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# RESEARCH INTO THE PROCESS OF MULTI-LEVEL MANAGEMENT OF ENTERPRISE PRODUCTION ACTIVITIES WITH TAKING RISKS INTO CONSIDERATION

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*Вирішено задачу динамічної оптимізації багаторівневого управління з урахуванням ризиків шляхом зведення до реалізації розв'язків скінченного числа задач лінійного й опуклого математичного програмування, а також скінченного числа задач дискретної оптимізації. Запропоновано метод на основі побудови множин досяжності. Це дає можливість отримати гарантований результат управління при впливі будь-яких можливих ризиків із визначеної множини*

*Ключові слова: динамічна оптимізація процесу управління, багаторівневе управління, множина досяжності, мінімаксий гарантований результат*

*Решена задача динамической оптимизации многоуровневого управления с учетом рисков путем сведения к реализации решений конечного числа задач линейного и выпуклого математического программирования, а также конечного числа задач дискретной оптимизации. Предложен метод на основе построения множеств достижимости. Это дает гарантированный результат управления при влиянии каких-либо возможных рисков из определенного множества*

*Ключевые слова: динамическая оптимизация процесса управления, многоуровневое управление, множество достижимости, минимаксный гарантированный результат*

## 1. Introduction

A decision-making process under conditions of uncertainty and the risk, which exists due to them, leads to the need to solve a problem of the best choice under conditions of incomplete information about a system under consideration. In this case, bases of existing approaches to solving similar problems are mostly static models and the application of apparatus of stochastic modeling. To apply such a mathematical tool, it is necessary to know probabilistic characteristics of main parameters of the model and special conditions for the implementation of the considered process.

In addition, using a stochastic modeling apparatus requires special conditions (for example, mass and homogeneity of a sample of values), which is usually difficult to execute in practice. It is necessary to take into account the specifics of a production activity (PA) of an enterprise, where, in

particular, risks do not depend on us and are uncontrolled parameters.

Consequently, to solve the problem of the management process (MP) of an enterprise PA, we propose to use a minimax approach or finding a guaranteed result. Its essence lies in the fact that a value of the worst (maximum) vector of heterogeneous risks is the smallest in comparison with similar values for others at the minimum guaranteed optimal management. Thus, we minimize the influence of risks in the MP problem, where risks are uncontrolled parameters, by choosing such optimal control that would guarantee the obtained result at the influence of any maximum of risks from a set of permissible ones.

We know that a modern enterprise is a complex multifactorial and multi-stage management object. Various types of risks can affect it. It consists of a large number of interconnected subsystems, which have relations of subordination in

the form of a hierarchical structure. The hierarchical system of a management of production activity (PA) of an enterprise leads to the need for further refinement of a concept of optimization of a management processes (MP). Conflict situations arise when solving optimization problems between subsystems of different levels in complex control systems with a given hierarchical structure. A decision leads to the need for a choice of management within a coherent strategy. A solution to this problem relates to the decision-making problem taking into account multilevel governance, which must take into account possibilities of its choice at each level of management. In addition, there is a problem of processing of large volumes of information when solving such problems. This fact creates considerable difficulties in automation of PA at an enterprise. Thus, optimization of a process of multilevel management of enterprise's PA taking into account risks is a complicated and relevant task.

## 2. Literature review and problem statement

We can distinguish the main approaches to solving a management problem. The first one is a dynamic programming method based on R. Bellman's optimality principle, which leads to the need to solve functional equations of a special form, and the second one is a variational approach.

Advantages and possibilities of dynamic programming developed on the base of mentioned approach are well known and sufficiently reflected in the literature. Papers [1, 2] show how the dynamic programming method makes possible to find an extremum of functional of many unknown functions by a replacement of the initial problem with a sequence of more simple tasks. A basis of such a replacement is the principle of optimality of the method, which underlies dynamic programming. Its essence is that "a next choice of managerial influences must be optimal in relation to a state, which a system has in a result of a first management influence, regardless of an initial state of a system and a first choice of managerial influence" [1].

Let us consider Bellman's method of dynamic programming. Fig. 1 shows that a technological process has  $N$  successive stages. A variable  $x_k$ , control  $u_k$  and some target function  $\varphi(u_k)$  characterize each time  $\varphi(u_k)$ . According to the principle of optimality, consideration of the process begins at the last stage.

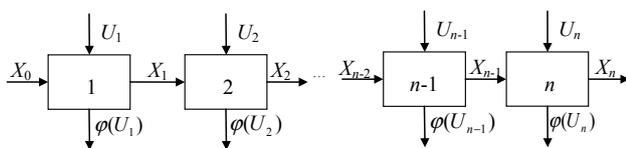


Fig. 1. Stages of technological process

Thus, the method of dynamic programming makes it possible to reduce a total  $N$ -dimensional problem of search for all  $u_k$  to a sequence of  $N$  one-dimensional problems, which greatly facilitates solving the problem.

However, the following question remains open: if we can limit our data to a number of iterations when solving a problem under study. That is, it requires careful, substantial control of results. Otherwise it is easy to obtain solutions that are far from optimal. But significant disadvantages of the method should include a need to memorize a large

amount of information at each stage. This creates considerable difficulties in the realization of tasks of large dimension in computational technology.

In general, the method of dynamic programming is better to use in combination with other methods, for example, mathematical programming. Mathematical programming is associated with tasks of efficient use and distribution of limited resources, which means to the search for a conditional extremum of functions of many variables with restrictions in the form of equalities and inequalities. There are effective computational methods in mathematical programming that make possible to solve extreme problems with a large number of available variables and corresponding limitations. This is especially true for problems of linear programming, the methods of which are widely used in economics and mathematical modeling. For example, this method promotes to optimize a management of innovative processes of enterprises for processing of agricultural raw materials in paper [3]. Work [4] studies modeling of production processes in small businesses in Ukraine. Authors of work [5] found the guaranteed result of a management of industrial technologies at an enterprise on the basis of the solution of an extreme problem with a large number of available variables and an influence of risks. Paper [6] investigates an influence of timeliness of reproduction processes on the efficiency of an enterprise development system with a use of economic and mathematical modeling. Authors of work [7] develop a model of a minimax adaptive management of innovation processes at an enterprise taking into account risks.

