Запропоновано метод аналізу нелінійних електричних кіл з використанням ортогональних складових миттєвої провідності та опору. Показан механізм формування ортогональних складових миттєвої провідності нелінійного електричного кола. Наведено та проаналізовано аналітичні вирази для складових миттєвої провідності та опору. Проведено аналіз рівнянь балансу складових миттєвої проводності та опору, на базі яких були визначені гармонічні складові струму нелінійного електричного кола

Ключові слова: миттєва провідність, миттєвий опір, частотна область, електричне коло, нелінійність, аналітичний метод

Предложен метод анализа нелинейных электрических цепей с использованием ортогональных составляющих мгновенной проводимости и сопротивления. Показан механизм формирования ортогональных составляющих мгновенной проводимости нелинейной электрической цепи. Приведены и проанализированы аналитические выражения для составляющих мгновенной проводимости и сопротивления. Проведен анализ уравнений баланса составляющих мгновенных проводимости и сопротивления, на основании которых были определены гармонические составляющие тока нелинейной электрической цепи

Ключевые слова: миттєва провідність, миттєвий опір, частотна область, електричне коло, нелінійність, аналітичний метод

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DEVELOPMENT OF THE ANALITICAL METHOD FOR NONLINEAR CIRCUITS ANALYSIS WITH THE USE OF THE ADMITTANCE AND RESISTANCE ORTHOGONAL COMPONENTS

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1. Introduction

The calculation of nonlinear electric circuits is known to be the basis of the analysis of various electric engineering devices [1–6], as nonlinearity is inherent in practically all electrical engineering devices without exception [7–14]. The calculation of nonlinear electric circuits comes to the solution of the direct (currents are determined by the known parameters of the circuit) and the inverse (the circuit parameters are determined by the known values of current and voltage) problems of electrical engineering [15, 16]. As is generally known, most classical, as well as modern, approaches to the analysis of energy processes [17–19], taking place in electrotechnical systems enable only numerical evaluation of the parameters. Also, it should be mentioned that the latter do not provide comprehensive information on the processes in nonlinear electric circuits [20–22].

In practice, researchers have to apply a combination of several methods, which, in turn, results in complication of the calculation dependences and provokes an increase in the calculation errors [1–3, 6, 23, 24]. To achieve sufficient accuracy of the determination of the parameters of the electric circuits of the electrotechnical devices, including various nonlinear elements, an efficient calculation method, simple in realization, is required.

As is known, the choice of the means for active and passive filtration is a topical problem nowadays [20, 33–35]. However, the correct choice of the means for active and passive filtration requires the analysis of the characteristics of the operation of the connected nonlinear load, which is possible due to the use of the analytical calculation methods. That is why it is expedient to develop the method for the analysis of the electrotechnical system nonlinear electric circuits in the analytical form, which will provide a possibility to determine the current harmonic composition, meet the requirements of acceptable accuracy and good adaptation to the automation of analytical calculations.

2. Literature review and problem statement

The calculation of nonlinear electric circuits is known to come to the calculation of nonlinear equations describing the physical phenomena in electric circuits [1, 4, 8–10, 12, 13, 15, 17]. The literature review revealed that the method of small parameter is a well-known representative of the classical methods for the solution of such equations [10, 13, 15]. The use of this method is restricted by the following drawbacks: the complication of mathematical and calculation dependences with each consecutive approximation and with the growth of the number of nonlinear elements, as well as limited accuracy [15].

Because of these drawbacks of the small parameter method, new methods enabling the calculation of both steady and transient processes in nonlinear electric circuits have recently appeared [25–29]. The methods for the calculation of the steady processes of nonlinear circuits belong to a wide class of calculation methods in the time [2–4, 10, 11, 16, 17, 25–27] or frequency domain [30–36]. In the calculation of nonlinear circuits in the time domain, three main restrictions can be singled out: the unavailability of the methods in the wide frequency range [2, 3]; the difficulty of use for the calculation of the circuits with several frequency exciters [4]; the deterioration of the calculation efficiency for circuits containing a large number of coils and capacitors.

The harmonic balance (HB) method is most widely used for the calculation of the steady modes of circuits in the frequency domain [10, 11, 15]. The essential advantage of this method consists in the fact that the harmonic composition of the parameters of the researched circuit is directly obtained due to the solution. Fourier series are used for the expansion of the unknown quantities in the researched circuit. This method has a number of modifications:

a) HB method based on Galerkin-Urabe's approach [4], widely used for the solution of various applied problems;

b) Newton's hybrid HB method for the solution of a combined circuit with the use of Newton-Raphson method [6];

c) relaxation hybrid HB method based on the relaxation method [5].

