Досліджено вплив фізичного зносу складного портового обладнання на динаміку показників ефективності його використання. Отримано оцінки коливань середніх витрат за одиницю часу служби обладнання, що функціонує в умовах нестійкого вантажопотоку. Запропоновано методику обґрунтування термінів оновлення обладнання, за яких досягається баланс між мінімальністю та стійкістю сумарних питомих витрат на обладнання

Ключові слова: знос обладнання, заміна обладнання, ефективність використання обладнання, стійкість показників ефективності

Исследовано влияние физического износа сложного портового оборудования на динамику показателей эффективности его использования. Получены оценки колебаний средних затрат на единицу времени службы оборудования, функционирующего в условиях неустойчивого грузопотока. Предложена методика обоснования сроков обновления оборудования, при которых достигается баланс между минимальностью и устойчивостью суммарных удельных расходов на оборудование

Ключевые слова: износ оборудования, замена оборудования, эффективность использования оборудования, устойчивость показателей эффективности

UDC 656.615: 658.589
DOI: 10.15587/1729-4061.2018.123367

ESTIMATION OF FLUCTUATIONS IN THE PERFORMANCE INDICATORS OF EQUIPMENT THAT OPERATES UNDER CONDITIONS OF UNSTABLE LOADING

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1. Introduction

Operational characteristics of equipment tend to worsen due to ageing and wear. Physical wear leads to an increase in the number of breakdowns and increases in current expenditures for equipment. As a result, there comes the time when it becomes appropriate to upgrade equipment. The determining factor when making a decision to replace equipment is typically how effective its further use could prove to be. In practice, it often happens that technical condition of the equipment would allow its further use over a long time under conditions of proper repairs and maintenance, but, nevertheless, this equipment is replaced with new machinery for efficiency reasons.

Equipment replacement can be caused by both physical and moral wear. Only physical depreciation of equipment is taken into consideration in this article. When planning reasonable terms of replacement of sophisticated equipment, it is important to take into account not only the average expected performance indicators, but also resistance of these indicators against possible fluctuations caused by random changes in the level of equipment loading. In recent years, the structure and intensity of freight traffic through Ukrainian ports has undergone significant changes and tends to fluctuate in the future. Thus, it is a relevant task to substantiate strategies for the renewal and development of fleet of loading equipment, which must operate under conditions of unstable level of loading.

2. Literature review and problem statement

Planning the terms of equipment replacement due to its physical wear is a relevant and, in many cases, difficult problem, the study of which is the focus of many scientific works. There are different approaches to investigate a given problem. For example, those techniques that are based on survival curves are rather effective for simple standard equipment [1]. More sophisticated methods are typically used for complex repairable equipment.

There are a number of models for technical maintenance and replacement of equipment. These models can be conventionally divided according to the types of maintenance policies: the policy of replacement due to age or service term, the policy of group changes, the policy of periodic preventive maintenance, etc. Each type of policy has its own specifics, their merits and drawbacks. In review [2], several existing maintenance policies both for one-unit and for multi-level systems are generalized, categorized, and compared. The relations between different maintenance policies are studied.

Paper [3] considered the task on choosing optimal preventive maintenance policy and a scheduled machine's sale date. It was assumed that performance of a machine did not depend on age, while a probability of failure of a machine increases over its operation period. In this case, preventive maintenance could be applied to reduce the probability of a machine's failure.

In article [4], certain policies of equipment replacement, which are implemented over assigned time intervals and at a preset total number of repairs, are compared and analyzed.

Papers [5, 6] explore the problems of CBM (condition-based maintenance) optimization. Thus, in article [5], authors addressed general problems of quality of the data used for analysis of CBM tasks, and proposed ways to solve them. Specifically, the article explores approaches to the estimation of data that are missing during analysis of decisions; the structure of data to control information on equipment maintenance, which is essential for making CBM-based decisions, was proposed. In paper [6], a task on finding the optimal strategy for equipment repairs and replacements was examined. The findings of research are based on using a dynamic model of equipment ageing.

Article [7] examines the issue about making decisions on the replacement of chains of machines over time employing an optimal control model, which takes into account random factors of machines' failures. The authors based their research on the known model [3]. This work also takes into consideration a possibility of technological improvements of the used equipment.

Studies [8, 9] tackled the issues of coordination of operations on maintenance equipment with the investment strategies in the presence of possible random failures of equipment. In these works, in contrast to the traditional approach, which was proposed in the classic paper [3], a stochastic process of failures of machines is modeled explicitly. Thus, the task on choosing the optimal strategies for equipment replacements comes down to analysis of the model of stochastic dynamic programming. In paper [8], the task on reserving production capacities was also studied, with comparison of deterministic politics of equipment replacements and policies that depend on a machines' condition. In addition, the authors studied influence of performance deterioration, technological improvements and possible delays in the implementations of decisions, as well as the impact of a discount rate on the optimal policy of control over a fleet of machines.

