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STUDY OF THE MATHEMATICAL MODELS OF OPTIMAL PARTITIONING FOR PARTICULAR CASES

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Досліджено особливості застосування теорії задач неперервного розбиття множин для випадку, коли вихідна множина є частиною плоскої кривої. Сформульована задача для окремого випадку відомої постановки. Ця задача розв'язана з урахуванням запропонованих обмежень. Виконано обчислювальний експеримент. Зроблено висновки про можливості прикладного застосування розв'язків поставленої задачі

Ключові слова: оптимальне розбиття, неперервна множина, мінімізація, довжина дуги, центр множини, метрика, розташування

Исследованы особенности применения теории задач непрерывного разбиения множеств для случая, когда исходное множество является частью плоской кривой. Сформулирована задача для частного случая известной постановки. Эта задача решена с учётом предложенных ограничений. Проведён вычислительный эксперимент. Сделаны выводы о возможности прикладного применения решения поставленной задачи

Ключевые слова: оптимальное разбиение, непрерывное множество, минимизация, длина дуги, центр множества, метрика, размещение

1. Introduction

Over the past decade, a number of fundamental results have been obtained in the theory of continuous optimal set partitioning problems (OSP). Methods and algorithms for solving multi-product, linear and nonlinear, stochastic and dynamical problems of optimal set partitioning problems with specified and unspecified coordinates of centers of subsets were developed. In addition, solutions to applied problems from the field of monitoring of ecology of industrial regions, territory planning of service spheres and control of social sphere, were obtained. The problems of control of technological processes and the problems, associated with construction of the elements of artificial intelligence systems were united in one direction [1, 4].

A variety of initial data, including information about properties of a set, restrictions on particular parameters of a problem and quality criteria, determines a wide range of applied partitioning problems. This, in turn, makes it possible to speak about the relevance of subsequent research both aimed at the development of the theoretical base, and at the search for solutions to specific problems. The problems of optimal partitioning of a plane curve, which are particular cases of a continuous OSP problem with placement of subsets' centers, are of special interest.

2. Literature review and problem statement

The theory of continuous OSP problems is based on a unified approach, which lies in reducing original infinite-dimensional optimization problems via Lagrange functional to non-smooth, as a rule, finite-dimensional optimization problems. For numerical solution of such problems, effective methods of non-differentiable optimization are used.

We can distinguish the following main directions of development of the theory of continuous OSP problems. By now, some of them have been sufficiently explored, such as linear OSP problems, nonlinear OSP problems, OSP problems under conditions of uncertainty [1].

Development of these results include studying problems of optimal coverage of continuous sets by balls [2], dynamical problems of optimal set partitioning with and without placement of subsets' centers, problems of sets' boundaries control [3–5]. Solutions to the problems of distributed systems control, which are special cases of continuous dynamical problems of optimal set partitioning, were obtained.

A fuzzy problem of optimal sets partitioning without restrictions was studied in paper [6].

One of the examples of conceptualizing of the OSP theory may be considered paper [7], which explored the problem how and when a discrete problem can be reduced to a continuous problem.

Application of the theory of optimal sets partitioning was found in the problems of artificial intelligence: image recognition, analysis and identification of systems, control of distributed systems, in which a permissible control region is determined by partitioning of a certain set into a finite number of subsets.

It is necessary to emphasize the studies based on sets partitioning, which typically use the Voronoi diagrams [8–10]. The theory of optimal partitioning considers the Voronoi diagrams as a particular case of OSP problems. Substantiation of theoretical fundamentals of plotting of the Voronoi diagram by using the methods of optimal sets partitioning is shown in a series of papers [11].

In [12], a generalization of the Voronoi partitioning, called EBVP (effectiveness-based Voronoi partitioning) by introducing the concept of node functions for distance measuring was proposed. With EBVP, a generalized environment for statement of optimal placement problems was proposed.

An example of development of algorithms for solving partitioning problems can be paper [13]. Application of multiagents was proposed in [14].

Modern applied results in the fields of medicine, technology and logistics are explored in works [15–17].

Despite a considerable number of theoretical and applied works for optimal partitioning, all of them are based on analysis of only one class of sets. There are examples of partitioning of the segment and examples of application of the OSP apparatus for the search for the minimum of the function. Accordingly, an interest arises from the point of view of theoretical prospects and practical application of partitioning of segments of a plane and a spatial curve.

3. The aim and objectives of the study

The aim of present research is to state and research one-dimensional problems of the theory of optimal sets partitioning, when a set is represented in the form of a curve on a plane with geometrical characteristics of the curve taken into account. This will make it possible to construct the models of elements of transport systems with detailed reference to the terrain.

To accomplish the set goal, the following tasks must be solved:

– to state and solve the optimal partitioning problem for a particular case, when a distance is assigned by the length of the radius-vector from the center to a point on a curve, to explore behavior of its solutions;

– to state and solve the optimal partitioning problem for a particular case, when a distance is assigned by the length of the path from the center to a point on a curve and to explore behavior of its solutions;

– to state and solve the optimal partitioning problem for a particular case, when a distance is assigned by the length of the path from the center to a point on a curve the curvature of the curve is taken into account as an additional parameter, to explore behavior of solutions.

