Аналітично знайдені двочастотні режими руху двомасної вібромашини з прямолінійним поступальним рухом платформ і віброзбудником у вигляді кульового, роликового або маятникового автобалансира. Із застосуванням методу малого параметра знайдені закони двочастотних вібрацій платформ і отримано рівняння для пошуку частот застрявання вантажів в автобалансирі. Оцінені величини складових, не врахованих в законах двочастотних вібрацій

Ключові слова: інерційний віброзбудник, двочастотні вібрації, резонансна вібромашина, автобалансир, двомасна вібромашина, ефект Зомерфельда

Аналитически найдены двухчастотные режимы движения двухмассной вибромашины с прямолинейным поступательным движением платформ и вибровозбудителем в виде шарового, роликового или маятникового автобалансира. С применением метода малого параметра найдены законы двухчастотных вибраций платформ и получено уравнение для поиска частот застревания грузов в автобалансире. Оценены величины составляющих, не учтенных в законах двухчастотных вибраций

Ключевые слова: инерционный вибровозбудитель, двухчастотные вибрации, резонансная вибромашина, автобалансир, двухмассная вибромашина, эффект Зоммерфельда

UDC 622.74: 621.928.235
DOI: 10.15587/1729-4061.2018.121737

SEARCH FOR THE DUAL-FREQUENCY MOTION MODES OF A DUAL-MASS VIBRATORY MACHINE WITH A VIBRATION EXCITER IN THE FORM OF PASSIVE AUTO-BALANCER

V. Yatsun
PhD, Associate Professor
Department of Road Cars and Building**
E-mail: yvkr@i.ua

G. Filimonikhin
Doctor of Technical Sciences, Professor,
Head of Department*
E-mail: filimonikhin@ukr.net

K. Dumenko
Doctor of Technical Sciences, Associate Professor
Department of Operation and Repair of Machines**
E-mail: dumenkokm@gmail.com

A. Nevdakha
PhD*
E-mail: aunevdaha@ukr.net

*Department of Machine Parts and Applied Mechanics**
**Central Ukrainian National Technical University
Universytetskyi ave., 8, Kropivnitskiy, Ukraine, 25006

1. Introduction

Among such vibratory machines as screeners, vibratory tables, vibratory conveyers, vibratory mills etc., the most promising ones are the multi-frequency, resonance, and multi-frequency-resonance machines.

Multi-frequency vibratory machines demonstrate better performance [1], resonance vibratory machines are the most energy efficient [2], and multi-frequency-resonance vibratory machines combine advantages of both multi-frequency and resonance vibratory machines [3].

The most effective and easiest technique to excite resonance dual-frequency oscillations is based on the use of a ball, a roller, or a pendulum auto-balancer as vibration exciter [4].

At present, workability of a new method of excitation of dual-frequency vibrations for single-mass vibratory machines with translational rectilinear movement of the vibratory platform has not been theoretically explored.

2. Literature review and problem statement

In [4], it was proposed to use a ball, a roller, or a pendulum auto-balancer for the excitation of dual-frequency resonance vibrations in vibratory machines with different kinematics in the motion of platforms. It is assumed that this technique is applicable for single-, dual-, and three-mass vibratory machines.

The technique employs a special motion mode of loads [5], which occurs at small forces of resistance to the motion of loads relative to the casing of an auto-balancer. Under this mode, the loads get together, but, failing to catch up with the rotor, unto which the auto-balancer is mounted, they get stuck at one of the resonance frequencies of the vibratory machine. Because the loads get stuck, slow resonance oscillations of platforms are excited. That is why the new technique is based on the Sommerfeld effect [6]. In addition, the unbalanced mass is mounted onto the casing of the auto-balancer. The unbalanced mass rotates synchronously with the rotor. This enables rapid oscillations of the platforms. Parameters of dual-frequency vibrations change by changing the rotor speed, the unbalanced mass, and total mass of loads.

Theoretical substantiation of the feasibility of the new method for the excitation of dual-frequency vibrations is important both for the theory of vibratory machines and for practical application.

In [7], generalized models of single-, dual-, and three-mass vibratory machines with translational motion of vibratory platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer were developed. Differential equations for the motion of vibratory machines were derived. Paper [8] analytically explores operability of the described technique for a single-mass vibratory machine with rectilinear translational motion of the platform. The scientific literature is also reviewed.

In this paper, we examine operability of the technique for a dual-mass vibratory machine with rectilinear translational motion of platforms.

The relevance of undertaking present research is also related to the fact that dual-mass vibratory machines have a number of advantages over single-mass machines. In the dual-mass vibratory machines:

– frequencies of platforms' oscillations are less dependent on load mass [9];

– excitation of anti-resonance oscillations is possible, at which platforms' oscillations are not transmitted to the foundation [10];

– resonance motion modes have larger areas of existence and stability [11];

– there is a possibility for the excitation of combined (poly-frequency) resonance oscillations of platforms with eigenfrequencies of the vibratory machine's oscillations [12];

– anti-resonance operating mode of the vibratory machine can be implemented over a wide range of parameters [13], it is less dependent on load mass [14], etc.

3. The aim and objectives of the study

The aim of present research is to study analytically the dual-frequency modes of motion of vibration platforms of a dual-mass vibratory machine with translational rectilinear motion of vibratory platforms, excited by a passive auto-balancer. It is necessary in order to substantiate the applicability of the new technique for vibration excitation in the dual-mass vibratory machines.

To accomplish the set goal, the following tasks must be solved:

– it is required, under condition that the loads get stuck, to find an approximated solution to differential equations of motion of the vibratory machine, and to estimate magnitudes of unaccounted (discarded) components of the solution;

– to obtain an equation to search for the frequencies at which loads get stuck, and to perform its general analysis.

4. Research methods

We apply differential equations of motion of a single-mass vibratory machine with translational rectilinear motion of vibratory platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer [9].

To search for the approximate solution to the system of differential equations of motion and frequencies at which loads get stuck, we employ perturbation methods and the elements of theory of nonlinear oscillations [15].