In [10], authors examine a task on the replacement of one machine and reduce it to a nonlinear integral equation for a variable optimal service life of a machine.

The tasks on the optimization of equipment fleet in ports, which operates under conditions of alternating intensity of cargo traffic, and the optimization of a system of cargo delivery under conditions of alternating intensity of cargo traffic, are covered in [11, 12]. In these papers, simulation models are proposed that allow the evaluation of effectiveness of various schemes of cargo delivery and optimization of the composition of port's equipment fleet.

Decisions on the terms of equipment replacement depend on such factors as the level of current operating expenses and replacement costs, as well as on coefficients of discount and performance difference, reliability, and safety of existing and new equipment. However, the predictions that are related to these factors are typically not fully defined and are not always easy to assess objectively. Article [13] discusses possible subjective approaches to the assessment of uncertainty, which could be used as a basis for making a decision on equipment replacement.

The issue that requires further research is the substantiation of terms of replacement of sophisticated equipment taking into consideration the instability of its loading level. In the studies given above, various options for stating the problem on the substantiation of terms of equipment replacement are proposed, employing different methods and models aimed at resolving it. However, each of the approaches, suggested in these papers, is aimed at examining one particular case, under certain conditions and constraints. The methods, proposed in the above studies, do not make it possible to substantially enough study the task on the substantiation of terms of replacement of sophisticated equipment taking into account the instability of its loading level. Specifically, all of them fail to establish quantitative relationships between the average level of equipment efficiency indicators, the level of fluctuations in these indicators at possible random fluctuations in loading and the term of equipment replacement.

3. The aim and objectives of the study

The aim of present research is to study dynamics of average indicators of equipment efficiency and fluctuations of these indicators over time, as well as to develop a procedure for the substantiation of terms of replacement of sophisticated equipment taking into account the instability of its loading intensity.

To accomplish the aim, the following tasks have been set:

– to develop a mathematical model for the estimation of dynamics of mean values and fluctuations of indicators of sophisticated equipment functioning, which would take into account random fluctuations in the loading of equipment;

– to calculate the limits of intervals for possible values of total average costs per unit of time of equipment operation in the period from the start of its operation to its write-off;

– to establish quantitative relationships between the average level of indicators of equipment efficiency, the level of fluctuations in these indicators at possible random loading fluctuations and the term of equipment replacement.

4. Materials and methods to study changes in performance indicators of equipment functioning at random fluctuations in loading

4. 1. Modeling of random changes in a coefficient of equipment loading

Let a coefficient of equipment loading $s=s(t)$ change randomly, accepting values between functions $s_t=s_t(t)$ and $s_b=s_b(t)$, $s_b(t)<s(t)<s_t(t)$, and the average level of coefficient of loading at every moment of time t equals to $s_c=s_c(t)$. Changes in coefficient of loading will be described with the help of a diffusion process $s=s(t,\omega)$, where $\omega\in\Omega$, and (Ω, A, P) is the probability space.

To model a random process of equipment loading, let us consider the function of displacement in the following form:

$$a(x,t)=l\cdot(s_c(t)-x)+s_c'(t). \tag{1}$$

Function of displacement expresses the rate of a change in the values of a random process [14]

$$a(x,t)=\lim_{\tau\to t}\frac{1}{\tau-t}\times \times E\big(s(\tau,\omega)-s(t,\omega)\,|\,s(t,\omega)=x\big). \tag{2}$$

Let us consider function of diffusion

$$b(x,t)=q^2\cdot(s_t(t)-x)\cdot(x-s_b(t)), \tag{3}$$

This function expresses the rate of change in conditional variance of random process $s=s(t, \omega)$:

$$b(x,t)=\lim_{\tau\to t}\frac{1}{\tau-t}\times$$
$$\times \mathrm{E}\left((s(\tau,\omega)-s(t,\omega))^2 \,|\, s(t,\omega)=x\right). \tag{4}$$

Thus, at existing functions of displacement and diffusion, random process $s=s(t, \omega)$ can be expressed with the help of the stochastic model of the state in the form of Ito [15]:

$$ds(t,\omega)=$$
$$=a(s(t,\omega),t)dt+\sqrt{b(s(t,\omega),t)}\,dW(t,\omega), \tag{5}$$

where $W(t, \omega)$ is the Wiener process that comes from zero under initial condition $s(t_0, \omega)=s_0(\omega)$.

It is possible to show that at such selection of function of displacement and function of diffusion, a random process of a change in coefficient of loading $s=s(t, \omega)$, which was obtained with the help of stochastic differential equation (5), is limited by functions $s_t=s_t(t)$ and $s_b=s_b(t)$ and has mathematical expectation $\mathrm{E}(s(t, \omega))=s_c(t)$.