4. Methods and results of research

Consider the basic problem of optimal sets partitioning with placements of centers [1]:

Problem A2. Let Ω be a bounded, measurable by Lebesgue set in n-dimensional Euclidean space E_n. We will designate through $P_N(\Omega)$ the class of all possible partitions of set Ω into N sub-sets

$$\hat{P}_N(\Omega)=\Big\{\overline{\omega}=(\Omega_1,...,\Omega_N):\ \Omega_i\subseteq\Omega,\ i=\overline{1,N};$$
$$\bigcup_{i=1}^{N}\Omega_i=\Omega;\ \text{mes}(\Omega_i\cap\Omega_j)=0,\ i\neq j;\ i,j=\overline{1,N}\Big\}.$$

It is required to determine partitioning $\overline{\omega}^*\in\hat{P}_N(\Omega)$ and a set of "centers" of subsets $\tau^*=(\tau_1, \tau_2,..., \tau_N)\in\Omega^N$, giving the minimum value to the functional

$$F(\overline{\omega},\tau)=\sum_{i=1}^{N}\int_{\Omega_i}\big(c(x,\tau_i)+a_i\big)\rho(x)\mathrm{d}x.$$

Here $c(x, \tau_i)$ are real, bounded, determined on $\Omega\times\Omega$, measurable by x at any fixed $\tau_i\in\Omega$ ($\forall i=1,..., N$) functions; $\rho(x)$ is the bounded, non-negative, measurable on Ω function; a_i ($\forall i=1,..., N$) are the assigned non-negative magnitudes.

Let functions $c(x, \tau_i)$ be one or another metrics in space E^2, $a_i=0$ ($\forall i=1,..., N$). Informally, we can state problem A2 in the following way. It is required to find such partitioning of the original set into an assigned number of sets and such coordinates of the centers of these subsets, at which the sum of weighed distances from the points of the set to the correspondent center should be minimal.

In physical problems, the minimized integral is treated as work, performed by a point (a physical body) at moving along the trajectory, leading from the center to each point of the subset. From the economic point of view, the quality criterion of problem A2 is a summary cost of transition to the center (or backwards) of the whole resource that is at each point of set Ω.

The aim of the present research is to study problem A2, when set Ω is a part of a plane curve, described by dependence $y=f(x)$ on an assigned segment. A segment of a curve, assigned both analytically and in a tabular way (the latter in the process of solving the problem is interpolated), can serve as analyzed areas. In addition, from practical considerations, the function is restricted by continuity and differentiability.

In a general form, the first problem (let us call it problem A2R) will be stated as follows. Suppose there is a segment of a plane curve. It is required to place an assigned number of sources on a resource on it and to link each point of the curve to a particular source. In this case, it is necessary to minimize costs of transportation of the whole resource from the sources to corresponding points of the curve along the shortest path. Function of value in this case will be considered proportional to the radius-vector, connecting point $(\tau_i, f(\tau_i))$, the source of resource, with the point of the corresponding subset $(x, f(x))$, so that $c_R(x, \tau_i)=((x-\tau_i)^2+(f(x)-f(\tau_i))^2)^{1/2}$ (Fig. 1). Obviously, coordinates of the so-called "centers" of subsets have the form $(\tau_i, f(\tau_i))$ and, in general, are determined by points $\tau_i\in[a, b]$.

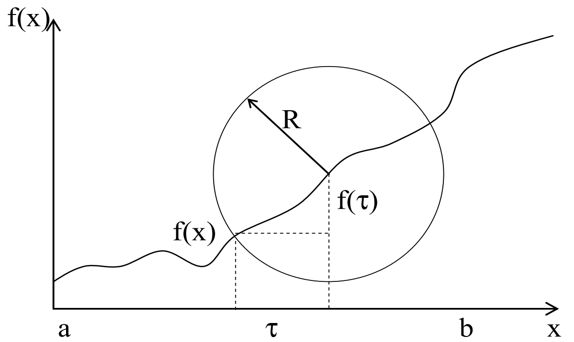


Fig. 1. Geometric interpretation of sub-integral function of problem A2R

Problem A2R. Let $\Omega=\{(x, y): a\le x\le b; y=f(x)\}$, where $f(x)$ is the real, bounded, differentiable determinable on $[a, b]$ function. It is required to find partitioning $\bar{\omega}^*\in\hat{P}_N(\Omega)$ and a set of "centers" of subsets, determined by the points of segment [a, b] $\tau^*=(\tau_1, \tau_2,..., \tau_N)\in\Omega^N$, giving minimal value to the functional.

$$F(\bar{\omega},\tau)=\sum_{i=1}^{N}\int_{\Omega_i}((x-t_i)^2+(f(x)-f(t_i))^2)^{1/2}\mathrm{d}x.$$

Once the basic problem has been stated, it is required to refine the problem, based on assumption that it is possible to take into consideration geometrical characteristics of the curve in this statement. Subsequently, it is necessary to show what geometrical characteristics correspond to the subject area.