4. 1. Description of a generalized model of the vibratory machine

A generalized model of the dual-mass vibratory machine is shown in Fig. 1 [9]. A vibratory machine is composed of two platforms with mass M_1 and M_2. Each platform is supported by external elastic-viscous supports with coefficients of rigidity k_i and viscosity b_i, $/i=1, 2/$. The platforms are connected by the inner elastic-viscous support with coefficients of rigidity k_{12} and viscosity b_{12}.

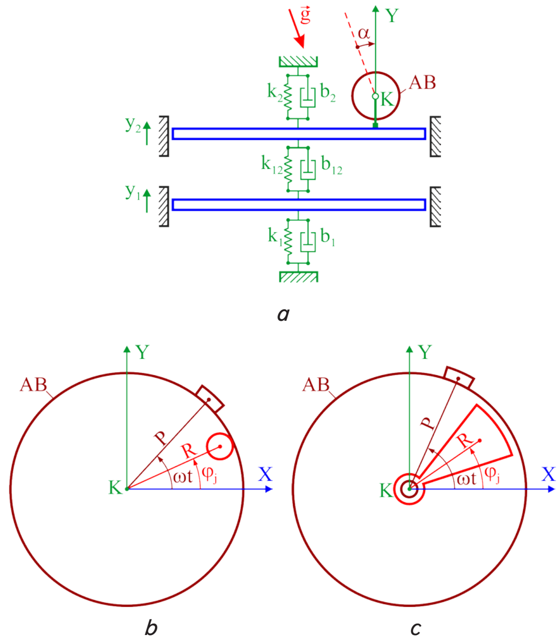

Fig. 1. Generalized model of the dual-mass vibratory machine: *a* – kinematics of platform's motion (the schematic is rotated at angle α); *b* – kinematics of motion of the unbalanced mass and a ball or a roller; *c* – kinematics of motion of the unbalanced mass and a pendulum

The platforms can move only in a rectilinear translational way due to the static guides. Direction of the platforms' motion forms angle α to the vertical. Coordinates y_1, y_2 of the platforms are counted from positions of the static equilibrium of the platforms.

A ball, a roller (Fig. 1, *b*), or a pendulum (Fig. 1, *c*) passive auto-balancer is placed on the second platform.

The casing of the auto-balancer rotates around the shaft – point K with a constant angular speed ω.

The point unbalanced mass μ is rigidly connected to the casing of the auto-balancer. It is located at distance P from point K. Two mutually perpendicular axes X, Y originate from point K and form the right-side coordinate system. The X axis is parallel to the platforms; the Y axis is parallel to the direction of platforms' motion. Location of the unbalanced mass relative to the casing determines angle ωt, where t is the time. The angle is counted from the X axis to the section that originates from point K and ends in the unbalanced mass.

The auto-balancer is composed of N identical loads. The mass of one load is m. The center of load's masses can move along circle with radius R centered at point K (Fig. 1, *b*, *c*). Position of load number j relative to the casing is determined by angle φ_j, $/j=\overline{1,N}/$. The angle is counted from the X axis to the section that originates from point K and ends in the center of masses of load number j. Load motion relative to the casing of the auto-balancer is prevented by the viscous resistance force with a module of

$$F_j = b_W v_j^{(r)} = b_W R\,|\varphi_j' - \omega|, \quad /j=\overline{1,N}/,$$

where b_W is the coefficient of viscous resistance force;

$$v_j^{(r)} = R\,|\,\phi'_j - \omega\,|$$

is the speed module of motion of the center of masses of load number j relative to the casing of the auto-balancer, with a dash behind the magnitude denoting time derivative t.

4. 2. Differential equation of motion of the dual-mass vibratory machine [9]

Differential equations of platform 1 and 2.

$$M_1 y_1'' + b_1 y_1' + k_1 y_1 + \\ + b_{12}(y_1' - y_2') + k_{12}(y_1 - y_2) = 0,$$

$$M_{2\Sigma} y_2'' + b_2 y_2' + k_2 y_2 + b_{12}(y_2' - y_1') + \\ + k_{12}(y_2 - y_1) + S_y'' = S_d \omega^2 \sin \omega t, \quad (1)$$

where $M_{2\Sigma} = M_2 + Nm + \mu$.

Equation of loads' motion:

$$\kappa m R \phi_j'' + b_W R(\phi_j' - \omega) + \\ + mg\cos(\phi_j - \alpha) + m y_2'' \cos\phi_j = 0, \ /\, j = \overline{1, N}\,/, \quad (2)$$

where for a ball, a roller, and a pendulum, respectively,

$$\kappa = \frac{7}{5}, \ \ \kappa = \frac{3}{2},$$

$$\kappa = 1 + J_C / (mR^2), \quad (3)$$

and J_C is the principal central axial moment of the pendulum's inertia.

We shall note that the form of differential equations of motion (1) and (2) does not depend on the type of an auto-balancer.

The sum of projections of all forces that impact the foundation

$$R_y = k_1 y_1 + b_1 y_1' + k_2 y_2 + b_2 y_2'. \quad (4)$$

At antiresonance $R_y \sim 0$.

5. Research results

5. 1. Reducing equations of motion to the dimensionless form

Let us introduce dimensionless variables and time

$$v_1 = y_1 / (\rho \tilde{y}), \ \ v_2 = y_2 / \tilde{y},$$

$$s_x = S_x / \tilde{s}, \ \ s_y = S_y / \tilde{s}, \ \ \tau = \tilde{\omega} t, \quad (5)$$

where $\tilde{y}$, ρ, $\tilde{s}$, $\tilde{\omega}$ are the characteristic scales, which will be selected later.