Indeed, mathematical expectation $m_s(t)=\mathrm{E}(s(t, \omega))$ of random process $s(t, \omega)$ is the solution to the Cauchy problem

$$\begin{cases}\dfrac{dm_s(t)}{dt}=l\cdot(s_c(t)-m_s(t))+s_c'(t),\\ m_s(t_0)=s_c(t_0),\end{cases} \tag{6}$$

which follows form the stochastic model of the state in the form of Ito (5), if it is influenced by operators of mathematical expectation (for example, [16]). Through the direct check, it is possible to make sure that function $s_c(t)$ is the solution of equation (6), so $m_s(t)=s_c(t)$.

In order to show that coefficient of loading $s(t, \omega)$, derived with the help of stochastic differential equation (5), is limited by functions $s_t=s_t(t)$ and $s_b=s_b(t)$, it is enough to explore the conditional probability density function of a stochastic process $s(t, \omega)$.

As it is known, for example from [16], conditional probability density function $f(t, x, \tau, y)$ of stochastic process $s(t, \omega)$, which is considered as function of final state (τ, y), satisfies the equation of Kolmogorov-Fokker-Planck

$$\frac{\partial f}{\partial \tau}+\frac{\partial}{\partial y}\left(a(y,\tau)\cdot f\right)-\frac{1}{2}\cdot\frac{\partial^2}{\partial y^2}\left(b(y,\tau)\cdot f\right)=0. \tag{7}$$

This differential equation is an equation in partial derivatives of parabolic type. There are various approaches to solving equations of this type with various initial and boundary conditions. Some of them are aimed at obtaining of a solution explicitly and are based on the use of integral transformations and the Fourier method of separation of variables. Some of them are oriented to the use of numerical methods.

Let us assume that s_t=const and s_b=const. Then in order to make sure that stochastic process $s(\tau, \omega)$ at any $\tau>0$ with probability 1 in between values of s_t and s_b, it is enough to verify that

$$\int_{s_b}^{s_t} W(\tau,y)\mathrm{d}y=1 \text{ for any } \tau>0, \tag{8}$$

where $W(\tau, y)$ is the conditional probability density function that at the initial moment $t_0=0$ of a stochastic process is in the point $s_0=s_c(t_0)$ and within time $(0, \tau)$ is not beyond the range of values (s_b, s_t). Function $W(\tau, y)$ is the solution to equation (7) with the correspondent initial and boundary conditions

$$\begin{cases}\dfrac{\partial W(\tau,y)}{\partial\tau}+\dfrac{\partial}{\partial y}\left(a(y,\tau)\cdot W(\tau,y)\right)-\\ -\dfrac{1}{2}\cdot\dfrac{\partial^2}{\partial y^2}\left(b(y,\tau)\cdot W(\tau,y)\right)=0,\ \tau>0,\ y\in(s_b,s_t),\\ W(0,y)=\delta(y-s_0),\\ W(\tau,s_b)=0,\\ W(\tau,s_t)=0,\end{cases} \tag{9}$$

where $\delta(x)$ is the Dirac δ-function; $W(0, y)=\delta(y-x_0)$ is the initial condition that reflects the fact that at the initial moment, stochastic process $s(\tau, \omega)$ is in the point s_0 $(s_b<s_0<s_t)$; $W(\tau, s_b)=0$ and $W(\tau, s_t)=0$ are the boundary conditions that determine the range of stochastic process $s(\tau, \omega)$.

For the case when functions $s_t=s_t(t)$ and $s_b=s_b(t)$ are not constant, the structure of equation (9) remains the same, however, expressions for boundary conditions become lengthier [16]. Solution to equation (9) and verification of condition (8) can be performed using, for example, numerical methods, which are implemented in the software packages Matlab or Maple.

Selection of parameters l, $q>0$ in displacement function (1) and diffusion function (3) makes it possible to take into account specifics of the process of equipment loading. Parameter l expresses the rate, at which trajectories of the stochastic process $s=s(t, \omega)$ approaches mathematical expectation $\mathrm{E}(s(t, \omega))$. Equipment loading, at which the value of coefficient of loading after deviations tend to approach quickly their mean values correspond to higher values of parameter l. Parameter q characterizes intensity of fluctuations of stochastic process $s=s(t, \omega)$. Higher values of q correspond to greater fluctuations of the level of equipment loading.

Freedom of selection of functions $s_t=s_t(t)$, $s_b=s_b(t)$ and $s_c=s_c(t)$ creates wide possibilities for taking into consideration the peculiarities of the process of a change in equipment loading, for example, like seasonal fluctuations and other specific factors, associated with the given cargo traffic.

