*Розглядається цілеспрямована територіальна система, що має сфери виробництва, споживання, управління, екологічну сферу, в процесі функціонування якої виникають проблемні ситуації. Розглянуто питання ідентифікації проблемних ситуацій на основі комплексу задач лінійного програмування, який віддзеркалює нерозривний зв'язок механізмів централізованого планування та ринкового ціноутворення. Сформована множина типових прецедентів, розроблена технологія ідентифікації проблемних ситуацій*

*Ключові слова: перехідна територіальна система, моделювання циклічної динаміки, технологія ідентифікації проблемних ситуацій*

*Рассматривается целенаправленная территориальная система, содержащая сферы производства, потребления, управления, экологическую сферу, в процессе функционирования которой возникают проблемные ситуации. Рассмотрены вопросы идентификации проблемных ситуаций на основе комплекса задач линейного программирования, отражающего неразрывную связь механизмов централизованного планирования и рыночного ценообразования. Сформировано множество типовых прецедентов, разработана технология идентификации проблемных ситуаций*

*Ключевые слова: переходная территориальная система, моделирование циклической динамики, технология идентификации проблемных ситуаций*

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# MODELING OF THE FUNCTIONING OF TERRITORIAL SYSTEMS WITH THE PURPOSE OF IDENTIFICATION OF PROBLEM SITUATIONS

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## 1. Introduction

The purposeful territorial system (TS) is considered as a form of spatial organization within the boundaries of the allocated area of socioeconomic life of society in the form of the TS of the region, district, state, world community. Active elements of the TS are producers and consumers, carrying out the functions of production and consumption of products and services that make up the sphere of production (SPR) and the sphere of consumption (SCO). There is the environmental sphere (ESP) – part of the natural environment associated with the TS. The microeconomic processes of the TS are based on almost unlimited chaos, subject to a number of constraining laws of market economy, determining the mechanism of market regulation (MMR), ordering the chaos prevailing in its depths. A necessary condition for the macroeconomic development of the TS is an increase in its intellectual potential (the array of active Mind, providing search for effective ways of increasingly complicated development of the TS) [1]. The management sphere (MS) of the TS integrates the mechanisms of market regulation (MMR) and the mechanism of public administration (MPA), distributes the production function among producers and consumption functions among consumers.

In accordance with this approach, the TS is considered as a set of the SPR, SCO, MS, ESP, which form some structure intended for the implementation of the functions of production, exchange, distribution, consumption. Effective management of these functions should provide the maximum satisfaction of the growing minimum needs of the TS population with the minimum costs of society, taking into account the existing restrictions.

Restriction of needs is caused by requirements of environmental safety of society. The target state of the TS is the permissible state of dynamic equilibrium in which the TS must remain or into which the TS must pass in the process of overcoming the crisis phenomena.

Management of the TS development is carried out from a single Center, providing system integration of the SPR, SCO, MS, ESP in order to obtain the maximum synergy effect. For this purpose, the Center generates management actions (indicative plans, state policy), coordinating the objectives and actions of the SPR, SCO, MS, ESP, integrating the MPA, MMR, mobilizing integrated intelligence of the society and capital of entrepreneurs to achieve the goal state (GS) of the TS.

This requires scientifically based technology for implementing scenarios and strategies to achieve the GS, ensuring effective overcoming of crisis phenomena by minimizing the amount of destructive work, maximizing the amount of creative work, timely adaptation of active participants to emerging macro- and micro-conditions.

It is assumed that the main management tools are hierarchy, market, culture. The market, defining horizontal relations in the TS, is the basis of the MMR. The hierarchy, defining vertical relations in the TS, is the basis of the MPA. The culture, defining the mentality of the TS population, is the basis of its rules of conduct. The MPA affects the MMR through the action of the system of indicative plans carried out by state policy on the mechanisms of self-regulation and self-organization. As aspects of the TS existence, morphology (structure) of the TS; functioning (cyclical dynamics, determined by circulation of goods, resources, income in a short time interval); development (historical dynamics, considering the processes of TS transformation on a continuous sequence of time intervals) are considered.

The reason for problem situations (PS) in existing TS is the violation of the basic national economic parities [3, 4]. In the state of dynamic equilibrium, these disparities of the TS are characterized by small deviations, determine the minimum losses of society. If there are significant deviations from the state of dynamic equilibrium, imbalances determine the TS, threatening the process of achieving the TS goal. Therefore, an important problem of effective management of the TS development is the reliable identification of the arisen PS, the formation of adequate management actions with a view to its resolution.

