Розроблено підхід, який поєднує семантичне навчання, гранулярне розбиття та розв'язання нечітких реляційних рівнянь для побудови точних та інтерпретабельних правил. Запропоновано сполучену нечітку модель прямого логічного виведення на основі первинних правил з гранулярними параметрами. Розроблено метод ієрархічного налаштування з лінгвістичною модифікацією на основі розв'язання нечітких реляційних рівнянь, що скорочує час навчання

Ключові слова: ієрархічне налаштування, класифікаційні нечіткі бази знань, розв'язання нечітких реляційних рівнянь

Разработан подход, который объединяет семантическое обучение, гранулярное разбиение и решение нечетких реляционных уравнений для построения точных и интерпретабельных правил. Предложена составная нечеткая модель прямого логического вывода на основе первичных правил с гранулярными параметрами. Разработан метод иерархической настройки с лингвистической модификацией на основе решения нечетких реляционных уравнений, что сокращает время обучения

Ключевые слова: иерархическая настройка, классификационные нечеткие базы знаний, решение нечетких реляционных уравнений

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CLASSIFICATION RULE HIERARCHICAL TUNING WITH LINGUISTIC MODIFICATION BASED ON SOLVING FUZZY RELATIONAL EQUATIONS

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1. Introduction

Hierarchical tuning is used to ensure the accuracy and interpretability of fuzzy models. The model of the highest level of the hierarchy is based on the primary terms that determine the semantic trend (*increase, decrease*). The model of the lowest level is constructed using linguistic modifiers (*strong, weak*) that reflect the semantic intensity of the primary terms. The modified candidate rules are generated for each primary rule, and the knowledge base is subject to further selection and reduction [1]. The linguistic modification is carried out by the concentration of the primary term with the subsequent shift. As a result, the problem of interpretability ensuring requires significant computing resources [1].

The reduction of complexity is provided by the method of hierarchical granular clustering, which carries out primary partition with the subsequent refinement of granules within primary classes [2]. The solution to the problem of rule selection may be the use of fuzzy relational equations [3], the solutions of which represent the linguistic modification of the primary terms. Therefore, it is important to develop a composite approach combining the benefits of semantic training, granular partition and fuzzy relational equations in simplification of the process of hierarchical tuning of fuzzy classification knowledge bases.

2. Literature review and problem statement

Hierarchical tuning requires the definition of conditions for modification of the primary rules, as well as the selection of the modified candidate rules [4, 5]. For this purpose, models of linguistic modifiers are developed. In semantic models, the linguistic modifier describes the significance measure of the primary term or hedging threshold [6–8]. Then the conditions of partition are associated with determining the hedging threshold, and selection is based on the relationships of semantic ordering [6].

The combination of advantages of the accuracy of granular models and interpretability of linguistic models has led to the emergence of the composite approach to tuning [9–12]. The linguistic modification is accomplished by the partition of the primary granules. The primary rules are consistently selected according to the contribution to the classification error [9]. The refined rules are formed using the methods of hierarchical or conditional fuzzy clustering [10, 11]. Such systems are regarded as partially granular, since the condition of the partition of the primary granule is determined by its description [11]. The interaction of the rules is provided by the granular parameters of the primary linguistic model, and the composite method of partition is provided by the flexible type of the primary membership function [12].

The incremental approach [9–12] accelerates the generation of candidate rules, but complicates selection. The hierarchical selection requires the choice of the best configuration of the primary rules, whose linguistic modification ensures the inference accuracy [9]. The common problem with hierarchical tuning methods is the lack of conditions for modification of the primary rules. As a result, both primary and modified rules are subject to selection [13].

In [14–16], the method for tuning of classification rules based on the inverse logic inference has been proposed. In [14–16], the primary relational model has been used that did not require primary selection. The hedging threshold of the primary terms has been determined by solutions of the system of fuzzy relational equations with extended max-min composition. In [17], the method of linguistic modification of the primary relational rules has been proposed. To do this, the transition to the primary system of equations with the hierarchical max-min/min-max composition has been carried out [3]. The solution of such a system of equations solves the problem of selection of the primary and modified rules, which simplifies the process of generation of candidate rules. However, the composite model [17] can yield in accuracy, since the primary model remains relational. The method of space partition of the input variables of such a model excludes the application of the primary rules with granular parameters. Finally, the model [17] needs to be tuned to experimental data.

Unlike [17], the composite fuzzy model of direct logic inference is developed on the basis of the primary rules with granular parameters. The method of tuning such model to experimental data is the method of classification rule hierarchical tuning with the linguistic modification based on solving fuzzy relational equations. In this case, the properties of the model [17] allow reducing the complexity of the problem of structural identification. In the first stage, the primary rules are tuned, and the modification conditions in the solutions of the primary system of equations are determined. In the second stage, the parameters of the granular solutions in the modified rules are tuned.

3. The aim and objectives of the study

The aim of the work is to develop the method of classification rule hierarchical tuning with the linguistic modification based on solving fuzzy relational equations. The method should ensure the construction of accurate and interpretable knowledge bases. In this case, the hierarchical selection of the primary and modified rules should simplify the tuning process.

To achieve this aim, the following objectives were accomplished:

– to develop a composite fuzzy model of direct logic inference based on the primary rules with granular parameters;

– to develop a genetic-neural algorithm of hierarchical tuning.

