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IDENTIFICATION OF SHIP MODEL AND DISTURBANCE PARAMETERS USING SPECTRAL ANALYSIS

S. Ivanov
Ph. D.
Research institute of telecommunications, head of the research department*
E-mail: marinex@inbox.ru

V.Teut
Postgraduate student
Department of technical systems and processes in navigation
Kyiv state academy of water transport, named by hetman Petro Konashevich Sagajdachny
Frunze st., 9, Kyiv, Ukraine,04071
E-mail: vm.teut@yandex.ua

P. Oliynik
Ph. D.
Research institute of telecommunications, research associate*
E-mail: poleinik@ukr.net
*National technical university of Ukraine "Kyiv polytechnic institute"
Peremohy ave.,37, Kyiv, Ukraine,03056

В роботі представлена методика ідентифікації параметрів моделі судна і хвилювання моря. Для визначення параметрів моделі судна і статичного збурюючого моменту запропоновано використати спектральний аналіз сигналу кутової швидкості. Запропонована методика дозволяє визначити параметри моделі судна, а також амплітуду статичного збурення з точністю, достатньою для синтезу і налаштування регулятора автостернового

Ключові слова: ідентифікація, модель судна, хитавиця, спектральний аналіз, збурення

В работе представлена методика идентификации параметров модели судна и волнения моря. Для определения параметров модели судна и статического возмущающего момента предложено использовать спектральный анализ сигнала угловой скорости. Предложенная методика позволяет определить параметры модели судна, а также амплитуду статического возмущения с точностью, достаточной для синтеза и настройки регулятора авторулевого

Ключевые слова: идентификация, модель судна, качка, спектральный анализ, возмущение

1. Introduction

When developing adaptive autopilots, one of methods to ensure their adaptation to navigation conditions is the use of a ship model to tune autopilot's controller up.

According to classification given in [1], adaptive autopilots can be divided into two groups: autopilots that use data about hydrodynamic characteristics of the ship in different navigation conditions, obtained during sea trials, and autopilots that use standard model of a ship (on a calm sea as a rule) to set parameters of the controller. The drawback of an autopilot of the first group is a necessity to store the table of ship parameters, obtained during its sea trials under different weather conditions and different loading conditions in a memory of the computing device.

Such trials are rather expensive because of objective reasons; moreover, it's impossible to carry out such trials for all possible combinations of weather and ship loading conditions. Therefore, parameters of a ship in specific navigation conditions have to be estimated using interpolation and extrapolation of the data present, and that leads to control quality degradation.

The result of a second group autopilot usage because of the concept of a construction itself is that setting of an autopilot parameters such a way, that the behavior of a ship-steering system conforms the standard model behavior, causes less then optimal ship reactions under specific navigation conditions, and unreasonably large number of rudder movements.

In order to simplify autopilot parameters tuning and to increase ship control quality one can use the adjustment of regulator parameters based on ship and disturbance model, which parameters are identified dynamically during navigation. Long-term sea trials for normal operation of such an autopilot are unnecessary.

2. Analysis of references and problem formulation

Active motion control is used most often in order to identify ship model and disturbance parameters; in most cases it is the identification during zigzag motion or turn. Such a procedure is described in [2-10]. But the procedure of identification during zigzag motion is useful only for sea trials or ship motion in open sea, when the limitations like fairway or corridor for ship motion are absent. During navigation in constrained conditions as a rule the usage of zigzag is unacceptable or even impossible. Therefore, a problem of ship model parameters identification under natural disturbances (e.g., under sea and wind disturbances) arises; moreover, ship deflection from the selected course should be minimal.

Attempts to solve that problem have been undertaken already, specifically A.K. Sheykhot in his work [11] offered the method of second order Nomoto model parameters identification using speed gradient method, developed by A.L. Fradkov in his works. The main point of the identification procedure proposed in [11] is the minimization of an objective function, that describes difference between state of Nomoto model and the ship itself given an input signal (rudder angle) and ship motion parameters (angular frequency and angular acceleration).

The drawback of this method is that although the identification (including one without an active control) during ship motion is possible, under disturbance of considerable level the algorithm, proposed by Sheykhot, may become inoperable.

3. The goal of present research

The goal of present research is the development of a method for ship model parameters identification under rough sea conditions that is free from defects mentioned above.