The next stage is the solution itself with the help of the known methods and interpretation of results.

We apply the procedure of solving continuous OSP problems [3, 5]. Then we introduce characteristic $\lambda_1(\cdot)$,..., $\lambda_N(\cdot)$ of subsets Ω_1,..., Ω_N, and pass from problem A2R to the equivalent problem of infinitely-dimensional programming.

$$I(\lambda(\cdot),\tau)=$$
$$=\sum_{i=1}^{N}\int_a^b\sqrt{(x-\tau_i)^2+(f(x)-f(\tau_i))^2}\lambda_i(x)\mathrm{d}x\to\min_{(\lambda(\cdot),\tau)\in\Lambda_1\times[a,b]^N},\quad(1)$$

where

$$\Lambda_1=\{\lambda(x)=(\lambda_1(x),...,\lambda_N(x)):\ \sum_{i=1}^{N}\lambda_i(x)=1$$

almost everywhere in $x\in\Omega$; $\lambda_i(x)=0\vee1$; almost everywhere for $x\in\Omega$; $i=\overline{1,N}\}$.

Analytic expression for the first component $\lambda^*(\cdot)$ of optimal solution $(\lambda^*(\cdot),\tau^*)$ of problem (1) can be obtained for each fixed $\tau^*=(\tau_1, \tau_2,..., \tau_N)$ in the form

$$\lambda^*_i(x)=\begin{cases}1, \sqrt{(x-\tau_i)^2+(f(x)-f(\tau_i))^2}=\\ \min\limits_{k=\overline{1,N}}\sqrt{(x-\tau_k)^2+(f(x)-f(\tau_k))^2},\quad i=\overline{1,N}\\ 0, \text{in other cases,}\end{cases}$$

To find the second component $\tau^*=(\tau_1, \tau_2,..., \tau_N)\in\Omega^N$ of the optimal solution (1), it is necessary to solve the finite-dimensional optimization problem in the form

$$G(\tau)=\int_a^b\min_{k=\overline{1,N}}\sqrt{(x-\tau_k)^2+(f(x)-f(\tau_k))^2}\rho(x)\mathrm{d}x\to\min_{\tau\in[0,1]^N},\quad(2)$$

the objective function of which is in the general case multi-extreme and non-differentiable. Solution to the problem (2) can be obtained, for example, using an algorithm, a part of which is r-algorithm by N. Z. Shor.

A detailed analysis of the properties of optimal solutions and properties of the minimized function $G(\tau)$ for a particular case of problem A2R, when $f(x)$=const, was studied earlier. It was shown that the problem (2) is multi-extreme. However, on each of the subsets of set $[a, b]^N$, formed by specifying of different relations of the order of sequence of centers $\tau_1, \tau_2,..., \tau_N$ on segment $[a, b]$:

a) function is one-extreme and convex by $\tau_1, \tau_2,..., \tau_N$;

b) has the point of a local minimum.

Minimal values of the function in points of a local minimum, each of which belongs to a subset of set $[a, b]^N$, formed by its relation of the sequence order of centers $\tau_1, \tau_2,..., \tau_N$ on segment $[a, b]$, coincide.

It is obvious that from the practical point of view it is interesting to study and solve the problem of partitioning of the segment of a special curve, modeling a real system. For example, when talking about road construction, there arises a problem of optimal placement of warehouses, work towns, asphalt factories along the construction route. It is required that costs of transportation of labor or material resources along the construction site should be minimal. That is why for computational experiments, we selected curves of the form $f(x)=x^2$, $f(x)=x^3$, $f(x)=\ln(x)$, $f(x)=\sin(x/\pi)$, $f(x)=$ $=A\sin^t(\Omega x)$, $A, t, \Omega\in Z$. The last is of particular interest, since it is known that any continuous function on the interval $[0,2\pi]$ can be represented in the form of a trigonometric series. Therefore, the properties of optimal solutions to problem A2R for any continuous functions, as well as applicability of the above algorithm, will be determined by the properties of problems A2R, functions of the form $f(x)=A\sin^t(\Omega x)$ at different parameters with precision of decomposition in trigonometric series are considered as a curve.

Fig. 2 shows examples of surfaces of minimized functions $G(\tau_1, \tau_2)$, to which problems A2R at N=2, [a, b]=[0,1] are reduced (it is possible to visualize the surface, assigned by the minimized function, only for two centers). As Fig. 2, *a–d*, shows, problems A2R can be conditionally divided into two classes. The first class includes the problems, in which objective function $G(\tau_1, \tau_2)$ has two minima, symmetric relative to the diagonal of a square, corresponding to area Ω and beginning at point (0; 0). Moreover, the values of the function at points of a minimum are the same. We will note that this class of problems includes those, in which the function, monotonous on $[a, b]$, serves as $f(x)$. The second one includes multi-extreme functions with a large number of local extrema that can have values close to optimal. In the case when the number of subsets is more than two, the number of local minima increases significantly. For functions from the first class, an optimum can be obtained by applying algorithm A2. For the second class of problems, algorithm A2 can lead to any point of a local extremum, not necessarily to the point where objective function of the problem has a global minimum. Therefore, in order to solve problems of the second class, it is possible to apply heuristic algorithms of search for a global extremum. Experience in application of genetic algorithms, in particular for similar functions, makes it possible to talk about their effectiveness.