4. Models and method of classification rule hierarchical tuning

4. 1. Composite fuzzy model of direct logic inference

For an object of the form $y=f(\mathbf{X})$ with n inputs $\mathbf{X}=(x_1,..., x_n)$ and the output y, the “input – output” relationship can be represented as a system of classification fuzzy IF-THEN rules:

$$\bigcup_{k=\overline{1,T}}\Big[\bigcap_{i=\overline{1,n}}(x_i=A_{ik})\Big]\rightarrow y=E_J,\quad J=\overline{1,M}, \tag{1}$$

$$\bigcup_{k=\overline{1,T}}\Big[\bigcup_{p=\overline{1,z_{jk}}}\Big\{\bigcap_{i=\overline{1,n}}(\mu^{A_{ik}}(x_i)=\alpha_i^{jk,p})\Big\}\Big]\rightarrow y=d_j,\quad j=\overline{1,m}, \tag{2}$$

where E_J and d_j are the primary and modified terms for estimating the variable y, $J=\overline{1,M}$, $j=\overline{1,m}$; M and m are the numbers of the output terms; A_{ik} is the primary term for estimating the variable x_i, $i=\overline{1,n}$, in the rule k, $k=\overline{1,T}$; T is the number of the primary rules; $\mu^{A_{ik}}$ is the significance measure of the primary term A_{ik}; $\alpha_i^{jk,p}$ is the fuzzy quantifier that describes the significance measure $\mu^{A_{ik}}$ in the rule with the number $p=\overline{1,z_{jk}}$ of the class d_j; z_{jk} is the number of the composite rules for the primary rule k in the class d_j.

The following system of fuzzy logic equations with hierarchical max-min/min-max composition corresponds to the primary fuzzy knowledge base (1) [17]:

$$\mu^{E_J}(y)=\max_{k=\overline{1,T}}(\min(\mu^{H_k}(\mathbf{X}),r_k^J)),\quad J=\overline{1,M}, \tag{3}$$

$$\mu^{H_k}(\mathbf{X})=\min_{i=\overline{1,n}}(\mu^{A_{ik}}(x_i)),\quad k=\overline{1,T}, \tag{4}$$

where $\mu^{E_J}(y)$ is the membership function of the variable y to the term E_J; $\mu^{H_k}(\mathbf{X})$ is the membership function of the vector $\mathbf{X}$ to the rule H_k; $\mu^{A_{ik}}(x_i)$ is the membership function of the variable x_i to the term A_{ik}; r_k^J is the weight of the primary rule H_k in the class E_J, $r_k^J\in[0,1]$.

The following system of fuzzy logic equations corresponds to the composite knowledge base (2):

$$\mu^{d_j}(y)=\max_{k=\overline{1,T}} v_{jk}[\max_{p=\overline{1,z_{jk}}} w_{jk,p}\{\min_{i=\overline{1,n}}(\mu_i^{jk,p}(x_i))\}],\quad j=\overline{1,m}, \tag{5}$$

where v_{jk} is the weight of the primary rule H_k in the class d_j, $v_{jk}=1(0)$; $w_{jk,p}$ is the weight of the composite rule with the number jk,p in the class d_j, $w_{jk,p}\in[0, 1]$; $\mu^{d_j}(y)$ is the membership function of the variable y to the class d_j; $\mu_i^{jk,p}(x_i)$ is the membership function of the variable x_i to the composite term $a_i^{jk,p}=(A_{ik},\alpha_i^{jk,p})$.

If the value of the variable x in (4) is given by the fuzzy term $\tilde{x}^*$, then the degree of membership $\mu^A(\tilde{x}^*)$ is defined as follows [18]:

$$\mu^A(\tilde{x}^*)=\sup_{x\in[\underline{x},\overline{x}]}[\min(\mu^A(x,\beta,\sigma),\mu^{\tilde{x}^*}(x))],$$

where $\mu^A(x, \beta, \sigma)$ and $\mu^{\tilde{x}^*}(x)$ are the membership functions of the fuzzy terms A and $\tilde{x}^*$; β is the coordinate of the maximum of the function μ^A; σ is the concentration parameter [17].

The relations (3)–(5) determine the composite fuzzy model of direct logic inference based on the primary rules in the form:

$$\boldsymbol{\mu}^{\mathrm{E}}(y, \mathbf{B}_{\mathrm{E}}, \boldsymbol{\Omega}_{\mathrm{E}})=f_R(\mathbf{X}, \mathbf{R}, \mathbf{B}_{\mathrm{A}}, \boldsymbol{\Omega}_{\mathrm{A}}), \tag{6}$$

$$y=f_r(\mathbf{X}, f_R, Z, q, \mathbf{V}, \mathbf{W}, \mathbf{B}_{\mathrm{a}}, \boldsymbol{\Omega}_{\mathrm{a}}, \mathbf{B}_{\mathrm{d}}, \boldsymbol{\Omega}_{\mathrm{d}}), \tag{7}$$

where $\boldsymbol{\mu}^E=(\mu^{E_1},...,\mu^{E_M})$ is the fuzzy effects vector;

$$\mathbf{R}\subseteq H_k\times E_J=[r_k^J, k=\overline{1,T}, J=\overline{1,M}]$$

is the weight matrix of the primary rules in the knowledge base (1);

$$\mathbf{B}_A=(\beta^{A_1},...,\beta^{A_P}),\ \boldsymbol{\Omega}_A=(\sigma^{A_1},...,\sigma^{A_P}),$$

$$\mathbf{B}_E=(\beta^{E_1},...,\beta^{E_M}),\ \boldsymbol{\Omega}_E=(\sigma^{E_1},...,\sigma^{E_M})$$

are the vectors of the β and σ parameters of the membership functions of the fuzzy terms A_I and E_J; P is the number of the primary input terms in the knowledge base (1); q and Z is the number of the composite input terms and rules in the knowledge base (2); $\mathbf{V}=(v_{11},..., v_{1T},..., v_{m1},..., v_{mT})$ and $\mathbf{W}=(w_1,..., w_Z)$ are the weight vectors of the primary and composite rules in the knowledge base (2);

$$\mathbf{B}_a=(\beta^{a_1},...,\beta^{a_q}),\ \ \mathbf{\Omega}_a=(\sigma^{a_1},...,\sigma^{a_q}),$$

$$\mathbf{B}_d=(\beta^{d_1},...,\beta^{d_m}),\ \ \mathbf{\Omega}_d=(\sigma^{d_1},...,\sigma^{d_m})$$

are the vectors of the β and σ parameters of the membership functions of the fuzzy terms a_l and d_j; f_R and f_r are the connection operators for the primary (1) and composite (2) rules.