Then

$$\frac{d\cdot}{dt} = \frac{d\cdot}{d\tau}\frac{d\tau}{dt} = \tilde{\omega}\frac{d\cdot}{d\tau}, \ \ \frac{d^2\cdot}{dt^2} = \tilde{\omega}^2 \frac{d^2\cdot}{d\tau^2}, \quad (6)$$

equations of motion (1) and (2) will also take the form

$$M_1 \tilde{\omega}^2 \tilde{y} \rho \ddot{v}_1 + b_1 \tilde{\omega} \tilde{y} \rho \dot{v}_1 + k_1 \tilde{y} \rho v_1 + \\ + b_{12} \tilde{\omega} \tilde{y} (\rho \dot{v}_1 - \dot{v}_2) + k_{12} \tilde{y} (\rho v_1 - v_2) = 0,$$

$$M_{2\Sigma} \tilde{\omega}^2 \tilde{y} \ddot{v}_2 + b_2 \tilde{\omega} \tilde{y} \dot{v}_2 + k_2 \tilde{y} v_2 - b_{12} \tilde{\omega} \tilde{y} (\rho \dot{v}_1 - \dot{v}_2) - \\ - k_{12} \tilde{y} (\rho v_1 - v_2) + \tilde{s} \tilde{\omega}^2 \ddot{s}_y = S_d \omega^2 \sin\frac{\omega}{\tilde{\omega}}\tau,$$

$$\kappa m R^2 \tilde{\omega}^2 \ddot{\phi}_j + b_W R^2 \tilde{\omega} (\dot{\phi}_j - \omega / \tilde{\omega}) + \\ + mRg\cos(\phi_j - \alpha) + mR\tilde{\omega}^2 \tilde{y} \ddot{v}_2 \cos\phi_j = 0, \ \ /\, j = \overline{1, N}\,/. \quad (7)$$

Where a dot above the magnitude denotes derivative from τ.

We shall divide equations 1 and 2 by $M_{2\Sigma} \tilde{\omega}^2 \tilde{y}$, and equation 3 – by $\kappa m R^2 \tilde{\omega}^2$, and obtain

$$\rho \frac{M_1}{M_{2\Sigma}} \left(\ddot{v}_1 + \frac{b_1}{M_1 \tilde{\omega}} \dot{v}_1 + \frac{k_1}{M_1 \tilde{\omega}^2} v_1 \right) + \\ + \frac{b_{12}}{M_{2\Sigma} \tilde{\omega}} (\rho \dot{v}_1 - \dot{v}_2) + \frac{k_{12}}{M_{2\Sigma} \tilde{\omega}^2} (\rho v_1 - v_2) = 0,$$

$$\ddot{v}_2 + \frac{b_2}{M_{2\Sigma} \tilde{\omega}} \dot{v}_2 + \frac{k_2}{M_{2\Sigma} \tilde{\omega}^2} v_2 - \frac{b_{12}}{M_{2\Sigma} \tilde{\omega}} (\rho \dot{v}_1 - \dot{v}_2) - \\ - \frac{k_{12}}{M_{2\Sigma} \tilde{\omega}^2} (\rho v_1 - v_2) + \frac{\tilde{s}}{M_{2\Sigma} \tilde{y}} \ddot{s}_y = \frac{S_d \omega^2}{M_{2\Sigma} \tilde{\omega}^2 \tilde{y}} \sin\frac{\omega}{\tilde{\omega}}\tau,$$

$$\ddot{\phi}_j + \frac{b_W}{\kappa m \tilde{\omega}} \left(\dot{\phi}_j - \frac{\omega}{\tilde{\omega}} \right) + \\ + \frac{g}{\kappa R \tilde{\omega}^2} \cos(\phi_j - \alpha) + \frac{\tilde{y}}{\kappa R} \ddot{v}_2 \cos\phi_j = 0, \ \ /\, j = \overline{1, N}\,/. \quad (8)$$

We shall introduce new dimensionless parameters and characteristic scale:

$$h_1 = \frac{b_1}{2 M_1 \tilde{\omega}}, \ \ h_{12} = \frac{b_{12}}{2 M_{2\Sigma} \tilde{\omega}}, \ \ h_2 = \frac{b_2}{2 M_{2\Sigma} \tilde{\omega}}, \ \ \rho = \frac{M_{2\Sigma}}{M_1},$$

$$n = \frac{\omega}{\tilde{\omega}}, \ \ \tilde{y} = \frac{\tilde{s}}{M_{2\Sigma}}, \ \ \varepsilon = \frac{\tilde{y}}{\kappa R} = \frac{\tilde{s}}{\kappa R M_{2\Sigma}},$$

$$n_1^2 = \frac{k_1}{M_1 \tilde{\omega}^2}, \ \ n_{12}^2 = \frac{k_{12}}{M_{2\Sigma} \tilde{\omega}^2}, \ \ n_2^2 = \frac{k_2}{M_{2\Sigma} \tilde{\omega}^2},$$

$$\varepsilon\beta = \frac{b_W}{\kappa m \tilde{\omega}}, \ \ \beta = \frac{b_W}{\varepsilon \kappa m \tilde{\omega}} = \frac{b_W R M_{2\Sigma}}{\tilde{s} m \tilde{\omega}}, \ \ \delta = \frac{S_d}{\tilde{s}}, \ \ \sigma = \frac{g}{\kappa R \tilde{\omega}^2}. \quad (9)$$

Then equations (8) will take the form:

$$\ddot{v}_1 + 2h_1 \dot{v}_1 + n_1^2 v_1 + 2h_{12}(\rho \dot{v}_1 - \dot{v}_2) + n_{12}^2 (\rho v_1 - v_2) = 0,$$

$$\ddot{v}_2 + 2h_2 \dot{v}_2 + n_2^2 v_2 - 2h_{12}(\rho \dot{v}_1 - \dot{v}_2) - \\ - n_{12}^2 (\rho v_1 - v_2) + \ddot{s}_y = \delta n^2 \sin n\tau,$$

$$\ddot{\phi}_j + \varepsilon\beta(\dot{\phi}_j - n) + \sigma\cos(\phi_j - \alpha) + \varepsilon \ddot{v}_2 \cos\phi_j = 0,$$

$$/\, j = \overline{1, N}\,/. \quad (10)$$

Let

$$\tilde{s} = NmR, \ \ \tilde{\omega} = \sqrt{\frac{k_2}{M_{2\Sigma}}}. \quad (11)$$

Then

$$s_x = \frac{1}{N} \sum_{j=1}^{N} \cos\phi_j, \ \ s_y = \frac{1}{N} \sum_{j=1}^{N} \sin\phi_j, \ \ n_2^2 = 1,$$

$$\tilde{y}=\frac{NmR}{M_{2\Sigma}},\quad \varepsilon=\frac{Nm}{\kappa M_{2\Sigma}},$$

$$\beta=\frac{b_W M_{2\Sigma}}{Nm^2\tilde{\omega}},\quad \delta=\frac{S_d}{NmR}=\frac{\mu P}{NmR}. \tag{12}$$

In this case, the form of equations (10) will be preserved. Transform R_y:

$$r_y=\frac{R_y}{M_{2\Sigma}\tilde{\omega}^2\tilde{y}}=n_1^2 v_1+2h_1\dot{v}_1+n_2^2 v_2+2h_2\dot{v}_2. \tag{13}$$

At antiresonance $r_y\sim 0$.