However, to achieve an accurate solution with the use of the mentioned modifications of the HB method in the process of its realization, it is necessary to perform a discrete Fourier transform and an inverse discrete Fourier transform more than once. This requires considerable time and results in errors [5]. Besides, the calculation efficiency sharply decreases with the increase of the number of nonlinear elements as the equation system for the determination of Fourier coefficients becomes very ponderous [17, 37]. Therefore, none of the existing analytical methods provides comprehensive information on the processes in the nonlinear circuits, so in practice one has to use a combination of several methods. This results in the complication of the calculation dependences and provokes an increase in calculation errors [3].

Taking into account the above, an approach based on the analysis of instantaneous power (IP) components, well described by the authors [18, 21, 30, 31, 36], was chosen for the analysis of nonlinear electric systems of various configurations and complexity. This method provides the possibility not only to determine the values of the harmonic components of the power signal parameters, but also makes it possible to analyze electric circuits on the basis of the analytical balance equations of IP components.

In the paper [15], the author proposes to determine the active and reactive power of electric circuits by the introduction of the value of instantaneous resistance into the calculation. The electric circuit instantaneous resistance is determined via instantaneous voltage u(t) and instantaneous current i(t):

$$z(t)=\frac{u(t)}{i(t)}. \tag{1}$$

It is known that instantaneous admittance is the reciprocal of instantaneous resistance:

$$y(t)=\frac{1}{z(t)}=\frac{i(t)}{u(t)}. \tag{2}$$

Instantaneous admittance can be used to determine the current in a nonlinear electric circuit $i(t)=y(t)u(t)$, i. e. to solve the direct problem of electric engineering

3. The aim and objectives of the study

The aim of the study consists in the development of the method of nonlinear electric circuit analysis with the use of the components of instantaneous admittance and resistance in the frequency domain.

To achieve the aim the following objectives were formulated:

– to analyze the mechanism of the formation of the components of admittance and resistance in the frequency and time domains;

– to develop an efficient and easily realized analytical method for the analysis of the nonlinear processes, which will allow evaluating the significance and influence of the circuit parameters on the current' spectrum;

– to improve the efficiency of the analysis of nonlinear electric circuits by the adaptation of the method of nonlinear electric circuits analysis to the automation of the analytical calculations in the frequency domain.

4. The research of the processes of the formation of admittance components in the frequency and time domains

4. 1. The formation of the admittance components in the frequency and time domains when the current is represented by two harmonics

Consider the formation of the admittance components for several cases.

Supply voltage $u(t)$ is assigned by the sine components of the first harmonic: $u(t)=U_{b1}sin(\omega t)$, current – by the first and the third harmonics:

$$i(t)=I_{a1}\cos(\omega t)+I_{b1}\sin(\omega t)+I_{a3}\cos(3\omega t)+I_{b3}\sin(3\omega t).$$

Then the expression for the instantaneous admittance will be of the form:

$$y(t)=\frac{I_{a1}\cos(\omega t)}{U_{b1}\sin(\omega t)}+\frac{I_{b1}\sin(\omega t)}{U_{b1}\sin(\omega t)}+ +\frac{I_{a3}\cos(3\omega t)}{U_{b1}\sin(\omega t)}+\frac{I_{b3}\sin(3\omega t)}{U_{b1}\sin(\omega t)}, \tag{3}$$

where I_a, I_b –the current cosine and sine harmonic components, respectively, U_{b1} – the voltage harmonic component represented by the first harmonic, ω – angular frequency.

To pass from the time domain into the frequency domain of assigning the instantaneous admittance, the expression (3) was transformed with the use of trigonometric transformations. To make it clearer, a mathematical transformation is shown below for each separate summand (Table 1).

Using the basic trigonometric identities and transformation formulae (addition formulae, the trigonometric functions of double argument, the transformation of the sine and cosine degrees), we will get:

$$y(t)=\frac{I_{a1}}{U_{b1}}ctg(\omega t)+\frac{I_{b1}}{U_{b1}}+\frac{I_{a3}}{U_{b1}}ctg(\omega t)-$$
$$-2\frac{I_{a3}}{U_{b1}}\sin(2\omega t)+\frac{I_{b3}}{U_{b1}}+2\frac{I_{b3}}{U_{b1}}\cos(2\omega t). \qquad (4)$$

Table 1

The transformation of instantaneous admittance components in the trigonometric form