## 2. Literature review and problem statement

Any national economy is a TS. Therefore, when modeling the TS functioning with the aim of identifying the emerging PS, it is advisable to use substantial experience accumulated in the development of the macroeconomic theory containing two basic concepts. Some scientists believe that the MMR works better if it is not influenced by the MPA [2, 3]. Others hold the opinion that the interference of the MPA can essentially improve the TS functioning [4]. Ongoing discussions between the supporters of the neoclassical theory and those of the Neo-Keynesian economics contribute to the development of the macroeconomic theory, enrich it with effective methods, models, tools for studying real TS. The development of the macroeconomic theory is also facilitated by its hidden interdisciplinarity. So the hypothesis of the spontaneous aspiration of territorial economic systems for a single state of economic equilibrium is the result of the mechanical view of economic processes, derived by Walras from Newton's physics. Speaking about the problem of modeling the functioning of territorial systems, it is necessary to mention the works [3, 4] of scientists who study the behavior of balanced market economies staying in a long-term dynamic equilibrium. However, at the present stage of dynamic development of the world system under the influence of a huge number of internal and external factors, problem situations (PS) may occur in the process of TS functioning. In this case, the TS can come close to the bifurcation point, when continuation of the existing technology of management of TS functioning and development is impossible and an alternative to further development arises [5, 6]. The choice of any alternative immerses the TS in a transition period. This is confirmed by the presence of a significant number of national economies in transition [7].

In the transition period, the MPA plays the main role in the management of functioning and development of the transitional territorial system (PTS) [8]. The effectiveness of the MPA is determined by its ability to use available human, production, intellectual capital to form a sample of alternatives for finding effective ways of increasingly complicated development. As a rule, the MPA does not know the management actions required by the MMR at each time point upon transition to a new state of dynamic equilibrium.

In this regard, there is a problem of increasing the efficiency of the MMR by creating the conditions for the fullest manifestation of its advantages. One of the most promising directions for determining these conditions is the use of methods for modeling organizational systems.

At the same time, there is an insufficient number of publications devoted to the modeling of functioning and development of transition territorial systems (TTS). This is due to the complexity of the systems and processes under study, the inability of the MMR to effectively manage the TTS functioning.

Herewith, when studying the processes of functioning of transition economies, many authors tend to use qualitative modeling methods to assess the level of regional development [9], determine the features of economic growth [10], and study the TTS stability [11]. On the basis of rich empirical material and deep analysis of it, a number of authors create econometric models to generate economic forecasts [12], create qualitative models of TTS functioning at different stages of the life cycle [6, 7, 13–16].

Quantitative models of interaction between the MPA and MMR in the management of functioning and development of the TTS are virtually absent [17]. Therefore, in order to coordinate the management actions of the MPA and MMR, the problem of system integration of the MPA and MMR is solved on the basis of complex quantitative modeling of TTS functioning. This allows identifying the emerging PS, forming adequate coordinated management actions of the MPA and MMR, required for the resolution of PS.

## 3. The aim and objectives of the study

The aim of the study is to improve the system characteristics of the TS by identifying the PS based on the modeling of TS functioning and formation of adequate management solutions for resolving the identified PS.

To achieve this aim, the following objectives were accomplished:

– to develop a set of interacting linear programming problems, modeling the circulation of products and resources during the TS functioning on the short-term interval;

– to identify the set of PS that arise during the TS functioning, indicate adequate methods for their resolution, using the analytical apparatus of linear programming;

– to develop the subject technology for the PS identification, an adequate management solution using the case-based reasoning technology.

## 4. Materials, methods of the study of the process of identification of problem situations of the functioning territorial system

To study the process of identification of PS emerging in the process of TS functioning, we propose the method for modeling the cyclical dynamics of the TS by a complex of dual pairs of LP problems. This complex reflects the underlying properties of the processes of production and consumption of products and services (PS), the supply and use of production factors (PF), as well as pricing processes in the commodity and resource markets of the TS on the macrolevel. This allows considering the inextricable relation between the mechanisms of centralized planning of production of PS, the supply of PF and market formation of their prices, using the developed analytical apparatus of LP, building the generalized Walras-Wald model of TS functioning.

The PS is further determined by the unacceptable deviation of the TS from the state of dynamic equilibrium. In this regard, it is expedient to study the conditions of the TS stay in the vicinity of the point of the admissible trajectory of its balanced growth, which determine the parameters of the state of dynamic equilibrium. The basis of the MMR of TS functioning is the law of the equilibrium price (optimum price).

Stable equilibrium prices of PS and PF allow each TS participant to compare costs and benefits with the interests of the whole society. In this case, what is beneficial to an individual participant is beneficial to the entire system, and vice versa. The unregulated market mechanism ensures the functioning of each producer and consumer in an equilibrium state in full accordance with the main objective of the TS. It provides a balance between the prices of the generalized producer (GPR) and the generalized consumer (GCO), as well as between the supply and demand of PS and PF.

In the modern theory of economic equilibrium of market economies, there are various approaches to the study of equilibrium and its stability, a significant number of models have been created that use different hypotheses of functioning of the market mechanism. In particular cases, the conditions for the existence of dynamic equilibrium have been proved. In the general case, there are objective reasons for the fact that imbalance is a usual state of the market economy, which, as a rule, causes severe socioeconomic consequences [2–4].

In the case of joint action of the mechanism of centralized regulation and the mechanism of market regulation with imperfect competition, the problem of determining the conditions for the economic equilibrium is very complex, difficult to formalize. Therefore, it is very difficult to find a satisfactory solution.