4. 2. Modeling of dynamics of physical wear of equipment depending on the degree of its loading

For the quantitative estimation of costs per unit of time of equipment operation from the beginning of its operation to its write-off at various scenarios of the change in coefficient of loading, we will consider wear indicator $u=u(t)$, $(0\leq u\leq 1)$. This indicator characterizes intensity of operating costs at moment t at full loading of equipment.

To model changes in the indicator of wear, we will consider a dynamic model that is described by differential equation

$$u'=(1-u)^q\cdot(u-L)^r\cdot(a\cdot s(t)+b) \tag{10}$$

under initial condition $u(0)=u_0$, where r is the parameter that determines intensity of an increase in wear of equipment at the initial stage of ageing; q is the parameter that determines intensity of an increase in wear of equipment at the final stage of ageing; $s(t)$ is the coefficient of equipment loading at moment t $(0\leq s(t)\leq 1)$; L is the parameter that determines the lower asymptote of the curve of wear indicator $(0\leq L<1)$;

u_0 is the initial value of wear indicator (u_0>L); a and b are the parameters that determine the total rate of an increase in wear within all time of operation equipment and influence of the level of equipment loading on the rate of its equipment.

We will discount all costs to the moment the operation of equipment starts. We will consider a year as the unit of time measurement by default. Average operating costs per unit of time of a machine's operation over the period from the beginning of operation till moment t can be found from formula

$$R_{oper}(t)=\frac{1}{t}\cdot\int_0^t\left(\frac{R_{\text{var}}\cdot \text{u}(\tau)}{\text{u}(0)}\cdot s(\tau)+R_{\text{const}}\right)\cdot e^{\frac{-\delta\tau}{100}}\text{d}\tau, \tag{11}$$

where $u(t)$ is the indicator of total wear at moment t, R_{const} is the constant operation costs of equipment per unit of time; R_{var} is the variable operation costs of the new equipment per unit of time at complete loading; δ is the annual interest rate at continuous accrual. The expression that is in brackets in formula (11) determines intensity of current costs for equipment per unit of time. Thus, expression (11) is the result of application of the known formula for finding present capital value at continuous accrual:

$$V(t)=\int_0^t f(\tau)\cdot e^{\frac{-\delta\tau}{100}}\text{d}\tau. \tag{12}$$

Formula (12) is often used when studying efficiency indicators of equipment operation [17].

Average capital costs per unit of time of operation of a machine in the period from the start of operation until the moment t will be derived from formula

$$R_{cap}(t)=\frac{R_0}{t}, \tag{13}$$

where R_0 is the price of new equipment.

Then total average costs per unit of time of equipment operation in the period from the start of operation till moment t will be equal to

$$R(t)=R_{cap}(t)+R_{oper}(t). \tag{14}$$

The optimal term of equipment service will be considered time t, which minimizes mathematical expectation of total average costs per unit of time of equipment operation in the period from the start of operation till write-off, i.e. minimizes the expression

$$\text{E}\big(R(s(t,\omega),t)\big)=\int_\Omega R(s(t,\omega),t)\,P(d\omega), \tag{15}$$

where $R(s, t)$ is the total average costs per unit of time of the given equipment operation in the period from the beginning of its operation till moment t at the function of a change in coefficient of loading s=$s(t)$. For statistic evaluation of the specific value $\text{E}(R(s(t, \omega), t))$ at every moment t, using (5), we will generate a sample of functions of equipment loading $s_1(t)$, $s_2(t)$, ..., $s_N(t)$ and use equation

$$E\big(R(s(t,\omega),t)\big)\sim\hat{R}(t)=\frac{1}{N}\sum_{k=1}^{N}R(s_k(t),t). \tag{16}$$

We will accept such value t that minimized function $\hat{R}(t)$ as the optimal time for replacement of equipment.

Prediction of the average level of costs of equipment at different possible trajectories of a change in loading volumes is of great importance. However, in many cases it is also important to estimate how much actual costs can deviate from their predicted mean. Standard deviation of total average costs per unit of time of equipment operation in the period from the beginning of its operation to write-off will be found from formula

$$\sigma\big(R(s(t,\omega),t)\big)=$$
$$=\left(\int_\Omega\Big(R(s(t,\omega),t)-\text{E}\big(R(s(t,\omega),t)\big)\Big)^2 P(d\omega)\right)^{1/2}, \tag{17}$$

or by the statistical estimation

$$\sigma\big(R(s(t,\omega),t)\big)\sim$$
$$\sim\hat{\sigma}(t)=\left(\frac{1}{N-1}\left(\sum_{k=1}^{N}R^2(s_k(t),t)-\frac{1}{N}\left(\sum_{k=1}^{N}R(s_k(t),t)\right)^2\right)\right)^{1/2}. \tag{18}$$

It is of practical interest to determine for every moment t the interval ($x_{(1-\alpha)/2}$, $x_{(1+\alpha)/2}$), within which values $R(s(t, \omega), t)$ will be found with the assigned probability α. The limits of this space are defined by equations

$$\frac{1-\alpha}{2}=\int_\Omega\theta\left(x_{\frac{1-\alpha}{2}}(t)-R(s(t,\omega),t)\right)P(d\omega), \tag{19}$$

$$\frac{1+\alpha}{2}=\int_\Omega\theta\left(x_{\frac{1+\alpha}{2}}(t)-R(s(t,\omega),t)\right)P(d\omega), \tag{20}$$

where $\theta(x)$ is the Heaviside function.