Reg. No.	The trigonometric transformation of the instantaneous admittance components
1	$\frac{I_{a1}}{U_{b1}}\frac{\cos(\omega t)}{\sin(\omega t)}=\frac{I_{a1}}{U_{b1}}ctg(\omega t)$
2	$\frac{I_{b1}}{U_{b1}}\frac{\sin(\omega t)}{\sin(\omega t)}=\frac{I_{b1}}{U_{b1}}$
3	$\frac{I_{a3}}{U_{b1}}\frac{\cos(3\omega t)}{\sin(\omega t)}=\frac{I_{a3}}{U_{b1}}\frac{\cos(2\omega t+\omega t)}{\sin(\omega t)}=$ $=\frac{I_{a3}}{U_{b1}}\frac{\cos(2\omega t)\cos(\omega t)-\sin(2\omega t)\sin(\omega t)}{\sin(\omega t)}=$ $=\frac{I_{a3}}{U_{b1}}\frac{(\cos^2(\omega t)-\sin^2(\omega t))\cos(\omega t)-2\sin^2(\omega t)\cos(\omega t)}{\sin(\omega t)}=$ $=\frac{I_{a3}}{U_{b1}}\left[\frac{\cos^3(\omega t)}{\sin(\omega t)}-3\sin(\omega t)\cos(\omega t)\right]=$ $=\frac{I_{a3}}{U_{b1}}\left[ctg(\omega t)\cos^2(\omega t)-3\sin(\omega t)\cos(\omega t)\right]=$ $=\frac{I_{a3}}{U_{b1}}\left[ctg(\omega t)\left(\frac{1}{2}+\frac{1}{2}\cos(2\omega t)\right)-3\sin(\omega t)\cos(\omega t)\right]=$ $=\frac{I_{a3}}{U_{b1}}\left[ctg(\omega t)(1-\sin^2(\omega t))-3\sin(\omega t)\cos(\omega t)\right]=$ $=\frac{I_{a3}}{U_{b1}}\left[ctg(\omega t)-ctg(\omega t)\sin^2(\omega t)-3\sin(\omega t)\cos(\omega t)\right]=$ $=\frac{I_{a3}}{U_{b1}}\left[ctg(\omega t)-4\sin(\omega t)\cos(\omega t)\right]=$ $=\frac{I_{a3}}{U_{b1}}ctg(\omega t)-2\frac{I_{a3}}{U_{b1}}\sin(2\omega t)$
4	$\frac{I_{b3}}{U_{b1}}\frac{\sin(3\omega t)}{\sin(\omega t)}=\frac{I_{b3}}{U_{b1}}\frac{\sin(2\omega t+\omega t)}{\sin(\omega t)}=$ $=\frac{I_{b3}}{U_{b1}}\frac{\sin(2\omega t)\cos(\omega t)+\cos(2\omega t)\sin(\omega t)}{\sin(\omega t)}=$ $=\frac{I_{b3}}{U_{b1}}\frac{2\sin(\omega t)\cos^2(\omega t)+2(\cos^2(\omega t)-1)\sin(\omega t)}{\sin(\omega t)}=$ $=\frac{I_{b3}}{U_{b1}}\left[\frac{4\sin(\omega t)\cos^2(\omega t)-\sin(\omega t)}{\sin(\omega t)}\right]=$ $=\frac{I_{b3}}{U_{b1}}\left[4\cos^2(\omega t)-1\right]=\frac{I_{b3}}{U_{b1}}\left[\frac{4}{2}(1+\cos(2\omega t)-1)\right]=$ $=\frac{I_{b3}}{U_{b1}}+2\frac{I_{b3}}{U_{b1}}\cos(2\omega t)$

To check the adequacy of the obtained trigonometric expression (4), the initial current signal curve (Fig. 1) was compared with the current curve built with the use of the value of instantaneous admittance $i(t)=y(t)u(t)$. The initial values for the analysis of the current and voltage: the amplitude values of voltage and current, respectively U_{b1}=10 V, I_{a1}=–2.5 A, I_{b1}=4.33 A, I_{a3}=–1.859 A, I_{b3}=3.542 A, the angle of the shift of the current first and third harmonic components, respectively $\varphi_1=\pi/6$, $\varphi_3=\pi/6.5$.

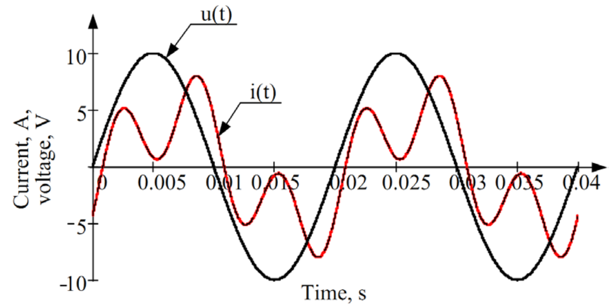

Fig. 1. Voltage $u(t)$ and current $i(t)$ signal curves, where (- - -) is the current curve calculated with the use of instantaneous admittance, (——) is the current initial curve

It should be noted that in (4) the components without the trigonometric function, as well as the coefficients at ctg(ω*t*) correspond to the cosine and sine components of the instantaneous admittance of the zero frequency, respectively. The coefficients at cos(*k*ω*t*) and sin(*k*ω*t*) –to the real and imaginary components of the instantaneous admittance of the corresponding harmonic *k*, respectively. According to the above and (4), the instantaneous admittance and supply voltage in the frequency domain will be written down in the form of arrays:

$$Y_{ak}=\begin{pmatrix}\frac{I_{b1}+I_{b3}}{U_{b1}}\\0\\\frac{I_{b3}}{U_{b1}}\end{pmatrix};\quad Y_{bk}=\begin{pmatrix}\frac{I_{a1}+I_{a3}}{U_{b1}}\\0\\\frac{I_{a3}}{U_{b1}}\end{pmatrix};$$

$$U_{an}=\begin{pmatrix}0\\0\\0\end{pmatrix};\quad U_{bn}=\begin{pmatrix}0\\U_{b1}\\0\end{pmatrix},$$

where *n*, *k* – the numbers of the harmonics of voltage and admittance, respectively.