4. Method of identification proposed

Consider a ship under regular sea disturbance. In that case, the disturbance that acts on a ship is the harmonic one. For this reason the ship, moving under such a disturbance will yaw (first of all because of a disturbance moment) and its way will be zigzag. Thus the equations of ship motion under stationary component of disturbance and regular disturbance in simplest form are (first order Nomoto model [12], that includes the steering gear dynamics)

$$\begin{cases} \dot{\Psi} = \omega \\ \dot{\omega} = -\mu\omega + \mu k \delta_c + m_d + m_{ds}\sin(wt) \\ \dot{\delta}_c = -\mu_g \delta_c + \mu_g u \end{cases}, \qquad (1)$$

where Ψ is the yaw angle, ω – yaw angular rate, δ_c – rudder angle, $\mu = 1/T$ – quantity, which is reciprocal to ship model time constant, k – transfer constant, μ_g – quantity, which is reciprocal to steering gear time constant (it is known from the vessel documentation), u – master control, m_d – disturbing moment that acts on the ship (we neglect the high-frequency disturbance, i.e. only time-independent and low-frequency disturbance in a white noise disturbance are taken into account), m_{ds} – apparent amplitude of a reduced disturbance caused by the regular sea disturbance, and w is the apparent frequency of the regular sea disturbance.

At first let's consider the simplified case of identification. Let the stationary part of the reduced disturbance moment m_d be absent, and the ship motion under disturbance occur in the range of a given corridor of course change. Due to that, the control action is absent, i.e. $\delta_c = 0$.

In this case, system (1) takes a form of

$$\begin{cases} \dot{\Psi} = \omega \\ \dot{\omega} = -\mu\omega + m_{ds}\sin(wt) \end{cases} \qquad (2)$$

Solving a system (2), one can obtain following formulae for the parameters of steady motion (assuming that transient process has decayed, and initial conditions are $\Psi(0) = \Psi_0$ (desired heading), $\omega(0) = \omega_0$):

$$\omega(t) = -\frac{m_{ds} w \cos(wt)}{w^2 + \mu^2} + \frac{m_{ds}\mu \sin(wt)}{w^2 + \mu^2},$$

$$\Psi(t) = \frac{m_{ds}}{w\mu} - \frac{m_{ds}\mu\cos(wt)}{w(w^2 + \mu^2)} - \frac{m_{ds}\sin(wt)}{w^2 + \mu^2} + \Psi_0 \qquad (3)$$

Determining the frequency w using the Fourier transform on output signal of the gyrocompass, measuring ship course and angular velocity at times t_2 and t_3, that are shifted by $\Delta t = t_3 - t_2 = \frac{3\pi}{2w}$ seconds (disturbance phase shift between those time moments is $\frac{3\pi}{2}$), and substituting respective data into (3), we get the system of equations, after solving which we obtain formulae for m_{ds} and μ:

$$m_{ds} = \frac{(\omega_2 + \omega_3)(-w^2(2\Psi_0 - \Psi_2 - \Psi_3)(\Psi_2 - \Psi_3) + \omega_2^2 - \omega_3^2)}{2w(\Psi_2 - \Psi_3)^2}$$

$$\mu = \frac{\omega_2 + \omega_3}{\Psi_2 - \Psi_3} \qquad (4)$$

Thus, if the ship under regular disturbance doesn't go out of the certain, preassigned corridor of course change, one can estimate amplitude of the stationary disturbance moment m_{ds} and ship model's time constant $T=1/\mu$ without use of an active control. So far as active control in this case is absent, the transfer constant estimation is impossible – because in the system of equations (2) transfer constant k is absent.

As the real sea disturbance is close to the regular one only in swell or in sufficiently heavy sea, such a simplified case is rarely seen in practice.

Therefore, in order to estimate all the parameters of a ship model in navigation conditions the method, proposed below, is developed. Consider the full system of equations (1) in the absence of the control action ($u=0$), and at that consider the steady motion of a ship. Initial conditions of the ship motion are: $\Psi(0)=\Psi_0$, $\omega(0)=0$, $\delta(0)=0$.