Paper [8] presents a variational approach. A base of it is a spread of ideas and methods of mathematical programming for multi-step problems. It closes up with the apparatus of the principle of the maximum of L. S. Pontryagin developed to solve optimal control problems in differential systems [9] (continuous time). The approach is usually called the "discrete principle of maximum". The principle of maximum extends variational methods to optimal problems for systems that describe ordinary differential equations with arbitrary restrictions on managerial influence, as well as some types of constraints on process variables. However, since the principle of maximum determines only the necessary condition for optimality, from the fact that a certain trajectory satisfies it, it does not follow that it is optimal. That is, the principle of maximum gives a trajectory only "suspicion" for optimality. It is necessary to conduct additional verification to determine an optimal trajectory from the number.

We know many applications of the principle of the maximum of L. S. Pontryagin. Recently, the principle of maximum has extended to discrete systems and systems with distributed parameters. In paper [10], it is extended to cluster models for the development of agricultural production of households. There were attempts to apply the method in order to control enterprises in international business [11]. But bases of the principle of Pontryagin's maximum are differential models for continuous processes, and in the tasks of MP the processes are essentially discrete. Thus, it is much better to use discrete economics and mathematical models immediately, all the more so that during the implementation of computing systems continuous models must still be discretized.

Thus, it is possible to conclude that solution to a problem on management of an enterprise PA requires using specialized methods. A corresponding mathematical apparatus must take into account a dynamic nature of problems under

study and their specificity. Namely – an influence of a risk factor, variability of technologies, etc., as well as possibility of effective computer realization, both simulation processes and real management. Thus, methods based on construction of predictive sets (areas of reach) of the considered dynamic model are suitable for the MP AD problem. They represent a set of all admissible states of a phase system vector at a given moment of time corresponding to a fixed management and all permissible risk vectors. A paper [12] considers a phase vector with a period of a quarter and a total annual term of management.

We should note that the application of such a method will reduce the initial multi-step problem to the realization of a finite sequence of one-step problems of discrete optimization. It is possible to solve such problems by methods of linear or nonlinear (depending on a type of target functions) mathematical programming with a use of modern computer technology. Thus, the main result of the application of the method of construction of areas of reach is that a problem of MP PA of an enterprise becomes solvable for a certain number of iterations.

Consequently, the process of optimal functioning of economic systems is multi-faceted. It includes organization of a process of optimal management and planning, control and operational management of the progress of plans, economic instruments of optimal economic development, methods for the implementation of vertical and horizontal connections in a system, etc. An example of business-models demonstrates this in paper [13], a developed concept of valuation of production assets – in paper [14]. Authors of work [15] investigated organizational and managerial connections on the example of an innovation factor of the business model. In paper [16] – in managing programs to improve business outcomes. Researchers of work [17] used a constructed concept of intellectualization of information support for enterprise management.

However, a starting point for the optimal functioning of an economic system, in particular, an enterprise PA, is a scientifically organized process for the development and implementation of MP. We can identify main features of this process on the basis of general principles of optimality. Also, we should take into account a hierarchical structure of enterprise PA, a conception of optimality criteria, modeling under conditions of uncertainty with complex structure of the investigated process.

One of examples of an economic system with a complex structure is the sphere of production of an enterprise. Its characteristics are dependence on raw materials, seasonality of production, energy intensity, lack of strict mathematical models that describe physical processes, presence of uncertainty factors that influence a dynamics of production. Thus, the production system is a set of interconnected elements, each of which carries out certain processing of some raw material or semi-finished product into a finished product. In addition, a management of an enterprise PA occurs under uncertainty due to an influence of various types of risks.

The decision-making process under uncertainty of the economic environment usually leads to the need to solve a certain task of the best choice under conditions of incomplete information on the investigated system. A typical decision-making situation in dynamic systems is a need to organize a management procedure under uncertainty, for example, in the process of taking into account production risks and risks of using production technology. The pro-

cedure aimed at achievement of one or another purpose of management often is necessary to accompany the process of optimization. This makes it possible to allocate guaranteed, best or acceptable result in a certain sense, that is, to use a minimax approach.

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### 3. The aim and objectives of the study

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The aim of present study is to investigate MP PA of an enterprise in the presence of risks and solution of the problem of optimization of multi-level management of enterprise PA in the light of risks. It means the development of a method that makes it possible to switch from a complex multi-level dynamic optimization problem to multi-level management taking into account risks for implementation of a finite sequence of one-step discrete optimization problems.

We set the following tasks to achieve the objective:

- formulation a substantial statement of a problem of multi-level management of an enterprise PA in the presence of risks;
- development of a formal statement of a model of multi-level management at an enterprise in the presence of risks;
- solution of the problem of optimization of multi-level management of an enterprise PA taking into account risks.

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### 4. Substantial statement of the problem of multi-level MP PA of an enterprise taking into account risks

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Actually, a dependence of the replenishment of resources (production and investment) on a management vector (volumes of products produced in each period of time) reflects a multi-level management of an enterprise provided in the model in the management of PA. Therefore, there are several levels of management for each fixed management for a selected period (that is, one of the permissible scenarios of MP PA). It goes from the upper level (first level) at an enterprise (for example, a planning department) to the next level (second level) of management. At this level, we calculate production resources corresponding to the management, as well as investment resources required for the implementation of this fixed management. The values are necessary then to perform necessary calculations in the model [3, 18]. Such a procedure at modeling is done for each possible management (hypothetically, during modeling). This is the real essence of the process of multi-level (in our case – two-level) management.

Given the above and relying on the principles of mathematical modeling and the theory of optimal management, we proceed to the modeling of MP PA problem.

Let us formulate a substantial statement. An enterprise plans its PA, that is, it makes a transition to the production of products based on the desired MP. This process takes into account different types of production factors, raw materials, variants of use and storage of raw materials, intermediate and final products, influence of various production and external factors, including risks, other components of the production process. It may consist of certain technological methods of organization of production, which involve a use of existing or replacement (partial or full) of technological equipment.

MP includes a value of volumes of production of new products, a vector of replenishment of material and labor

resources for production and a vector of current investments for the implementation of PA, which form a scenario of management of a corresponding MP. There is possibility of a use of different MP scenarios depending on the variation of values of corresponding components of a production process.

It is necessary to carry out an optimal management of a process according to a corresponding scenario at a given time interval of its life cycle. This must be done by selection among a plurality of alternatives of possible managerial influences so that an overall performance criterion for MP is maximal [19]. Moreover, if there are several variants of the implementation of different MPs considered on the basis of the corresponding technologies [20], then it is also necessary to make a choice between them and to find optimal management for the chosen criterion.