It is shown that if the main goal of the centralized TS is to increase public welfare, and the local goal of each participant is to maximize profits, and at the same time the prices of the optimum plan are established in the TS, then the behavior of all participants of the TS of the economic system automatically corresponds to the national economic optimum.

Taking into account the above circumstances, one should expect that an effective pricing mechanism can be built for each TS, which can provide an objective basis for the coordinated functioning of all producers and consumers, as well as the TS as a whole. Following Walras, the basis of such a pricing mechanism can be trial and error process. In this process, the prices of PS and PF are determined by the Center. The production and consumption spheres, on the basis of these prices, determine the levels of supply and demand for each PS and PF, providing the maximum income to GPR and GCO. If, as a result of adopted decisions, a supply and demand balance is formed in each market of PS and PF, or a supply and demand balance in PS and PF market systems, these prices satisfy the Center and determine the system of equilibrium prices of PS and PF. If the prices do not satisfy the Center, it determines new prices, and the GPR and GCO identify new levels of supply and demand of PS and PF, etc. The trial and error process continues until the equilibrium prices that determine the economic equilibrium in the system are established. It should be borne in mind that equilibrium prices may not exist. To study the conditions for the existence of economic equilibrium, it is proposed to use the complex of the models described in [18] as a theoretical platform. This complex simultaneously models the spontaneous pricing mechanism and the mechanism of centralized planning of activities of the GPR and GCO at fixed prices.

To construct the generalized Walras-Wald model, simplifying assumptions are introduced.

1. The set of PS and the set of technologies for their production are equinumerous sets.

2. The set of PF and the set of technologies of their supply are equinumerous sets.

3. There are no interchangeable among the PS produced and PF supplied.

4. The aggregated consumer (GCO) gains income L2 from the implementation of the formed supplies of PF, which are his property. All available PF belong to the sphere of consumption. In addition, the GCO spends all the income to purchase PS, and there are restrictions

$$(P, Z_1)=(W, Z_2), \tag{1}$$

here and below:
$Z_1=(Z_1^1,..., Z_r^1)$ is the column vector of PS production volumes;
$P=(P_1,..., P_r)$ is the column vector of PS prices, $P^T=(P_1...P_r)$ is the row vector;
$Z_2=(Z_1^2,..., Z_m^2)$ is the column vector of PF supply volumes;
$W=(w_1,..., w_m)$ is the column vector of PF prices, $W^T=(w_1...w_m)$ is the row vector;

5. Let Z10 be the vector of the minimum needs of the TS in the PS produced; Z20 – the vector of PF available in the TS in current time.

It is assumed that demand Z10 is satisfied for any production volumes $Z_1$, and the available PF Z20 are able to provide any production volumes $Z_1$ of PS. In this case, the inequalities

$$Z_1 \geq Z10,\ Z_1 \geq 0,\ Z_2 \leq Z20,\ Z_2 \geq 0 \tag{2}$$

always hold and can be neglected when constructing an ideal model of cyclical dynamics.

6. Sustainable development involves the restoration of the biosphere properties, significant improvement in the quality of life of people. Therefore, in the ideal case, the requirements for environmentally friendly development are met automatically

$$G_1 \cdot Z_1 + G_2 \cdot Z_2 \leq d,\ d_1 + d_1 = d,\ d_{1,2} \geq 0, \tag{3}$$

where $G_1$ is the matrix of pollution of the environmental sphere of the TS by industrial waste; $G_2$ is the matrix of pollution of the environmental sphere of the TS by household waste; $d_1$ is the vector determining the environmental pollution by industrial waste; $d_2$ is the vector determining the environmental pollution by household waste; d is the vector determining the maximum permissible standards of environmental pollution.

The compliance with the requirements (2), (3) is ensured at the stage of formation of parameters of the goal state and current management actions.

Taking into account the assumptions made, the ideal economic-mathematical model (EMM) of PS and PF circulation contains: the ideal model of PS production:

$$L_1=(P, Z_1)\to \max,\ A_n\times Z_1\le Z_2,\ Z_1\ge 0, \qquad (4)$$

where $A_n$ is the matrix of size m×r – the cost matrix of PF in PS production; $L_1$ is the total income of the GPR.

If $y_1$ is the column vector of the estimates $y_i^1$, $i$=1,..., m, PF supply in the factor market, $\tilde{L}_1$ is the total estimate of PF supply, then the dual problem to the problem (4)

$$\tilde{L}_1=(y_1, Z_2)\to \min,\ A_n^T\cdot y_1\ge P,\ y_1\ge 0, \qquad (5)$$

is the problem of objective estimation (($y_1=y_1^1,...,y_m^1$) – the column vector) of PF supply in the factor market. In addition, the ideal EMM also contains an ideal model of PF supply

$$L_2=(W, Z_2)\to \max,\ A_p\times Z_2\le Z_1,\ Z_2\ge 0, \qquad (6)$$

where $A_p$ is the matrix of size r×m – the cost matrix of PS in the generation of PF supply, and the dual problem

$$\tilde{L}_2=(y_2, Z_1)\to \min,\ A_p^T\times y_2\ge W,\ y_2\ge 0, \qquad (7)$$

is the problem of objective estimation ($y_2$) (( $y_2=y_1^2,...,y_r^2$ ) – the column vector) of PS supply in the commodity market. The system of mathematical expressions (1)–(7) models the interaction of production and consumption spheres through the PS and PF market systems without considering the mechanism of state macroeconomic regulation. The constructed ideal model is a generalization of the Walras-Wald model and the basis for the situational analysis of the cyclical dynamics of the TS.