After transient process has decayed, oscillations of vessel angular velocity ω have the form of

$$\omega_1(t)=\frac{m_d}{\mu}-\frac{m_{ds}w}{w^2+\mu^2}\cos(wt)+\frac{m_{ds}\mu}{w^2+\mu^2}\sin(wt)\,. \qquad (5)$$

If one feed a certain value of master control $u=u_p$ to the input of a steering booster, oscillation of the angular velocity after transient process has decayed will have the form of

$$\omega_2(t)=ku_p+\frac{m_d}{\mu}-\frac{m_{ds}w}{w^2+\mu^2}\cos(wt)+\frac{m_{ds}\mu}{w^2+\mu^2}\sin(wt)\,. \qquad (6)$$

In order to determine the transfer constant k, one can use the Fourier transform, sampling the signal beforehand and analyzing the constant component of spectrum after the transform. The spectrum constant component of a signal (5) is $\omega_{10}=\frac{m_d}{\mu}$, and of a signal (6) is $\omega_{20}=ku_p+\frac{m_d}{\mu}$. So, if one knows the value of u_p, one can write down that

$$k=\frac{\omega_{20}-\omega_{10}}{u_p}\,. \qquad (7)$$

If the value of ship model time constant T (or $\mu=1/T$) is known, then in order to determine the stationary disturbance moment, which is necessary to design ship control system, one can use the formula

$$m_d=\mu\omega_{10}=\omega_{10}/T\,, \qquad (8)$$

that directly follows from (5).

In order to determine the time constant of a ship model, one can again use the active control of ship motion and spectral analysis. At that in order to exclude the influence of steering gear (steering booster) dynamics from the analysis, it's appropriate to measure not the master control u, but directly the rudder angle sensor signal δ_c. As it follows from (1), the transfer function of a ship with respect to its angular velocity has the form

$$\frac{\omega(s)}{\delta_c(s)}=\frac{k}{Ts+1}\,. \qquad (9)$$

If one change δ_c harmonically, i.e. if $\delta_c=\delta_{cmax}\sin(\Omega t+\phi_0)$, then taking the Fourier transform of (9) and taking the modulus of result, one find out that the amplitude of an angular velocity oscillation ω_{max} after transient process has decayed is expressed by the formula

$$\omega_{max}=\frac{kT\delta_{cmax}}{\sqrt{T^2\Omega^2+1}}\,. \qquad (10)$$

From (10) it is easy to get the formula for estimation of the ship model's time constant:

$$T=\frac{1}{\Omega}\sqrt{\frac{k^2\delta_{cmax}^2}{\omega_{max}^2}-1}\,. \qquad (11)$$

Therefore, the proposed method of identification is as follows:

1. Put the rudder into a diametral plane, and, after angular velocity transient process has decayed, measure N-point sample of the ship angular velocity ω. Calculate the amplitude spectrum, and store the value of the constant component ω_{10} («zero line» of the spectrum mentioned).

2. Shift the rudder with master control u_p so that the ship could return to its desired heading, and, after angular velocity transient process has decayed, measure N-point sample of the ship angular velocity ω. Calculate the amplitude spectrum, and store the value of the constant component ω_{20} («zero line» of the spectrum mentioned).

3. Feed a harmonic or other periodic action with constant period that is not equal to $1/w$ right to the input of a steering booster, and when oscillations in system become stationary (time that is equal to transient process decay time in p.1 passes), measure N-point samples of rudder angle δ_c and ship angular velocity ω.Calculate their spectrums and determine values of δ_{cmax} and ω_{max} at the frequency, where the amplitude of control spectrum is maximal.

4. Calculate estimations of k, T and m_d with formulas (7), (8) and (11).

According to the proposed method with active control under the natural disturbance one can estimate parameters of the model (1) that are necessary for synthesis of autopilot regulator – k, T and m_d – using spectral analysis. Unlike the known methods (for example, [13]), the proposed one does not require the use of input action in the form of a white noise and estimation of ship gain-frequency characteristic (GFC); as a result the computation intensity of identification procedure is decreased.

It should be noted that as the disturbance parameters are not included in formulae (7), (8) and (11), this method can be used also under a non-regular sea disturbance, i.e. under conditions when there's no prevailing harmonic in the disturbance spectrum and sea disturbance should be considered to be the random process with appropriate characteristics.

Certain complication in the use of a proposed method is the necessity to provide the accurate measurement of angles and angular velocities for identification of the ship model parameters.