Results of numerical calculations of the intervals within which indicators of the total average costs per unit of operation time of equipment are given in the next chapter.

5. Results of studying indicators of performance of equipment operation at random fluctuations in loading

Generation of a random sample from implementations of the random process, which is the solution of the stochastic differential equation (5), was implemented in the package of applied programs for engineering calculations Matlab. Using Matlab, we performed calculation of indicators from formulas (11)–(20) and numerical calculations of trajectories of a change of coefficient of equipment wear, which are assigned by the dynamic model (10).

Fig. 1 shows a random sample of 100 trajectories of a change in average costs per unit of time of equipment operation $R(s(t, \omega), t)$. Numeric values of input parameters for the dynamic model (1)–(20), which were used, were obtained based on statistical data of operation of port container loaders. In this work, results of point estimations of parameters for the proposed dynamic model of equipment ageing (1)–(20) are used. This can be substantiated by the fact of using dynamic models as an approach that is widely and successfully used for modeling of various technical systems. The problem of statistical estimation of parameters of stochastic dynamic models is the focus of many studies, for example, [16]. It should also be noted that the method of maximum likelihood is effective for the estimation of input parameters of dynamic models.

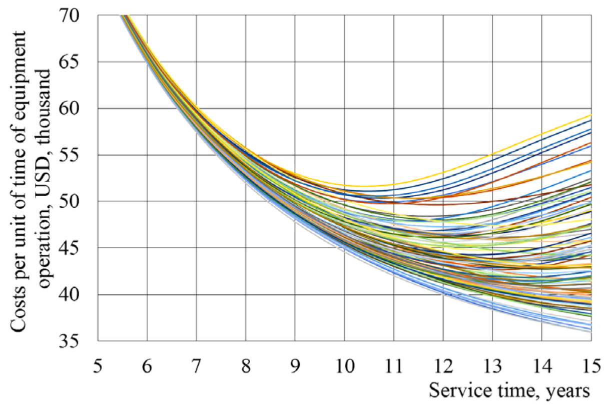

Fig. 1. Random sample of 100 trajectories of a change in average costs per unit of time of equipment service

The curves, shown in Fig. 1, are different trajectories of random process $R(s(t,\omega),t)$. When calculating, it was considered that the curve of a change in wear $u(t)$ is described with the help of a dynamic model (10) with initial conditions $u(0)=0{,}1$ and parameters $q=1$, $r=2.01$, $L=0$, $a=1.4$, $b=0.3$. This shape of the curve of a change in wear is typical for port container loaders. It was accepted that $R_{\text{var}}=200$, $R_{\text{const}}=8$, $R_0=240$, $\delta=10$. When modeling a stochastic process of a change in the coefficient of loading with the help of differential equation (5), it was assumed that $s_b=0.1$, $s_c=0.4$ and $s_t=0.9$. Function of wear and function of diffusion are, respectively, equal to $a(t)=0.1\times(0.4-x)$ and $b(t)=0.09\times(0.9-x)\times(x-0.1)$.

Table 1 gives estimations of indicators of average costs per unit of time of equipment operation, obtained by random sample of function $\left\{R(s(t,\omega_i),t)\right\}_{i=1}^{400}$. Estimations of quantiles of distributions of average costs per unit of time of equipment operation for different terms of equipment service at random trajectories of intensities of equipment loading are shown in Table 2. It is possible to follow more visually the changes in the law of distribution of average costs per unit of time of equipment operation over time in Fig. 2, 3.

