities of project configuration management in development of distributed systems which can use identification of similarities between projects according to the set of indicators that characterize these projects are considered in [26].

A promising attempt is combination of traditional prediction models and the method of identifying similarities in retrospection of time series and constructing combined prediction models. These models should be endowed with adaptive properties to dynamically adapt to the mechanisms that generate time series. It is assumed that such models will be effective in predicting time series in conditions of uncertainty, in particular, data from stock markets, commodity time series, etc.

### 3. The aim and objectives of the study

The study objective was to create combined models of prediction of time series in conditions of uncertainty with adaptive characteristics at a maximum efficiency. Under the maximum efficiency of a model, it is understood its achieving the minimum average prediction errors.

To achieve this objective, the following tasks were set:

- construct adaptive combined models of hybrid and selective types for prediction of time series;
- construct adaptive combined models of hybrid and selective types for prediction of time series taking into account the results of identification of similarities in the retrospection of these time series.

### 4. Formal statement of the problem of prediction of time series and estimation of prediction results

Let  $\{z_i\}_{i=0}^n$  is some discrete time series of length  $n$  with no spaces:

$$\begin{aligned} \{z_i\}_{i=0}^n &= \{z_0, z_1, \dots, z_n\} = \\ &= \{z(t_0), z(t_1), \dots, z(t_n)\}, \end{aligned} \tag{1}$$

where  $t_i \in S$  are the discrete moments of time in which the values of time series are fixed,  $i = \overline{0, n}$ ,  $S$  is a discrete set,  $t_0$  is the initial time moment [7].

Based on the retrospective values  $z_n, z_{n+1}, \dots, z_{n-m+1}$  of the time series  $\{z_i\}_{i=0}^n$ ,  $n \geq m$ , estimate most accurately behavior of this time series in the future at time moments  $t_{n+1}, t_{n+2}, \dots, t_{n+\theta}$ , i. e. construct a sequence of predictive values:

$$\{\hat{z}_i\}_{i=n+1}^{n+\theta} = \{\hat{z}_{n+1}, \hat{z}_{n+2}, \dots, \hat{z}_{n+\theta}\}, \tag{2}$$

where  $\theta$  is the prediction horizon and  $m$  is the volume of the retrospective sample.

Let  $\hat{z}_\tau(n)$  be the prediction which is calculated at the moment  $t_n$  (at the point  $n$ ),  $\tau$  points ahead,  $\tau = \overline{1, \theta}$ . Functional dependence which allows one to describe behavior of the time series is called the prediction model. Denote by  $F$  the model that describes behavior of the time series  $\{z_i\}_{i=0}^n$ . Prediction of this time series for one point ahead ( $\tau=1$ ) based on the model  $F$  can be written as:

$$\hat{z}_{n+1} = \hat{z}_1(n) = F(z_{n-m+1}, z_{n-m}, \dots, z_n), \tag{3}$$

where  $\hat{z}_1(n)$  is the prediction of the time series calculated at the point  $n$ , one point ahead.

In the case of prediction with the horizon  $\tau > 1$ , we can write:

$$\begin{aligned} \hat{z}_{n+1} &= \hat{z}_1(n) = F(z_{n-m+1}, z_{n-m+2}, \dots, z_n), \\ \hat{z}_{n+2} &= \hat{z}_2(n) = F(z_{n-m+2}, z_{n-m+3}, \dots, z_n, \hat{z}_{n+1}), \\ &\vdots \\ \hat{z}_{n+\theta} &= \hat{z}_\theta(n) = F(z_{n-m+\theta}, \dots, \hat{z}_{n+\theta-2}, \hat{z}_{n+\theta-1}), \end{aligned} \tag{4}$$

where  $m$  is the volume of the retrospective sample,  $n \geq m$ .

As a rule, prediction models have a number of parameters to be evaluated before calculating the prediction. These parameters can be of adaptive nature, i. e., they change dynamically taking into account the prediction errors in the previous steps. The model that satisfies the respective criterion of prediction quality estimation will be considered the most accurate. Such criteria can be a mean square error, a mean absolute deviation, a standard deviation, a relative error, etc.

Most models require a certain amount of retrospective information to calculate the prediction. If sufficient historical data are available for time series observation, it is expedient to estimate quality of the prediction model in a given time series before calculating the prediction. Estimates can be used to construct confidence intervals of prediction or make more accurate parameters of the prediction model.

To evaluate quality of the prediction model  $F$ , consider a retrospective time series  $Z' = \{z_i\}_{i=n-m}^n$  of length  $m+1$ . Construct predictive values of the retrospective time series based on a series  $\{z_{n-m-h+1}, z_{n-m-h+2}, \dots, z_{n-m-1}\}$  of  $h+1$  length, denoting it as:  $Z'' = \{z_i\}_{i=n-m-h+1}^{n-m-1}$ . To evaluate prediction quality,  $1, 2, \dots, \theta$  points ahead, iteratively apply the model  $F$  according to the scheme (4).

For  $\tau=1$ , this can be written formally as follows:

$$\begin{aligned} \hat{z}_1(n-m-1) &= F(z_{n-m-h+1}, z_{n-m-h+2}, \dots, z_{n-m-1}), \\ \hat{z}_1(n-m) &= F(z_{n-m+h+2}, z_{n-m+h+3}, \dots, z_{n-m}), \\ &\vdots \\ \hat{z}_1(n-1) &= F(z_{n-m-1}, \dots, z_{n-2}, z_{n-1}), \quad m+l \leq n, \end{aligned} \tag{5}$$

where  $\hat{z}_1(i-1)$  is the prediction made at the moment  $i-1$  for one point ahead, that is, prediction of the  $i$ -th element of the retrospective series  $Z'$ ,  $i = n-m, n$ . The sequence of such predictions is denoted by  $\hat{Z}'_1 = \{\hat{z}_1(i-1)\}_{i=n-m}^n$ .