4. 2. The problem of tuning the composite fuzzy model

Let the training data set be given in the form of L pairs of experimental data: $\langle \hat{\mathbf{X}}_p, \hat{d}_p \rangle$, $\hat{d}_p \in \{d_1,...,d_m\}$ – for tuning the primary fuzzy model; $\langle \hat{\mathbf{X}}_p, \hat{y}_p \rangle$ – for tuning the composite fuzzy model, where $\hat{\mathbf{X}}_p=(\hat{x}_1^p,...,\hat{x}_n^p)$ and $\hat{d}_p$ ($\hat{y}_p$) is the vector of the values of the input variables and the output class (the value of the output variable) in the experiment with the number p, $p=\overline{1,L}$.

The essence of tuning the fuzzy model (6) is as follows. It is necessary to find the weight matrix of the rules **R**, the parameter vectors of the membership functions of the inputs $\mathbf{B}_A$, $\mathbf{\Omega}_A$ and the output $\mathbf{B}_E$, $\mathbf{\Omega}_E$, $\mathbf{B}_d$, $\mathbf{\Omega}_d$, which provide the minimum distance between the model and experimental fuzzy effects vectors:

$$\sum[f_R(\hat{\mathbf{X}}_p,\mathbf{R},\mathbf{B}_A,\mathbf{\Omega}_A)-\hat{\mu}\ (\hat{d}_p,\mathbf{B}_E,\mathbf{\Omega}_E,\mathbf{B}_d,\mathbf{\Omega}_d)]\ =\min_{\mathbf{R},\mathbf{B},}. \quad (8)$$

The essence of tuning the fuzzy model (7) is as follows. It is necessary to find the weight vectors of the primary and composite rules **V**, **W** and the parameter vectors of the membership functions of the inputs $\mathbf{B}_a$, $\mathbf{\Omega}_a$, which provide the minimum distance between the model and experimental outputs of the object:

$$\sum[f_r(\hat{\mathbf{X}}_p,f_R,Z,q,\mathbf{V},\mathbf{W},\mathbf{B}_a,\mathbf{\Omega}_a)-\hat{y}_p]\ =\min_{Z,q,\mathbf{V},\mathbf{W},\mathbf{B},}. \quad (9)$$

The number of rules Z and terms q of the composite model is determined by solving the primary system of fuzzy logic equations.

Statement. The β parameters of classification rules of the form:

$$\bigcup_{k=\overline{1,T}}[\bigcup_{p=\overline{1,z_{jk}}}\{\bigcap_{i=\overline{1,n}}(x_i\in[\underline{\beta}_i^{jk,p},\overline{\beta}_i^{jk,p}])\}]\to y=d_j,\quad j=\overline{1,m}, \quad (10)$$

are the solutions of the primary system of equations (3), (4) for the given output classes, that is, provide the minimum distance between the observed and model significance measures of effects [3, 17]:

$$\sum_{J=1}^{M}[\mu^{E_J}(d_j)-\max_{k=\overline{1,T}}(\min(\min_{i=\overline{1,n}}(\mu^{A_{ik}}(\beta_i^{jk,p})),r_k^J))]^2=\min_{\beta}. \quad (11)$$

Here $\underline{\beta}_i^{jk,p}$ ($\overline{\beta}_i^{jk,p}$) are the lower (upper) bounds of the coordinates of the maximum of membership functions of the composite terms $a_i^{jk,p}$ in the rule with the number jk, p.

Proof. Formula (10) follows from the properties of the set of solutions of the system of fuzzy logic equations with hierarchical max-min/min-max composition [3, 17]. Let:

$\boldsymbol{\mu}_j^H=(\mu_j^{H_1},...,\mu_j^{H_T})$ – the weight vector of the primary rules H_k in the class d_j;

Z_j – the number of the primary rules, selected for modification in the class d_j;

$\mathbf{B}_j=(\beta_1^j,...,\beta_n^j)$ – the coordinate vector of the maximum of membership functions in the composite rule in the class d_j, $j=\overline{1,m}$.

The weights of the primary rules in the class d_j are determined by a single maximum solution $\overline{\boldsymbol{\mu}}_j^H=(\overline{\mu}_j^{H_1},...,\overline{\mu}_j^{H_T})$ and a set of minimum solutions $\underline{\boldsymbol{\mu}}_{jl}^H=(\underline{\mu}_{jl}^{H_1},...,\underline{\mu}_{jl}^{H_T})$, $l=\overline{1,Z_j}$, of the system (3).