To verify the correctness of the formation of the arrays of instantaneous admittance components, using the automated method of IP components formation [36] on the basis of the algorithm of the discrete convolution [1] of two series, we will determine the current harmonic components. The obtained arrays of the required current orthogonal components correspond to the initial ones:

$$I_{am}=\begin{pmatrix}0\\-I_{a1}\\0\\-I_{a3}\end{pmatrix};\quad I_{bm}=\begin{pmatrix}0\\I_{b1}\\0\\I_{b3}\end{pmatrix},$$

where *m* – the numbers of the current harmonics. This confirms the correctness of the method of the formation of the instantaneous admittance components arrays.

4. 2. The formation of the admittance components in the frequency and time domains when the current is represented by three harmonics

Supply voltage $u(t)$ is specified similarly to the first case: $u(t)=U_{b1}\sin(\omega t)$, a and the current – by the first, third and fifth harmonics:

$i(t)=I_{a1}\cos(\omega t)+I_{b1}\sin(\omega t)+I_{a3}\cos(3\omega t)+$
$+I_{b3}\sin(3\omega t)+I_{a5}\cos(5\omega t)+I_{b5}\sin(5\omega t).$

The expression for the instantaneous admittance is written down as follows:

$$y(t)=\frac{I_{a1}\cos(\omega t)}{U_{b1}\sin(\omega t)}+\frac{I_{b1}\sin(\omega t)}{U_{b1}\sin(\omega t)}+\frac{I_{a3}\cos(3\omega t)}{U_{b1}\sin(\omega t)}+ +\frac{I_{b3}\sin(3\omega t)}{U_{b1}\sin(\omega t)}+\frac{I_{a5}\cos(5\omega t)}{U_{b1}\sin(\omega t)}+\frac{I_{b5}\sin(5\omega t)}{U_{b1}\sin(\omega t)}. \quad (5)$$

After trigonometric transformations (5), we will obtain:

$$y(t)=\frac{I_{a1}}{U_{b1}}ctg(\omega t)+\frac{I_{b1}}{U_{b1}}+\frac{I_{a3}}{U_{b1}}ctg(\omega t)- -2\frac{I_{a3}}{U_{b1}}\sin(2\omega t)+\frac{I_{b3}}{U_{b1}}+2\frac{I_{b3}}{U_{b1}}ctg(2\omega t)+ +\frac{I_{a5}}{U_{b1}}ctg(\omega t)+\frac{I_{b5}}{U_{b1}}-2\frac{I_{a5}}{U_{b1}}\sin(2\omega t)+2\frac{I_{b5}}{U_{b1}}\cos(2\omega t)- -2\frac{I_{a5}}{U_{b1}}\sin(4\omega t)+2\frac{I_{b5}}{U_{b1}}\cos(4\omega t). \quad (6)$$

The arrays of the orthogonal components of the instantaneous admittance in the frequency domain are presented below:

$$Y_{ak}=\begin{pmatrix}\frac{I_{b1}+I_{b3}+I_{b5}}{U_{b1}}\\ 0\\ \frac{I_{b3}+I_{b5}}{U_{b1}}\\ 0\\ \frac{I_{b5}}{U_{b1}}\end{pmatrix};\quad Y_{bk}=\begin{pmatrix}\frac{I_{a1}+I_{a3}+I_{a5}}{U_{b1}}\\ 0\\ \frac{I_{a3}+I_{a5}}{U_{b1}}\\ 0\\ \frac{I_{a5}}{U_{b1}}\end{pmatrix}.$$

The correctness of the obtained expression is confirmed by the coincidence of the current signal curves shown in Fig. 2. The curves shown below are built using the following data: the amplitude values of voltage and current, respectively U_{b1}=10 V, I_{a1}=–2.5 A, I_{b1}=4.33 A, I_{a3}=–1.859 A, I_{b3}=3.542 A, I_{a5}=–1.302 A, I_{b5}=2.703 A, the angle of shift of the current first, third and fifth harmonic components, respectively $\varphi_1=\pi/6$, $\varphi_3=\pi/6.5$, $\varphi_5=\pi/7$.