For  $\tau=2$ , the process of finding prediction estimates will have the form:

$$\begin{aligned} \hat{z}_1(n-m-1) &= F(z_{n-m-h+1}, z_{n-m-h+2}, \dots, z_{n-m-1}), \\ \hat{z}_2(n-m-1) &= F(z_{n-m+h+2}, z_{n-m+h+3}, \dots, \hat{z}_1(n-m-1)), \end{aligned}$$

$$\begin{aligned}\hat{z}_1(n-m) &= F(z_{n-m-h+2}, z_{n-m-h+3}, \dots, z_{n-m}), \\ \hat{z}_2(n-m) &= F(z_{n-m+h+3}, z_{n-m+h+4}, \dots, \hat{z}_1(n-m)), \\ &\vdots \\ \hat{z}_1(n-2) &= F(z_{n-m-2}, z_{n-m-1}, \dots, z_{n-2}), \\ \hat{z}_2(n-2) &= F(z_{n-m-1}, z_{n-m}, \dots, \hat{z}_1(n-2)), \quad m+l \leq n, \quad (6)\end{aligned}$$

where  $\hat{z}_2(i-2)$  is the prediction made at the moment of  $i-2$ , 2 points ahead, that is, the prediction of the  $i$ -th element of the retrospective series  $Z'$ ,  $i = n-m+1, n$ . The sequence of such elements is denoted by  $\hat{Z}'_2 = \{\hat{z}_2(i-2)\}_{i=n-m+1}^n$ . Using the above scheme, other sequences of predictions  $\hat{Z}'_3, \hat{Z}'_4$ , etc. can be constructed.

To construct a criterion of assessing quality of the prediction model, it is necessary to determine which of the predicted values of each of the sequences  $\hat{Z}'_\tau$ ,  $\tau = \overline{1, \theta}$  should be taken into account in the target function of the criterion. Call the subsequences of the sequences  $\hat{Z}'_\tau$ ,  $\tau = \overline{1, \theta}$ , used to estimate accuracy of prediction evaluative and denote by

$$\hat{Z}'_\tau = \{\hat{z}_\tau(i-\tau)\}_{i \in J_\tau},$$

where  $J_\tau$  are the sets of indexes of elements of sequences  $\hat{Z}'_\tau$ , chosen to construct the evaluative sequence  $\hat{Z}'_\tau$ ,  $\tau = \overline{1, \theta}$ ,  $n-m+\tau-1 \leq i \leq n$ ,  $\text{card}(J_\tau) \leq m$ ,  $\text{card}(J_\tau)$  is the number of elements in sequences  $\hat{Z}'_\tau$ . The sequence  $\hat{Z}'_\tau$ ,  $\tau = \overline{1, \theta}$  can be constructed of the most significant predictions using expert judgment. Besides, a quantitative estimate of significance of each individual prediction can be indicated with the help of weighting factors. That is, all prediction values  $\hat{z}'_\tau = \hat{z}'_\tau$ ,  $\tau = \overline{1, \theta}$ , will be included in the resulting target function, but with the weighting factors that can take zero values.

After construction of the evaluative series, criterion of quality of the prediction model is calculated which enables determining of the degree of satisfaction of the objectives that were set. In this case, the objective is to achieve maximum accuracy of predictions.

Consider the main criteria of quality assessment:

$$E^0 = E^0(Z', \hat{Z}'_\tau) = \frac{1}{v_\tau} \sum_{i=1, v_\tau} \omega_i |\hat{z}'_\tau - z'_j| \quad (7)$$

– prediction on the basis of the mean absolute deviation taking into account significance of predictions using the normalized weights  $\omega_i$ ,  $i = \overline{1, v_\tau}$ ,

$$\sum_{i=1}^{v_\tau} \omega_i = 1,$$

here and thereafter,  $n-m+\tau-1 \leq j \leq n$ ,  $v_\tau = \text{card}(J_\tau)$ ,  $v_\tau \leq m$ ,  $\tau = \overline{1, \theta}$ . The notation

$$E^0 = E^0(Z', \hat{Z}'_\tau)$$

means that the criterion  $E^0$ , is used for estimation according to which the members of the retrospective series  $Z'$  are compared with the corresponding members of the estimation series  $\hat{Z}'_\tau$ , with the prediction calculated,  $\tau$  steps ahead  $\tau = \overline{1, \theta}$ . Also, mean square error (8), standard deviation (9), average relative error (10) and maximum absolute error

(11) respectively which are calculated according to the formulas  $\tau = \overline{1, \theta}$ ,  $\sum_{i=1}^{v_\tau} \omega_i = 1$ :

$$E^1_\tau = E^1(Z', \hat{Z}'_\tau) = \frac{1}{v_\tau} \sum_{i=1, v_\tau} \omega_i (\hat{z}'_\tau - z'_j)^2, \quad (8)$$

$$E^2_\tau = E^2(Z', \hat{Z}'_\tau) = \sqrt{\frac{1}{v_\tau} \sum_{i=1, v_\tau} \omega_i (\hat{z}'_\tau - z'_j)^2}, \quad (9)$$

$$E^3_\tau = E^3(Z', \hat{Z}'_\tau) = \frac{1}{v_\tau} \sum_{i=1, v_\tau} \omega_i \frac{|\hat{z}'_\tau - z'_j|}{z'_j} \times 100\%, \quad (10)$$

$$E^4_\tau = E^4(Z', \hat{Z}'_\tau) = \max_{i=1, v_\tau} \omega_i |\hat{z}'_\tau - z'_j| \quad (11)$$

can be used for estimation.

Let us estimate quality of  $L$  prediction models of  $F$  type that were tested in a retrospective series. Denote by  $\hat{Z}'_{\tau, p}$  the evaluative series obtained by the  $i$ -th model in predicting a retrospective series  $Z'$ ,  $\tau$  steps ahead at  $p = \overline{1, L}$ . Then, the model that corresponds to the minimum deviation of the prediction values from the real

$$E^j(Z', \hat{Z}'_{\tau, p}) \rightarrow \min, \quad j = \overline{0, 4}, \quad p = \overline{1, L}$$

will be considered to be optimal one for each model. One of the approaches that can take into account the benefits of the models from a program set at different sections of the time series, is construction of combined models for prediction of selective and hybrid types.