5. 2. Transformation of equations of loads' motion

We shall add equations of loads' motion from (10) and obtain

$$\sum\nolimits_{j=1}^{N}\ddot{\phi}_j+\varepsilon\beta\sum\nolimits_{j=1}^{N}(\dot{\phi}_j-n)+$$
$$+\sigma\sum\nolimits_{j=1}^{N}\cos(\phi_j-\alpha)+\varepsilon\ddot{v}_2\sum\nolimits_{j=1}^{N}\cos\phi_j=0. \tag{14}$$

We shall introduce a mean angle for consideration

$$\phi=\frac{1}{N}\sum_{j=1}^{N}\phi_j. \tag{15}$$

Conduct transformations

$$\sum\nolimits_{j=1}^{N}\cos(\phi_j-\alpha)=\sum\nolimits_{j=1}^{N}(\cos\phi_j\cos\alpha-\sin\phi_j\sin\alpha)=$$
$$=\cos\alpha\sum\nolimits_{j=1}^{N}\cos\phi_j-\sin\alpha\sum\nolimits_{j=1}^{N}\sin\phi_j=$$
$$=N(s_x\cos\alpha-s_y\sin\alpha). \tag{16}$$

Then, considering (15), (16), equation (14) will take the form

$$\ddot{\phi}+\varepsilon\beta(\dot{\phi}-n)+$$
$$+\sigma(s_x\cos\alpha-s_y\sin\alpha)+\varepsilon\ddot{v}_2 s_x=0. \tag{17}$$

This equation will be used to search for the frequencies at which loads get stuck.

In the research that follows, the influence of gravity forces is not taken into consideration $(\sigma=0)$.

5. 3. Dual-frequency motion mode at zero approximation

At $\varepsilon=0$, the last N equations in the system (10) take the form:

$$\ddot{\phi}_j=0,\quad /j=\overline{1,N}/. \tag{18}$$

From these equations, we find

$$\phi_j^{(0)}=\Omega_j\tau+\psi_j,\quad \Omega_j,\psi_j-\text{const},\quad /j=\overline{1,N}/. \tag{19}$$

Then

$$\phi^{(0)}=\tau\frac{1}{N}\sum_{j=1}^{N}\Omega_j+\frac{1}{N}\sum_{j=1}^{N}\psi_j=\Omega\tau+\psi. \tag{20}$$

Hence, we shall find

$$\Omega=\frac{1}{N}\sum_{j=1}^{N}\Omega_j,\quad \psi=\frac{1}{N}\sum_{j=1}^{N}\psi_j. \tag{21}$$

Since the balls or rollers are located on one track, then:

$$\Omega_j=\Omega,\quad /j=\overline{1,N}/;$$

$$s_x=\frac{1}{N}\sum_{j=1}^{N}\cos\phi_j=\frac{1}{N}\sum_{j=1}^{N}\cos(\Omega\tau+\psi_j)=$$
$$=\frac{1}{N}\sum_{j=1}^{N}(\cos\Omega\tau\cos\psi_j-\sin\Omega\tau\sin\psi_j)=$$
$$=\frac{\cos\Omega\tau}{N}\sum_{j=1}^{N}\cos\psi_j-\frac{\sin\Omega\tau}{N}\sum_{j=1}^{N}\sin\psi_j;$$

$$s_y=\frac{1}{N}\sum_{j=1}^{N}\sin\phi_j=\frac{1}{N}\sum_{j=1}^{N}\sin(\Omega\tau+\psi_j)=$$
$$=\frac{1}{N}\sum_{j=1}^{N}(\sin\Omega\tau\cos\psi_j+\cos\Omega\tau\sin\psi_j)=$$
$$=\frac{\sin\Omega\tau}{N}\sum_{j=1}^{N}\cos\psi_j+\frac{\cos\Omega\tau}{N}\sum_{j=1}^{N}\sin\psi_j. \tag{22}$$

We shall demand that

$$s_x=A\cos(\Omega\tau+\gamma_0)=$$
$$=A(\cos\Omega\tau\cos\gamma_0-\sin\Omega\tau\sin\gamma_0);$$

$$s_y=A\sin(\Omega\tau+\gamma_0)=$$
$$=A(\sin\Omega\tau\cos\gamma_0+\cos\Omega\tau\sin\gamma_0). \tag{23}$$

Then

$$A\cos\gamma_0=\frac{1}{N}\sum_{j=1}^{N}\cos\psi_j,\quad A\sin\gamma_0=\frac{1}{N}\sum_{j=1}^{N}\sin\psi_j,$$

$$A^2=\frac{1}{N^2}\left[\left(\sum_{j=1}^{N}\cos\psi_j\right)^2+\left(\sum_{j=1}^{N}\sin\psi_j\right)^2\right],$$

$$\operatorname{tg}\gamma_0=\sum_{j=1}^{N}\sin\psi_j\Bigg/\sum_{j=1}^{N}\cos\psi_j. \tag{24}$$

From (23), we find $\ddot{s}_y=-A\Omega^2\sin(\Omega\tau+\gamma)$. Then the first two equations in system (10) will take the form

$$\ddot{v}_1+2h_1\dot{v}_1+n_1^2 v_1+2h_{12}(\rho\dot{v}_1-\dot{v}_2)+n_{12}^2(\rho v_1-v_2)=0,$$

$$\ddot{v}_2+2h_2\dot{v}_2+n_2^2 v_2-2h_{12}(\rho\dot{v}_1-\dot{v}_2)-$$
$$-n_{12}^2(\rho v_1-v_2)=A\Omega^2\sin(\Omega\tau+\gamma_0)+\delta n^2\sin n\tau. \tag{25}$$