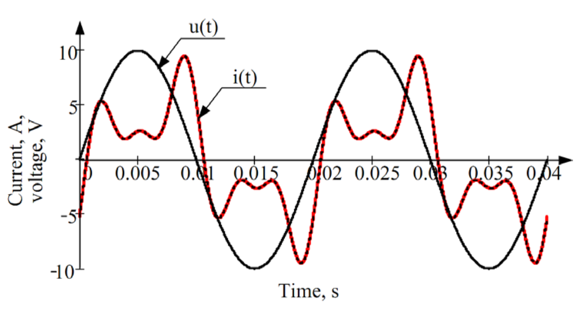

Fig. 2. Current $i(t)$ and voltage $u(t)$ signal curves, (- - -) – current curve calculated with the use of instantaneous admittance, (——) – current initial curve

To verify the correctness of the formation of the instantaneous admittance components arrays, we will determine the current harmonic components. The obtained arrays of the current desired orthogonal components correspond to the initial ones:

$$I_{am}=\begin{pmatrix}0\\ -I_{a1}\\ 0\\ -I_{a3}\\ 0\\ -I_{a5}\end{pmatrix};\quad I_{bm}=\begin{pmatrix}0\\ I_{b1}\\ 0\\ I_{b3}\\ 0\\ I_{b5}\end{pmatrix}.$$

The equality of the obtained and the initial orthogonal components of current for the two cases analyzed above confirm the correctness of the formation of the arrays of the instantaneous admittance orthogonal components and the applicability of the proposed method for the determination of current.

On the basis of (4) and (6), it is possible to write down the expression for the instantaneous admittance in the trigonometric form more generally:

$$y(t)=\sum\frac{I_{bm}}{U_{b1}}+\sum\frac{I_{am}}{U_{b1}}ctg(\omega t)+ +2\sum\frac{I_{bm}}{U_{b1}}\cos(k\omega t)-2\sum\frac{I_{am}}{U_{b1}}\sin(k\omega t). \quad (7)$$

Thus, according to (7), the instantaneous admittance orthogonal components can be presented in the general form:

$$Y_0=\frac{1}{U_{b1}}\left(\sum_{m=1}^{M}I_{bm}\right);\quad Y_{ak}=\frac{1}{U_{b1}}\left(\sum_{m=k+1}^{M}I_{bm}\right);$$

$$Y_{b0}=\frac{1}{U_{b1}}\left(\sum_{m=1}^{M}I_{am}\right);\quad Y_{bk}=\frac{1}{U_{b1}}\left(\sum_{m=k+1}^{M}I_{am}\right),$$

where Y_0 – admittance constant component, M – the number of the current harmonic components.

5. The research of the nonlinear electric circuit by means of the analysis of the components of the instantaneous admittance and resistance

To demonstrate the practical use of the proposed approach to the analysis of the components of instantaneous admittance $Y(t)$ and resistance $Z(t)$ to determine the harmonic components of the electric circuits current, we will consider the simplest electric circuit of linear and nonlinear resistances connected in series. The following parameters were chosen for the calculation R_0=1 Ohm, R_2=2 Ohm for the case of one sine harmonic component in the voltage signal $u(t)=U_{b1}\sin(\omega t)$, U_{b1}=10 V. The nonlinear resistance is described by the expression $R(I)=R_2 i^2(t)$. The equivalent circuit is shown in Fig. 3.

It follows from the energy conservation law that $p_S(t)=$ $=p_C(t)$ [36], where $p_S(t)$ – the power supply IP, $p_C(t)$ – the consumers' IP.

The power supply IP is determined by the product of the corresponding components of voltage and current $p_S(t)=u(t)i(t)$ [36]. In turn, for the considered case (Fig. 3), the consumers' IP is equal to the sum of all the elements IPs $p_C(t)=p_{R0}(t)+p_{R(I)}(t)$, where $p_{R0}(t)=R_0 i^2(t)$, $p_{R(I)}(t)=R_2 i^4(t)$ – IP at the linear and nonlinear resistances, respectively.

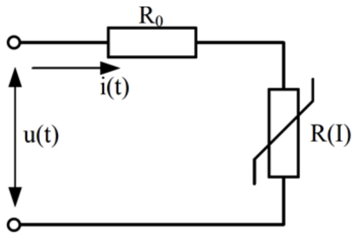

Fig. 3. The researched electric circuit

Taking into account expressions (1) and (2), it is possible to state that the discrete convolution of arrays I^2 and Z is equal to the convolution of arrays U^2 and Y, also, it is equal to the power supply IP P_S:

$$I^2 * Z = U^2 * Y = P_S, \qquad (8)$$

where $*$ – the discrete convolution operation, I^2, U^2 – the orthogonal components of the squared current and voltage, respectively, Z, Y – the orthogonal components of the impedance and admittance, respectively.