Ideally, the management object forms the spheres of production and consumption, communicating among themselves through the PS and PF market systems, the prices P and W of which can be regulated by the Center ("Invisible Hand", the state).

The current situation Q for the management object is hereinafter referred to as the tuple $<A_n, A_p, Z10, Z20, Z_1, Z_2>$, containing information about the structure of the management object and its functioning at a given time. The current situation U for the management system is the tuple <P, W, ω>, containing information about the state of the management system (P; W), as well as information on the technological scheme ω∈τ for regulating the cyclical dynamics of the TS, τ is the set of available schemes. The current situations Q and U determine the overall situation S.

It is assumed that the management system of the TS has a finite set τ of various methods of influence on the management object, determined by the existing pricing mechanism in the system. The formed price system P, W determines and gives an opportunity to implement the priorities of economic development. Among the elements ω of the set τ, one can distinguish three classes of pricing technologies:

$\tau_1$ – establishment of fixed prices for PS and PF in the form of state-listed prices, fixed monopoly prices, "frozen" market open prices by the Center (state);

$\tau_2$ – open pricing based on the law of supply and demand ("Invisible hand"). The state limits the independence of producers and consumers by setting the rules of the game in the PS and PF markets by introducing various prohibitions;

$\tau_3$ – state regulation of prices by setting PS and PF pricing conditions, fixing the overall price level, allocating the maximum premiums of fixed prices, setting the maximum level of a one-time price increase, exercising state control over monopoly prices, setting prices for products of manufacturers of the public sector of the TS, etc.

The use of management ω∈τ causes a change ΔP in PS prices and ΔW in PF prices. When the prices P+ΔP, W+ΔW change, the consumption menu of the PU is changed by the consumption sphere and the production plan of the PU determined by $Z_1$. In addition, the menu of PS consumption by the consumption sphere and the plan of PF generation by the consumption sphere determined by Z2 are changed. These changes lead to changes in production and consumption technologies and, as a consequence, cause the evolution of the resource cost matrix $A_n$ and the PS cost matrix $A_p$, the evolution Z10, Z20. Thus, under the influence of ω∈τ, the current situation Q is transformed into a new situation Q'.

$$Q'=<A_n+\Delta A_n,\ A_p+\Delta A_p,\ Z10+\Delta Z10,\ Z20+\Delta Z20,\ Z_1+\Delta Z,\ Z_2+\Delta Z_2>.$$

Due to the finiteness of the number of pricing technologies, a lot of possible complete situations break up into a finite number of classes, each corresponding to one of the possible technological schemes for regulating the cyclical dynamics of the TS. In this regard, one of the main objectives of the situational analysis of the cyclical dynamics is the selection of classes of complete situations for which the technological regulation scheme is common in terms of the implementation of the pricing technology.

In the process of the TS movement from the initial state to the goal one, the following problems are solved:

– stabilization (transition to a state of dynamic equilibrium and ensuring continued movement along an admissible main);

– current restrictions determined by the initial state Z10, Z20;

– extremity (maximizing the volume of GDP produced in the TS).

These problems define the classes of complete situations, types of pricing technologies. The problem of stabilization involves the formation of the necessary main properties of the system, allowing it to be in a state of economic equilibrium, determined by the conditions of market equilibrium in the PS markets

$$(P, (A_pZ_2-Z_1))=0,\ P\ge 0,\ A_pZ_2\ge Z_1 \qquad (8)$$

and PF market.

$$(W, (A_nZ_1-Z_2))=0\ W\ge 0,\ A_nZ_1\le Z_2. \qquad (9)$$

The equilibrium of the TS is given by the vectors $P^*$, $W^*$, $Z_1^*$, $Z_2^*$, which satisfy the relations (2)–(9). To determine the conditions for the existence of market equilibrium, it is necessary to verify the following statements.

*Theorem 1.* If $P^*$ is the equilibrium price vector of PS, then the equilibrium vector $Z_1^*$ can be found by solving the problem (4) for $P=P^*$, $Z_2=Z_2^*$, and the equilibrium price vector of PF $W^*$ – by solving the problem (5) for $P=P^*$, $Z_2=Z_2^*$.

Proof.