Table 1

Estimations of indicators of average costs per unit of time of equipment operation at random changes in intensity of loading

Service term, years	Estimation of mathematical expectation of average costs per unit of time of equipment operation, USD, thousands	Estimation of variance of average costs per unit of time of equipment operation	Confidence interval for estimation of mathematical expectation
1	330.12	0.02	0.002
2	170.54	0.07	0.007
3	117.59	0.13	0.013
4	91.35	0.22	0.021
5	75.80	0.32	0.032
6	65.65	0.46	0.045
7	58.61	0.65	0.063
8	53.58	0.89	0.087
9	49.92	1.21	0.119
10	47.31	1.64	0.161
11	45.53	2.22	0.218
12	44.47	2.97	0.292
13	44.02	3.86	0.379
14	44.11	4.82	0.474
15	44.64	5.76	0.567

The data in Tables 1, 2 and diagrams in Fig. 1–4 were obtained based on calculations of trajectories of random process $R(s(t,\omega),t)$, which is determined from formulas (1)–(15). For Fig. 1, the sample of 100 trajectories of random process $R(s(t,\omega),t)$ was used. From statistical estimations in Tables 1, 2, the sample of 400 implementations was used. Confidence intervals for assessment of mathematical expectations of cross-sections of the random process, which are presented in the last column of Table 1, were calculated based on the sample of 400 implementations of process $R(s(t,\omega),t)$ with the use of Student's distribution.

It is possible to observe the way the shape of the law of distribution of indicators of average costs per unit of time of equipment operation changes at replacement of equipment after 3, 7 or 11 years, in Fig. 2–4, which represent histograms of distributions of values of cross sections of random process $R(s(t,\omega),t)$ at the specified moment. Fig. 2–4 were plotted using the software Statistica based on the sample of 400 implementations of random process $R(s(t,\omega),t)$. In these figures, densities of normal distributions, mathematical expectation and standard deviations of which correspond to the indicators of the sample are shown in red color, in addition, results of verification of the Kolmogorov-Smirnov test and χ^2 test for normality of distribution are specified. Thus, at small t, the law of distribution of average costs per unit of time of equipment operation almost does not differ from the normal (Fig. 2, 3), but over time, deviation from the normal law of distribution is becoming more noticeable (Fig. 4).

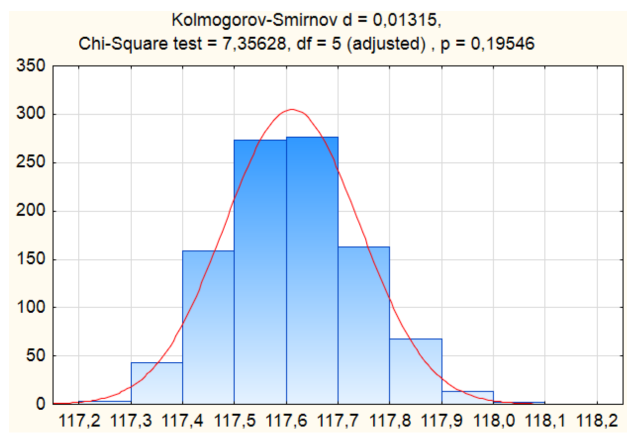

Fig. 2. Histogram of distribution of values of cross-section of random process $R(s(t,\omega),t)$ at $t=3$

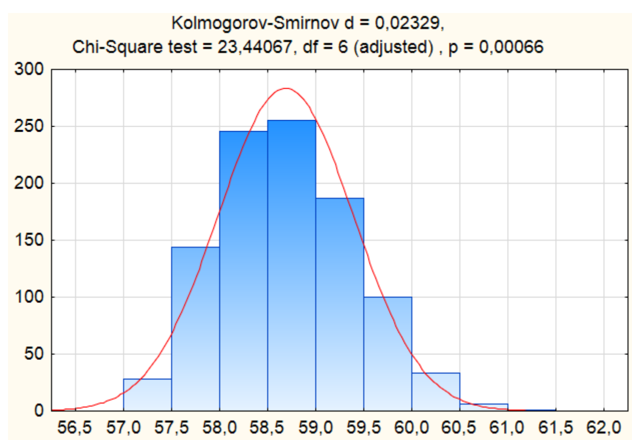

Fig. 3. Histogram of distribution of values of cross-section of random process $R(s(t,\omega),t)$ at $t=7$

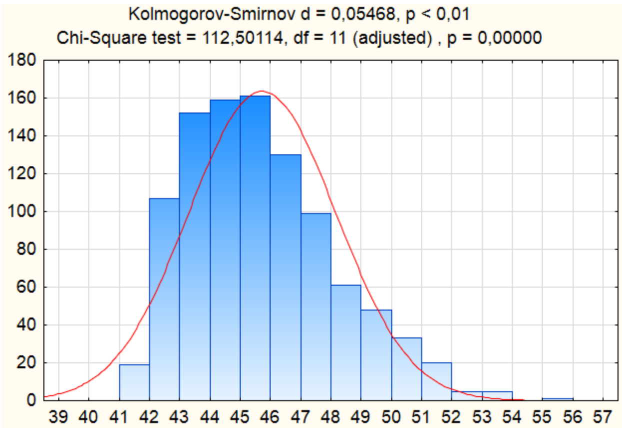

Fig. 4. Histogram of distribution of values of cross-section of a random process $R(s(t, \omega), t)$ at t=11