To confirm the above, we provide the numerical values obtained with the use of the automated method of formation of IP components based on the algorithm of discrete convolution (Table 2). The values of the amplitude components of current and voltage of the equivalent circuit of the researched electric circuit were obtained as a result of the calculation of the mathematical model in a mathematical package MATLAB. During the analysis of the electric circuit (Fig. 3), containing only active linear and nonlinear resistances, the currents will be presented only by the sine components. To simplify the calculation, we will limit ourselves with the most important (the first, the third and the fifth) current harmonics.

Table 2

The numerical values of the balance of the IP components for the researched electric circuit, obtained in different ways

IP cosine components	P_{Sa}, W	P_{Ca}, W	YU^2, W	ZI^2, W
Numerical values	9.22	9.2	9.22	9.308
	0	0	0	0
	–3.79	–3.87	–3.79	–3.76
	0	0	0	0
	–0.45	–0.78	–0.45	–0.43
	0	0	0	0
	–0.38	–0.41	–0.16	–0.99
	0	0	0	0
	0	0.12	–0.22	0.31
	0	0	0	0
	0	0.05	0	0.11

The data shown in Table 2 prove that equality (8) is true. The accuracy of the values obtained for the zero, second, and fourth harmonic components of IP is sufficiently high. The discrepancy of the results in the higher harmonics can be explained by the calculation error resulting from the limited number of the analyzed current harmonics.

Taking into account (1) and (2), it is possible to write down $U=I*Z$, $I=U*Y$, then the expression $P_S=I*U$ can be written down in the following way $P_S= I*Z* U*Y$. Hence, the convolution of the spectra of the instantaneous admittance Y and instantaneous resistance Z equals to one:

$$Z * Y = 1. \qquad (9)$$

Expression (9) makes it possible to write down in an analytical form and numerically find the solution to the system of equations with the desired harmonic components of the electric circuit current (Fig. 3). Below there are analytical expressions for the orthogonal components of instantaneous admittance and resistance:

$$Z_{ax} = \begin{pmatrix} \left(2I_{b1}^2 + 2I_{b3}^2 + 2I_{b5}^2\right)R_2 + R_0 \\ 0 \\ \left(-I_{b1}^2 + 2I_{b3}I_{b1} + 2I_{b3}I_{b5}\right)R_2 \\ 0 \\ \left(-2I_{b3}I_{b1} + 2I_{b5}I_{b1}\right)R_2 \\ 0 \\ \left(-2I_{b5}I_{b1} + I_{b3}^2\right)R_2 \\ 0 \\ \left(-2I_{b3}I_{b5}\right)R_2 \\ 0 \\ \left(I_{b5}^2\right)R_2 \end{pmatrix};$$

$$Z_{bx} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

$$Y_{ak} = \begin{pmatrix} \dfrac{I_{b1}+I_{b3}+I_{b5}}{U_{b1}} \\ 0 \\ \dfrac{I_{b3}+I_{b5}}{U_{b1}} \\ 0 \\ \dfrac{I_{b5}}{U_{b1}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad Y_{bk} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where Z – the orthogonal components of resistance, x – the number of the harmonic components of resistance.

Using the above expressions, we will present (9) in the frequency domain:

$$\begin{pmatrix} \left(2I_{b1}{}^{2}+2I_{b3}{}^{2}+2I_{b5}{}^{2}\right)R_2+R_0 \\ 0 \\ \left(-I_{b1}{}^{2}+2I_{b3}I_{b1}+2I_{b3}I_{b5}\right)R_2 \\ 0 \\ \left(-2I_{b3}I_{b1}+2I_{b5}I_{b1}\right)R_2 \\ 0 \\ \left(-2I_{b5}I_{b1}+I_{b3}{}^{2}\right)R_2 \\ 0 \\ \left(-2I_{b3}I_{b5}\right)R_2 \\ 0 \\ \left(I_{b5}{}^{2}\right)R_2 \end{pmatrix} * \begin{pmatrix} \dfrac{I_{b1}+I_{b3}+I_{b5}}{U_{b1}} \\ 0 \\ \dfrac{I_{b3}+I_{b5}}{U_{b1}} \\ 0 \\ \dfrac{I_{b5}}{U_{b1}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Using the automated method of the formation of IP components, we will obtain an equation system (10) in the analytical form. Its solution will allow the determination of the desired harmonic components of the analyzed electric circuit current.