## 5. Combined adaptive prediction models taking into account similarities in the retrospection of time series

Let the program set  $\mathfrak{S}_{PS}$  of the prediction models  $f_1, f_2, \dots, f_L$  be given on the basis of which estimates of future elements of the time series can be constructed for the series  $\{z_i\}_{i=0}^n$  at the point  $n$

$$\{\hat{z}_{n+1}^p, \hat{z}_{n+2}^p, \dots, \hat{z}_{n+\theta}^p\}, \quad (12)$$

where  $\hat{z}_{n+\tau}^p$  is the prediction calculated at the point  $n$ ,  $\tau$  points ahead, according to the  $p$ -th model.

It is necessary to build the most accurate sequence of prediction values based on the set  $\mathfrak{S}_{PS}$  and retrospective values of the time series  $\{z_i\}_{i=0}^n$ .

$$\hat{Z} = \{\hat{z}_i\}_{i=n+1}^{n+\theta} = \{\hat{z}_{n+1}, \hat{z}_{n+2}, \dots, \hat{z}_{n+\theta}\}.$$

Let a set of non-reference histories of the same length  $m$  is constructed for the time series  $\{z_i\}_{i=0}^n$ . Denote this set through  $\mathfrak{R}$ . Non-reference  $(N, m)$ -histories are such sections of the time series  $\{z_i\}_{i=0}^n$  of fixed length which are constructed by the method of the flow window according to the rule:

$$z_{(m)}^N = \{z_N, z_{N+1}, \dots, z_{N+m-1}\} = \{z_{N+j}\}_{j=0}^{m-1},$$

$$N = \overline{0, n - m}, \tag{13}$$

where the upper index in  $z_{(m)}^N$  defines the point from which construction of the history begins and the lower index in the brackets is the length of the history or the length of the section of the time series. In other words, the history of the time series refers to its subsequence of retrospective values or retrospection of a fixed length. Details on the construction of non-reference and reference histories are described in [7].

Denote predictions calculated on the basis of each model  $f_p$  from the program set  $\mathfrak{S}_{PS}$ ,  $p = \overline{1, L}$  at points  $m, m+1, \dots, n-1$ ,  $m < n$  of series  $\{z_i\}_{i=0}^n$ , corresponding to the last elements of non-reference histories  $z_{(m)}^N$  (13),  $N = \overline{0, n - m}$ , are the lengths  $m$   $\tau = \overline{1, \theta}$  points ahead by:

$$\begin{aligned} \hat{z}_1^p(m + N - 1) &= f_p \left( \{z_{N+j}\}_{j=0}^{m-1} \right), \\ \hat{z}_2^p(m + N - 1) &= f_p \left( \{z_{N+j}\}_{j=0}^{m-1}, \hat{z}_1^p(m + N - 1) \right), \\ &\vdots \\ \hat{z}_\theta^p(m + N - 1) &= \\ &= f_p \left( \{z_{N+j}\}_{j=0}^{m-1}, \hat{z}_1^p(m + N - 1), \hat{z}_2^p(m + N - 1), \dots, \hat{z}_{\theta-1}^p(m + N - 1) \right), \end{aligned} \tag{14}$$

where  $\hat{z}_\tau^p(m + N - 1)$  is the predictive value of the point  $z_{m+N-1+\tau}$ , following after  $(N, m)$ -history obtained on the basis of the  $p$ -th prediction model. Record  $f_p \left( \{z_{N+j}\}_{j=0}^{m-1} \right)$  means that the corresponding non-reference  $(N, m)$ -history belonging to the set of non-reference histories  $z_{(m)}^N \in \mathfrak{R}$  was used to construct the prediction according to the model  $f_p$ .

Construct the reference  $(N, m)$ -history for the input time series  $\{z_i\}_{i=0}^n$ :

$$z_{(m)}^{n-m+1} = \{z_{n-m+1}, z_{n-m+2}, \dots, z_n\} = \{z_j\}_{j=n-m+1}^n \tag{15}$$

and basing on some degree of proximity, for example, using the Euclid distance by the method of the nearest neighbor, find from the set of non-reference histories one that is most similar to the reference one.

An non-reference history  $z_{(m)}^x \in \mathfrak{R}$  is called the most similar to the reference history  $z_{(m)}^{n-m+1}$ , if there are no other non-reference histories  $z_{(m)}^k$ ,  $k \in [1, n - m - 1]$ ,  $k \neq x$ , for which

$$d \left( z_{(m)}^{n-m+1}, z_{(m)}^k \right) < d \left( z_{(m)}^{n-m+1}, z_{(m)}^x \right),$$

where  $d$  is some metric distance.

In accordance with the principles of constructing combined models, there are two approaches to calculation of predictions: selective and hybrid. Let us first consider the usual combined models of selective and hybrid types and then the same models but using similarity identifications.

*Adaptive Combined Selective Model by B-criterion of selection (ACSM-B)*

Let  $\mathfrak{S}_{PS}$  be a program set of prediction models. The selective approach consists in a selection for each value of  $\tau$  from the program set of models  $\mathfrak{S}_{PS}$ , a single model which provides high accuracy of prediction according to a certain selection criterion: B-criterion, K-criterion [21], etc. As a rule, parameters of selection criteria are adaptive. Besides, to improve accuracy of prediction, selection criteria are often applied not to the program but to the so-called main set.