In addition the proposed identification procedure may take a long time to complete depending on the ship's time constant (because of the transient process decay time in the "autopilot-ship" system, and the necessity to measure data during at least one master control period to get correct results) and may also require more powerful controller that is capable to compute Fast Fourier Transform.

5. Results of the method modeling

Before we start modeling, let's consider some points, connected with signal processing features and possibility of practical realization of the method on a ship.

1. One should calculate spectra without usage of any spectral window, because such a window distorts the value of spectrum constant component.

2. Sampling frequency during data measurement should be chosen such a way, that angular frequency of $1/T$ is surely covered. Sampling frequency of 256 Hz is enough for practical application on the ship. At that, number of samples N should be enough to distinguish master control and disturbance at frequency w, where its amplitude is maximal. With a sample of 8192 points and sampling frequency of 256 Hz (at that frequency range is 100 Hz) the resolution of a Fast Fourier Transform is $\Delta f = 0{,}03125$ Hz.

3. Frequency w, at which the disturbance amplitude is maximal, should differ from the master control frequency – otherwise it's impossible to separate them in output signal, and that will decrease accuracy of identification. At the same time, the master control frequency $f_{act} = \Omega / 2\pi$ should correspond to the spectrum filter (line) medium frequency in order to prevent energy "leakage" to the neighbor filters. For example, when we have $\Delta f = 0{,}03125$ Hz, master control period can be taken equal to $1/\Delta f = 32$ s, $1/(2\Delta f) = 16$ s, etc.

4. As a rule, it's impossible to feed sinusoidal control action directly to the steering booster input. Thus one have to use the rectangular impulse control action; because the spectrum of a rectangular impulse with period T_{pulse} has significant harmonic at the frequency, which is close to $1/T_{pulse}$, and that has no influence on calculations.

Modeling of an identification with the method proposed above is done for the ship with parameters of Nomoto model $k = 0{,}08\ s^{-1}$, $T = 12$ s, $m_d = 5 \cdot 10^{-5}\ s^{-1}$.

For the first cycle of identification the amplitude of master control (and, accordingly, of the rudder angle) is of 1°. In the Fig. 1 the reduced disturbing moment that acts on the ship is shown. In the Fig. 2 one can see the change of a rudder angle when the rectangular pulses with period 32 s are fed to the input of a steering booster, and in the Fig. 3 the change of angular velocity during identification process is shown. The smoothed front of pulses in the Fig. 2 is the result of steering booster inertia.

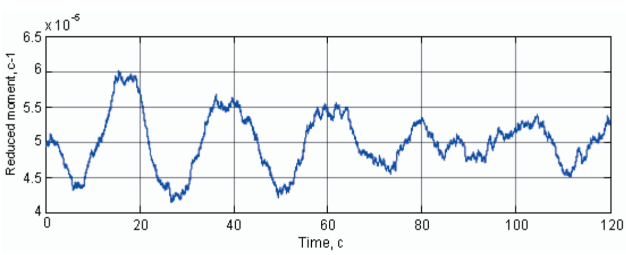


Fig. 1. Reduced moment, acting on a ship versus time

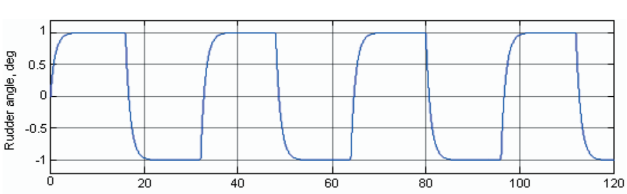


Fig. 2. Rudder angle δ_c versus time

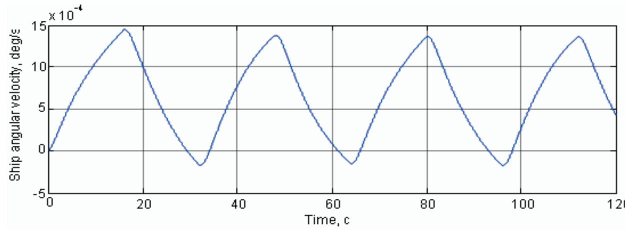
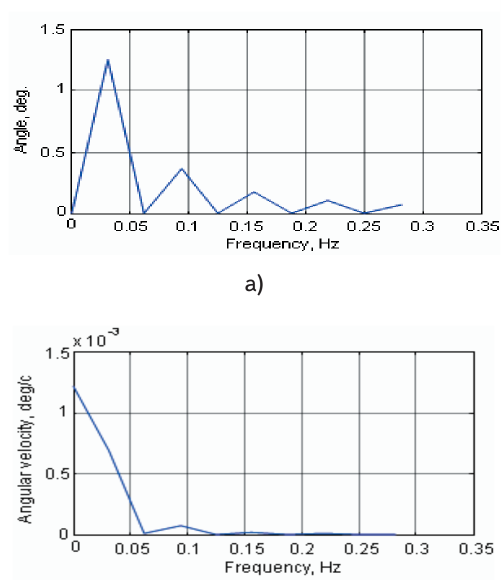