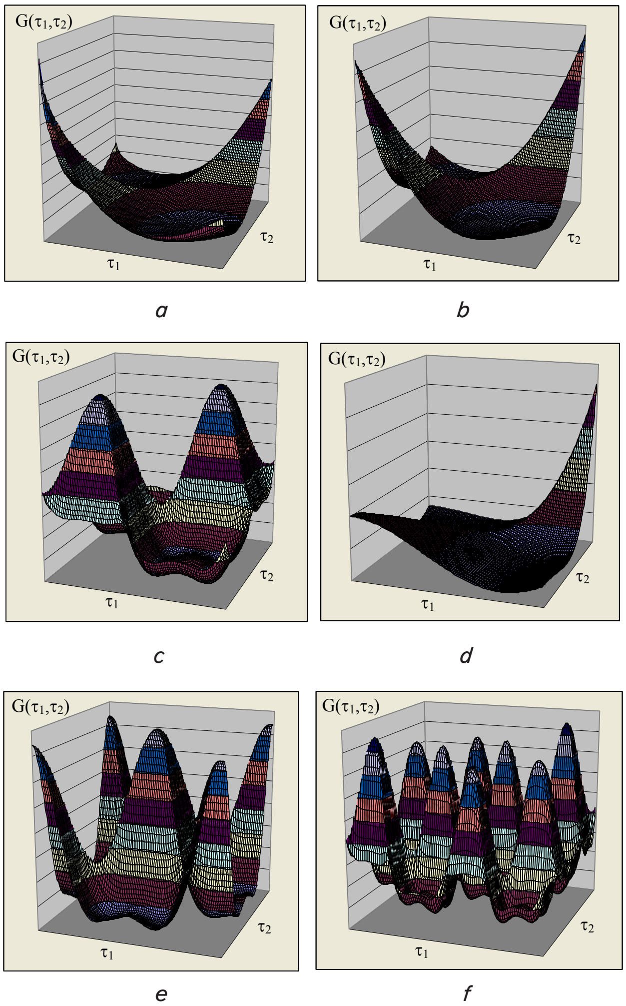


Fig. 2. Surfaces, constructed by values of function $G(\tau_1, \tau_2)$. The set to be partitioned is a segment of a curve: *a* – $f(x)$ =const; *b* – $f(x)$ =ln(x); *c* – $f(x)=x^2$; *d* – $f(x)$=sin($\pi x/2$); *e* – f(x)=sin($2\pi x$); *f* – $f(x)$=sin($4\pi x$); $x\in[0, 1]$. Values of F are normalized

Problem A2R is applicable in practice, when it is possible to get from the center to any point of the area along the corresponding radius vector. This situation is possible, for example, when developing the reservoir, deforestation or work on an assembly line.

Let us specify the model and state the following problem of optimal partitioning of a plane curve (let us call it problem A2L). Let us impose additional restrictions on the movement trajectory. It is obvious that in the case of transport communications, movement of resources will be implemented only along them. For example, materials supply on the constructed road is most likely to be implemented on its ready segments.

Let us state the problem in a general form. It is necessary to minimize the costs for transportation of the entire resource to each point along the curve at a specified segment from a predetermined number of sources along the same curve. In this case, it is required to find an optimal placement of the sources.

Problem A2L. Let $\Omega=\{(x, y): a\le x\le b; y=f(x)\}$, where $f(x)$ is the real, bounded, differentiable function, determined on $[a, b]$. It is necessary to find partitioning $\bar{\omega}^*\in\hat{P}_N(\Omega)$ and a set of "centers" of subsets, determined by the points of segment $[a, b]$ $\tau^*=(\tau_1, \tau_2,...,\tau_N)\in\Omega^N$, giving minimal value to the functional.

$$F(\bar{\omega},\tau)=\sum_{i=1}^{N}\int_{\Omega_i}\left|\int_{\tau_i}^{x}\sqrt{1+f'^2(\xi)}\rho(x)\mathrm{d}\xi\right|\mathrm{d}x,$$

where

$$\int_{\tau_i}^{x}\sqrt{1+f'^2(\xi)}\mathrm{d}x$$

is the length of the curve's arc from center τ_i to point x, $\rho(x)$ is the assigned real function, bounded on $[a, b]$ (hereinafter, consider without loss of generality that $\rho(x)=1$).

Evidently, for $f(x)$=const, problems A2R and A2L coincide.