We shall find a particular solution to system (25). Let us consider auxiliary system

$$\ddot{v}_1+2h_1\dot{v}_1+n_1^2 v_1+2h_{12}(\rho\dot{v}_1-\dot{v}_2)+n_{12}^2(\rho v_1-v_2)=0,$$

$$\ddot{v}_2+2h_2\dot{v}_2+n_2^2 v_2-2h_{12}(\rho\dot{v}_1-\dot{v}_2)-$$
$$-n_{12}^2(\rho v_1-v_2)=Fq^2\sin(q\tau). \tag{26}$$

We search for a particular solution to this system in the form of

$$v_1(\tau,q)=D(q,F)\sin(q\tau)+E(q,F)\cos(q\tau),$$

$$v_2(\tau,q)=K(q,F)\sin(q\tau)+L(q,F)\cos(q\tau). \tag{27}$$

Substitute (27) in (26) and collect coefficients before $\sin(q\tau)$, $\cos(q\tau)$. We shall obtain the following system of equations for the search $D(q,F)$, $E(q,F)$, $K(q,F)$, $L(q,F)$:

$$\begin{pmatrix} a_{11}(q) & a_{12}(q) & a_{13}(q) & a_{14}(q) \\ -a_{12}(q) & a_{11}(q) & -a_{14}(q) & a_{13}(q) \\ \rho a_{13}(q) & \rho a_{14}(q) & a_{33}(q) & a_{34}(q) \\ -\rho a_{14}(q) & \rho a_{13}(q) & -a_{34}(q) & a_{33}(q) \end{pmatrix}\begin{pmatrix} D(q) \\ E(q) \\ K(q) \\ L(q) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ b_3(q,F) \\ 0 \end{pmatrix}, \quad (28)$$

where

$$a_{11}(q) = n_1^2 + \rho n_{12}^2 - q^2,\ a_{12}(q) = = -2q(h_1 + \rho h_{12}),\ a_{13} = -n_{12}^2,\ a_{14}(q) = 2qh_{12},$$

$$a_{33}(q) = n_2^2 + n_{12}^2 - q^2,\ a_{34}(q) = = -2q(h_2 + h_{12}),\ b_3(q,F) = Fq^2. \quad (29)$$

We shall introduce determinants

$$\Delta(q) = \begin{vmatrix} a_{11}(q) & a_{12}(q) & a_{13}(q) & a_{14}(q) \\ -a_{12}(q) & a_{11}(q) & -a_{14}(q) & a_{13}(q) \\ \rho a_{13}(q) & \rho a_{14}(q) & a_{33}(q) & a_{34}(q) \\ -\rho a_{14}(q) & \rho a_{13}(q) & -a_{34}(q) & a_{33}(q) \end{vmatrix} =$$
$$= \{a_{11}(q)a_{33}(q) - \rho[a_{13}^2(q) - a_{14}^2(q)] - a_{12}(q)a_{34}(q)\}^2 + +[2\rho a_{13}(q)a_{14}(q) - a_{12}(q)a_{33}(q) - a_{11}(q)a_{34}(q)]^2,$$

$$\Delta_1(q,F) = \begin{vmatrix} 0 & a_{12}(q) & a_{13}(q) & a_{14}(q) \\ 0 & a_{11}(q) & -a_{14}(q) & a_{13}(q) \\ b_3(q) & \rho a_{14}(q) & a_{33}(q) & a_{34}(q) \\ 0 & \rho a_{13}(q) & -a_{34}(q) & a_{33}(q) \end{vmatrix} =$$
$$= b_3(q,F)\langle a_{13}(q)\{\rho[a_{13}^2(q) + a_{14}^2(q)] - -a_{11}(q)a_{33}(q) + a_{12}(q)a_{34}(q)\} - -a_{14}(q)[a_{11}(q)a_{34}(q) + a_{12}(q)a_{33}(q)]\rangle,$$

$$\Delta_2(q,F) = \begin{vmatrix} a_{11}(q) & 0 & a_{13}(q) & a_{14}(q) \\ -a_{12}(q) & 0 & -a_{14}(q) & a_{13}(q) \\ \rho a_{13}(q) & b_3(q,F) & a_{33}(q) & a_{34}(q) \\ -\rho a_{14}(q) & 0 & -a_{34}(q) & a_{33}(q) \end{vmatrix} =$$
$$= b_3(q,F)\langle a_{14}(q)\{\rho[a_{13}^2(q) + a_{14}^2(q)] + +a_{11}(q)a_{33}(q) - a_{12}(q)a_{34}(q)\} - -a_{13}(q)[a_{11}(q)a_{34}(q) + a_{12}(q)a_{33}(q)]\rangle,$$

$$\Delta_3(q,F) = \begin{vmatrix} a_{11}(q) & a_{12}(q) & 0 & a_{14}(q) \\ -a_{12}(q) & a_{11}(q) & 0 & a_{13}(q) \\ \rho a_{13}(q) & \rho a_{14}(q) & b_3(q,F) & a_{34}(q) \\ -\rho a_{14}(q) & \rho a_{13}(q) & 0 & a_{33}(q) \end{vmatrix} =$$
$$= b_3(q,F)\langle \rho\{a_{11}(q)[a_{14}^2(q) - a_{13}^2(q)] - -2a_{12}(q)a_{13}(q)a_{14}(q)\} + a_{33}(q)[a_{11}^2(q) + a_{12}^2(q)]\rangle,$$

$$\Delta_4(q,F) = \begin{vmatrix} a_{11}(q) & a_{12}(q) & a_{13}(q) & 0 \\ -a_{12}(q) & a_{11}(q) & -a_{14}(q) & 0 \\ \rho a_{13}(q) & \rho a_{14}(q) & a_{33}(q) & b_3(q,F) \\ -\rho a_{14}(q) & \rho a_{13}(q) & -a_{34}(q) & 0 \end{vmatrix} =$$
$$= b_3(q,F)\langle \rho\{a_{12}(q)[a_{13}^2(q) - a_{14}^2(q)] - -2a_{11}(q)a_{13}(q)a_{14}(q)\} + a_{34}(q)[a_{11}^2(q) + a_{12}^2(q)]\rangle. \quad (30)$$