Since $P^*$, $W^*$, $Z_1^*$, $Z_2^*$ are equilibrium vectors, then, taking into account (9) and (1), we can write

$$W^{*T}A_n Z_1=\left(W^*, Z_2^*\right)=\left(P^*, Z_1^*\right), \quad (10)$$

where $Z_1^*$ is the plan of the problem (4), and $Z_1^*$ is the plan of the problem (6). Generally speaking, the vector $W^*$ may not be a plan of the problem (5) for $P=P^*$, $Z_2=Z_2^*$. However, since $A_n$ has at least one positive element in each row or column, then the vector $KW^*$ with a sufficiently large factor $K>0$ will be the plan of the problem (5). Since $Z_1^*$ is the plan of the problem (4), and $KW^*$ is the plan of the problem (5), according to the main lemma of the duality theory of linear programming, the following chain of inequalities holds

$$\left(P^*, Z_1^*\right) \le KW^{*T}A_n Z_1^* \le K\left(W^*, Z_2^*\right). \quad (11)$$

It follows from the simultaneous validity of the relations (10) and (11) that K=1 and $W^*$ is the plan of the problem (5) for $P=P^*$, $Z_2=Z_2^*$. Thus, $Z_1^*$ is the plan of the problem (4), $W^*$ is the plan of the problem (5) and the equality $\left(P^*, Z_1^*\right)=\left(W^*, Z_2^*\right)$ holds. Then, according to the known lemma of the duality theory, $Z_1^*$ is the solution of the problem (4) for $P=P^*$, and $W^*$ is the solution of the problem (5) for $P=P^*$.

*Theorem 2*. If $W^*$ is the equilibrium price vector of PF, then the equilibrium vector $Z_2^*$ can be found by solving the problem (6) for $W=W^*$, $Z_1=Z_1^*$, and the equilibrium price vector of PS $P^*$ – by solving the problem (7) for $W=W^*$, $Z_1=Z_1^*$.

The proof of Theorem 2 is similar to the proof of Theorem 1.

*Theorem 3*. If for $m \le r$ the matrix $A_{np}=(A_n \cdot A_p)$, and for $m \ge r$ the matrix $A_{pn}=(A_p \cdot A_n)$ are nonnegative, indecomposable matrices, and their majorizing roots $\lambda_{np}^*$ or $\lambda_{pn}^*$ are equal to unity, then the economic equilibrium in the system is possible. In this case, the equilibrium vector $Z_2^*$ will coincide in direction with the right eigenvector, and the equilibrium vector $W^{*T}$ – with the left eigenvector of the matrix $A_{np}$, corresponding to its majorizing root. The equilibrium vector $Z_1^*$ will coincide in direction with the right eigenvector, and the equilibrium vector $P^{*T}$ – with the left eigenvector of the matrix $A_{pn}$, corresponding to its majorizing root.

*Proof.* Let $m \le r$ and $P^*$ be the equilibrium price vector of PS, then the equilibrium $Z_1^*$ can be obtained by solving the problem (4) for $P=P^*$. Since $m \le r$, in the optimal case the following relation holds

$$A_n \cdot Z_1^* = Z_2, \; Z_1^* \ge 0. \quad (12)$$

The equilibrium vector $W^*$ can be found by solving the problem (5) for $P=P^*$. Taking into account the solutions $Z_1^*$, $W^*$, the problem (6) takes the form

$$L_2=(W^*, Z_2) \to \max, \; A_p \cdot Z_2 \le Z_1^*, \; Z_2 \ge 0, \quad (13)$$

The problem (13) taking into account the constraints (1), relation (12) takes the form

$$L_2=(P^*, Z_1^*) \to \max, \; A_{pn} \cdot Z_1^* \le Z_1^*,$$

$$Z_1^* \ge 0, \; A_{pn}=A_p \cdot A_n. \quad (14)$$

Since $A_{pn}$ is a square matrix and $Z_1>0$, the solution $Z_1^*$ is obtained from the system of equations

$$\left(E_r - A_{pn}\right) \cdot Z_1^* = 0, \; Z_1^* \ge 0. \quad (15)$$

The problem (7) for $W=W^*$, $Z_1=Z_1^*$, $y_2=P^*$ takes the form

$$\tilde{L}_2 = \left(P^*, Z_1^*\right) \to \min, \; A_p^T \cdot P^* \ge W^*, \; P^* \ge 0. \quad (16)$$

Since $m \le r$, in the optimal case the following relation holds

$$W^* = A_p^T \cdot P^*, \; P^* \ge 0. \quad (17)$$

The problem (5) for $P=P^*$, $y_1=W^*$, taking into account (17), is represented as

$$\tilde{L}_1 = \left(W^*, Z_2\right) = \left(P^*, Z_1^*\right) \to \min,$$

$$A_n^T \cdot A_p^T \cdot P^* \ge P^*, \; P^* \ge 0, \quad (18)$$

and the solution $P^*$ of the problem (18) can be obtained from the system of equations

$$P^{*T}(E_r - A_{pn})=0, \; P^* \ge 0, \quad (19)$$

From the Perron-Frobenius theorem, it follows that the solution $Z_1^* > 0$ of the system (15) and the solution $P^{*T}>0$ of the system (19) exist if the majorizing root $\lambda_{pn}^*$ of the matrix $A_{pn}$ is equal to unity.