Denote it with  $\mathfrak{S}_{BS}^*$ . This set is formed at each point of the time series in the process of prediction and consists of such models that give the most accurate predictions at the current section of the  $\mathfrak{S}_{BS}^* \subseteq \mathfrak{S}_{PS}$ . time series. Selection of models for the main set can be done, for example, on the basis of the D-criterion [21]:

$$D_p(\tau) \leq \lambda D_{\min}(\tau), \tag{16}$$

where  $c$  is the prehistory period,  $\lambda$  is the criterion parameter,

$$\begin{aligned} D_{\min}(\tau) &= \min_{p=\overline{1, L}} D_p(\tau), \\ D_p(\tau) &= \frac{1}{c} \sum_{j=0}^c \left( \hat{z}_\tau^p(m - \tau - j) - z_{m-j} \right)^2. \end{aligned}$$

Calculate the value of the B-criterion for each model for the time moment  $n$  by the formula:

$$B_{j,\tau}^p = (1 - \alpha_B) \cdot B_{j-1,\tau}^p + \alpha_B \cdot \left| \hat{z}_\tau^p(j - \tau) - z_j \right|, \tag{17}$$

where  $B_{j,\tau}^p$  is the value of the B-criterion calculated at the moment  $j = n - c - 1, n$ ,  $c$  is the prehistory period for each model  $f_p$ ,  $p = \overline{1, L}$  from the set  $\mathfrak{S}_{PS}$  and for each value of the prediction period  $\tau = \overline{1, \theta}$ ,  $\alpha_B$  is the smoothing parameter. The problem of determining optimal parameter  $\alpha_B$  in the B-criterion does not differ from the estimation of the smoothing parameter of the exponential model [20].

Compare the resulting finite values  $B_{n,\tau}^p$  with each other. For each  $\tau = \overline{1, \theta}$ , find the minimum value of the B-criterion

$\min_{p=\overline{1, L}} B_{n,\tau}^p$  for each of the models of the program set. Denote by  $\tilde{f}^\tau$  the models to which minimum values of B-criteria for each  $\tau = \overline{1, \theta}$ ,  $\tilde{f}^\tau \in \mathfrak{S}_{PS}$  correspond. Then, the prediction at the point  $n$  according to the model ACSM-B is calculated by the formula:

$$\hat{z}_\tau(n) = \hat{z}_\tau^*(n), \tag{18}$$

where  $\hat{z}_\tau^*(n)$  are the predictions according to the models that are calculated at the point  $n$ ,  $\tau = \overline{1, \theta}$  steps ahead.

*Adaptive Combined Selective Model according to the R-criterion of selection (ACSM-R)*

Introduce coefficient  $\lambda_p$  for each model from the program set. Let  $z_n$  be the last point of the time series  $Z$  at which selection of models is performed. Consider the interval  $[n - c, n]$  where  $c$  is the period of prehistory. At each point in this section, selection of the most accurate models is performed according to the D-criterion (16). If the model  $f_p$  satisfies the D-criterion, then the coefficient  $\lambda_p$  of the model  $f_p - \lambda_p$  increments by one. The model with coefficient  $\lambda_p$  at the time  $n$  being maximal is chosen to calculate the prediction according to the adaptive composite model. If there are several models with the same maximum coefficients, then one of them is selected that gets the coefficient increment later than others. After coming of the new  $z_{n+1}$  point and transition to the next interval  $[n+1 - c, n+1]$ , all coefficients  $\lambda_p$ ,  $p = \overline{1, L}$  are zeroed and calculation starts from the beginning.

Let the prediction period  $\tau=1$  and the moment for which values of the D-criterion are calculated is  $t_{n-p}$ . The sequence of coefficients  $\lambda_p$  for each model  $f_p$  from the program set  $p = \overline{1, L}$  is given. Assume that before the algorithm is execut-

ed,  $\lambda_p=0$ . The general formula for calculating the weighted D-criterion at an arbitrary time moment  $t$  for the prediction period  $\tau$  is  $D_p^t(\tau)$  is as follows:

$$D_p^t(\tau) = \left( \sum_{k=1}^c k \right)^{-1} \cdot \sum_{j=0}^{c-1} (c-j) \cdot (\hat{z}_\tau^p(t-\tau-j) - z_{t-j})^2, \quad (19)$$

where  $c$  is the period of prehistory.

The first step of the algorithm is calculation of the value  $D_p^t(\tau)$  for each model  $p=1, \bar{L}$ . Selection from the models  $f_p$  those for which the following condition is fulfilled.

$$D_p^t(\tau) \leq (1.2 + (\tau-1) \cdot h) \cdot D_{\min}^t(\tau), \quad (20)$$

where  $D_{\min}^t(\tau) = \min_p D_p^t(\tau)$ , is the threshold

$$h = \frac{1.9 - 1.2}{c - 1}.$$

The coefficients of the models  $f_p$  for which condition (20) is fulfilled increment by one:  $\lambda_p^t = \lambda_p^{t-1} + 1$ ,  $\lambda_p^t$  is the coefficient of the model  $f_p$  at time  $t$ . If  $r \neq 0$ , accept  $r=r-1$  and proceed to the first step of the algorithm, that is, continue to calculate the D-criterion for the moment  $t=n-r-1$ . If  $r=0$ , then the algorithm is complete and the prediction value for  $\tau$  can be calculated. Denote by  $\tilde{f}^\tau$  the model to which the maximum value corresponds among the coefficients  $\lambda_p^n$ , for the given  $\tau$ ,  $\tilde{f}^\tau \in \mathfrak{S}_{PS}$ , and by  $\hat{z}_\tau^*(n)$  the prediction according to this model which is calculated at the point  $n$ . Like for the previous model, the prediction is determined by formula (18).

If several models with the same maximum coefficients are selected, then the values of the coefficients for the points preceding the point  $n$ :  $n-1, n-2, \dots$  are compared. This is necessary to find the model with its coefficient incremented at the time closest to  $n$ .

Next, zero coefficients  $\lambda_p=0$ , increase the value of  $\tau$ ,  $\tau=2$  and proceed to the first step. Continue to execute the algorithm until predictions for each given  $\tau=1, \theta$  are calculated at time  $n$ .

It should be noted that the parameter  $r$  in this model should be small ( $r=3$ ) but can be adjusted before implementation of prediction. Use of this selection criterion can increase prediction accuracy compared to other approaches.