Fig. 3. Ship angular velocity versus time

On the base of Fig. 3 one can conclude that the time of transient process is not more than 40 s, and after that the oscillation of angular velocity with constant amplitude becomes stationary. Thus in this case for reliable identification one should start measurements from the moment of 40 s after feeding the first impulse, and measure data during a period of 32 s (one master control period) at least. In practice the fact that transient process is over (has decayed), can established by comparing the maximal amplitudes of angular velocity oscillation: if the difference of magnitudes over two periods is insignificant (for example less than 1%), the oscillation can be considered as a stationary.

In Fig. 4 spectrums of rudder angle and ship angular velocity after transient process has decayed are shown.



a)

b)

Fig. 4. Fourier spectrums of rudder angle (a) and ship angular velocity (b)

As one can see from the Fig.4, the maximum of rudder angle spectrum is at the frequency of 0,03125 Hz ($1/32\ s^{-1}$). In the spectrum of vessel angular velocity at this frequency the significant harmonic, connected with control action is present, and disturbance is apparent at higher frequencies. This corroborates the possibility of spectral analysis application for the identification of ship model parameters.

Then the influence of master control amplitude on identification accuracy has been studied. In the table 1 results of time constant and statistical disturbance moment identification at different values of master control amplitude u_p with master control in the form of sinusoidal signal and rectangular impulses with the same period of 32 s are presented. The identified value of transfer constant in both cases is $k = 0{,}0779\ s^{-1}$, relative error of identification is ε_k = -2,65%.

Table 1

Results of ship model parameters identification with different master control signals

u_p, °	Sinusoidal signal				Rectangular impulse			
	T, s	$m_d \times 10^{-5}$, s^{-1}	ε_T, %	ε_{m_d}, %	T, s	$m_d \times 10^{-5}$, s^{-1}	ε_T, %	ε_{m_d}, %
1,0	11,5334	5,06285	3,89	1,26	11,4668	5,09228	-4,44	1,85
2,0	11,6034	5,03233	3,31	0,65	11,5759	5,04427	-3,53	0,89
3,0	11,6268	5,02218	3,11	0,44	11,6126	5,02831	-3,23	0,57
4,0	11,6386	5,01711	3,01	0,34	11,6311	5,02035	-3,07	0,41
5,0	11,6456	5,01407	2,95	0,28	11,6422	5,01557	-2,98	0,31

As one can see from the Table 1, the difference in accuracy of identification when rectangular pulse and sinusoid is applied is insignificant. Furthermore, accuracy that is adequate for the practical use of the method can be achieved when master control is of 1°. Consequently, during the application of the proposed method, unlike during the identification with a method for sea trials [10], rudder angle can be small (about 1 degree). Deviation of a ship from the desired heading because of a control action will be also small. According to demands of Russian navigation Register the ship deviation under autopilot control shouldn't exceed 1° at sea number of 3 and 3° at sea number of 6. In order to ensure ship deviation from the desired heading in the range of a standard, the rectangular impulses should be symmetric not with respect to zero but with respect to the value of u_0, at which the rudder fully compensates the action of stationary disturbing moment m_d. The value of u_0 can be selected experimentally or estimated from the signal of rudder angle sensor before the start of identification process.

During practical application of the method proposed in order to determine that prescribed identification accuracy is achieved it's proposed to estimate the difference of values which are identified in two consecutive cycles. Iterative identification process with the increase of master control amplitude value is over when the given value of difference is obtained.

In this paper the regression analysis of dynamic processes [13] is proposed to use as an alternative method of identification under the irregular sea disturbance. In the table 2 the results of ship model parameters identification using regression analysis – estimates of model parameters and their relative errors are presented. As a control action that is fed to autopilot input, the impulse of specified deviation from course $\Delta\Psi$ of 20 s length is used.