Equations (15) and (19) imply the relations

$$A_p \cdot Z_2 = Z_1^* \text{ and } A_n^T \cdot W^* = P^*,$$

represented taking into account equalities (12) and (17) as the system of equations

$$\left(E_m - A_{np}\right) \cdot Z_2^* = 0, \; Z_2^* > 0,$$

$$W^{*T}(E_m - A_{np})=0, \; W^*>0, \quad (20)$$

the solution of which determines the equilibrium vectors $W^*$, $Z_2^*$. Here $E_m$ is the identity matrix of size m×m. The system of equations (20) has the solution $Z_2^*>0$, $W^{*T}>0$, if the majorizing root $\lambda_{np}^*$ of the matrix $A_{np}$ is equal to unity. Thus, for $\lambda_{pn}^* = \lambda_{np}^* = 1$, the solutions $P^*$, $Z_1^*$, $W^*$, $Z_2^*$ of the systems of equations (15), (19), (20) ensure the fulfillment of market equilibrium conditions (8), (9).

Let $m \le r$ and $W^*$ be the equilibrium price vector of PF. The equilibrium vector $Z_2^*$ is found by solving (6), and $P^*$ – solving the problem (7) for $W=W^*$. Since $r \le m$, in the optimal case the following relation holds

$$A_p \cdot Z_2^* = Z_1^*, \; Z_2^* > 0. \quad (21)$$

The problem (4), for $y_1=P^*$, $Z_2=Z_2^*$ considering the constraints (1), conditions (21) is represented in the form

$$L_1=\left(W^*, Z_2^*\right) \to \max, \; A_{np} \cdot Z_2^* \le Z_2^*,$$

$$Z_2^*>0, \; A_{pn}=A_p \cdot A_n. \quad (22)$$

Since $A_{pn}$ is a square matrix and $Z_2^* \ge 0$, then the solution of the problem (22) can be obtained by solving the system of equations

$$A_{np} \cdot Z_2^* = Z_2^*, \; Z_2^* > 0. \quad (23)$$

The problem (5) for $P=P^*$, $Z_2=Z_2^*$ has the form

$$\tilde{L}_1 = \left(W^*, Z_2^*\right) \to \min, \;\; A_n^T \cdot W^* \geq P^*, \;\; W^* \geq 0. \qquad (24)$$

In the optimum case, the conditions of the problem (24) are satisfied as exact equalities

$$P^* = A_n^T \cdot W^*, \;\; W^* \geq 0. \qquad (25)$$

Taking into account the obtained results, the problem (7) takes the form

$$\tilde{L}_2 = \left(W^*, Z_2^*\right) \to \min, \;\; A_n^T W^* \geq W^*, \;\; W^* \geq 0. \qquad (26)$$

If the problem (26) is solvable, then its solution can be obtained by solving the system of equations

$$A_{np}^* \cdot W^* = W^*, \;\; W^* \geq 0. \qquad (27)$$

Thus, in the case m≥r, equilibrium exists if the following systems of equations have the solution

$$\left(E_m - A_{np}\right) \cdot Z_2^* = 0, \;\; Z_2^* > 0,$$

$$W^{*T}(E_m - A_{np}) = 0, \; W^* > 0 \qquad (28)$$

and

$$\left(E_r - A_{pn}\right) \cdot Z_1^* = 0, \;\; Z_1^* > 0,$$

$$P^{*T}(E_r - A_{pn}) = 0, \; P^* > 0. \qquad (29)$$

The resulting systems of equations (28) and (29) are solvable if the majorizing roots $\lambda_{np}^*$ and $\lambda_{pn}^*$ are equal to unity. Thus, Theorem 3 holds for any m and r.

However, the economic equilibrium is realized if conditions (2) are also fulfilled in the system along with conditions (8) and (9), where the vector $Z10 = Z_1(0) \cdot \ell_{10}$, and $Z20 = Z_2(0) \cdot \ell_{20}$. Here the vector $\ell_{10}$ specifies the structure of the minimum needs of society, and the vector $\ell_{20}$ specifies the structure of the resources available in the system. $Z_1(0)$ defines the minimum number of the sets $\ell_{10}$ of PS, $Z_2(0)$ – the maximum level of the supply of the sets $\ell_{20}$ of PF.

The TS is called productive if there are plans of the problems (4), (5) – vectors $Z_1$ and $Z_2$, for which the following inequalities hold

$$A_{pn} \cdot Z_1 \leq Z_1, \; Z_1 \geq 0,$$

$$A_{np} \cdot Z_2 \leq Z_2, \; Z_2 \geq 0. \qquad (30)$$

The TS is called profitable if there are plans of the problems (4), (5) – vectors $Z_1$ and $Z_2$, price vectors P, W, for which the following inequalities hold

$$(P, Z_1) - (P, A_{pn} Z_1) \geq 0, \; P \geq 0, \; W \geq 0, \; Z_1 \geq 0,$$

$$(W, Z_2) - (W, A_{np} Z_2) \geq 0, \; P \geq 0, \; W \geq 0, \; Z_2 \geq 0. \qquad (31)$$