Then

$$D(q,F) = \Delta_1(q,F)/\Delta(q),$$

$$E(q,F) = \Delta_2(q,F)/\Delta(q),$$

$$K(q,F) = \Delta_3(q,F)/\Delta(q),$$

$$L(q,F) = \Delta_4(q,F)/\Delta(q). \quad (31)$$

Dual-frequency mode of the platforms' motion at zero approximation $(\varepsilon = 0)$ takes the form

$$v_1(\tau) = D(\Omega,A)\sin(\Omega\tau + \gamma_0) + +E(\Omega,A)\cos(\Omega\tau + \gamma_0) + +D(n,\delta)\sin(n\tau) + E(n,\delta)\cos(n\tau),$$

$$v_2(\tau) = K(\Omega,A)\sin(\Omega\tau + \gamma_0) + +L(\Omega,A)\cos(\Omega\tau + \gamma_0) + K(n,\delta)\sin(n\tau) + +L(n,\delta)\cos(n\tau). \quad (32)$$

In it, the value of constant parameter Ω, which determines the frequency at which loads get stuck, was not defined.

5. 4. Condition for existence of the dual-frequency modes of motion

We search for the mean angle at steady motion in the first approximation. Assume that

$$\phi = \Omega\tau + \gamma_0 + \varepsilon\gamma_1, \quad (33)$$

where Ω=const, and γ_1 is the periodic function. Then, with accuracy to the magnitudes of first-order smallness inclusive

$$\dot\phi = \Omega + \varepsilon\dot\gamma_1,\quad \ddot\phi = \varepsilon\ddot\gamma_1,$$

$$s_x \approx A\cos(\Omega\tau + \gamma_0) - \varepsilon\gamma_1\sin(\Omega\tau + \gamma_0).$$

At the same accuracy, equation (17) takes the form

$$\varepsilon\ddot\gamma_1 + \varepsilon\beta(\Omega - n) + \varepsilon\ddot v_2 A\cos(\Omega\tau + \gamma_0) = 0,$$

hence, we find

$$\ddot\gamma_1 = -\beta(\Omega - n) - \varepsilon\ddot v_2 A\cos(\Omega\tau + \gamma_0). \quad (34)$$

At zero approximation, v_2 takes the form (32). Find the second derivative

$$\ddot v_2(\tau) = -\Omega^2[K(\Omega,A)\sin(\Omega\tau + \gamma_0) + +L(\Omega,A)\cos(\Omega\tau + \gamma_0)] - -n^2[K(n,\delta)\sin(n\tau) + L(n,\delta)\cos(n\tau)].$$

Substituting it in (34), we shall obtain

$$\ddot{\gamma}_1=-\beta(\Omega-n)+\{\Omega^2[K(\Omega,A)\sin(\Omega\tau+\gamma_0)+ \\ +L(\Omega,A)\cos(\Omega\tau+\gamma_0)]+ \\ +n^2[K(n,\delta)\sin(n\tau)+L(n,\delta)\cos(n\tau)]\}\times \\ \times\cos(\Omega\tau+\gamma_0)=0. \quad (35)$$

The right-hand side of this equation includes a constant that generates the secular component:

$$-\beta(\Omega-n)+\Omega^2L(\Omega,A)/2=0. \quad (36)$$

If this constant is equal to zero, then γ_1 is the periodic function.

With respect to (31), condition (36) takes the form

$$P(\Omega,n)=2\beta(n-\Omega)\Delta(\Omega)+\Omega^2\Delta_4(\Omega,A)=0. \quad (37)$$

Equation (37) is the polynomial of degree 9 relative to Ω. Its real roots determine frequencies at which loads can get stuck. The quantity of frequencies at which loads get stuck depends on the rotor speed.

We shall note that in the first approximation corrections to v_1, v_2 will equal to the order of ε. For actual vibratory machines $\varepsilon<1/50$, and, therefore, the correction will not exceed 2 % of the dual-frequency mode of motion already found. That is why a given correction is not determined in subsequent calculations.

Estimation of the magnitudes of discarded (unaccounted) components shows that despite strong asymmetry of supports, the platforms execute almost perfect dual-frequency oscillations.

5. 5. Analysis of equation for the search for a frequency at which loads get stuck

We shall substitute (29) in (30) and obtain

$$\Delta_4(\Omega,A)=-2A\Omega^3\{\rho h_1(4h_{12}^2\Omega^2+n_{12}^4)+ \\ +h_{12}[(n_1^2-\Omega^2)^2+4h_1^2\Omega^2]+ \\ +h_2[(n_1^2+\rho n_{12}^2-\Omega^2)^2+4\Omega^2(h_1+\rho h_{12})^2]\}. \quad (38)$$

(30) and (38) show that:

$$\forall\Omega<0\cdot\Delta_4(\Omega,\mathrm{A})>0,$$

$$\forall\Omega>0\cdot\Delta_4(\Omega,\mathrm{A})<0,\ \forall\Omega\cdot\Delta(\Omega)>0.$$

That is why

$$\forall\Omega\le0\cdot P(\Omega,n)>0,\ \forall\Omega\ge n\cdot P(\Omega,n)<0,$$

and:

– all real roots of the polynomial (37) are in the open interval (0, n);

– ∀ n>0 at least one real positive root exists, $\Omega_1\in(0,n)$, is the frequency, at which loads get stuck.

If the resistance forces in supports do not exist (h_1, h_{12}, h_2=0)

$$\Delta(q)=[(n_1^2+\rho n_{12}^2-q^2)(n_2^2+n_{12}^2-q^2)-\rho n_{12}^4]^2. \quad (39)$$

Two different twofold roots of this equation

$$q_{1/2}=\frac{1}{\sqrt{2}}\sqrt{n_1^2+n_2^2+(1+\rho)n_{12}^2\mp\sqrt{[n_1^2-n_2^2-(1+\rho)n_{12}^2]^2+4\rho n_{12}^4}} \quad (40)$$

determine natural frequencies of oscillations of the system at loads that are motionless relative to the auto-balancer.