Table 2

Model parameters, identified using the regression analysis, and estimation of identification accuracy

$\Delta\Psi$, °	T, s	k, s^{-1}	$m_d \times 10^{-5}$, s^{-1}	ε_T, %	ε_k, %	ε_{m_d}, %
1,00	11,8208	0,086345	5,5447	-	-	-
1,50	11,7199	0,082623	5,3545	-0,86	-4,50	-3,55
2,00	11,7188	0,081281	5,2694	-0,01	-1,65	-1,61
2,50	11,7380	0,080672	5,2222	0,16	-0,75	-0,90
3,00	11,7600	0,080356	5,1925	0,19	-0,39	-0,57
3,50	11,7807	0,080176	5,1721	0,18	-0,22	-0,39
4,00	11,7989	0,080068	5,1574	0,15	-0,14	-0,29

As one can see from this table, when the magnitude of specified deviation from course $\Delta\Psi$ increases, the identification accuracy also increases. Knowing the model parameters values, we can state that when $\varepsilon_T < 1\%$, $\varepsilon_k < 1\%$ the accuracy of ship model parameters estimation using the regression will be no worse than $\frac{11{,}73280-12}{12} = -2{,}18\%$. If we compare this result with the data obtained above, we can also state that the proposed identification method with spectral analysis allows one to estimate ship model parameters with the same order of accuracy (or even a higher one using the higher master control amplitude) So, the method proposed offers the accuracy not worse than the classical object parameters identification method under disturbance conditions.

6. Conclusions

The method, proposed in this paper, allows one to identify the ship model parameters and stationary disturbance moment using spectral analysis. Accuracy of identification is adequate for practical application of parameters obtained in synthesis and adjustment of ship course control system (autopilot) controller. Further research will be dedicated to practical realization of the proposed method in an adaptive autopilot.

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МИНИМАЛЬНЫЕ ПОВЕРХНОСТИ

Актуальність даної роботи полягає в тому, що результати роботи можуть бути покладені в основу написання інших робіт і застосовані в різних сферах діяльності. Подальше використання навичок, набутих в процесі написання даної роботи, можуть бути реалізовані в подальшому, при виконанні робіт, що стосуються побудови поверхонь

Ключові слова: Шерк, мінімальні поверхні, рівняння Лагранжа, циліндричні поверхні, теорема, катеноїд

Актуальность данной работы заключается в том, что результаты работы могут быть положены в основу написания других работ и применены в различных сферах деятельности. Последующее использование навыков, приобретенных в процессе написания данной работы, могут быть реализованы в дальнейшем, при выполнении работ, касающихся построения поверхностей

Ключевые слова: Шерк, минимальные поверхности, уравнение Лагранжа, цилиндрические поверхности, теорема, катеноид

И. А. Кирноз*
E-mail: inst@adidonntu.org.ua

Е. А. Королёв
Кандидат физико-математических наук, доцент
Кафедра высшей математики*
E-mail: inst@adidonntu.org.ua

М. Е. Королёв
Кандидат физико-математических наук, доцент
Кафедра прикладной математики и информатики*
E-mail: inst@adidonntu.org.ua

*Автомобильно-дорожный институт "Донецкий национальный технический университет"
ул. Кирова, 51, г. Горловка, Украина, 84624

1. Введение

Исследования, о которых идёт речь в статье, относятся к области фундаментальных и прикладных аспектов математики и кибернетики. Актуальность рассматриваемой темы нашей статьи является обоснованной и нами доказанной, так как данная работа является составляющей процесса обучения в сфере минимальных поверхностей. В процессе написания статьи будут получены навыки работы с построением минимальных, цилиндрических поверхностей, а так же поверхностей переноса. Рациональное использование полученных знаний даст возможность для дальнейшего исследования и написания других работ, касающихся построения поверхностей. Наша работа является наглядным пособием того, как знания о минимальных поверхностях из одной области могут стать надёжным помощником во многих других отраслях нашей деятельности, в случае умелого использования приобретённых навыков.

2. Определение минимальной поверхности

Пусть $M^2 \subset R^3$ - двумерная гладкая поверхность, где R^3 отнесено к декартовым координатам x, y, z.