*Adaptive Combined Selective Model according to the P-criterion of selection (ACSM-P)*

Consider the P-criterion of selection. Let  $z_n$  be the last point of the time series. Like for the R-criterion, consider the section  $[n-c, n]$  and calculate for  $\forall j = n-c, n$  the value

$$S_j^p = \text{sign}(\hat{z}_1^p(j) - z_j), \quad (21)$$

where  $\hat{z}_1^p(j)$  is the prediction calculated at the point  $j$ , one point ahead, according to the model  $f_p$ ,  $p=1, \bar{L}$ . Then, according to this criterion at time  $n$ , the model taking the maximum value of  $\eta_p$  is selected:

$$\eta_p \xrightarrow{p=1, \bar{L}} \max, \quad (22)$$

where

$$\eta_p = \prod_{j=n-c}^n \left( \frac{z_{j+1}}{z_j} \right)^{S_j^p}. \quad (23)$$

According to the ACSM-P model, prediction is calculated like for the previous models by formula (18) where  $\hat{z}_\tau^*(n)$  is the prediction,  $\tau$  steps ahead, at the point  $n$  according to the model for which the value  $\eta_p$  is maximal.

*Adaptive Combined Hybrid Model (ACHM)*

Let formula (16) be used to select the most exact models at the point  $n$ . That is, for each  $\tau$ , the main set  $\mathfrak{S}_{BS}^\tau$  was constructed. Denote by  $f_1^\tau, f_2^\tau, \dots, f_{L_B^\tau}^\tau$  the prediction models included in the set  $\mathfrak{S}_{BS}^\tau$ ,  $L_B^\tau$  is the number of models for each fixed  $\tau$ ,  $L_B^\tau \leq L$ .

Let the values  $B_{j,\tau}^p$  be calculated at the point  $n$  by formula (17) for each of the models  $f_p^\tau$ ,  $p=1, L_B^\tau$  from the set  $\mathfrak{S}_{BS}^\tau$  and for each value of the prediction  $\mathfrak{S}_{BS}^\tau$  period. Prediction by the model of ACHM will be determined by the formula:

$$\hat{z}_\tau(n) = \sum_{p=1}^{L_B^\tau} \omega_\tau^p \cdot \hat{z}_\tau^p(n), \quad (24)$$

where the weighted coefficients  $\omega_\tau^p$  are determined on the basis of values  $B_{j,\tau}^p$ , and the following condition is fulfilled

$$\sum_{i=1}^{L_B^\tau} \omega_\tau^i = 1.$$

Algorithm for choosing weight coefficients in the ACHM model:

1. Form a matrix  $G = (g_{ij})_{i,j=1}^{L-1}$ , consisting of numbers of all models of prediction of a program set  $\mathfrak{S}_{PS}$ ,  $L$  is the number of models. For example, if  $L=6$ , matrix  $G$  has the form:

$$G = \begin{pmatrix} 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 4 & 5 & 6 \\ 1 & 2 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 6 \\ 1 & 2 & 3 & 4 & 6 \end{pmatrix}.$$

2. Form the basic sets of models at the point  $n$  for each value  $\tau$ . Re-enumerate all models of these sets from 1 to  $L_B^\tau$ , where  $L_B^\tau = \text{card}(\mathfrak{S}_{BS}^\tau)$ . Calculation of weight coefficients is performed according to the formulas:

$$\phi_{j,\tau} = \prod_{j=1}^{L_B^\tau} \prod_{i=1}^{L_B^\tau-1} B_{n,\tau}^{g_{ij}}, \quad j = \overline{1, L_B^\tau}, \quad \tau = \overline{1, \theta}, \quad (25)$$

$$\omega_\tau^p = \frac{\phi_p}{\sum_{j=1}^{L_B^\tau} \phi_j}, \quad p = \overline{1, L_B^\tau}, \quad (26)$$

where  $g_{ij}$  are the elements of the  $G$  matrix.

*Adaptive Combined Selective Model with Similarity Identification (ACSMwSI)*

Suppose that on the basis of some degree of proximity, the history  $z_{(m)}^{x-m+1} \in \mathfrak{R}$ ,  $x \in [m, n-\theta-1]$ , most similar to the reference history  $z_{(m)}^{n-m+1}$  was determined by the method of the nearest neighbor. The last element of history  $z_{(m)}^{x-m+1}$  will be the element  $z_x$ . Construct the main sets of prediction models  $\mathfrak{S}_{BS}^\tau$ , at this point using the D-criterion. That is, models  $f_1^\tau, f_2^\tau, \dots, f_{L_B^\tau}^\tau$ ,  $L_B^\tau \leq L$  are obtained. For details of choice of similar histories see the work [7].

For each model of the basic set  $\mathfrak{S}_{BS}^\tau$ , calculate the value of the B-criterion by the formula:

$$B_{x,\tau}^{q_\tau} = (1 - \alpha_B) B_{x-1,\tau}^{q_\tau} + \alpha_B e_{x,\tau}^{q_\tau} (x - \tau), \tag{27}$$

where  $0 < \alpha_B \leq 1$  is the smoothing parameter,

$$e_{x,\tau}^{q_\tau} (x - \tau) = \left| \hat{z}_{x,\tau}^{q_\tau} (x - \tau) - z_x \right| \tag{28}$$

is the absolute error of the prediction calculated at the moment  $t_{x-\tau}$ ,  $\tau$  steps ahead, according to the models

$$f_{q_\tau}^\tau, \tau = \overline{1, \theta}, \quad q_\tau = \overline{1, L_B^i}.$$

Denote by  $f^{\tau^*}$  the model selected from the main set according to some selection criterion for a fixed value  $\tau$  (27), (28). The prediction calculated on the basis of models  $f^{\tau^*}$  at the point  $n$ ,  $\tau$  steps ahead, denote by  $\hat{z}_\tau^*(n)$ .

If a model  $f^{\tau^*}$  with a minimum value of the B-criterion  $\min_{q_\tau=1, L_B^i} \{ B_{x,\tau}^{q_\tau} \}$  corresponds to each  $\tau$ , then the prediction according to the combined model of selective type according to the B-criterion of selection (27) using the identification of similarities by the method of the nearest neighbor will be calculated by the formula:

$$\hat{z}_\tau(n) = \alpha \hat{z}_\tau^*(n) + (1 - \alpha) z_{x+\tau}, \tag{29}$$

where  $z_{x+\tau}$  is the value of the time series which follows after the history  $z_{(m)}^{x-m+1}$ , most similar to the reference history  $z_{(m)}^{n-m+1}$ ,  $\alpha \in [0, 1]$  is a certain parameter.