We developed a substantial economic and mathematical model of an enterprise PA. Now let us proceed to the formal setting and development in several stages.

### 5. Formal statement of the problem of the multi-level MP PA of an enterprise taking into account risks

Let us consider a multi-step dynamic system of enterprise PA on a given integer-valued time interval

$$\overline{0, T} = \{0, 1, \dots, T\} \quad (T > 0).$$

It includes an enterprise (an object  $I$ ) controlled by  $P$  player (a subject of management). The state of MP is a linear discrete recurrent vector equation of the following form (dynamic model):

$$\begin{aligned} \bar{x}(t+1) &= A(t)\bar{x}(t) + B(t)\bar{u}(t) + \\ &+ C(t)\bar{v}(t) + D(t)\bar{w}(t), \\ \bar{x}(0) &= \{x_0, I_0\}, \end{aligned} \quad (1)$$

where

$$t \in \overline{0, T-1} = \{0, 1, \dots, T-1\} \quad (T > 0);$$

$$\bar{x}(t) = (\bar{x}_1(t), \bar{x}_2(t), \dots, \bar{x}_{\bar{n}}(t))' \in \mathbf{R}^{\bar{n}}$$

is a vector of phase variables or a phase vector – a set of basic parameters describing a MP state at  $t$  time;  $\mathbf{R}^{\bar{n}}$  is  $\bar{n}$ -dimensional Euclidean space of vector-columns,  $\bar{n} \in \mathbf{N}$  is a set of positive natural numbers;

$$\bar{u}(t) = (\bar{u}_1(t), \bar{u}_2(t), \dots, \bar{u}_{\bar{p}}(t))' \in \mathbf{R}^{\bar{p}}$$

is a vector of MP management (managerial influence) of an enterprise satisfying the given limitation:

$$\begin{aligned} \bar{u}(t) &\in U_1(t) \subset \mathbf{R}^{\bar{p}}, \\ U_1(t) &= \{\bar{u}(t) : \bar{u}(t) \in \{\bar{u}^{(1)}(t), \bar{u}^{(2)}(t), \dots, \bar{u}^{(N_t)}(t)\} \subset \mathbf{R}^{\bar{p}}\}, \end{aligned} \quad (2)$$

where  $U_1(t)$ , for each  $t \in \overline{0, T-1}$  is a finite set of vectors, that is, a finite set consisting of  $N_t$  ( $N_t \in \mathbf{N}$ ) vectors in  $\mathbf{R}^{\bar{p}}$ , that define all possible realizations of different MP scenarios at  $t$  time; ( $\bar{p} \in \mathbf{N}$ );

$$\bar{w}(t) = (\bar{w}_1(t), \bar{w}_2(t), \dots, \bar{w}_m(t))' \in \mathbf{R}^{\bar{m}}$$

is a vector of replenishment of material and labor resources at a period of time  $t$  ( $t \in \overline{0, T-1}$ ), which depends on the permissible realization of management and must satisfy the following given limitation:

$$\begin{aligned} \bar{w}(t) &\in W_1(\bar{u}^{(i)}(t)) \subset \mathbf{R}^{\bar{m}}, \\ W_1(\bar{u}^{(i)}(t)) &\{ \bar{w}(t) : \bar{w}(t) \in \\ &\in \{\bar{w}^{(1)}(t), \bar{w}^{(2)}(t), \dots, \bar{w}^{(M_i)}(t)\} \subset \mathbf{R}^{\bar{m}} \}, \end{aligned} \quad (3)$$

where  $W_1(\bar{u}^{(i)}(t))$  for each time point  $t \in \overline{0, T-1}$  and management  $\bar{u}^{(i)}(t) \in U_1(t)$  is a finite set of vectors, that is, a finite set consisting of  $M_i(i)$ , ( $M_i(i) \in \mathbf{N}$ ,  $i \in \overline{1, N_t}$ ) of vectors in  $\mathbf{R}^{\bar{m}}$ , that determine all possible realizations of different scenarios of replenishment of material and labor resources and investment resources at  $t$  time in the investigated process; a matrix  $D(t)$  at the vector  $\bar{w}(t)$  determines the intensity of its effect on the vector  $\bar{w}(t)$  at each time point.

Each realization of a phase vector is permissible for all  $t \in \overline{0, T-1}$ .

$$\bar{x}(t) = (x_1(t), x_2(t), \dots, x_{\bar{n}}(t)) \in \mathbf{R}^{\bar{n}}$$

satisfies the following given phase limitation:

$$\bar{x}(t) = (x_1(t), x_2(t), \dots, x_{\bar{n}}(t)) \in X_1(t) \subset \mathbf{R}^{\bar{n}}, \quad (4)$$

where  $X_1(t)$  is a convex, closed and bounded polyhedron of space  $\mathbf{R}^{\bar{n}}$ , that is, a set, which limits permissible values of the realization of a phase vector at  $t$  time;

$$\bar{v}(t) = (\bar{v}_1(t), \bar{v}_2(t), \dots, \bar{v}_{\bar{q}}(t))' \in \mathbf{R}^{\bar{q}}$$

is a vector of risks affecting MP realization, which in each period of time  $t$  ( $t \in \overline{0, T-1}$ ) depends on the permissible realization of management  $\bar{u}(t) \in U_1(t)$ , that satisfies the given limitation:

$$\bar{v}(t) \in V_1(\bar{u}(t)) \subset \mathbf{R}^{\bar{q}}, \quad (5)$$

where  $V_1(\bar{u}(t))$  is a convex, closed and bounded polyhedron of space  $\mathbf{R}^{\bar{q}}$ , that is, a set that limits possible realizations of a risk vector during MP at  $t$  time  $t$ ;  $\bar{q} \in \mathbf{N}$ .

Matrices  $A(t)$ ,  $B(t)$ ,  $C(t)$  and  $D(t)$  in the vector recurrence equation (1) describing MP dynamics are real matrices of orders  $(\bar{n} \times \bar{n})$ ,  $(\bar{n} \times \bar{p})$ ,  $(\bar{n} \times \bar{m})$  and  $(\bar{n} \times \bar{q})$ , respectively, are such that for all  $t \in \overline{0, T-1}$  matrices  $A(t)$  are non-degenerate, that is, for a matrix  $A^{-1}(t)$ , corresponding to it, and a rank of a matrix  $B(t)$  is equal to  $\bar{p}$  (the dimension of a vector  $\bar{u}(t)$ ).