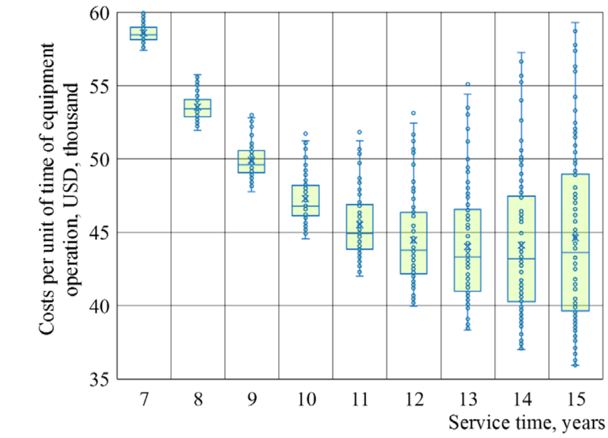

Fig. 5. "box and whisker" diagram, constructed by the sample of 100 random trajectories of a change in average costs per unit of time of equipment service

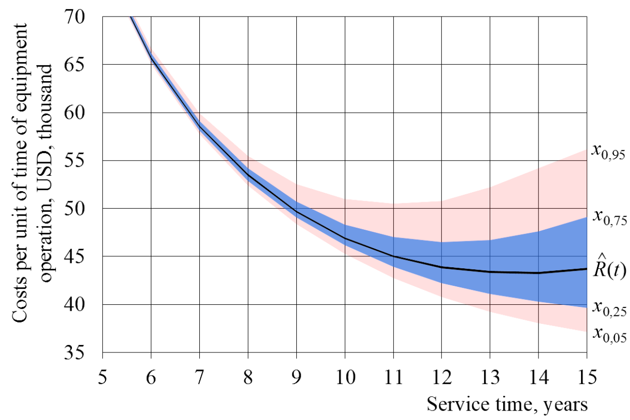

Fig. 6. Estimations of a change in mathematical expectation and quantiles for total average costs per unit of time of equipment operation

Table 2

Estimations of quantiles of distributions of average costs per unit of time of equipment operations for various terms of equipment operation at random functions of equipment loading intensity

Service time, years	$x_{0.05}$	$x_{0.25}$	$x_{0.50}$	$x_{0.75}$	$x_{0.95}$
1	330.10	330.11	330.12	330.14	330.16
2	170.45	170.49	170.52	170.57	170.66
3	117.40	117.51	117.57	117.69	117.86
4	91.04	91.21	91.29	91.52	91.75
5	75.34	75.59	75.72	76.05	76.43
6	64.98	65.31	65.56	65.97	66.49
7	57.73	58.15	58.49	58.98	59.79
8	52.37	52.90	53.42	54.03	55.31
9	48.37	49.08	49.60	50.53	52.34
10	45.24	46.15	46.81	48.15	50.79
11	42.72	43.95	44.93	46.85	50.35
12	40.75	42.23	43.80	46.29	50.57
13	39.22	41.06	43.30	46.54	52.07
14	38.02	40.28	43.18	47.43	54.00
15	37.10	39.65	43.63	48.95	55.99

It can be noted that the degree of scattering of values of average costs per unit of time of equipment operation substantially increases over time, and function $\hat{R}(t)$ takes the smallest value approximately in 13.5 years after the beginning of equipment operation (Fig. 5, 6).

The boundaries of the color areas, which are shown in Fig. 6, are the curves of a change in quantiles $x_{0.05}$, $x_{0.25}$, $x_{0.75}$, $x_{0.95}$ depending on the time of equipment operation. These curves are plotted according to data given in Table 2. Thus, the blue color area determines the boundaries of the intervals, within which with probability of 0.5 there will be total average costs per unit of time of the equipment operation. The upper and the lower boundaries of the red color areas determine the limits of the intervals, within which there are total average costs per unit of time of equipment operation with probability of 0.9. Given a significant increase in fluctuations of total average costs, there is a need to substantiate the measures, aimed at an increase in the level of stability of average costs of equipment.

6. Discussion of results of studying the dynamics of stability level of equipment performance indicators

In Fig. 7, a continuous bold line shows the curve of values of estimation of mathematical expectation of total average costs per unit of time of equipment operation $\hat{R}(t)$.