*Adaptive Combined Hybrid Model with Similarity Identification (ACHMwSI)*

Apply a hybrid approach to construct a prediction. The prediction according to the hybrid approach is calculated as a weighted sum of predictions for all models that make up the main set  $\mathfrak{S}_{BS}^\tau$ . Let following the identification of similarities at the  $z_x$  point, basic sets  $\mathfrak{S}_{BS}^\tau$  were formed for each  $\tau$ , and the value of the B-criteria  $B_{x,\tau}^{q_\tau}$ ,  $q_\tau = \overline{1, L_B^i}$  was calculated. Denote by  $\hat{z}_\tau^{q_\tau}(n)$  the prediction calculated at the point  $n$ ,  $\tau$  points ahead, according to the models  $f_{q_\tau}^\tau$  from the main set  $\mathfrak{S}_{BS}^\tau$ ,  $q_\tau = \overline{1, L_B^i}$ ,  $\tau = \overline{1, \theta}$ . Then, the prediction according to the hybrid model with identification of similarities by the method of the nearest neighbor is determined by the formula:

$$\hat{z}_\tau(n) = \alpha \sum_{q_\tau=1}^{L_B^i} \omega_{q_\tau}^{q_\tau} \hat{z}_\tau^{q_\tau}(n) + (1 - \alpha) z_{x+\tau}, \tag{30}$$

where  $\alpha \in [0, 1]$ , weights can be determined on the basis of the B-criterion taking into account the coefficient of proportionality determined from the equality of the weights to one

$$\sum_{q_\tau=1}^{L_B^i} \omega_{q_\tau}^{q_\tau} = 1,$$

that is, by the formulas.

In simple words, prediction in the described combined models is calculated in accordance with the appropriate hybrid or selective principle taking into account the retrospective values of the time series which followed after the corresponding, most similar history.

In the case of dynamic prediction, a new reference history is constructed after calculation of the prediction at point  $z_n$  and the arrival of a new point  $z_{n+1}$ .

$$z_{(m)}^{n-m+2} = \{ z_{n-m+2}, z_{n-m+3}, \dots, z_{n-1}, z_n, z_{n+1} \},$$

and the old reference history  $z_{(m)}^{n-m+1}$  becomes non-reference, that is, it is included in the set  $\mathfrak{R}$ , and the calculation process starts from the beginning. That is, based on a certain degree of proximity, the history most similar to the reference history is found, sets  $\mathfrak{S}_{BS}^\tau$  are formed for each  $\tau$ , the value of the B-criterion is calculated for each prediction model that uses the most similar non-reference history as retrospective information. Next, a combined prediction is constructed.

**6. Discussion of the results obtained in the study of predicting time series based on the adaptive combined models**

For the experiment, time series of raw material prices (daily data) were taken for the period from 2014 to 2017 (by 900 points). The developed models were implemented and tested on these time series. Tables 1–4 show the results of testing some of the selected time series: prices for zinc, copper, platinum, nickel, silver, aluminum. These time series were loaded and pre-processed using sequential R/S analysis to determine the Hurst index. More information on the procedure of fractal sequential R/S analysis can be found in [27]. Moreover, the theoretical index which corresponds to the truth of the basic hypothesis of randomness of these time series,

$$E \left( \frac{R_m}{S_m} \right) \approx 0.540$$

and was calculated by formula [28]:

$$E \left( \frac{R_m}{S_m} \right) = \frac{\Gamma \left( \frac{m-1}{2} \right)}{\sqrt{\pi} \Gamma \left( \frac{m}{2} \right)} \sum_{i=1}^{m-1} \sqrt{\frac{m-i}{i}}, \tag{31}$$

where  $m$  is the length of the time series,  $R_m$  is the swing, and  $S_m$  is the mean root square deviation of the input time series  $Z$  of length  $m$ . The results of prediction are shown in the Tables 1–6 (the minimum average relative prediction errors for each  $\tau$  value are shown in bold with underlining). The average relative errors were calculated by formula (10). Designation of models in Tables 1–6: SESM-0, -1, -2 are Brown adaptive polynomial models of the 0.1 and 2 orders (Simple exponential smoothing model), ACSM-B, -R, -P are adaptive combined models of selective type with B, R, P selection criteria, ACHM is adaptive combined hybrid model, ACSMwSI-B, -R is adaptive combined selective model with similarity identification in the retrospection of the time series and selection according to B and R criteria, ACHMwSI is adaptive hybrid model with identification of similarities in the retrospection of the time series. The program set of combined models was formed on the basis of SESM models of 0, 1 and 2 orders with various parameters of smoothing.

It can be concluded that in the case of short-term prediction of time series, the maximum accuracy is obtained by the adaptive hybrid model with formation of the basic set  $\mathfrak{S}_{BS}$  according to the D-criterion with adaptation of the value  $\lambda$  and the combined selective models. In the case of prediction for a medium period  $\tau > 3$ , an increase in the accuracy of combined models with identification of similarities is observed. In addition, the more persistent the input time series, the more precise the ACHM model. For less persistent

series, prediction accuracy of ACSMwSI-B, ACSMwSI-R and ACHMwSI models is greatest already for  $\tau > 3$ . For example, the Hurst index for time series of aluminum prices is  $H=0.818$ , that is, this time series is persistent according to the results of a sequential R/S analysis. For other time series, whose prediction results are shown in Tables 1–5, the Hurst index varies in the range of  $H \in [0.570, 0.670]$  which indicates

a weak persistence of these time series. The lowest Hurst index is observed for the time series of prices for zinc  $H=0.570$  (Table 5). Also, as a result of the study, it was found that in the case of sufficiently persistent time series, at  $H > 0.75$  it would be sufficient to use conventional adaptive combined models without identification of similarities in the retrospection of these time series.