Depending on the choice of $f(x)$, objective function for problem A2L will take a particular form. For example, if $f(x)=x^2$, then

$$c(x,\tau_i)=\left|\begin{array}{l}\tau_i\sqrt{1+\tau_i^2}/2+\ln\left|\tau_i+\sqrt{1+\tau_i^2}\right|/2-\\ -x\sqrt{1+x^2}/2+\ln\left|x+\sqrt{1+x^2}\right|/2\end{array}\right|,$$

respectively, for $f(x)=\ln(\sin(x))$, we obtain

$$c(x,\tau_i)=|\ln(\mathrm{tg}(x/2)/\mathrm{tg}(\tau_i/2))|.$$

Examples of surfaces, formed by values of function F, constructed for set $\Omega=\{(x, y): 0.02\le x\le 1; y=f(x)\}$, are shown in Fig. 3.

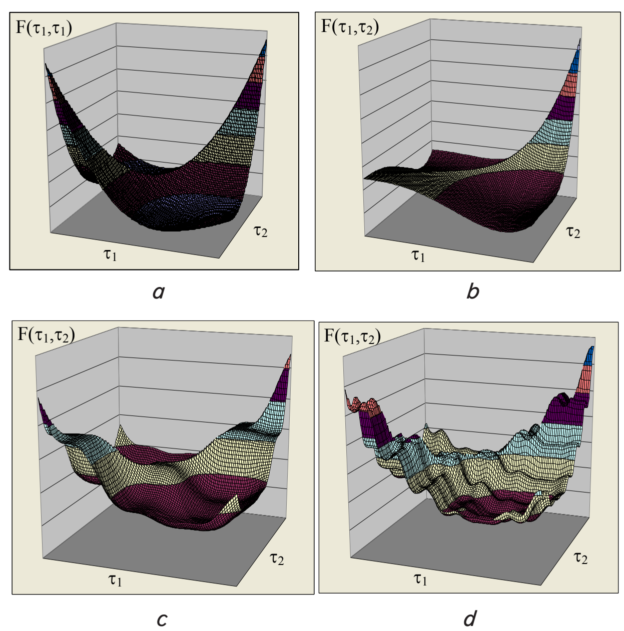


Fig. 3. Surface of objective function F of problem A2L, depending on $\tau_1, \tau_2\in[0, 1]$. A set to be partitioned is a segment of a curve: *a* – $f(x)$=const, *b* – $f(x)=x$, *c* – $f(x)$=sin(x), *d* – $f(x)$=sin($2x$)

For problem A2L with placement of two centers on the interval [0; 1], solution can be obtained in analytical form, previously assuming that f(x) is a monotonous function,

$\tau_1 \le \tau_2$. We open module and rewrite the functional of problem A2L in the form

$$\hat{F}(p,\tau_1,\tau_2)=\int_0^{\tau_1}\int_x^{\tau_1}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi\mathrm{d}x+\int_{\tau_1}^{p}\int_{\tau_1}^{x}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi\mathrm{d}x+$$
$$+\int_p^{\tau_2}\int_x^{\tau_2}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi dx+\int_{\tau_2}^{1}\int_{\tau_2}^{x}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi\mathrm{d}x.$$

Write down the necessary conditions of an unconditional extremum for function $\hat{F}(p,\tau_1,\tau_2)$ (calculation of partial derivatives of function $\hat{F}(p,\tau_1,\tau_2)$ is performed from the Leibniz formula for integral differentiation by the parameter):

$$\frac{dF}{d\tau_1}=\int_0^{\tau_1}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi-\int_{\tau_1}^{p}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi=0;$$

$$\frac{dF}{d\tau_2}=\int_p^{\tau_2}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi-\int_{\tau_2}^{1}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi=0; \qquad (3)$$

$$\frac{dF}{dp}=\int_{\tau_1}^{p}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi-\int_p^{\tau_2}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi=0.$$

Among the stationary points, we will choose the ones that will satisfy conditions: $0\le\tau_1\le p\le\tau_2\le 1$.

Analyzing system (3), it is possible to make a conclusion that point $(p, f(p))$ is the boundary between areas Ω_1 and Ω_2 and lies on curve $y=f(x)$ so that the distances from this point along the curve to points $(\tau_1, f(\tau_1))$ and $(\tau_2, f(\tau_2))$ are the same. Point $(\tau_1, f(\tau_1))$ is in the middle of the distance along the curve between $(0, f(0))$ and $(p, f(p))$, and $(\tau_2, f(\tau_2))$ is in the middle of the distance along the curve between $(p, f(p))$ and $(1, f(1))$.

In the case when $f(x)$ is periodic, or the number of the located centers is more than two, analytical solution to the problem is virtually impossible to obtain. In these cases, problem A2L is multi-extreme. Examples of the surfaces, formed by objective functions of the problem are shown in Fig. 3, *c, d*.

For the numerical solution, it is possible to apply algorithm A2 [3] or any other heuristic approach.