$$\begin{pmatrix} \left(\dfrac{I_{b1}+I_{b3}+I_{b5}}{U_{b1}}\right)R_0+\left(\dfrac{2I_{b3}^3+2I_{b1}^3+6I_{b3}^3I_{b1}+2I_{b5}^3+6I_{b3}^2I_{b5}+6I_{b5}^2I_{b1}+6I_{b5}^2I_{b3}}{U_{b1}}\right)R_2 \\ 0 \\ \left(\dfrac{I_{b3}+I_{b5}}{U_{b1}}\right)R_0-\left(\dfrac{-3I_{b1}^2I_{b3}-6I_{b3}I_{b5}^2-2I_{b5}^3-2I_{b3}^3-3I_{b3}^2I_{b5}+I_{b1}^3-6I_{b1}I_{b3}I_{b5}}{U_{b1}}\right)R_2 \\ 0 \\ I_{b5}\dfrac{R_0}{U_{b1}}+\left(\dfrac{3I_{b1}^2I_{b5}+3I_{b3}^2I_{b5}+2I_{b5}^2-3I_{b1}^2I_{b3}-I_{b3}^3}{U_{b1}}\right)R_2 \\ 0 \\ -\left(\dfrac{3I_{b1}^2I_{b5}+3I_{b3}^2I_{b1}+I_{b3}^3+3I_{b3}^2I_{b5}+I_{b5}^3}{U_{b1}}\right)R_2 \\ 0 \\ -\left(\dfrac{6I_{b1}I_{b3}I_{b5}+I_{b3}^3+3I_{b3}^2I_{b5}+3I_{b3}I_{b5}^2+I_{b5}^3}{U_{b1}}\right)R_2 \\ 0 \\ -\left(\dfrac{3I_{b1}I_{b5}^2+3I_{b3}^2I_{b5}+3I_{b3}I_{b5}^2+I_{b5}^3}{U_{b1}}\right)R_2 \\ 0 \\ \left(\dfrac{-3I_{b3}I_{b5}^2-I_{b5}^3}{U_{b1}}\right)R_2 \\ 0 \\ \left(\dfrac{-I_{b5}^3}{U_{b1}}\right)R_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (10)$$

The solution of the obtained equation system, i. e. the determination of the first, third and fifth harmonic components of the current, was performed by the numerical method of Levenberg-Marquardt. The calculated values I_{b1}=–0.93 A, I_{b3}=–0.164 A, I_{b5}=–0.084 A were compared with the ones obtained as a result of the calculation of the mathematical model of the considered circuit in the mathematical package MathCAD. The relative errors by the first $\delta(I_{b1})$, third $\delta(I_{b3})$ and fifth $\delta(I_{b5})$ harmonic components were calculated; they were 0.8 %, 0 %, 10.5 %, respectively. The obtained values confirm the adequacy and sufficient accuracy of the proposed method of the solution of the direct problem of electric engineering.

To solve the inverse problem (the circuit parameters are determined by the known currents and voltages), one can use the expression of the researched circuit voltages balance:

$$I*Z=U. \quad (11)$$

Let us write down (11) in the frequency domain:

$$\begin{pmatrix} \left(2I_{b1}{}^{2}+2I_{b3}{}^{2}+2I_{b5}{}^{2}\right)R_2+R_0 \\ 0 \\ \left(-I_{b1}{}^{2}+2I_{b3}I_{b1}+2I_{b3}I_{b5}\right)R_2 \\ 0 \\ \left(-2I_{b3}I_{b1}+2I_{b5}I_{b1}\right)R_2 \\ 0 \\ \left(-2I_{b5}I_{b1}+I_{b3}{}^{2}\right)R_2 \\ 0 \\ \left(-2I_{b3}I_{b5}\right)R_2 \\ 0 \\ \left(I_{b5}{}^{2}\right)R_2 \end{pmatrix} * \begin{pmatrix} 0 \\ I_{b1} \\ 0 \\ I_{b3} \\ 0 \\ I_{b5} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ U_{b1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Using the automated method of IP components formation, we will get the equation system in the analytical form:

$$\begin{pmatrix} 0 \\ I_{b1}R_0+\left(3I_{b1}^3+6I_{b3}^2I_{b1}+6I_{b5}^2I_{b1}-3I_{b1}^2I_{b3}-6I_{b1}I_{b3}I_{b5}+3I_{b3}^2I_{b5}\right)R_2 \\ 0 \\ I_{b3}R_0+\left(6I_{b1}^2I_{b3}+3I_{b3}^2+6I_{b5}^2I_{b3}-I_{b1}^3+6I_{b1}I_{b3}I_{b5}-3I_{b1}^2I_{b5}\right)R_2 \\ 0 \\ I_{b5}R_0+\left(-3I_{b1}^2I_{b3}+6I_{b1}^2I_{b5}+3I_{b3}^2I_{b1}+6I_{b3}^2I_{b5}+3I_{b5}^3\right)R_2 \\ 0 \\ \left(-3I_{b1}^2I_{b5}+6I_{b1}I_{b3}I_{b5}+3I_{b5}^2I_{b3}-3I_{b3}^2I_{b1}\right)R_2 \\ 0 \\ \left(-6I_{b1}I_{b3}I_{b5}+3I_{b5}^2I_{b1}-I_{b3}^3\right)R_2 \\ 0 \\ \left(-3I_{b1}I_{b5}^2-3I_{b3}^2I_{b5}\right)R_2 \\ 0 \\ -3I_{b5}^2I_{b3}R_2 \\ 0 \\ -I_{b5}^3R_2 \end{pmatrix} = \begin{pmatrix} 0 \\ U_{b1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (12)$$