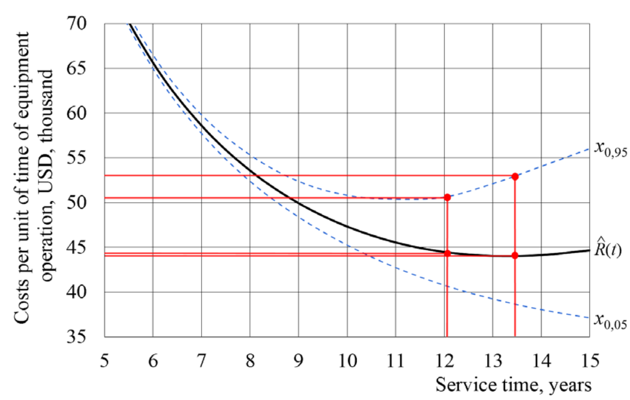

Fig. 7. Change in mathematical expectation and boundaries of the interval, within which there are values of total average costs per unit of time of equipment operation with a probability of 0.9

The diagrams in Fig. 5–7 are plotted based on data given in Table 1 and 2. Dotted lines in Fig. 7 represent the boundaries of the intervals, within which there are the values of total average costs per unit of time of equipment operation with probability of α=0.9. Fig. 7 shows that in terms of minimizing mathematical expectation of total average costs per unit of time of equipment operation, the optimal service time of equipment is approximately 13.5 years. At equipment replacement at any time in the interval between 12 and 13.5 years, mathematical expectation of average costs is almost minimal. Along with this, over time, there is a noticeable increase in degree of dispersion and, consequently, an increase in risks of getting excessive costs.

Thus, in this case it is advisable to reduce the planned service time of equipment from approximately 13.5 years to about 12 years and even less. Then expected total average costs per unit of time of operation equipment almost do not change, but in this case, the level of their stability will increase significantly.

Most of the research, devoted to planning of terms of equipment replacements, consider the level of loading to be permanent (for example, [1–5]). But in some cases, this assumption is not true. A change in the level of equipment loading significantly affects the rate of physical deterioration and, as a result, equipment performance indicators. The advantage of the method for determining of the terms of equipment replacements, proposed in this paper, is the fact that it takes into account possible fluctuations of the level of equipment loading. This makes it possible to determine the terms of equipment replacement, based not only on mean values of equipment performance indicators, but also taking into consideration the level of stability of these indicators. Specifically, the applied aspect of the use of the obtained scientific outcome is the possibility of operation efficiency of the port equipment that operates under conditions of unstable cargo traffic.

Assume that there are data about results of the use of a certain number of machines, in addition, the history of changing in intensity of the loading level for each of these machines is individual. Then using standard statistical methods, it is possible to determine the values of parameters of the proposed dynamic model by these statistical data. Subsequently, it is possible to use this model for prediction of changes in performance indicators of operation of these new machines. Moreover, this model enables us to make predictions even in the case, where dynamics of the intensity of loading of new machines will be significantly different from the level of loading of machines that were studied before.

However, it should be noted that the proposed model does not take into account the impact of obsolescence on planning the terms of replacement of sophisticated equipment. Sophisticated equipment usually has a long term of service. That is why during its operation, new more efficient models can appear on the market. The appearance of new models of equipment is also a factor that determines the terms of replacement of obsolete equipment. Thus, it is advisable to continue research, directed at studying the joint influence of physical and moral depreciation on determining of the optimal terms of replacement of sophisticated equipment that operates under conditions of unstable loading.

7. Conclusions

1. The dynamic model of a change in performance indicators of operation of sophisticated equipment was proposed. The structure of the proposed model consists of two parts. The first part concerns modeling of a random process of changes in the level of equipment loading and is described by the stochastic equation in the form of Ito (5), and by formulas (1)–(4). The second part concerns modeling of dynamics of equipment wear depending on changing in the levels of its loading and is described by differential equation (10) and formulas (11)–(14). As a result, we obtained the stochastic dynamic model of a change in performance indicators of sophisticated equipment (1)–(20), which takes into account random fluctuations in the level of equipment loading. This model makes it possible to calculate the limits of the intervals of possible values of total average costs per unit of time of equipment operation in the period from the beginning of its operation to write-off. It was proposed to determine a random process of changing the level of equipment loading, described by the stochastic model of state in the form of Ito (5), with the use of displacement function (1) and diffusion function (3). This approach created wide possibilities for taking into account the specific features of the process of changing in the level of equipment loading through selection of values of parameters in functions of displacement and diffusion. Numeric values of input parameters for the dynamic model (1)–(20), which were used in this article, and verification of adequacy of the model, were obtained based of statistical data on operation of port container loaders.

2. We established quantitative ratios of the average level of equipment performance indicators, the level of fluctuations in these indicators at possible random fluctuations in loading and the term of equipment replacement. Specifically, studies showed that changes in average total specific costs of equipment can be insignificant over a certain period, whereas the range of variability in the level of costs of equipment during the same period can significantly increase.

3. Based on analysis of the developed mathematical model, a procedure for planning the terms of replacement of sophisticated equipment taking into account instability of loading intensity level was proposed. A given procedure makes it possible to substantiate the terms of equipment replacement, taking into considerationt both average expected indicators of equipment performance efficiency and the level of possible fluctuations in these indicators.

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