Further, we will refine the model, for optimization of which problem A2L is solved. Until now, it was assumed that costs are proportional to the length of the trajectory between the center and each point on it. Now let us take into account the fact that movement along the curve is complicated by its form, or rather curvature, which causes additional costs, therefore, we will add the costs in proportion to a curvature indicator to the full costs. Let us consider two cases:

1. We will take into account the effect of a curvature on the costs of moving through each intermediate point, then the total costs will be determined by a weighted sum (β_1, $\beta_2 \ge 0$, $\beta_1+\beta_2\ne 0$):

$$c(x,\tau_i)=\beta_1\left|\int_{\tau_i}^{x}\sqrt{1+f'^2(x)}dx\right|+\beta_2\left|\int_{\tau_i}^{x}f''(x)/(\sqrt{1+f'^2(x)})^3dx\right|, i=1,..,n,$$

where $f''(x)/(\sqrt{1+f'^2(x)})^3$ is the curvature in point x.

Problem A2(L+K). Let $\Omega=\{(x, y)$: $a\le x\le b$; $y=f(x)\}$, where $f(x)$ is the real, bounded, differentiable, determined on $[a,b]$. It is necessary to find partitioning $\bar{\omega}^*\in\hat{P}_N(\Omega)$ and a set of "centers" of subsets, determined by points of segment $[a,b]$ $\tau^*=(\tau_1^*,\tau_2^*,...,\tau_N^*)\in[a,b]^N$, giving minimal value to the functional

$$F(\bar{\omega},\tau)=$$
$$=\sum_{i=1}^{N}\int_{\Omega_i}\left(\left|\int_{\tau_i}^{x}\sqrt{1+f'^2(\xi)}d\xi\right|+\left|\int_{\tau_i}^{x}f''(\xi)/(\sqrt{1+f'^2(\xi)})^3d\xi\right|\right)\mathrm{d}x.$$

As in the previous case, for each function $f(x)$, we will obtain specific function $c(x, \tau_i)$.

Separately, we will integrate the second summand:

$$K_{(\tau_i,x)}=\int_{\tau_i}^{x}f''(x)/(\sqrt{1+f'^2(x)})^3\mathrm{d}x=$$
$$=f'(\tau_i)/\sqrt{1+f'^2(\tau_i)}-f'(x)/\sqrt{1+f'^2(x)};$$

then:

$$F(\bar{\omega},\tau)=\sum_{i=1}^{N}\int_{\Omega_i}\left(\begin{array}{l}\left|\int_{\tau_i}^{x}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi\right|+\\+\left|f'(\tau_i)/\sqrt{1+f'^2(\tau_i)}-f'(x)/\sqrt{1+f'^2(x)}\right|\end{array}\right)\mathrm{d}x.$$

As an example, we will consider the type of objective function for the problem with two centers, when $f(x)=x^2/2$, $x\in[0, 1]$.

Integral of the length of this curve in this case:

$$L_{(\tau_i,x)}=\int_{\tau_i}^{x}\sqrt{1+f'^2(\xi)}\mathrm{d}\xi=\tau_i\sqrt{1+\tau_i^2}/2+$$
$$+\ln\left|\tau_i+\sqrt{1+\tau_i^2}\right|/2-\sqrt{1+x^2}/2+\ln\left|x+\sqrt{1+x^2}\right|/2;$$

then the function will take the form:

$$F=\int_0^{\tau_1}(L_{(x,\tau_1)}+K_{(x,\tau_1)})\mathrm{d}x+\int_{\tau_1}^{p}(L_{(\tau_1,x)}+K_{(\tau_1,x)})\mathrm{d}x+$$
$$+\int_p^{\tau_2}(L_{(x,\tau_2)}+K_{(x,\tau_2)})\mathrm{d}x+\int_{\tau_2}^{1}(L_{(\tau_2,x)}+K_{(\tau_2,x)})\mathrm{d}x.$$

For the case, when $f(x)$=const, $f(x)=x$, the second derivative is equal to zero, that is why the problem is reduced to problem A2R or A2L. Examples of surfaces, formed by function F for the problem with two centers at different kinds of function $f(x)$, are shown in Fig. 4.

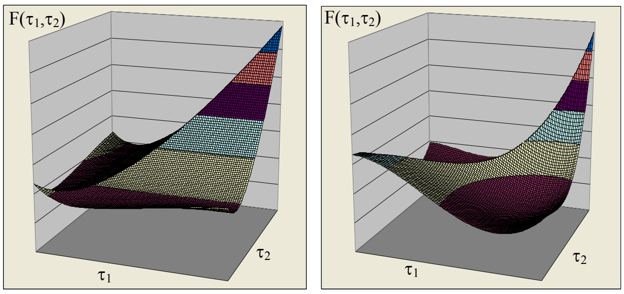
For $f(x)=x^2$ on the interval [0; 1], the curvature decreases monotonously and an increment of the function by the unit of length increases monotonously. Function F reaches a minimum at point $\tau^{(1)}=(0.03, 0.24)$ and at point $\tau^{(2)}=(0.24, 0.03)$, symmetric to it relative to straight line $y=x$. The boundary between the subsets corresponds to value $p=0.1215$.