For each interval solution of the system (3), that is, for each primary rule with the weight $[\underline{\mu}_{jl}^{H_k},\overline{\mu}_j^{H_k}]$, $k=\overline{1,T}$, $l=\overline{1,Z_j}$, the system (4) has the set of solutions S_{jl}^k, which is determined by a single minimum solution $\underline{\mathbf{B}}_{jl}^k$ and a set of maximum solutions $\overline{S}_{jl}^k=\{\overline{\mathbf{B}}_{jl,h}^k,\ h=\overline{1,z_{jkl}}\}$:

$$S_{jl}^k=\bigcup_{\overline{\mathbf{B}}_{jl,h}^k\in\overline{S}_{jl}^k}[\underline{\mathbf{B}}_{jl}^k,\overline{\mathbf{B}}_{jl,h}^k]. \quad (12)$$

Here $\underline{\mathbf{B}}_{jl}^k=(\underline{\beta}_1^{jk,l},...,\underline{\beta}_n^{jk,l})$ and $\overline{\mathbf{B}}_{jl,h}^k=(\overline{\beta}_1^{jk,lh},...,\overline{\beta}_n^{jk,lh})$ are the vectors of the lower and upper bounds of the coordinates of the maximum β_i^j.

By taking the union over the subsets (12) with the primary rules, we obtain the set of solutions $S_j^*(\mathbf{R},d_j)$ of the system (3), (4), which is determined by the set of minimum solutions $\underline{S}_j^k=\{\underline{\mathbf{B}}_{jl}^k, l=\overline{1,Z_j}\}$:

$$S_j^*(\mathbf{R},d_j)=\bigcup_{\underline{\mathbf{B}}_{jl}^k\in\underline{S}_j^k}S_{jl}^k=\bigcup_{\underline{\mathbf{B}}_{jl}^k\in\underline{S}_j^k}\ \bigcup_{\overline{\mathbf{B}}_{jl,h}^k\in\overline{S}_{jl}^k}[\underline{\mathbf{B}}_{jl}^k,\overline{\mathbf{B}}_{jl,h}^k]. \quad (13)$$

Then the set of interval solutions (13) for the class d_j has the form:

$$S_j^*(\mathbf{R},d_j)=\bigcup_{k=\overline{1,T}}\left[\bigcup_{p=\overline{1,z_{jk}}}[\underline{\mathbf{B}}_{jp}^k,\overline{\mathbf{B}}_{jp}^k]\right],\quad j=\overline{1,m}. \quad (14)$$

Since for the hierarchical composition, the solution $\mathbf{B}_{jp}^k$ in the set (14) is interpreted as $\bigcap_{i=\overline{1,n}}[\underline{\beta}_i^{jk,p},\overline{\beta}_i^{jk,p}]$, we obtain the formula (10). The hedging threshold of the primary terms A_{ik} and E_J is determined by the bounds of the significance measures $[\underline{\alpha}_i^{jk,p},\overline{\alpha}_i^{jk,p}]$ in the solutions of the system of equations (3), (4), and the significance measures $(\mu^{E_1},...,\mu^{E_M})$ in the fuzzy effects vector $\boldsymbol{\mu}^E$.

4. 3. Genetic-neural tuning algorithm

The genetic-neural method is developed in accordance with [19, 20] for tuning the primary rules and solving the system of fuzzy logic equations, as well as with [21] for tuning the composite rules.

To solve the optimization problem (8), the chromosome encodes the structure and parameters of the primary rules; optimization problem (11) – the structure of the composite rules; optimization problem (9) – the parameters of the composite rules. The fitness function is constructed on the basis of the criteria (8), (11), (9).

The cross-over operation consists in the exchange of parts of the chromosomes in the weight matrix $\mathbf{R}$ and the parameter vectors of the membership functions $\mathbf{B}_A$, $\mathbf{\Omega}_A$, $\mathbf{B}_E$, $\mathbf{\Omega}_E$, $\mathbf{B}_d$, $\mathbf{\Omega}_d$; in the weight vectors of the primary rules μ_j^H and coordinate vectors of the maximum $\mathbf{B}_{jp}^k$; in the weight vector of the rules $\mathbf{W}$ and the parameter vectors of the membership functions $\mathbf{B}_a$, $\mathbf{\Omega}_a$.

For the neural tuning of the model, the primary and composite fuzzy rules were implanted into a composite neuro-fuzzy network (Fig. 1).

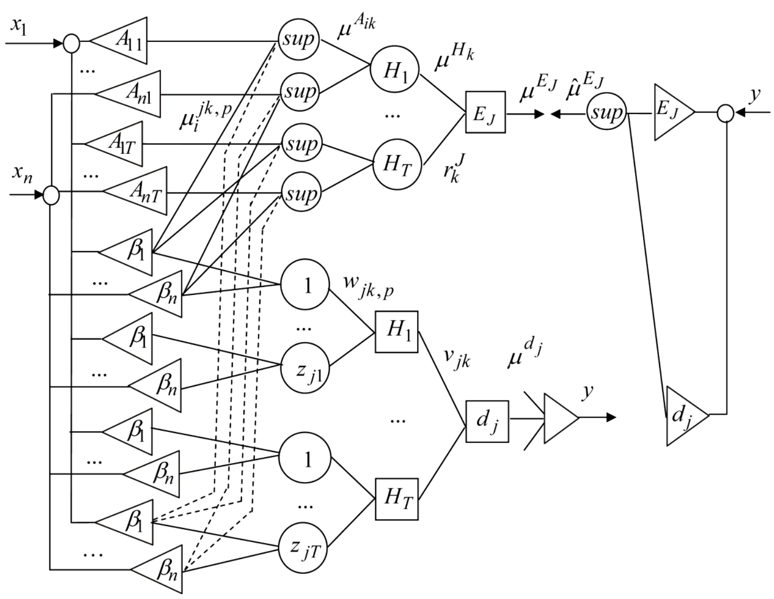

Fig. 1. Composite neuro-fuzzy model

For tuning the structure and parameters of the primary fuzzy rules, the recurrence relations are used:

$$r_k^J(t+1)=r_k^J(t)-\eta\frac{\partial\varepsilon_t^R}{\partial r_k^J(t)};$$

$$\beta^{A_I}(t+1)=\beta^{A_I}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\beta^{A_I}(t)};$$

$$\sigma^{A_I}(t+1)=\sigma^{A_I}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\sigma^{A_I}(t)};$$

$$\beta^{E_J}(t+1)=\beta^{E_J}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\beta^{E_J}(t)};$$

$$\sigma^{E_J}(t+1)=\sigma^{E_J}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\sigma^{E_J}(t)};$$

$$\beta^{d_j}(t+1)=\beta^{d_j}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\beta^{d_j}(t)};$$

$$\sigma^{d_j}(t+1)=\sigma^{d_j}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\sigma^{d_j}(t)}, \quad (15)$$

which minimize the criterion

$$\varepsilon_t^R=\frac{1}{2}(\hat{\mu}^E(t)-\mu^E(t))^2, \quad (16)$$

where $\hat{\mu}^E(t)$ ($\mu^E(t)$) is the experimental (theoretical) fuzzy effects vector on the t-th training step; $r_k^J(t)$ are the weights of the primary rules on the t-th training step; $\beta^{A_I}(t)$, $\sigma^{A_I}(t)$, $\beta^{E_J}(t)$, $\sigma^{E_J}(t)$ are the parameters of the membership functions of the primary terms on the t-th training step; $\beta^{d_j}(t)$, $\sigma^{d_j}(t)$ are the parameters of the membership functions of the composite output terms on the t-th training step; η is the training parameter.

For tuning the structure of the composite fuzzy rules, the recurrence relations are used:

$$\mu_j^{H_k}(t+1)=\mu_j^{H_k}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\mu_j^{H_k}(t)},$$

$$\beta_i^{jk,p}(t+1)=\beta_i^{jk,p}(t)-\eta\frac{\partial\varepsilon_t^R}{\partial\beta_i^{jk,p}(t)}, \quad (17)$$

which minimize the criterion (16), where $\mu_j^{H_k}(t)$ are the weights of the primary rules in the class d_j on the t-th training step; $\beta_i^{jk,p}(t)$ are the coordinates of the maximum of membership functions of the composite input terms on the t-th training step.

For tuning the parameters of the composite fuzzy rules, the recurrence relations are used:

$$w_{jk,p}(t+1)=w_{jk,p}(t)-\eta\frac{\partial\varepsilon_t^r}{\partial w_{jk,p}(t)},$$

$$\beta_i^{jk,p}(t+1)=\beta_i^{jk,p}(t)-\eta\frac{\partial\varepsilon_t^r}{\partial\beta_i^{jk,p}(t)},$$

$$\sigma_i^{jk,p}(t+1)=\sigma_i^{jk,p}(t)-\eta\frac{\partial\varepsilon_t^r}{\partial\sigma_i^{jk,p}(t)}, \quad (18)$$

which minimize the criterion

$$\varepsilon_t^r=\frac{1}{2}(\hat{y}_t^r-y_t^r)^2,$$

where $\hat{y}_t^r$ (y_t^r) is the experimental (theoretical) output of the object on the t-th training step; $w_{jk,p}(t)$ are the weights of the composite rules on the t-th training step; $\sigma_i^{jk,p}(t)$ are the concentration parameters of membership functions of the composite input terms on the t-th training step.

The partial derivatives included in (15), (17), (18) are calculated according to [19–21].

5. Example: the problem of quality control of the wastewater treatment process

The problem of quality control of the process of primary wastewater treatment is considered [22]. Rules to be tuned are interpreted as solutions to the inverse problem of restoring the reasons for pollution. Observation data for 527 days of operation of treatment facilities were obtained from [23].

Input parameters are: x_1 – acidity, $x_1\in[7.3, 8.5]$; x_2 – biological demand of oxygen, $x_2\in[32, 517]$; x_3 – suspended substances, $x_3\in[104, 692]$; x_4 – volatile substances, $x_4\in[7.1, 93.5]$; x_5 – sediments, $x_5\in[1.0, 16.0]$; x_6 – conductivity, $x_6\in[0.64, 3.17]*10^3$. The output parameter is: y – the performance of suspended organics sedimentation, $y\in[5.3, 96.1]$.

The primary rules with weights are presented in Table 1, where the variables x_i and y were described by the *decrease*

(D) and *increase* (I) terms. The primary output terms in Table 1 were specified using linguistic modifiers: strongly (s), moderately (m), weakly (w). The tuned membership functions of the fuzzy terms E_J allowed obtaining the method of partition of the primary output granules into the modified granules $d_j=[\underline{\beta}^{d_j},\overline{\beta}^{d_j}]$ (Table 2).

Table 1

Primary fuzzy knowledge base

IF								THEN	
X	Weight	x_1	x_2	x_3	x_4	x_5	x_6	y	
H_1	0.51	D	D	D	I	D	D	D	sD mD wD
H_2	0.68	I	D	D	I	D	D		
H_3	0.80	D	I	D	I	D	D		
H_4	0.93	I	I	D	I	D	D		
H_5	0.45	D	D	D	I	D	I		
H_6	0.59	I	D	D	I	D	I		
H_7	0.67	D	I	D	I	D	I		
H_8	0.89	I	I	D	I	D	I		
H_9	0.75	D	D	I	D	D	D	I	sI mI wI
H_{10}	0.62	D	D	I	I	D	D		
H_{11}	0.91	D	D	I	D	I	D		
H_{12}	0.56	D	D	I	I	I	D		