Table 1

Average relative error of prediction of prices for copper with the horizon  $\tau = \overline{1,10}$

Models \ $\tau$	1	2	3	4	5	6	7	8	9	10
SESM-0	1.683	2.568	3.246	3.785	4.224	4.545	4.826	5.056	5.201	5.258
SESM-1	1.479	2.355	3.068	3.726	4.253	4.654	4.880	5.156	5.469	5.614
SESM-2	1.667	2.353	2.998	3.599	4.135	4.585	4.889	5.186	5.551	5.736
ACSM-B	1.667	2.579	3.330	3.703	4.173	4.615	4.729	5.008	5.390	5.526
ACSM-R	1.670	2.482	3.114	3.566	4.132	4.604	4.856	5.136	5.446	5.631
ACSM-P	1.655	2.349	2.977	3.540	4.075	4.525	4.827	5.124	5.538	5.754
ACHM	<b>1.351</b>	<b>2.270</b>	<b>2.966</b>	3.603	4.101	4.501	4.776	5.045	5.356	5.510
ACSMwSI-B	1.696	2.594	3.057	3.740	4.013	<b>4.167</b>	4.502	4.761	<b>4.941</b>	5.163
ACSMwSI-R	1.916	2.571	3.129	3.560	3.951	4.231	4.548	4.847	5.008	<b>4.996</b>
ACHMwSI	1.491	2.426	2.993	<b>3.502</b>	<b>3.822</b>	4.168	<b>4.448</b>	<b>4.751</b>	4.981	5.079

Table 2

Average relative error of prediction of prices for platinum with the horizon  $\tau = \overline{1,10}$

Models \ $\tau$	1	2	3	4	5	6	7	8	9	10
SESM-0	1.203	1.783	2.251	2.457	2.715	3.061	3.353	3.570	3.701	3.806
SESM-1	1.190	1.887	2.349	2.545	2.703	3.005	3.400	3.665	3.838	3.935
SESM-2	1.434	2.165	2.578	2.819	2.913	3.066	3.473	3.790	3.975	4.044
ACSM-B	1.370	2.007	2.431	2.650	2.811	3.204	3.567	3.691	3.709	3.764
ACSM-R	1.373	1.870	2.398	2.578	2.920	3.132	3.467	3.674	3.849	3.887
ACSM-P	1.363	2.021	2.426	2.661	2.842	3.103	3.475	3.773	4.006	4.096
ACHM	<b>1.082</b>	<b>1.776</b>	<b>2.245</b>	2.506	2.651	2.987	3.360	3.596	3.766	3.842
ACSMwSI-B	1.840	2.271	2.418	<b>2.378</b>	2.559	2.842	<b>2.992</b>	<b>3.099</b>	<b>3.178</b>	3.312
ACSMwSI-R	1.918	2.248	2.605	2.659	2.605	2.812	3.129	3.175	<b>3.178</b>	<b>3.270</b>
ACHMwSI	1.470	2.117	2.345	2.431	<b>2.494</b>	<b>2.780</b>	3.022	3.186	3.227	3.306

Table 3

Average relative error of prediction of prices for nickel with the horizon  $\tau = \overline{1,10}$

Models \ $\tau$	1	2	3	4	5	6	7	8	9	10
SESM-0	2.507	3.711	4.366	4.827	5.077	5.360	5.540	5.643	5.813	6.002
SESM-1	2.353	3.707	4.550	5.034	5.411	5.737	5.944	6.160	6.259	6.425
SESM-2	2.692	4.098	4.974	5.503	5.833	6.016	6.194	6.398	6.523	6.752
ACSM-B	2.832	3.837	4.415	5.098	5.642	5.866	5.787	5.887	6.046	6.250
ACSM-R	2.621	3.604	4.525	4.968	5.575	5.639	5.742	6.087	6.232	6.567
ACSM-P	2.627	3.970	4.727	5.334	5.822	6.016	6.134	6.331	6.319	6.570
ACHM	<b>2.167</b>	<b>3.559</b>	4.332	4.890	5.301	5.599	5.733	5.934	5.995	6.188
ACSMwSI-B	3.025	3.730	<b>4.273</b>	4.773	5.102	5.320	5.514	5.620	5.751	5.919
ACSMwSI-R	3.003	3.859	4.569	4.776	<b>4.847</b>	<b>5.076</b>	<b>5.193</b>	<b>5.213</b>	5.911	6.106
ACHMwSI	2.337	3.769	4.343	<b>4.693</b>	4.980	5.277	5.388	5.551	<b>5.727</b>	<b>5.901</b>

Table 4

Average relative error of prediction of prices for silver with the horizon  $\tau = 1,10$

Models \ $\tau$	1	2	3	4	5	6	7	8	9	10
SESM-0	1.705	2.566	3.228	3.747	4.119	4.422	4.682	5.038	5.345	5.615
SESM-1	1.444	2.434	3.144	3.618	4.035	4.349	4.658	4.904	5.153	5.485
SESM-2	1.726	2.589	3.240	3.679	3.993	4.313	4.670	4.986	5.212	5.395
ACSM-B	1.723	2.666	3.093	3.651	4.033	4.276	4.608	5.065	5.271	5.287
ACSM-R	1.620	2.580	3.101	3.578	3.998	4.297	4.683	5.091	5.344	5.399
ACSM-P	1.626	2.500	3.160	3.635	3.922	4.262	4.650	4.952	5.170	5.420
ACHM	<b>1.311</b>	2.348	3.079	3.566	3.968	4.284	4.605	4.893	5.211	5.456
ACSMwSI-B	1.593	2.506	3.153	3.562	3.967	4.299	4.569	4.860	5.109	5.202
ACSMwSI-R	1.612	2.457	3.047	3.549	3.892	4.285	4.569	4.860	5.109	5.323
ACHMwSI	1.233	<b>2.345</b>	<b>3.005</b>	<b>3.438</b>	<b>3.827</b>	<b>4.167</b>	<b>4.457</b>	<b>4.705</b>	<b>4.979</b>	<b>5.170</b>

Table 5

Average relative error of prediction of prices for zinc with the horizon  $\tau = 1,10$