Let us describe informational capabilities of subject of management ( $P$  player) while MP management in the discrete dynamic system (1)–(5).

We assume that the subject of control has certain information capabilities during a course of MP realization and a fixed natural number  $s \gg T > 0$  at each point of time  $t \in \overline{1, T}$ . They correspond to realizations of a phase vector of a system, managerial influence, and a risk vector at an integer-valued interval of time  $\overline{-s, t}$ . The phase vector precedes a considered one during MP management:

1) we know a history of the realization of a phase system vector

$$\begin{aligned} \bar{x}_t(\cdot) &= (\bar{x}_1(\cdot)_t, \bar{x}_2(\cdot)_t, \dots, \bar{x}_n(\cdot)_t) = \\ &= \{(\bar{x}_1(\tau), \bar{x}_2(\tau), \dots, \bar{x}_n(\tau))\}_{\tau \in \overline{0, T-1}} = \{\bar{x}(\tau)\}_{\tau \in \overline{0, T-1}}; \end{aligned}$$

2) we know a history of the realization of managerial influence of a system

$$\begin{aligned} \bar{u}_t(\cdot) &= (\bar{u}_1(\cdot)_t, \bar{u}_2(\cdot)_t, \dots, \bar{u}_p(\cdot)_t) = \\ &= \{(\bar{u}_1(\tau), \bar{u}_2(\tau), \dots, \bar{u}_p(\tau))\}_{\tau \in \overline{0, T-1}} = \{\bar{u}(\tau)\}_{\tau \in \overline{0, T-1}}; \end{aligned}$$

3) we know a history of the realization of a vector of intensity of replenishment of production and investment resources

$$\begin{aligned} \bar{w}_t(\cdot) &= (\bar{w}_1(\cdot)_t, \bar{w}_2(\cdot)_t, \dots, \bar{w}_m(\cdot)_t) = \\ &= \{(\bar{w}_1(\tau), \bar{w}_2(\tau), \dots, \bar{w}_m(\tau))\}_{\tau \in \overline{0, T-1}} = \{\bar{w}(\tau)\}_{\tau \in \overline{0, T-1}}; \end{aligned}$$

4) we know a history of the realization of a vector of system risks

$$\begin{aligned} \bar{v}_t(\cdot) &= (\bar{v}_1(\cdot)_t, \bar{v}_2(\cdot)_t, \dots, \bar{v}_q(\cdot)_t) = \\ &= \{(v_1(\tau), v_2(\tau), \dots, v_q(\tau))\}_{\tau \in \overline{0, T-1}} = \{\bar{v}(\tau)\}_{\tau \in \overline{0, T-1}}. \end{aligned}$$

We should note that we can solve the problem of a posterior identification of all main elements of a discrete dynamic system based on these data (1). It is necessary to determine elements of the matrices  $A(t)$ ,  $B(t)$ ,  $C(t)$  and  $D(t)$  in the vector recurrence equation (3). It describes the dynamics of  $I$  object, that is, the object of MP [18].

Let us assume that the subject of management (a person who makes decisions on MP) –  $P$  player also knows equations (1) and limitations (2)–(5).

A value of the convex functional  $\tilde{F}: \mathbf{R}^n \rightarrow \mathbf{R}^1$ , defined on possible realizations of the phase vector  $\bar{x}(T) \in \mathbf{R}^n$  of the system (1) at the final moment of time  $T$  determines a quality of selection of optimal MP management from a standpoint of  $P$  player.

Then for the system (1)–(5), the objective of a minimax multi-level MP from a point of view of  $P$  player can be formulated in the following way. It is necessary for  $P$  Player to form a management

$$\bar{u}_T^{(e)}(\cdot) = \{\bar{u}_T^{(e)}(t)\}_{t \in \overline{0, T-1}}$$

(for all  $t \in \overline{0, T-1}$ :  $\bar{u}_T^{(e)}(t) \in U_1(t)$ ) at a given time interval  $0, T$ . The value of the convex functional defined on realizations of the vector  $\bar{x}(t) \in \mathbf{R}^n$  must be minimal. Where  $\bar{x}(T)$  is the realization of a phase system vector at  $T$  time and a vector  $\bar{w}_T(\cdot)$  at the worst (that is, those that maximize the value of the functional  $\tilde{F}$ ) of permissible realizations of a risk vector

$$\bar{v}_T(\cdot) = \{\bar{v}_T(t)\}_{t \in \overline{0, T-1}}$$

(for all  $t \in \overline{0, T-1}$ :  $\bar{v}_T(t) \in V_1(\bar{u}_T^{(e)}(t))$ ). At the same time, realization

$$\bar{w}_T(\cdot) = \{\bar{w}_T(t)\}_{t \in \overline{0, T-1}}$$

(for all  $t \in \overline{0, T-1}$ :  $\bar{w}_T(t) \in W_1(\bar{u}_T^{(e)}(t))$ ) of a vector of intensity of replenishment of production and investment resources contributes to the achievement of objectives of  $P$  player. That is, an aim of selection (according to the  $P$  player's task) is minimization of this functional in accordance with the management chosen by him.

## 6. Solution of the problem of multi-level MP PA of an enterprise taking into account risks

We turn to the solution of the problem of multi-level MP taking into account risks after determination of all the parameters of the model:

1. We form a set of alternatives of possible MP managements  $U(t)$ . The components of the first group of management vectors of the set represent output product volumes over a period of time (in accordance with the corresponding MP) (first level of management).

2. We build a set of replenishment of material, labor resources and investment resources  $W(\bar{u}(t))$  based on values of the first group of management vectors of MP (second level of management).

3. We determine a set of permissible risks  $V(t)$  with appropriate limitations.

4. We fix the management with the corresponding vector of replenishment of material and labor resources, investment resources consistently and “take away” all risks from the set of permissible ones. We construct the corresponding predictable sets of reach regions  $G$  for the final vectors  $x(T)$  of the state of the system at the time  $T$ .

5. On the sets of reach domains, we solve the discrete optimization problem with the help of minimax and find the optimal control, which provides a guaranteed result of the solution of MP PA problem of an enterprise under the influence of any risks of the set of permissible ones. Fig. 2 presents the scheme of multi-level management during MP production and commercial cycle.