Equation system (12), as well as (10), was calculated by Levenberg-Marquardt method [1]. When solving (12), it is reasonable to neglect some higher harmonics in the array $I*Z$. Taking into account the limited number of current harmonics, it is possible to explain the increase of the error with the growth of the harmonic number during the solution of the equation system. The calculated values of the parameters R_0=1.064 Ohm and R_2=1.959 Ohm were compared with the initial ones. The obtained values of the relative errors of the value of resistance were $\delta(R_0)$=6.4 %, $\delta(R_2)$=2.04 %, which demonstrates the efficiency of the proposed method for the solution of the nonlinear electric circuit with the use of the components of instantaneous admittance and resistance.

6. The discussion of the results of the research of the developed analytical method for the analysis of nonlinear electric circuits

The research results presented in the paper made it possible to solve the problem of achieving sufficient accuracy in the determination of the parameters of the nonlinear electric circuits of electrical engineering devices due to the use of the developed efficient and easily realized analytical method with the use of admittance and resistance components.

It is proposed to realize the trigonometric transformations and numerical calculations in the frequency domain with the use of the developed automated method of the formation of electric values components. In comparison with the methods described in the second part, this method essentially facilitates the mathematical analysis and shortens the time even if the nonlinear electric circuit configuration is complicated.

The proposed analytical method, unlike the known ones, is well adapted to the automation of the calculations in the frequency domain. Besides, this method provides the possibility to assess the researched circuit parameters influence on the current spectrum components.

The advantage of this method consists in the possibility to obtain the predicted result independently of the degree of the approximating polynomial and the number of the analyzed harmonics.

The presented method makes it possible to analyze qualitatively the connected nonlinear load characteristics and the current spectrum. This is a good basis for the active and passive filtering means choosing method' development

The disadvantage of the method is the need to neglect the higher harmonic components during solving the generated equations sets. This entails a decrease in accuracy when solving the direct or inverse electrical engineering tasks. Also, using the presented method, it may be difficult to describe a nonlinear characteristic by a polynomial function. This is due to the fact that to achieve an accurate approximation, it is necessary to use a high-degree polynomial, which will increase the cumbersomeness of analytical expressions.

7. Conclusions

1. The mechanism of the formation of the orthogonal components of the instantaneous admittance on the basis of current and voltage components in the frequency domain has been presented. An automated method for the electrical quantities frequency components formation, realized on the basis of the discrete convolution is used for the possibility of process automation.

2. A method for the analysis of nonlinear electric circuits with the use of the components of instantaneous admittance and resistance has been developed. Along with the determination of the numerical quantities of the desired values, this method makes it possible to perform the analytical analysis of forming the balance equations for the components of instantaneous admittance and resistance.

3. The results of the numerical solution of the equations system of the balance of the components of instantaneous admittance and resistance confirm the high accuracy and efficiency of the proposed method in relation to the analysis of nonlinear electric circuits. In this case, the relative error of the values of the higher harmonic components of the desired current does not exceed 10 % and 7 % during the solution of the direct and inverse problems of electric engineering, respectively. This level of accuracy is provided even at a low degree of the approximating polynomial, which indicates the sufficient accuracy of the obtained results.

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ANALYSIS OF RELATIONSHIP BETWEEN THE DYNAMICS OF A THERMOELECTRIC COOLER AND ITS DESIGN AND MODES OF OPERATION

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Розглянуто модель взаємозв'язку основних показників динаміки і надійності з врахуванням конструктивних елементів охолоджуючого пристрою для різних режимів роботи. Одержані співвідношення дозволяють визначити час виходу термоелектричного охолоджуючого пристрою на стаціонарний режим і температуру теплопоглинаючого спаю. Показано, що врахування теплофізичних, конструктивних і енергетичних показників дозволяє управляти часом виходу охолоджувача в стаціонарний режим

Ключові слова: термоелектричний охолоджувач, стаціонарний режим, температура тепло поглинаючого спаю, показники надійності

Рассмотрена модель взаимосвязи основных показателей динамики и надежности с учетом конструктивных элементов охлаждающего устройства для различных режимов работы. Полученные соотношения позволяют определить время выхода термоэлектрического охлаждающего устройства на стационарный режим и температуру теплопоглощающего спая. Показано, что учет теплофизических, конструктивных и энергетических показателей позволяет управлять временем выхода охладителя в стационарный режим

Ключевые слова: термоэлектрический охладитель, стационарный режим, температура теплопоглощающего спая, показатели надежности

1. Introduction

Determining the time that it takes for a thermoelectric cooling device (TED) to enter a stationary working mode over the preset temperature range is an interesting task. This is related to the fact that dynamic indicators for the means that enable heat regimes of thermally loaded elements largely define both functional and reliable capabilities of critical