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DEVELOPMENT OF THE METHOD FOR THE FORMATION OF ONE-DIMENSIONAL CONTOURS BY THE ASSIGNED INTERPOLATION ACCURACY

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Запропоновано метод формування одновимірних обводів виходячи з заданої точності інтерполяції. Максимальна абсолютна похибка інтерполяції визначається з урахуванням геометричних властивостей вихідної кривої лінії. Розглядається два різновиди похибки. По-перше, похибка, з якою сформована дискретно представлена крива, що інтерполює вихідний точковий ряд, представляє вихідну криву. По-друге, похибка, з якою інтерполююча крива представляє будь-яку криву з заданими геометричними характеристиками

Ключові слова: похибка інтерполяції, упорядкована множина точок, осциляція, монотонна зміна диференціально-геометричних характеристик

Предлагается метод формирования одномерных обводов исходя из заданной точности интерполяции. Максимальная абсолютная погрешность интерполяции определяется с учетом геометрических свойств исходной кривой линии. Рассматривается две разновидности погрешности. Во-первых, погрешность, с которой сформированная дискретно представленная кривая, интерполирующая исходный точечный ряд, представляет исходную кривую. Во-вторых, погрешность, с которой интерполирующая кривая представляет любую кривую с заданными геометрическими характеристиками

Ключевые слова: погрешность интерполяции, упорядоченное множество точек, осцилляция, монотонное изменение дифференциально-геометрических характеристик

1. Introduction

Geometric modeling is one of the tools for investigation of objects, phenomena and processes. The task of geometric modeling is to determine properties of an object being modeled using characteristics of a geometric model. Output data are geometric images assigned by a set of points. Their location reflects properties of the examined object. Geometric characteristics of a discretely represented geometric image (line or surface) can be given at the output points. We can obtain output by calculations or measurements at physical objects.

There are difficulties in modeling discretely presented curves and surfaces because we know characteristics of curves at the output points only. It is possible to determine a character of a change in characteristics between the output points using additional information about properties of the object of modeling.

One of the methods of modeling based on discrete sets is interpolation. The task of interpolation is to restore an unknown function (curve) by the value of ordinates given on a discrete set of points with a given accuracy [1]. In the general case, it is impossible to restore an original curve accurately. An important step in solution of the task is a choice of interpolation methods that provide the required accuracy. There are two components of the error occurrence: sampling error and interpolation error.

A sampling error occurs as a result of representation of a source object by a discrete set of points. A sampling error is inevitable, it does not depend on a method chosen for further interpolation, and we cannot eliminate it during the modeling. A sampling error increases with unsuccessful location of the output points on a geometric image, when a point series does not reflect presence of special areas (change of convexity-concavity, reversal of stroke, etc.). Such error reduces in case of increase in the number of output points of a discrete set. An increase in the number of output points increases an aggregate measurement error, a volume of calculations, and may increase a calculation error. It is expedient to select a distance between output points (sampling step) as large as possible, but it should satisfy requirements for the accuracy of the modeling. Known interpolation methods do not make it possible to determine a sampling step based on the given accuracy of representation of an output curve.

We can estimate an interpolation error by a deviation of a model from the original geometric image. Reduction of a sampling step reduces an interpolation error when forming a model with methods characterized by convergence and stability. The development of interpolation methods that provide the given accuracy is an important task of modeling.

2. Literature review and problem statement

The most developed interpolation methods at the moment are based on analytically given functions (continuous interpolation methods). The methods include methods of global modeling and methods of piece-smooth approximations. The methods of global modeling determine a geometric image with one equation. For example, papers [2, 3] describe a contour by algebraic Hermitian and Newton's polynomials, and paper [4] – by a trigonometric function. The methods of piece-smooth approximations form a contour of sections of analytically given curves, which link up at output points. Sections of curves of the second order form a contour in a work [5], Beziers curves – in work [6], B-splines – in paper [7].

The accuracy of a formed model represents the original object. We evaluate it as a deviation of a model from a known function that interpolates the same point series.

For example, paper [2] proposes a method for the determination of polynomial equation $P_n(x)$. It interpolates the entire point series, which belongs to a discretely represented geometric image. An output function is known and has a continuous n+1-y derivative. Authors evaluate an interpolation error as a maximum deviation of the resulting polynomial $P_n(x)$ of the output $f(x)$ function.

$$\varepsilon = \left| f(x) - P_n(x) \right| \le \frac{\left|(x-x_0)(x-x_1)\ldots(x-x_n)\right|}{(n+1)!} \cdot M_{n+1}, \qquad (1)$$

where $x_0, x_1, \ldots x_n$ are the abscissas of points of the output discrete set; n is a number of points of the initial discrete set; $M_{n+1} = \max\left| f^{(n+1)}(x) \right|$ is the maximum value of n+1-th derivative of $f(x)$ function on the investigated segment.