Let us introduce a series of definitions, which are necessary for the formalization of the problem of a minimax multi-level MP for the discrete dynamic system under consideration (1)–(5).

We denote a metric space of functions of an integer-value argument  $\phi: i, j \rightarrow R^k$  for  $k \in \mathbf{N}$  and any integer-value interval  $i, j$  ( $i \leq j$ ). We determine the metric  $\rho_k$  by the ratio:

$$\begin{aligned} \rho_k(\phi_1(\cdot), \phi_2(\cdot)) &= \\ &= \max_{t \in i, j} \|\phi_1(t) - \phi_2(t)\|_k \quad ((\phi_1(\cdot), \phi_2(\cdot)) \in_k S_k(i, j) \times S_k(i, j)), \end{aligned}$$

and a symbol  $(S_k(i, j))$  is a set of all nonempty and compact subsets of  $S_k(i, j)$  space in the contents of this metric.

Here and for any of sets  $X$  and  $Y$ , the set  $X \times Y$  is a product of  $X$  and  $Y$ , that is, a set of all pairs  $(x, y)$  such that  $x \in X, y \in Y$  (we use the same notation for a greater number of sets).

Using the limitation (2), we define a set  $U(\tau, \vartheta) \subset \mathbf{R}^{(p \times (\vartheta - \tau))}$  of managements  $\bar{u}(\cdot) = \{\bar{u}(t)\}_{t \in \tau, \vartheta - 1}$  of  $P$  player at time period  $\tau, \vartheta \subseteq 0, T$  ( $\tau < \vartheta$ ) by the ratio

$$\begin{aligned} U(\tau, \vartheta) &= \{\bar{u}(\cdot): \bar{u}(\cdot) \in \mathbf{R}^{(p \times (\vartheta - \tau))}, \\ \forall t \in \tau, \vartheta - 1, \bar{u}(t) &\in \bar{U}_1(t)\}, \end{aligned} \tag{6}$$

which is a set of all permissible realizations of managements  $\bar{u}(\cdot)$  (all possible scenarios for the realization of multi-level MP) at an integer-valued period of time  $0, T$  and, taking into account the assumption, is a finite set of vectors of space  $\mathbf{R}^{(p \times (\vartheta - \tau))}$ .

For each fixed management  $\bar{u}(\cdot) \in U(\tau, \vartheta)$ , using the limitations (3), we define a set  $W(\tau, \vartheta; \bar{u}(\cdot)) \subset \mathbf{R}^{(m \times (\vartheta - \tau))}$  of intensities  $\bar{w}(\cdot) = \{\bar{w}(t)\}_{t \in \tau, \vartheta - 1}$  of replenishment of production and investment resources at a time interval  $\tau, \vartheta \subseteq 0, T$  ( $\tau < \vartheta$ ), which correspond  $\bar{u}(\cdot)$ , by the following ratio:

$$\begin{aligned} \mathbf{W}(\tau, \vartheta; \bar{u}(\cdot)) &= \{\bar{w}(\cdot): \bar{w}(\cdot) \in \mathbf{R}^{(\bar{m} \times (\vartheta - \tau))}(\tau, \vartheta - 1), \\ \forall t \in \tau, \vartheta - 1, \bar{w}(t) &\in \bar{W}_1(\bar{u}(t))\}, \end{aligned} \quad (7)$$

which is a set of all permissible realizations of intensities of replenishment and investment resources  $\bar{w}(\cdot)$  (all possible scenarios of the realization of such vector-functions) at an integer-valued period of time  $\overline{0, T}$  and, given the assumption, is a finite set of space vectors  $\mathbf{R}^{(\bar{m} \times (\vartheta - \tau))}$ .

$$\begin{aligned} \mathbf{V}(\tau, \vartheta; \bar{u}(\cdot)) &= \{\bar{v}(\cdot): \bar{v} \in \mathbf{S}_q(\tau, \vartheta - 1), \\ \forall t \in \tau, \vartheta - 1, \bar{v}(t) &\in \bar{V}_1(\bar{u}(t))\}, \end{aligned} \quad (8)$$

which is a set of all permissible realizations of a risk vector  $\bar{v}(\cdot)$  (all permissible scenarios for the realization of a risk vector during MP management) at an integer-valued interval of time  $\overline{0, T}$ .

We call the set

$$\begin{aligned} g(\tau) &= \{\tau, \bar{x}(\tau)\} \in \overline{0, T} \times \mathbf{R}^{\bar{n}} \\ (g(0) &= g_0 = \{0, \bar{x}_0, I_0\}) \end{aligned}$$

as  $\tau$ -position (a current state at the appropriate stage of the studied time interval, taking into account initial parameters of a phase vector and initial investments) of  $P$  player in the discrete dynamic system (1)–(5) during MP. It is necessary to note that we use all information, which  $P$  player has in MP process at a time interval  $\overline{0, \tau}$ , during formation of  $\tau$ -position.

We also define a set for all  $\tau \in \overline{0, T}$

$$\begin{aligned} \hat{\mathbf{G}}(\tau) &= \{\tau\} \times \mathbf{R}^{\bar{n}} \\ (\hat{\mathbf{G}}(0) &= \hat{\mathbf{G}}_0 = \\ &= \{g(0) = g_0: g_0 = \{0, \bar{x}_0, I_0\} \in \{0\} \times \mathbf{R}^{\bar{n}}\}) \end{aligned}$$

of all the permissible  $\tau$ -positions of  $P$  player. Next, for a fixed time interval  $\tau, \vartheta \in \overline{0, T}$  ( $\tau < \vartheta$ ),  $\tau$ -position

$$g(\tau) = \{\tau, \bar{x}(\tau)\} \in \hat{\mathbf{G}}(\tau)$$

of  $P$  player, its management

$$\bar{u}(\cdot) \in U(\tau, \vartheta)$$

and the permissible realization of a vector

$$\bar{w}(\cdot) \in \mathbf{W}(\tau, \vartheta; \bar{u}(\cdot)),$$

we define the following set:

$$\begin{aligned} \mathbf{G}(\tau, g(\tau), \vartheta, \bar{u}(\cdot), \bar{w}(\cdot)) &= \\ &= \{g(\vartheta): g(\vartheta) = \{\vartheta, \bar{x}(\vartheta)\} \in \hat{\mathbf{G}}(\vartheta), \\ \bar{x}(\vartheta) &= \bar{x}_{\tau, \vartheta}(\vartheta; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)), \bar{v}(\cdot) \in \\ &\in \mathbf{V}(\tau, \vartheta; \bar{u}(\cdot))\}, \end{aligned} \quad (9)$$

which we call a set of permissible  $\vartheta$ -positions of  $P$  player, which corresponds to its  $\tau$ -position  $g(\tau)$  and a pair  $(\bar{u}(\cdot), \bar{w}(\cdot))$ .