It can be shown that if the matrices $A_{np}$ and $A_{pn}$ satisfy the conditions of the Perron-Frobenius theorem and their majorizing roots are less than unity, then the following relations hold

$$P^*\left(A_p Z_2^* - K_1^* Z_1^*\right) = 0, \;\; Z_1^* \geq 0, \;\; Z_2^* \geq 0,$$

$$A_p Z_2^* \leq K_1^* Z_1^*, \;\; K_1^* = \sqrt[2]{\lambda_{np}^*},$$

$$W^*(A_n Z_1^* - K_2^* Z_2^*) = 0, \;\; Z_1^* \geq 0, \;\; Z_2^* \geq 0,$$

$$A_n Z_1^* \leq K_2^* Z_2^*, \;\; K_2^* = \sqrt[2]{\lambda_{pn}^*}, \qquad (32)$$

providing the possibility of economic equilibrium in the TS in the excess production of PS and in the excess generation of resource supply. Multipliers $K_1^*$ and $K_2^*$ determine the degree of utilization of PS and PF by the territorial system in the equilibrium state.

Conditions (32) determine the validity of Theorem 4.

*Theorem 4.* If the matrices $A_{np}$ and $A_{pn}$ are nonnegative, indecomposable matrices and their majorizing roots $\lambda_{np}^*$, $\lambda_{pn}^*$ are less than unity, then the TS is productive and profitable, in addition, the economic equilibrium in the system is possible.

This economic equilibrium can be implemented if conditions (2) are satisfied. If conditions (2) are not met, then management technologies from the class $\tau_7$ are needed, providing the formation of the main properties of the TS based on the evolution of the technological matrices $A_p$ and $A_n$ [19].

The TS is called unproductive if there are no plans of the problems (4), (6) – vectors $Z_1$, $Z_2$, satisfying the condition (30).

Theorem 5 holds. If the matrices $A_{pn}$ and $A_{np}$ are nonnegative, indecomposable matrices and their majorizing roots $\lambda_{pn}^*$, $\lambda_{np}^*$ are greater than unity, then the TS is unproductive.

The main technology of management of the cyclical dynamics of the TS in this case is the technology of the class $\tau_7$ [19].

## 5. Results of studies of identification of problem situations by modeling the functioning of the territorial system

The results of the study of the PS identification process on the basis of modeling the TS functioning by the complex of interacting LP problems allow selecting and describing the classes of possible PS. The current state of the TS is determined by the tuple STATE=<$A_n$, $A_p$, Z10, Z20, P, W, $\alpha \in \tau$>, where $\tau$ is the set of management technologies of the TS functioning, $\alpha$ is the implemented technology.

The tuple SIT=<$\alpha$, $Z_1^*$, Z10, $Z_2^*$, Z20, W, $W^*$, P, $P^*$, $\lambda_{np}^*$, $\lambda_{pn}^*$> determines the current situation that has occurred during the functioning and development of the TS, is the basis for forming a system of informative features used in the identification of PS. The informative feature $D_1$,

$$D_1 = \begin{cases} -1, \text{if } ((1-\lambda_{np}^*) < 0) \text{ or } ((1-\lambda_{pn}^*) < 0), \\ a = 2 - \lambda_{np}^* - \lambda_{pn}^*, \text{ otherwise,} \end{cases}$$

determines the degree of deviation of the TS structure from the equilibrium, divides the TS into structurally stable ($D_1 \geq 0$) and structurally unstable ($D_1 < 0$). For structurally unstable TS, structural transformation using the management technologies $\tau_7 \in \tau$ is necessary [19]. For $D_1 = 0$, the tuple <$Z_1^*$, $Z_2^*$, $P^*$, $W^*$> determines the state of economic equilibrium in the TS. Herewith, $Z_1^*$ is found as the right, and $P^*$ – the left eigenvector of the matrix $A_{pn}$, corresponding to its majorizing root $\lambda_{pn}^*$. The vector $Z_2^*$ is found as the right, and $W^*$ – the left eigenvector of the matrix $A_{np}$, corresponding to its majorizing root $\lambda_{np}^*$. This economic equilibrium can be implemented if the conditions (2) hold, where $Z10 = Z_1(0) \cdot \ell_{10}$, $Z20 = Z_2(0) \cdot \ell_{20}$. The value $D_1 > 0$ determines the productivity and profitability of the TS. The informative feature $D_2$ determines the resource disparity between the structure of available resources of Z20 and their consumption $Z_2^*$. $D_2$=0 determines the absence of resource disparity ($Z_2^* \le Z20$), $D_2$=1 – the presence of resource disparity, in which there are scarce resources, insufficient for the implementation of the equilibrium vector $Z_2^*$. The informative feature $D_3$ defines the product disparity between the structure Z10 and supply $Z_1^*$ of PS. $D_3$=0 determines the absence of product disparity ($Z_1^* \ge Z10$), $D_3$=1 – the presence of product disparity, in which there are PS, the production volume of which is insufficient to meet the minimum needs of society in this product. The informative feature $D_4=D_5 \vee D_6$, where

$$D_5 = \begin{cases} 0, \text{ if } (1-(P,P^*)/|P|\cdot|P^*| = 0 \pm \varepsilon), \\ 1, \text{ otherwise}, \end{cases}$$

$$D_6 = \begin{cases} 0, \text{ if } (1-(W,W^*)/|W|\cdot|W^*| = 0 \pm \varepsilon), \\ 1, \text{ otherwise}, \end{cases}$$