This problem can be generalized in the case of the curve lying on the surface (the model of a road segment). Considering a curvature in this case may occur due to both lateral and vertical deviations.

2. Let us assume that delivery costs are proportional to the length of the curve and curvature at each point (curvature acts as density function):

$$c(x,\tau_i)=\left|\int_{\tau_i}^{x}\sqrt{1+f'^2(x)}\cdot f''(x)/(\sqrt{1+f'^2(x)})^3\mathrm{d}x\right|,$$

$$i=1,\ldots,n.$$



a *b*

Fig. 4. The surface of objective functions F of problem A2(L+K), depending on. $\tau_1,\tau_2\in[0, 1]$. A set to be partitioned is a segment of the curve: a – $f(x)=x^2$, b – $f(x)=\sin(x)$, $x\in[0, 1]$

Problem A2(L·K). Let $\Omega=\{(x, y): a\le x\le b; y=f(x)\}$, where $f(x)$ is the real, bounded, differentiable, determined on $[a, b]$ function. It is necessary to find pair,

$$(\overline{\omega}^*,\tau)\in\hat{P}_N(\Omega)\times[a,b]^N,$$

giving minimal value to the functional

$$F(\overline{\omega},\tau)=\sum_{i=1}^{N}\int_{\Omega_i}\left|\int_{\tau_i}^{x}\sqrt{1+f'^2(x)}\cdot f''(x)/(\sqrt{1+f'^2(x)})^3\mathrm{d}x\right|\mathrm{d}\xi.$$

Let us integrate sub-module expression:

$$\int_{\tau_i}^{x}\sqrt{1+f'^2(x)}\cdot f''(x)/(\sqrt{1+f'^2(x)})^3\mathrm{d}x=$$

$$=\mathrm{arctg}(f'^2(\tau_i))-\mathrm{arctg}(f'^2(x));$$

then

$$F(\{\Omega_1,..,\Omega_N\})=$$

$$=\sum_{i=1}^{N}\int_{\Omega_i}\left|\mathrm{arctg}(f'^2(\tau_i))-\mathrm{arctg}(f'^2(x))\right|\mathrm{d}x.$$

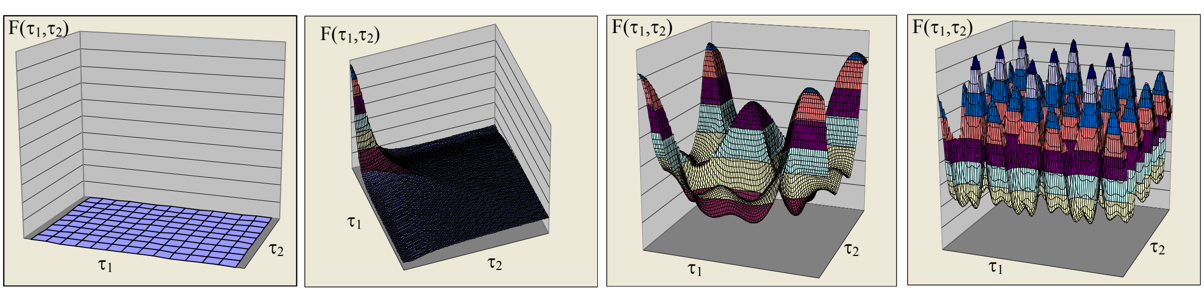
The problem degenerates, if $f(x)$=const, $f(x)=x$ (Fig. 5, *a*). In the general case, the scheme of the solution remains the same. Examples of surfaces, made up by objective functions of problem A2(L·K) for the problem with two centers at various $f(x)$, are shown in Fig. 5, *b–d*. It is not difficult to notice that the properties of objective functions for some problems A2(L·K) coincide with the properties of objective functions of problems A2(L+K), A2L (at the same assignments of $f(x)$). That is why algorithms of solutions to these problems are the same.

5. Discussion of results of one-dimensional OSP problems

The basic results of the theory of continuous sets partitioning, such as problem statements, methods of their solution and substantiation, were obtained for the case when a set is plane. Different methods of calculating the distances between points on a plane are used as metrics. This makes it possible to construct optimization models for economic and social problems. Taking into account geometric properties qualitatively complicates the basic problems, because instead of using the Euclidean or Manhattan metrics, the distance along the curve is used. Consequently, carrying out numerical computation itself also gets complicated. But this approach gives new possibilities for modeling, in particular introduces in consideration the topography of placement of centers τ and points of the placement of a given resource. It is obvious that in the case of road transportation, delivery costs depend on the distance between destinations, which is measured along the road. The second component of the costs is non-unified fuel consumption, set by the engine operation modes, which in turn depends on the relief of the road.

The studies were performed for problems of lesser dimensionality, unlike the known ones, which is caused by complexity of analysis of solutions for the curves in space. In addition, visualization of solutions' surfaces is possible only for a one-dimensional case with two centers. Consideration of properties of the curves adds one more stage of searching for a solution to the known scheme. As a result, it appears that within the basic problem, we obtain a particular problem, requiring a separate solution, for every case of consideration. The classic method for solving OSP problems in general case allows us immediately to apply the algorithm of global search for a minimum, which simplifies the search for a solution.