Here, a vector

$$\bar{x}(\vartheta) = \bar{x}_{\tau, \vartheta}(\vartheta; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) \in \mathbf{R}^{\bar{n}}$$

defines a cutover of the motion of  $I$  object at a time interval  $\tau, \vartheta$  at the time point  $\vartheta$  during the MP generated by a set  $(\bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))$ .

We introduce a scalar target function

$$\mathbf{F}_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)),$$

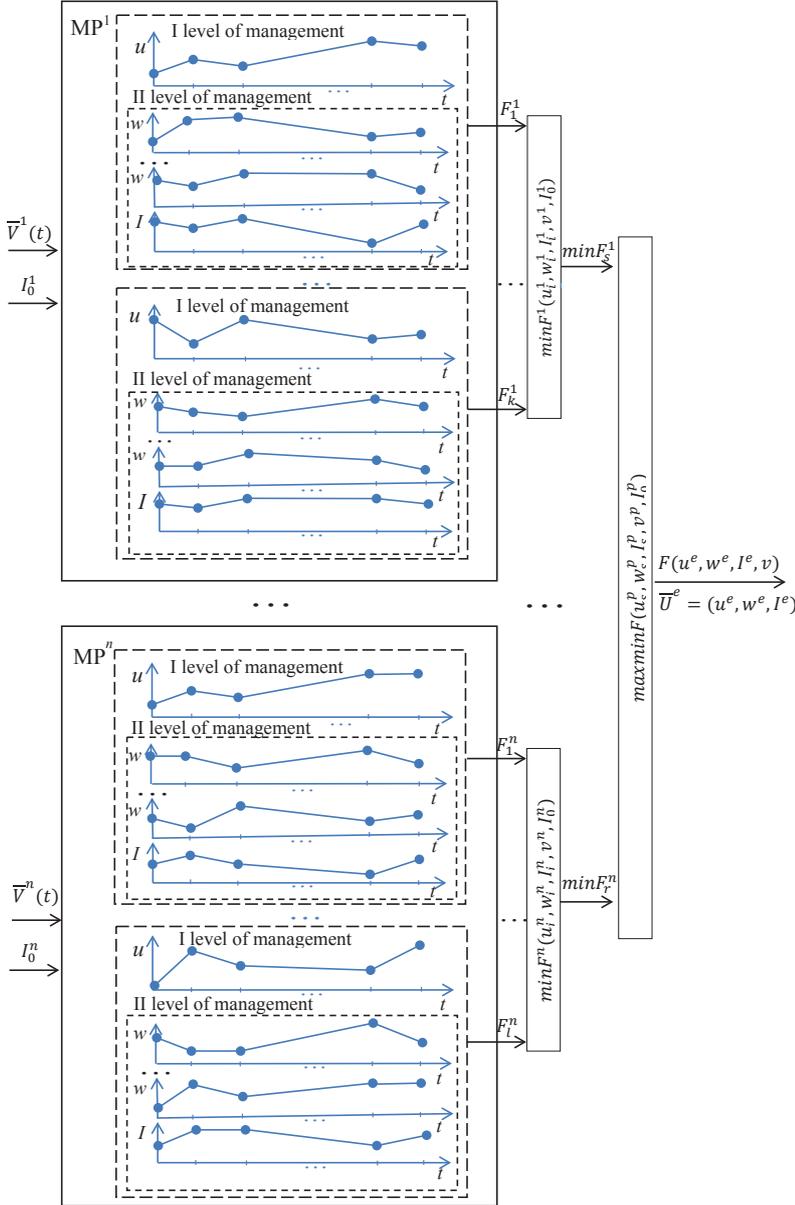


Fig. 2. Scheme of multi-level process of management

For each fixed management  $\bar{u}(\cdot) \in U(\tau, \vartheta)$ , using the limitations (3), we define a set  $\mathbf{V}(\tau, \vartheta; \bar{u}(\cdot))$  of risks

$$\bar{v}(\cdot) = \bar{v}(t)_{t \in (\tau, \vartheta - 1)}$$

at the time interval

$$\tau, \vartheta \subseteq \overline{0, T} \quad (\tau < \vartheta)$$

by the ratio:

a value of which  $\bar{x}$  for all permissible realization of sets at the time interval  $\tau, T$ , into consideration to evaluate a quality of the considered process of minimax MP

$$(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) \in \hat{G}(\tau) \times U(\tau, T) \times W(\tau, T; \bar{u}(\cdot)) \times V(\tau, T; \bar{u}(\cdot)),$$

where

$$g(\tau) = \{\tau, \bar{x}(\tau)\} \in \hat{G}(\tau),$$

$$\bar{u}(\cdot) = \{\bar{u}(t)\}_{t \in \tau, T-1} \in U(0, T),$$

$$\bar{w}(\cdot) = \{\bar{w}(t)\}_{t \in \tau, T} \in W(\tau, T; \bar{u}(\cdot)),$$

$$\bar{v}(\cdot) = \{\bar{v}(t)\}_{t \in \tau, T-1} \in V(0, T; \bar{u}(\cdot)),$$

we determine in accordance with the following ratio:

$$F_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = \sum_{k=1}^r \mu_k \cdot \Phi_{\tau, T}^{(k)}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = \sum_{k=1}^r \mu_k \cdot F_{\tau, T}^{(k)}(\bar{x}_{\tau, T}(T; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))) = \sum_{k=1}^r \mu_k \cdot F_{\tau, T}^{(k)}(\bar{x}(T)) = \tilde{F}(\bar{x}(T)),$$
(10)

$$\forall k \in \overline{1, r}: \mu_k \geq 0, \sum_{k=1}^r \mu_k = 1,$$

where

$$\bar{x}(T) = \bar{x}_{\tau, T}(T; \bar{x}(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)),$$

and  $\tilde{F}$  is a convex functional.