ε is the permissible error, determines the price disparity between the existing system of prices P, W and equilibrium P*, W*. If $D_4$=0, price disparity is absent (P=P*, W=W*), if $D_4$=1 – price disparity is present. In case of $D_1 \ge 0$, we introduce

$$\overline{D_1} = \begin{cases} 1, & D_1 > 0, \\ 0, & D_1 = 0 \end{cases}$$

and

$$\mathbf{X}=(D_4, D_3, D_2, \overline{D}_1)$$

is the vector of informative features, which determines the number s of the problem situation in the range [0, 15] written in a binary code. The situation with number 0 determines the dynamic equilibrium, with number 7 – resource and product disparities, with number 15 – resource, product and price disparities, etc. Thus, specification of numerical values of the informative features $D_j$, $j$=1, 2, ..., 6, uniquely determines the number s of the existing PS, which makes it possible to uniquely specify a set of Π cases.

## 6. Discussion of the results of the study of identification of problem situations in the functioning of territorial systems

The results of the process of identification of problem situations on the basis of modeling the TS functioning by the complex of interacting LP problems create a scientifically grounded basis for the practical identification of PS using the case-based reasoning technology. The case $\Pi_s$ is determined by the pair (s, α), where s is the PS number, α∈τ is the functioning management technology when dealing with the current PS. The cardinal number of the set Π is 16. Thus,

$\tau_1, \tau_2, \tau_3$ – classes of technologies of management through pricing in the commodity and resource markets;

$\tau_4$ – technology of management by adjusting the structure of the minimum needs of society in domestic goods on the basis of integration of the TS with other TS;

$\tau_5$ – technology of management by adjusting the structure and volumes of the minimum needs of society in domestic goods on the basis of self-isolation of the TS;

$\tau_6$ – technology of management by adjusting the structure and volumes of available resources on the basis of integration of the TS with other TS.

The technologies $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7$ require an effective mechanism of state regulation of cyclical dynamics of the TS.

The process of PS identification involves the following data processing stages:

1. Assignment of the range of PS products and PF resources of the TS.
2. Construction of the tuple <$A_n$, $A_p$, z10, z20, P, W> based on the data of the reporting period.
3. Calculation of the generalized technological matrices $A_{np}=A_n \cdot A_p$ and $A_{pn}=A_p \cdot A_n$.
4. Computation of the majorizing roots $\lambda_{pn}^*$, $\lambda_{np}^*$ and corresponding left and right eigenvectors of the generalized technological matrices.
5. Calculation of numerical values of informative features D={$D_j$, $j$=1, 2,..., 6}. Formation of the vector **X**= =($D_4$, $D_3$, $D_2$, $\overline{D}_1$).
6. Selection of the number s of the case, which corresponds to the current situation based on **X**.
7. Formation of a diagnosis indicating the technology to overcome the arisen problem situation.

The advantage of the scientific study is the formation of a scientifically grounded system of informative features for the detection and identification of PS of transitional TS. The disadvantage is that the management actions of the MPA, the effects of the environment are taken into account only by an exogenous change in the parameters of the generalized Walras-Wald model. This significantly affects the applicability of the model, determines the effective use of the model only in the non-violent (evolutionary) development of transitional TS. In this case, in the presence of reliable reporting data on TS functioning and development, the proposed identification process can be useful in the creation of the information technology (IT) of functioning management of the TTS (national economy, region). The developed subject technology can serve as a theoretical basis for the creation of IT for the detection and identification of PS, the formation of effective management solutions to overcome the arisen PS. Further improvement of the conducted studies will be related to the development of the subject technology of combined synthesis of management of TTS functioning and development.

## 7. Conclusions

1. The method for modeling the functioning of the territorial system by the complex of interacting linear programming problems is developed. The developed complex reflects the inextricable link between the mechanism of centralized planning of production and consumption of goods, resources and the mechanism of market formation of their prices.

2. The dependence of system characteristics of the cyclical dynamics of the TS on spectral properties of the generalized technological matrices of circulation of goods and resources is proved.

3. To identify problem situations, it is proposed to form the generalized technological matrixes of circulation of goods and resources on the basis of reporting data, to determine their majorizing roots that characterize the structural stability of the system. It is shown that the right and left eigenvectors found, corresponding to the majorizing matrix roots, determine the effective structures of supply of goods, resources and market prices for them. The system of informative features that determine the deviations of the actual characteristics of the territorial system from efficient ones is constructed. The proposed system of features makes it possible to detect national economic disparities that give rise to problem situations.

4. Classes of problem situations are identified. Each class contains problem situations for which the same technology for their resolution is effective. This allowed assigning a standard case to each class that sets the reference problem situation and the way to resolve it (using an adequate pricing technology, adjusting the structure of the minimum needs of the population, the structure of available resources, forming the main properties of the territorial system).

5. The obtained research results can serve as a platform for *creating* the information technology of detection and identification of PS, which implements the case-based reasoning technology.

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