Note that

$$[n_1^2-n_2^2-(1+\rho)n_{12}^2]^2+4\rho n_{12}^4= \\ =[n_1^2+n_2^2+(1+\rho)n_{12}^2]^2- \\ -4(n_1^2n_2^2+n_1^2n_{12}^2+\rho n_2^2n_{12}^2)>0.$$

That is why these frequencies always exist and $0<q_1<q_2$.

Equation (37) always has at least one root Ω^*, close to n. Using the method of expansion of polynomial roots by the powers of small parameter [15], it is possible to obtain that at low or very high rotor speeds there is only one root; in this case:

$$\forall n:0<n<<1\quad \Omega^*\approx \\ \approx n-An^5\frac{\rho h_1n_{12}^4+h_{12}n_1^4+h_2(n_1^2+\rho n_{12}^2)^2}{\beta(n_1^2n_2^2+n_1^2n_{12}^2+\rho n_2^2n_{12}^2)^2},$$

$$\forall n>>1\quad \Omega^*\approx\frac{\beta n}{\beta+A(h_{12}+h_2)}. \quad (41)$$

In the case of low rotor speeds, this is the single frequency at which loads get stuck.

In the case of a rapidly rotating rotor, there can exist other frequencies at which loads get stuck.

In the absence of resistance forces in supports, summand $2\beta(n-\Omega)\Delta(\Omega)$ has five real positive roots: q_1, q_1, q_2, q_2, n. If there are forces of viscous resistance in supports

$$\forall\Omega\in(0,n)\quad 2\beta(n-\Omega)\Delta(\Omega)>0,\quad \Omega^2\Delta_4(\Omega,A)<0.$$

That is why, in the case of small viscous resistance forces in supports, other frequencies at which loads get stuck:

– are close to the eigenfrequencies of vibratory machine's oscillations;

– occur in pairs in the vicinity of each eigenfrequency;

– one frequency at which loads get stuck from the pair is slightly lower than the corresponding eigenfrequency of vibratory machine's oscillations; while the other is slightly higher.

That is why, at small viscous resistance forces in the supports of a vibratory machine, an increase in the rotor speed leads to a sequential growth in the number of frequencies at which loads get stuck. In this case, the quantity of such frequencies is: 1 – for $n<q_1^*$; 3 – for $n\in(q_1^*,q_2^*)$; 1, 5 – for $n>q_2^*$. Here, q_1^*,q_2^* are some characteristic rotor speeds, such that $q_1<q_1^*<q_2<q_2^*$. Magnitudes of these characteristic speeds were not determined.

Arbitrary viscous resistance forces in the supports may interfere with the emergence of new frequencies at which loads get stuck. That is why, in the most general case, the quantity of such frequencies can amount to: 1 – for $n<q_1$; 1 or 3 – for $n\in(q_1, q_2)$; 1, 3, or 5 – for $n>q_2$.

6. Discussion of results of studying dual-frequency motion modes of the dual-mass vibratory machines

The theoretical study conducted allowed us to establish that a dual-mass vibratory machine with rectilinear translational motion of platforms and a vibration exciter in the form of a passive auto-balancer always possesses steady-state operation modes that are close to dual-frequency regimes. During these motions, loads in the auto-balancer create constant imbalance, cannot catch up with the rotor, and get stuck at a certain frequency. In this way, loads serve as the first vibration exciter, inducing vibrations with the frequency at which loads get stuck. The second vibration exciter is formed by the unbalanced mass on the casing of the auto-balancer. The mass rotates at the rotor speed and excites faster vibrations.

Despite the strong asymmetry of supports, the auto-balancer excites almost perfect dual-frequency vibrations of platforms. Deviations from the dual-frequency law are proportional to the ratio of loads' mass to the mass of the entire machine. That is why they do not exceed 2 % for actual machines.

A dual-frequency vibratory machine has its own two oscillation frequencies – q_1, q_2 ($q_1<q_2$). Loads can get stuck only at speeds close to: the eigenfrequencies of vibratory machine's oscillations; the rotor rotation frequency.

A vibratory machine has always one, and only one, frequency at which loads get stuck, which is slightly lower than the rotor speed.

In the case of small viscous resistance forces in the supports, an increase in the rotor speed of vibratory machine leads to an increase in the quantity of frequencies at which loads get stuck, first to 3, then to 5. In this case, new frequencies at which loads get stuck:

– occur in pairs in the vicinity of each eigenfrequency of the vibratory machine's oscillations;

– one of the frequencies is slightly lower, while the other is slightly higher, than the eigenfrequency of vibratory machine's oscillations.

Arbitrary viscous resistance forces in the supports may interfere with the emergence of new frequencies at which loads get stuck. That is why, in the most general case, the quantity of such frequencies can reach: 1 – for $n<q_1$; 1 or 3 – for $n\in(q_1, q_2)$; 1, 3, or 5 – for $n>q_2$.

Thus, at small viscous resistance forces in the supports, a dual-mass vibratory machine has more dual-frequency motion modes than the single-mass machine [10]. In this case, the dual-frequency motion modes in the dual-mass vibratory machine do not disappear at an increase in the rotor speed. This opens up new possibilities for designing vibratory machines with different dynamic characteristics.

We shall note that the stability of different dual-frequency motion modes and dynamic properties of the vibratory machine under these motions were not explored. It should be noted that differential equations of motion for the vibratory machine have solutions that correspond to the onset of auto-balancing. These solutions also remained unexplored.

The obtained results (the laws of platform motion, equation for finding the frequencies at which loads get stuck, etc.) could be applied both for analytical research and for a computational experiment. In the future, it is planned to investigate dynamic properties of the vibratory machine under a dual-frequency motion mode by employing a computational experiment.