We should note that an objective function (functional)  $F_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))$  is a convex scalar convolution of a vector functional

$$\Phi_{\tau, T} = (\Phi_{\tau, T}^{(1)}, \Phi_{\tau, T}^{(2)}, \dots, \Phi_{\tau, T}^{(r)}).$$

That is, we construct a function in accordance with the method of scalarization of vector objective functions with inseparable weight coefficients  $\mu_k, k \in \overline{1, r}$ . We can determine coefficients, for example, expertly or on the basis of knowledge of statistical information about the history of the realization of main parameters of the considered MP.

Then, on the basis of the above, we can formulate the problem of a minimax multi-level MP for the dynamic system (1)–(5), (10) in this way.

Let us assume that by influencing it by a possible selection of its permissible managements  $\bar{u}(\cdot) \in U(\tau, T)$ ,  $P$  player is interested in such a result of MP at the interval of time  $\tau, T \subseteq 0, T$  ( $\tau < T$ ) when a functional  $F_{\tau, T}$ , is defined by the ratio (10) and takes the least possible value, considering that the worst risk value for it can be realized. Thus, it maximizes the functional, and realizations  $\bar{w}(\cdot) \in W(\tau, T; \bar{u}(\cdot))$  contribute to the achievement of the objective of  $P$  player.

The achievement of the objective by  $P$  player realizes within a framework of the solution of a nonlinear multi-level problem of the minimax multi-level terminal management for the dynamic system (1)–(5), (10), which describes MP in the presence of risks [3, 5].

For a fixed time interval  $\tau, T \subseteq 0, T$  ( $\tau < T$ ) and the realization of the  $\tau$ -position  $g(\tau) = \{\tau, \bar{x}(\tau)\} \in \hat{G}(\tau)$  ( $g(0) = g_0$ ) of  $P$  player in the dynamical system (1)–(5), (10), which describes MP, we should find a set

$$U_F^{(e)}(\tau, T, g(\tau)) \subseteq U(\tau, T)$$

of minimax managements  $\bar{u}^{(e)}(\cdot) \in U(\tau, T)$  of  $P$  player, which we determine by the ratio:

$$U_F^{(e)}(\tau, T, g(\tau)) = \{\bar{u}^{(e)}(\cdot): \bar{u}^{(e)}(\cdot) \in U(\tau, T),$$

$$F_{\tau, T}^{(e)} = \max_{\bar{u}(\cdot) \in V(\tau, T; \bar{u}^{(e)}(\cdot))} F_{\tau, T}(g(\tau), \bar{u}^{(e)}(\cdot), \bar{w}^{(e)}(\cdot), \bar{v}(\cdot)) = \min_{\bar{u}(\cdot) \in W(\tau, T; \bar{u}^{(e)}(\cdot))} \max_{\bar{v}(\cdot) \in V(\tau, T; \bar{u}^{(e)}(\cdot))} F_{\tau, T}(g(\tau), \bar{u}^{(e)}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = \min_{\bar{u}(\cdot) \in U(\tau, T)} \min_{\bar{w}(\cdot) \in W(\tau, T; \bar{u}(\cdot))} \max_{\bar{v}(\cdot) \in V(\tau, T; \bar{u}(\cdot))} F_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot)) = \min_{\bar{u}(\cdot) \in U(\tau, T)} \min_{\bar{w}(\cdot) \in W(\tau, T; \bar{u}(\cdot))} \max_{\bar{v}(\cdot) \in V(\tau, T; \bar{u}(\cdot))} \tilde{F}(\bar{x}_{\tau, T}(g(\tau), \bar{u}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))) = \min_{\bar{u}(\cdot) \in W(\tau, T; \bar{u}^{(e)}(\cdot))} \max_{\bar{v}(\cdot) \in V(\tau, T; \bar{u}^{(e)}(\cdot))} \tilde{F}(\bar{x}_{\tau, T}(g(\tau), \bar{u}^{(e)}(\cdot), \bar{w}(\cdot), \bar{v}(\cdot))) = \max_{\bar{v}(\cdot) \in V(\tau, T; \bar{u}^{(e)}(\cdot))} \tilde{F}(\bar{x}_{\tau, T}(g(\tau), \bar{u}^{(e)}(\cdot), \bar{w}^{(e)}(\cdot), \bar{v}(\cdot))) = c_F^{(e)}(\tau, T, g(\tau))$$
(11)

as the realization of a finite sequence of one-step operations only.

Here we determine the functional  $F_{\tau, T}^{(e)}$  by ratio (10).

We call the number  $c_F^{(e)}(\tau, T, g(\tau)) = F_{\tau, T}^{(e)}$  a guaranteed (minimax) result of a process of the minimax multi-level MP for  $P$  player at the time interval  $\tau, T$  for the discrete dynamic system (1)–(5), (10) with respect to its  $g(\tau)$  position and a functional  $F_{\tau, T}$ .

We consider the finiteness of sets of permissible managements  $U(\tau, T)$  and permissible intensities of replenishment of production and investment resources  $W(\tau, T; \bar{u}(\cdot))$ , which correspond to fixed management  $\bar{u}(\cdot) \in U(\tau, T)$ , and the ratios (10), (11). Then we can show that a solution to the problem of MP PA of an enterprise taking into account risks exists. It is reduced to the solution of a finite number of problems of linear and convex mathematical programming, as well as a finite number of discrete optimization problems.

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## 7. Implementation of the study results

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We implemented results of the study for MP of bioethanol production. Bioethanol is a product of sugar fermentation by the technology used in the production of beer and food spirits. Ethanol is separated from brew in distillation columns, nest it is purified in rectification columns additionally, at the output of which we obtain a mixture of ethanol and water. At the dewatering stage, the residue of water is removed from the mixture and anhydrous bioethanol is obtained. In order to obtain fuel ethanol, raw alcohol should be removed from water. Water can be removed with a help of alternative technologies: molecular sieves or diffusion evaporation through a membrane. We chose an effective technological method with a help of the proposed method.

Let us consider MP modeling on the example of the state enterprise (SE) “Ivashkivsky Alcohol Plant” (Ukraine). Previously, the plant specialized in the production of molasses spirits, which were supplied to alcoholic beverage plants.

But the market for such alcohol was limited, therefore, management decided to re-profile the plant for the production of technical products, in particular bioethanol. Raw material for its production is molasses – wastes from the production of sugar factories. It was decided to supplement the technological chain of the plant with equipment for dehydration of alcohol. It is necessary to determine which alternative MP of bioethanol production is more effective – a process based on the use of molecular sieves or diffusion evaporation through a membrane.