Models \ $\tau$	1	2	3	4	5	6	7	8	9	10
SESM-0	2.507	3.711	4.366	4.827	5.077	5.360	5.540	5.643	5.813	6.002
SESM-1	2.353	3.707	4.550	5.034	5.411	5.737	5.944	6.160	6.259	6.425
SESM-2	2.692	4.098	4.974	5.503	5.833	6.016	6.194	6.398	6.523	6.752
ACSM-B	2.832	3.837	4.415	5.098	5.642	5.866	5.787	5.887	6.046	6.250
ACSM-R	2.621	3.604	4.525	4.968	5.575	5.639	5.742	6.087	6.232	6.567
ACSM-P	2.627	3.970	4.727	5.334	5.822	6.016	6.134	6.331	6.319	6.570
ACHM	<b>2.167</b>	<b>3.559</b>	4.332	4.890	5.301	5.599	5.733	5.934	5.995	6.188
ACSMwSI-B	3.025	3.730	<b>4.273</b>	4.773	5.102	5.320	5.514	5.320	5.751	5.919
ACSMwSI-R	3.003	3.859	4.869	5.262	<b>4.847</b>	<b>5.076</b>	<b>5.193</b>	<b>5.213</b>	5.911	6.106
ACHMwSI	2.337	3.769	4.343	<b>4.693</b>	4.980	5.277	5.388	5.551	<b>5.727</b>	<b>5.901</b>

Table 6

Average relative error of prediction of prices for aluminum with the horizon  $\tau = 1,10$

Models \ $\tau$	1	2	3	4	5	6	7	8	9	10
SESM-0	1.538	2.254	2.779	3.190	3.527	3.855	4.266	4.604	4.903	5.224
SESM-1	1.357	2.076	2.630	3.127	3.471	3.807	4.134	4.533	4.884	5.209
SESM-2	1.578	2.217	2.700	3.175	3.531	3.810	4.161	4.496	4.876	5.159
ACSM-B	1.512	2.028	2.556	<b>3.077</b>	3.616	4.060	4.450	4.675	4.824	<b>5.065</b>
ACSM-R	1.512	2.028	<b>2.547</b>	3.220	3.513	4.031	4.287	4.537	4.830	5.179
ACSM-P	1.541	2.238	2.704	3.200	3.546	3.794	4.144	<b>4.469</b>	4.852	5.143
ACHM	<b>1.293</b>	<b>2.017</b>	2.575	3.094	<b>3.441</b>	<b>3.775</b>	<b>4.120</b>	4.476	<b>4.815</b>	5.094
ACSMwSI-B	2.213	2.650	3.148	3.429	3.748	4.063	4.333	4.700	5.071	5.167
ACSMwSI-R	2.198	2.708	3.144	3.400	3.684	4.036	4.334	4.682	5.009	5.144
ACHMwSI	1.757	2.554	3.050	3.349	3.644	4.005	4.291	4.618	4.950	5.197

The described adaptive combined models with similarity identification can be used to predict both stationary and non-stationary equidistant time series without spaces,  $\tau$  steps ahead. It was found that greater accuracy of these models was observed for  $\tau > 3$ , that is, in the case of medium-term prediction. For these models, the basic requirement is a sufficient amount of retrospective information, that is, the input series must have at least 700 points.

### 7. Conclusions

For solution of the problem of prediction of time series, formalization was made and new adaptive combined predic-

tion models of non-stationary time series were proposed, as well as methods for estimating accuracy of prediction. The results of the study are as follows:

1. Adaptive combined selective models of prediction according to B-, R-, P-selection criteria with automatic formation of the basic set of models based on the adaptive D-criterion (ACSM-B, ACSM-R, ACSM-P models) were constructed. The adaptive combined hybrid model of prediction of time series (ACHM model) was constructed.

2. Adaptive combined selective models of prediction according to the R- and B-criteria of selection with identification of similarities in the retrospection of time series by the method of the nearest neighbor (ACSMwSI-R, ACSMwSI-B models) were constructed. The adaptive combined

hybrid model of prediction with identification of similarities in the retrospection of time series (ACHMwSI model) was constructed.

Adaptive characteristics of combined models with similarity identification in retrospection of time series, compared with conventional combined models, can improve accuracy in the case of medium-term prediction of non-stationary time series, in particular financial indicators. As a result of the experiment, it was found that in the case of short-term prediction for  $\tau \leq 2$ , the ACHM model has the highest

accuracy. ACSM models with various selection criteria are effective in predicting persistent time series with the Hurst index  $H > 0.75$  for  $\tau > 2$ . In the case of prediction of time series with the Hurst index

$$E\left(\frac{R_m}{S_m}\right) < H \leq 0.75$$

for  $\tau > 2$ , the ACSMwSI models with various selection criteria and ACNmwSI are more precise.

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*Робота відноситься до області верифікації кібернетичних оціночних показників, зокрема до вирішення завдання визначення обмеженого класу операцій з розподіленими параметрами, об'єкти якого мають наперед визначену рейтингову ефективність. В рамках дослідження визначаються правила формування операційних об'єктів, які можуть використовуватися для тестування оціночних показників на предмет можливості їх використання в якості формули ефективності*

*Ключові слова: верифікація оціночного показника, операція з розподіленими параметрами, клас операцій, метод верифікації*

*Работа относится к области верификации кибернетических оценочных показателей, в частности к решению задачи определения ограниченного класса операций с распределенными параметрами, объекты которого имеют predetermined рейтинговую эффективность. В рамках исследования определяются правила формирования операционных объектов, которые могут использоваться для тестирования оценочных показателей на предмет возможности их использования в качестве формулы эффективности*

*Ключевые слова: верификация оценочного показателя, операция с распределенными параметрами, класс операций, метод верификации*

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## DEVELOPMENT OF TEST OPERATIONS WITH DIFFERENT DURATION IN ORDER TO IMPROVE VERIFICATION QUALITY OF EFFECTIVENESS FORMULA

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### 1. Introduction

The importance of solving the problems of identification of operations in terms of their effectiveness is related to the

need to maximize the pace of an enterprise development and investment capabilities of its owners [1].

In turn, the use of the effectiveness formula as an optimization criterion [2] makes it possible to formalize and