Table 2

Method of partition of primary output granules

d_j	Hedging threshold	Modified granules	d_j	Hedging threshold	Modified granules
sD	(0.86, 0.21)	[18.25, 35.11]	sI	(0.18, 0.95)	[67.70, 89.14]
mD	(0.65, 0.29)	[27.54, 47.61]	mI	(0.28, 0.74)	[52.17, 77.75]
wD	(0.48, 0.36)	[36.23, 54.30]	wI	(0.30, 0.53)	[49.85, 67.04]

The total number of the primary input terms (D (I)) in Table 1 is: $A_{1(2)}$, $A_{3(4)}$, $A_{5(6)}$, $A_{7(8)}$, $A_{9(10)}$, $A_{11(12)}$. The tuned membership functions of the fuzzy terms A_I, I=1,...,12, allowed obtaining the method of partition of the primary input granules into the modified granules $a_{il}=[\underline{\beta}^{a_{il}},\overline{\beta}^{a_{il}}]$, l=1,...,7 (Table 3). Granular solutions of the system of equations (3), (4) are presented in Table 4.

For the relational primary rules from Table 1, the mean square error is 5.2918, and for the rules with granular parameters – 3.9295. The resulting granular solutions provide the approximation of productivity y to the experimental data presented in Fig. 2.

The modified rules are given in Table 5.

Table 3

Method of partition of primary input granules

x_i	q_i	Hedging threshold	Modified granules	q_i	Hedging threshold	Modified granules
1	2	3	4	5	6	7
x_1	A_1^1	[0.91, 1]	[7.30, 7.41]	A_2^1	[0.86, 1]	[8.27, 8.50]
	A_1^2	[0.74, 0.91]	[7.41, 7.55]	A_2^2	[0.65, 0.86]	[8.15, 8.27]
	A_1^3	[0.65, 0.86]	[7.50, 7.62]	A_2^3	[0.48, 0.65]	[8.04, 8.15]
	A_1^4	[0.53, 0.74]	[7.55, 7.72]	A_2^4	[0, 0.65]	[7.30, 8.15]
	A_1^5	[0.48, 0.65]	[7.62, 7.77]			
	A_1^6	[0.36, 0.48]	[7.77, 7.91]			
	A_1^7	[0, 0.53]	[7.72, 8.50]			
x_2	A_3^1	[0.74, 1]	[32.00, 135.70]	A_4^1	[0.65, 1]	[390.60, 517.00]
	A_3^2	[0.48, 0.74]	[135.70, 196.12]	A_4^2	[0.65, 0.86]	[390.60, 432.84]
	A_3^3	[0.36, 0.48]	[196.12, 235.49]	A_4^3	[0.48, 0.65]	[352.19, 390.60]
	A_3^4	[0, 0.53]	[183.27, 517.00]	A_4^4	[0, 0.48]	[32.00, 352.19]
x_3	A_5^1	[0.86, 1]	[104.00, 168.41]	A_6^1	[0.91, 1]	[534.92, 692.00]
	A_5^2	[0.65, 0.86]	[168.41, 223.58]	A_6^2	[0.74, 0.91]	[485.02, 534.92]
	A_5^3	[0.41, 0.65]	[223.58, 298.11]	A_6^3	[0.53, 0.74]	[422.80, 485.02]
	A_5^4	[0, 0.48]	[272.60, 692.00]	A_6^4	[0.36, 0.53]	[354.16, 422.80]
				A_6^5	[0, 0.53]	[104.00, 422.80]

Continuation of Table 3

1	2	3	4	5	6	7
x_4	A_7^1	[0.91, 1]	[7.10, 21.90]	A_8^1	[0.86, 1]	[75.04, 93.50]
	A_7^2	[0.74, 0.91]	[21.90, 29.08]	A_8^2	[0.65, 0.86]	[67.28, 75.04]
	A_7^3	[0.53, 0.74]	[29.08, 39.15]	A_8^3	[0.41, 0.65]	[56.12, 67.28]
				A_8^4	[0, 0.48]	[7.10, 59.74]
x_5	A_9^1	[0.86, 1]	[1.00, 3.17]	A_{10}^1	[0.91, 1]	[13.26, 16.00]
	A_9^2	[0.65, 0.86]	[3.17, 4.86]	A_{10}^2	[0.74, 0.91]	[12.03, 13.26]
	A_9^3	[0.48, 0.65]	[4.86, 6.45]	A_{10}^3	[0.53, 0.74]	[10.51, 12.03]
	A_9^4	[0.41, 0.74]	[4.15, 7.28]	A_{10}^4	[0.36, 0.53]	[8.82, 10.51]
	A_9^5	[0, 0.48]	[6.45, 16.00]			
x_6	A_{11}^1	[0.74, 1]	[0.64, 1.21]	A_{12}^1	[0.65, 0.86]	[2.33, 2.57]
	A_{11}^2	[0.65, 0.86]	[1.08, 1.30]	A_{12}^2	[0.48, 0.65]	[2.10, 2.33]
	A_{11}^3	[0.53, 0.74]	[1.21, 1.44]	A_{12}^3	[0.30, 1]	[1.74, 2.98]
	A_{11}^4	[0.48, 0.65]	[1.30, 1.52]	A_{12}^4	[0, 0.48]	[0.64, 2.10]
	A_{11}^5	[0, 0.53]	[1.44, 3.17]			