Let us build a model of MP PA of the State Enterprise “Ivashkivsky Alcohol Plant”. As the company works for one year only, we will consider a management interval equal to one year to study MP. We broke it up quarterly for four periods with a period of three months. We studied bioethanol production with a use of molecular sieves technology and diffusion evaporation technology as alternative MPs.

Thus, we consider separate MP vectors of a set of possible alternatives as a model example:

1) for the application of the technology of molecular sieves (first MP);

2) on the basis of diffusion evaporation technology (second MP).

The first component of a management vector is the output volumes at each time period based on the application of diffusion evaporation technology.

On the basis of normative-technological specifications (GOST 667-73, GOST 2184-77, GOST 857-95, GOST 3118-77, TU 6-01-193-80, etc.) we found that the following material and labor resources  $w(t)$  were used during the production of bioethanol: molasses ( $t$ ), gasoline ( $t$ ), MTBE ( $t$ ), sulfuric acid (monohydrate) (kg), ammophos (kg), carbamide (kg), oleic acid (kg), NABAK (kg), labor of workers, (thousand people per year), gas (thousand  $m^3$ ), electricity (thousand KW per year).

Based on the analysis of information from journals of the violation of MP technological regulations, information from service centers for used technological equipment, current financial accounting, the execution of contracts for the supply of raw materials and materials of the enterprise “Ivashkivsky Alcohol Plant” with suppliers of resources, we revealed that the most significant risks for the investigated enterprise and the corresponding MP were a breakdown of technological equipment during its debugging to start MP, insufficiency of needed volumes of molasses and an increase in prices for it, as well as violation of rules of the technological process due to the instability of energy supply.

On the basis of the analysis of the financial accounting of the enterprise “Ivashkivsky Alcohol Plant” we found that financial and economic risks affecting the financial result of the enterprise during the introduction of the considered technologies during the researched year were not significant for each MP, except for an increase of prices for raw materials (molasses), which is also the same for both the first and the second MP.

We calculated a complex functional  $F$  for each pair of vectors  $u(t)$  and  $v(t)$  using the scalar convolution method with the weighted coefficients found on the basis of a factor load calculations of a partial target criteria:  $F(u_1(T), v_1(T))=0,47$ ;  $F(u_2(T), v_2(T))=0,59$ .

Thus, the result of MP will be guaranteed (minimax) for a vector of management  $u_2(T)$  with a risk vector  $v_2(T)$ . Table 1 presents complex modeling results.

Table 1

Results of the modeling of MP PA management of the State Enterprise “Ivashkivsky Alcohol Plant”

Period	Value of a vector of PA management: output product volumes $u(t)$ , vectors of replenishment of material and labor resources $w(t)$ and investments $I(t)$
1	$u(1)=3000$ ; $w(1)=(10,000; 5; 0.5; 70; 5; 3; 3; 0.2; 71.52; 1,367.07; 721.5)$ ; $I(1)=2,000$
2	$u(2)=3,100$ ; $w(2)=(14,350; 40; 5; 75; 4.1; 3.5; 3.5; 0; 73.904; 1,412.64; 745.55)$ ; $I(2)=1,800$
3	$u(3)=3,050$ ; $w(3)=(14,500; 38.3; 4; 75.5; 6.5; 2.1; 3.4; 0; 72.712; 1,344.29; 733.525)$ ; $I(3)=500$
4	$u(4)=2,950$ ; $w(4)=(14,000; 35.7; 4; 71.5; 5; 2.7; 4; 0; 70.328; 1,389.86; 709.475)$ ; $I(4)=100$

Thus, it is possible to conclude that for the dehydration of alcohol in the production of bioethanol taking into account the permissible risks, a more effective MP will be MP with a use of molecular sieve technology than diffusion evaporation. This option of the plant’s operation makes possible to reduce the technological scheme, to reduce the cost of maintenance of MP PA and to increase the profitability of production.

## 8. Discussion of the study results

The advantage of the conducted researches is that they make possible to solve the problem of dynamic optimization of the process of multi-level management of MP of an enterprise taking into account risks. We applied an optimization tools, in which, unlike the methods of dynamic programming and the principle of Pontryagin’s maximum, we used a method of construction of predictive sets (areas of reach) of states of MP management at the end point of the management interval.

The proposed method makes possible to reduce the multi-step problem of MP management of an enterprise taking into account risks to the realization of a finite sequence of one-step optimization problems. Thus, the obtained research results give possibility to avoid difficulties connected with the large dimensionality of the original problem.

The disadvantages of the study include the fact that risks taken into account in the model are deterministic values only. This fact causes difficulties in determination of the magnitude of potential losses associated with influence of risks. Moreover, situation with risks with probabilistic values may arise under real-life conditions of MP of an enterprise.

Thus, the development of the study may consist in taking into account risks of a stochastic nature. In this case, it would be appropriate to introduce deterministic and stochastic risks into the model of a multi-level management of enterprise MP.

In the future, the results of the study can become a basis for development of a software management system for an enterprise MP. This will make it possible to automate the

process of multi-level management of MP and to adjust specific enterprises to working conditions.

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## 9. Conclusions

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1. We investigated problems of MP PA. We formulated a substantial problem of multi-level management of enterprise MP in the presence of risks. We determined that the following features complicate the model of MP PA: incompleteness and lack of information, heterogeneity and non-stationary nature of parameters of the model, variability of production technologies, etc.

2. We formalized the model of multi-level management of a production process of an enterprise for the organization of the minimax multi-level management in the chosen class of permissible management strategies.

3. We found the solution to the formulated problem of the minimax multi-level management. The solution makes

possible to get the optimal guaranteed (minimax) result of MP PA of an enterprise taking into account risks. We proposed a method based on the construction of predictive sets (areas of reach) of the dynamical model under consideration. Application of such a method will make possible to reduce the initial multistep problem of dynamic optimization to the realization of a finite sequence of one-step problems of discrete optimization. We carried out approbation of the model conditions of real production. We implemented the obtained results in MP management of “Ukrspirt” Concern on the example of the state enterprise “Ivashkivsky Alcohol Plant”. The application of the results made it possible to reduce the technological scheme, to reduce costs for MP provision and influence of risks. The most significant of them were a breakdown of technological equipment during its adjustment to launch MP, a lack of necessary quantities of raw materials (molasses) and an increase in prices for it, violation of rules of the technological process due to instability of energy supply.

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