Exploration of the properties of stated problems showed that it is possible to obtain analytically a solution of some problems of partitioning of curves into two subsets with placement of the centers, using the necessary optimality conditions. If the number of subsets, and consequently, the number of centers is more than two, in order to solve these problems, it is necessary to apply the algorithms of solutions of continuous OSP problems or the heuristic methods. In the case, where $f(x)$=*const*, results of the studies coincide with the known results for a single interval of a real straight line.



a *b* *c* *d*

Fig. 5. Surfaces, constructed by values of function F of problem A2(L·K). Partitioned set is the segment of a curve: a – $f(x)$=const, b – $f(x)=x^2$, c– $f(x)=\sin(x/3)$, d – $f(x)=\sin(x)$, $x\in[0,1]$

The proposed problems can be stated for n-dimensional Euclidean space, as well as for complex non-linear spaces, modeling economic and physical structures. A generalization of these problems is the problems of partitioning of surface curves or the surface itself, assigned by the function of n-variables. Such problems will make the object for subsequent studies, because they have a wide range of practical applications (in construction, economy, geodesy, and transport technologies). An interesting continuation of the studies will be consideration of the ability of the centers to move along the assigned relief with the problem of optimization of the influence spread area of each of the centers with minimal costs. This will require, first and foremost, creation of new software with support for geo-information and data on modes of objects' motion.

6. Conclusions

1. We stated the problem A2R of optimal partitioning of a continuous set, assigned by function $f(x)$ for the case, when the distance is assigned by the length of the radius-vector from the center to the point on the curve. Functions of the form $f(x)=A\sin^t(x)$ at different parameters with precision of decomposition in a trigonometric series were considered as a curve. It was shown that for problems with placements of two centers on the interval [0; 1], it is possible to obtain a solution in the analytical form. In the case, where there are more than two centers, the search for a more exact solution becomes difficult, that is why it is proposed to use heuristics.

2. The problem A2(L+K) of optimal partitioning for the case, where the distance is assigned by the length of the path from the center to a point on the curve, was stated. For problem A2(L+K) with placement of two centers on the interval [0; 1], the solution was given in the analytical form, in the case, where $f(x)$, solution is a monotonic function. It was shown when the problem degenerates. The surfaces, formed by solutions to the problem, were constructed.

3. We stated the problem A2(L·K) of optimal partitioning for the case, where the distance is assigned by the length of the path from the center to a point on the curve and the curvature of the curve is taken into account as an optional parameter. The properties of objective functions for some problems A2(L·K) coincide with the properties of objective functions A2(L+K), A2L (at the same assignments of $f(x)$). That is why the algorithms of solutions to these problems are the same.

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Запропоновано метод формування одновимірних обводів виходячи з заданої точності інтерполяції. Максимальна абсолютна похибка інтерполяції визначається з урахуванням геометричних властивостей вихідної кривої лінії. Розглядається два різновиди похибки. По-перше, похибка, з якою сформована дискретно представлена крива, що інтерполює вихідний точковий ряд, представляє вихідну криву. По-друге, похибка, з якою інтерполююча крива представляє будь-яку криву з заданими геометричними характеристиками

Ключові слова: похибка інтерполяції, упорядкована множина точок, осциляція, монотонна зміна диференціально-геометричних характеристик

Предлагается метод формирования одномерных обводов исходя из заданной точности интерполяции. Максимальная абсолютная погрешность интерполяции определяется с учетом геометрических свойств исходной кривой линии. Рассматривается две разновидности погрешности. Во-первых, погрешность, с которой сформированная дискретно представленная кривая, интерполирующая исходный точечный ряд, представляет исходную кривую. Во-вторых, погрешность, с которой интерполирующая кривая представляет любую кривую с заданными геометрическими характеристиками

Ключевые слова: погрешность интерполяции, упорядоченное множество точек, осцилляция, монотонное изменение дифференциально-геометрических характеристик

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DEVELOPMENT OF THE METHOD FOR THE FORMATION OF ONE-DIMENSIONAL CONTOURS BY THE ASSIGNED INTERPOLATION ACCURACY

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1. Introduction

Geometric modeling is one of the tools for investigation of objects, phenomena and processes. The task of geometric modeling is to determine properties of an object being modeled using characteristics of a geometric model. Output data are geometric images assigned by a set of points. Their location reflects properties of the examined object. Geometric characteristics of a discretely represented geometric image (line or surface) can be given at the output points. We can obtain output by calculations or measurements at physical objects.

There are difficulties in modeling discretely presented curves and surfaces because we know characteristics of curves at the output points only. It is possible to determine a character of a change in characteristics between the output points using additional information about properties of the object of modeling.

One of the methods of modeling based on discrete sets is interpolation. The task of interpolation is to restore an un-