7. Conclusions

1. A dual-mass vibratory machine with rectilinear translational motion of platforms and a vibration exciter in the form of a passive auto-balancer always has steady-state motion modes that are close to dual-frequency regimes. Under these motions, loads in the auto-balancer create constant imbalance, cannot catch up with the rotor, and get stuck at a certain frequency. In this way, loads serve as the first vibration exciter, inducing vibrations with the frequency at which loads get stuck. The second vibration exciter is formed by the unbalanced mass on the casing of the auto-balancer. The mass rotates at rotor speed and excites faster vibrations of this frequency.

Despite the strong asymmetry of supports, the auto-balancer excites almost perfect dual-frequency vibrations of the platforms. Deviations from the dual-frequency law are proportional to the ratio of loads' mass to the mass of the entire machine. That is why they do not exceed 2 % for actual machines.

2. A dual-frequency vibratory machine has two eigenfrequencies of oscillations. Loads can get stuck only at speeds close to eigenfrequencies of the vibratory machine's oscillations, or to the rotor rotation frequency.

A vibratory machine has always one, and only one, frequency at which loads get stuck, which is slightly lower than the rotor speed.

At low rotor speeds, there is only one frequency at which loads get stuck.

In the case of small viscous resistance forces in the supports, at an increase in the rotor speed, the quantity of frequencies at which loads get stuck in a vibratory machine increases, first to 3, then to 5. In this case, new frequencies at which loads get stuck:

– occur in pairs in the vicinity of each eigenfrequency of the vibratory machine's oscillations;

– one of the frequencies is slightly lower, while the other is slightly higher, than the eigenfrequency of vibratory machine's oscillations.

Arbitrary viscous resistance forces in the supports may interfere with the emergence of new frequencies at which loads get stuck. That is why, in the most general case, the quantity of such frequencies can be 1, 3, or 5, depending on the rotor speed and the magnitudes of viscous resistance forces in supports.

References

1. Intensification of technological processes through the implementation of vibrators biharmonic modes / Bukin S. L., Maslov S. G., Lyutiy A. P., Reznichenko G. L. // Enrichment of minerals. 2009. Issue 36 (77)-37 (78).
2. Kryukov B. I. Dinamika vibratsionnyih mashin rezonansnogo tipa [Dynamics of vibratory machines of resonance type]. Kyiv: Naukova dumka, 1967. 210 p.
3. Lanets O. S. Vysokoefektyvni mizhrezonansni vibratsiyni mashyny z elektromagnitnym pryvodom (teoretychni osnovy ta praktyka stvorennia) [High-Efficiency Inter-Resonances Vibratory Machines with an Electromagnetic Vibration Exciter (Theoretical Bases and Practice of Creation)]: monograph. Lviv: Publishing house of Lviv Polytechnic National University, 2008. 324 p.

4. Filimonikhin G. B., Yatsun V. V. Method of excitation of dual frequency vibrations by passive autobalancers // Eastern-European Journal of Enterprise Technologies. 2015. Vol. 4, Issue 7 (76). P. 9–14. doi: 10.15587/1729-4061.2015.47116
5. Artyunin A. I. Research of motion of the rotor with autobalance. Proceedings of the higher educational institutions // Mechanical Engineering. 1993. Issue 1. P. 15–19.
6. Sommerfeld A. Beitrage zum dinamischen Ausbay der Festigkeislehre // Zeitschriff des Vereins Deutsher Jngeniere. 1904. Bd. 48 (18). P. 631–636.
7. Equations of motion of vibration machines with a translational motion of platforms and a vibration exciter in the form of a passive auto-balancer / Yatsun V., Filimonikhin G., Dumenko K., Nevdakha A. // Eastern-European Journal of Enterprise Technologies. 2017. Vol. 5, Issue 1 (89). P. 19–25. doi: 10.15587/1729-4061.2017.111216
8. Search for two-frequency motion modes of single-mass vibratory machine with vibration exciter in the form of passive auto-balancer / Yatsun V., Filimonikhin G., Dumenko K., Nevdakha A. // Eastern-European Journal of Enterprise Technologies. 2017. Vol. 6, Issue 7 (90). P. 58–66. doi: 10.15587/1729-4061.2017.117683
9. Fedorenko I. Ya., Gnezdilov A. A. Dinamicheskiye svoystva dvukhmassnoy vibratsionnoy tekhnologicheskoy mashiny [The dynamic properties of a two-mass vibration technological machine] // Vestnik Altayskogo gosudarstvennogo agrarnogo universiteta. 2016. Issue 3 (137). P. 179–183.
10. Obgruntuvannya konstruktsiyi ta modelyuvannya roboty rezonansnoho dvomasovoho vibrostola z inertsiynym pryvodom [Justification of the design and simulation of the operation of a resonant two-mass vibration table with an inertia drive] / Lanets O. S., Hurskyi V. M., Lanets O. V., Shpak Ya. V. // Visnyk Natsionalnoho universytetu “Lvivska politekhnika”. Dynamika, mitsnist ta proektuvannia mashyn i pryladiv. 2014. Issue 788. P. 28–36.
11. Makarenkov O. Yu. Asymptotic stability of fluctuations of two mass of a resonant roar // Applied mathematics and mechanics. 2013. Vol. 77, Issue 3. P. 398–409.
12. Antipov V. I., Palashova I. V. Dynamics of a two-mass parametrically excited vibration machine // Journal of Machinery Manufacture and Reliability. 2010. Vol. 39, Issue 3. P. 238–243. doi: 10.3103/s1052618810030052
13. Influencing Factors of Anti-Resonant Inertial Resonant Machine Vibration Isolation System / Zhao J., Liu L., Song M., Zhang X. // 2015 3rd International Conference on Computer and Computing Science (COMCOMS). 2015. doi: 10.1109/comcoms.2015.22
14. Xiaohao L., Tao S. Dynamic performance analysis of nonlinear anti-resonance vibrating machine with the fluctuation of material mass // Journal of Vibroengineering. 2016. Vol. 18, Issue 2. P. 978–988.
15. Nayfeh A. H. Introduction to Perturbation Techniques. New York, United States: John Wiley and Sons Ltd., 1993. 533 p.