Table 4

Granular solutions of the system of fuzzy logic equations

X	μ^H	IF						THEN
		x_1	x_2	x_3	x_4	x_5	x_6	y
1	2	3	4	5	6	7	8	9
H_3 H_4 H_8	0.86	A_1^3	$A_4^{2,3,4}$	A_5^1	$A_8^{2,3,4}$	$A_9^{2,3,5}$	$A_{11}^{2,3,5}$	*sD*
		A_2^1	$A_4^{2,3,4}$	A_5^1	A_8^1	A_9^1	$A_{11}^{2,3,5}$	
		$A_2^{2,4}$	$A_4^{2,3,4}$	A_5^2	A_8^1	$A_9^{2,3,5}$	A_{12}^1	
H_2 H_3 H_4 H_7 H_8	0.65	$A_2^{1,2,3}$	$A_3^{1,2}$	$A_5^{1,2}$	$A_8^{1,2}$	$A_9^{1,2}$	$A_{11}^{1,2}$	*mD*
		A_1^5	$A_4^{3,4}$	A_5^2	A_8^2	$A_9^{3,5}$	A_{11}^2	
		A_2^2	A_4^2	A_5^2	$A_8^{3,4}$	A_9^2	$A_{11}^{4,5}$	
		$A_1^{5,7}$	A_4^1	$A_5^{1,2}$	$A_8^{1,2}$	$A_9^{3,5}$	A_{12}^2	
		A_2^4	$A_4^{3,4}$	A_5^2	A_8^2	A_9^2	$A_{12}^{2,4}$	
H_1 H_2 H_3 H_4 H_5 H_6 H_7 H_8	0.48	$A_1^{1,3,5,6}$	A_3^3	$A_5^{1,2,3}$	$A_8^{1,2,3}$	$A_9^{1,2,3}$	$A_{11}^{1,2,4}$	*wD*
		A_2^4	$A_3^{1,2,3}$	A_5^3	A_8^3	A_9^3	A_{11}^4	
		A_1^6	A_4^4	A_5^3	A_8^3	A_9^5	$A_{11}^{1,2,4}$	
		A_2^3	A_4^4	A_5^3	A_8^4	A_9^5	A_{11}^4	
		$A_1^{1,3,5,6}$	$A_3^{1,2}$	$A_5^{1,2,3}$	$A_8^{1,2,3}$	$A_9^{1,2,3}$	A_{12}^4	
		$A_2^{1,2,3}$	$A_3^{1,2}$	$A_5^{1,2,3}$	$A_8^{1,2,3}$	$A_9^{1,2,3}$	A_{12}^3	
		A_1^7	A_4^3	A_5^4	A_8^3	A_9^3	A_{12}^4	
		A_2^4	A_4^3	A_5^4	A_8^4	A_9^3	A_{12}^3	
H_{10} H_{12}	0.36	$A_1^{1,3,5,6}$	A_3^3	A_6^4	$A_8^{1,2,3}$	$A_9^{1,2,3}$	A_{11}^5	
		A_1^7	$A_3^{1,2,3}$	A_6^4	$A_8^{1,2,3}$	A_{10}^4	$A_{11}^{1,2,4}$	

Continuation of Table 4

1	2	3	4	5	6	7	8	9
H_1 H_5	0.41	$A_1^{1,2,4}$	A_3^4	A_5^4	$A_8^{1,2,3}$	A_9^5	A_{11}^5	*wI*
H_9 H_{10} H_{11} H_{12}	0.53	A_1^7	A_3^4	A_5^3	$A_8^{1,2,3}$	A_9^5	A_{12}^3	
		A_1^4	A_3^2	A_6^3	A_7^3	$A_9^{1,2,4}$	A_{11}^3	
		$A_1^{1,2,4}$	$A_3^{1,2}$	A_6^5	$A_8^{1,2,3}$	$A_9^{1,2,4}$	$A_{11}^{1,3}$	
		A_1^4	A_3^2	A_6^3	A_7^3	A_{10}^3	A_{11}^5	
		A_1^7	$A_3^{1,2}$	A_6^5	$A_8^{1,2,3}$	$A_{10}^{1,2,3}$	$A_{11}^{1,3}$	
H_9 H_{11}	0.74	$A_1^{4,7}$	$A_3^{2,4}$	A_6^3	A_7^2	A_9^4	$A_{11}^{3,5}$	*mI*
		A_1^2	A_3^1	A_6^2	A_7^2	A_{10}^2	A_{11}^1	
H_{11}	0.91	A_1^1	A_3^1	A_6^1	A_7^1	A_{10}^1	A_{11}^1	*sI*

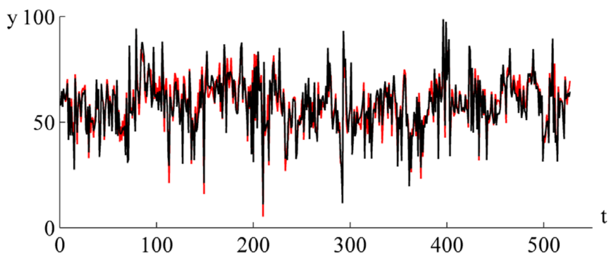

Fig. 2. Model (——) and experimental (——) productivity

Table 5

Modified fuzzy knowledge base

IF							THEN
X	x_1	x_2	x_3	x_4	x_5	x_6	y
H_3	*mD*	*wI-sI*	*sD*	*mI-sI*	*mD-sD*	*wD-sD*	*sD*
H_4	*sI*	*wI-sI*	*sD*	*sI*	*sD*	*wD-sD*	
H_8	*wI-sI*	*wI-sI*	*mD-sD*	*sI*	*mD-sD*	*mI*	
H_2	*mI-sI*	*wD-sD*	*mD-sD*	*mI-sI*	*mD-sD*	*mD-sD*	*mD*
H_3	*mD*	*wI-mI*	*mD*	*mI*	*wD-mD*	*mD*	
H_4	*mI*	*mI-sI*	*mD*	*wI-mI*	*mD*	*wD-mD*	
H_7	*wD-mD*	*mI*	*mD-sD*	*mI-sI*	*wD-mD*	*mI*	
H_8	*wI-mI*	*wI-mI*	*mD*	*mI*	*mD*	*wI-mI*	
H_1	*wD-sD*	*wD*	*mD-sD*	*mI-sI*	*mD-sD*	*mD-sD*	*wD*
H_2	*wI-mI*	*wD-sD*	*mD*	*mI*	*mD*	*mD*	
H_3	*wD*	*wI*	*mD*	*mI*	*wD*	*mD-sD*	
H_4	*mI*	*wI*	*mD*	*wI*	*wD*	*mD*	
H_5	*wD-sD*	*mD-sD*	*mD-sD*	*mI-sI*	*mD-sD*	*wI*	
H_6	*wI-sI*	*mD-sD*	*mD-sD*	*mI-sI*	*mD-sD*	*wI-mI*	
H_7	*wI-wD*	*mI*	*wD*	*mI*	*mD*	*wI*	
H_8	*wI-mI*	*mI*	*wD*	*wI*	*mD*	*wI-mI*	
H_{10}	*wD-sD*	*wD*	*wI*	*mI-sI*	*mD-sD*	*wD*	
H_{12}	*wD-wI*	*wD-sD*	*wI*	*mI-sI*	*mI*	*mD-sD*	
H_1	*mD-sD*	*wD-wI*	*wD-wI*	*wI-mI*	*wD-wI*	*wD-wI*	*wI*
H_5	*wD-wI*	*wD-wI*	*wD-mD*	*wI-mI*	*wD*	*wI-mI*	
H_9	*mD*	*mD*	*mI*	*mD*	*wD-sD*	*mD*	
H_{10}	*mD-sD*	*mD-sD*	*wI-mI*	*wI-sI*	*wD-sD*	*mD-sD*	
H_{11}	*mD*	*mD*	*mI*	*mD*	*mI*	*wD-wI*	
H_{12}	*wD-wI*	*mD-sD*	*wI-mI*	*wI-mI*	*mI-sI*	*mD-sD*	
H_9	*wD-mD*	*wD-mD*	*mI*	*mD-sD*	*wD-mD*	*wD-mD*	*mI*
H_{11}	*mD-sD*	*mD-sD*	*mI-sI*	*mD-sD*	*mI-sI*	*mD-sD*	
H_{11}	*sD*	*mD-sD*	*sI*	*sD*	*sI*	*mD-sD*	*sI*

The refined terms were described by the modifiers: *s*, *m*, *w* (*D*, *I*) for x_1–x_3; *s*, *m* (*D*) and *w*, *m*, *s* (*I*) for x_4; *s*, *m*, *w* (*D*) and *m*, *s* (*I*) for x_5; *s*, *m*, *w* (*D*) and *w*, *m* (*I*) for x_6.

6. Discussion of the results of effectiveness evaluation of classification rule hierarchical tuning

In [14–17], the method for tuning fuzzy classification knowledge bases based on the linguistic modification of the primary relational models has been proposed. This method develops these results for hierarchical tuning with the linguistic modification of the primary rules. The fundamental feature of the method is the transition to the primary granular model with the subsequent tuning to experimental data. As a result, the tuning time is reduced due to simplification of the hierarchical selection of the primary and modified rules.

The primary selection requires solving the optimization problem with Z_jT variables for the weights of the primary rules in the class d_j, $j=\overline{1,m}$ [13]. The solution of the system of equations with max-min composition allows reducing the complexity of the primary selection by solving Z_j optimization problems with T variables. Tuning of the primary model is an optimization problem with $2(M+m)+2nT$ variables for the weights of the rules and two-parameter membership functions.

Modification of each primary rule in the class d_j is carried out by solving the optimization problem with $2nz_{jk}$, $j=\overline{1,m}$, $k=\overline{1,Z_j}$, variables for the β parameters of the rules [9–12]. The solution of the system of equations with min-max composition allows reducing the complexity of rule generation by solving z_{jk} optimization problems with $2n$ variables for each primary rule in the class d_j.

Monitoring of the condition of water bodies is carried out by automated measuring systems online. In such systems, multispectral control methods of integrated parameters of water pollution and biotesting methods are used [24–27]. Emergency situations of different levels of danger require that the time of tuning a fuzzy model as new data arrive did not exceed the time of primary treatment. To prevent polluted water from entering the biofilters, this time is limited to 45 min.

Tuning by the method [9–13] takes 57 min, which exceeds the allowable time. The tuning time for this method is 33 min (Intel Core 2 Duo P7350 2.0 GHz processor).

The limitation of this method is the classification format of the primary and modified fuzzy models.

7. Conclusions

1. The approach that combines semantic training, granular partition and solution of fuzzy relational equations for constructing accurate and interpretable rules is developed. The composite fuzzy model of direct logic inference based on the primary rules with granular parameters is proposed. It is shown that the weights of the primary rules, which are subject to modification, as well as the hedging threshold of the primary terms, are solutions of the primary system of fuzzy logic equations with the hierarchical max-min/min-max composition, which solves the problem of the hierarchical selection of the primary and modified rules for the given output classes. For a particular technological process quality control problem contained in the experimental part, the primary model with granular parameters allows reducing the tuning error by 25 % compared with the primary relational model [17].

2. The method of classification rule hierarchical tuning with the linguistic modification is developed based on solving fuzzy relational equations, which allows reducing the training time. The genetic-neural approach for tuning the primary rules and solving the system of equations, as well as tuning the composite rules was used. The effectiveness of the approach is illustrated by the example of tuning and interpreting the solutions to the technological process quality control problem for the specified productivity classes. Compared to the hierarchical selection methods [9–13], this method allows reducing the tuning time by half.

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