In work [3], authors estimate an error of interpolation by the value of a maximum deviation of polynomial $P_n(x)$, which interpolates n+1 point, from the polynomial $P_{n-1}(x)$, which interpolates n points that are included in the point series, which defines the polynomial $P_n(x)$. If the value of the deviation does not exceed the given ε value, then we consider that the polynomial $P_n(x)$ represents the output curve with ε error.

One can use the mentioned methods for estimation of interpolation error to solve test cases in order to verify effectiveness of the interpolation method. There is an assumption that the order of error will be the same as in solution of test cases when solving other problems. In the general case, when an output function is unknown, such an assumption may be incorrect. The interpolation method should ensure convergence and stability of the solution in order to set an interpolating curve representing an output curve with given accuracy [8]. The interpolation polynomial is stable if small errors in the output data lead to a small change in the result. The use of stable interpolation methods reduces requirements for the accuracy of determination of the output data and enables their correction if conditions of the task lets it. The use of stable interpolation methods reduces the error resulting from inaccurate determination of the output point series. Most interpolation polynomials are unstable at solution of practical problems.

The convergence of interpolation is to reduce the interpolation error when the number of output points increases [1]. In the general case, methods of global modeling do not ensure the convergence of the interpolating function and the output curve at interpolation of a point series of arbitrary configuration, which consists of a large number of nodes.

An example is the Runge's function in the form:

$$R(x) = 1 \Big/ \left(1 + 25x^2\right).$$

Fig. 1 shows its graph. Work [9] investigates the phenomenon of Runge. The polynomial $P_5(x)$, which interpolates a point series belonging to $R(x)$, does not reflect a change in the convexity-concavity of the output curve between points 3 and 9. It is possible to refine the solution in this section due to an increase in the number of points belonging to the output curve. This increases a degree of interpolating polynomial, which leads to an increase in its deviation from the output function in other sections. For example, the deviation of the polynomial $P_{10}(x)$ from the output function $R(x)$ on the edges of the segment [–1; 1] exceeds the deviation from $R(x)$ of the polynomial $P_5(x)$ (Fig. 1).

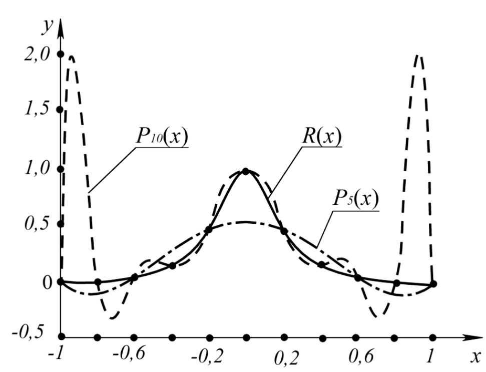

Fig. 1. Investigation of the convergence of interpolation

The deviation of curve line from its initial position appears as a result of the change in the position of the output points or the increase in the number of points due to the uncontrolled occurrence of special points of the curve. Uncontrolled occurrence of special points is inevitable with the increase of a parametric number of the curve. For a polynomial curve line, they are, above all, points of inflection. Stability and convergence are not possible in the case of interpolation by polynomials, a degree of equation of which depends on the number of outlets directly.

The method of interpolation based on trigonometric functions developed in a paper [4] can prevent uncontrolled occurrence of points of inflection (occurrence of oscillations). The disadvantage of the method is that it is designed to interpolate a small number of points. This feature of the method reduces the area of its practical application. In addition, the method does not provide for the control of regularity of a change of a curvature along sections of the contour.

The absence of oscillations at the interpolation of any number of output nodes provides methods of piece-smooth approximations. Separate sections, which are linked up at the output points, form the interpolating curve.

Oscillation on sections of curves of the second order is impossible at formation of contours by the method proposed in work [5]. The main disadvantage of the method is violation of regularity of values of the curvature at the linking points. A second degree of the equation of the curve determines this defect.

The method developed in paper [6] makes possible to ensure regularity of values of the curvature along the contour. The method involves a use of Bezier curves of the third or higher order. The main disadvantage of formation of contours based on Bezier splines is that they do not provide a locality of control of the shape of the curve in contour sections.

Authors of work [7] developed a method for the formation of contours based on B-splines. The use of the B-spline provides the maximum locality of control of curve shape in comparison with other known methods of continuous geometric modeling. The locality of the control of a shape of B-spline decreases with an increase in the degree of the alignment of the curve in order to improve a quality of joints of sections of a contour. The control of a shape of a polygon, which a spline sets, provides prevention of oscillation at the use of Bezier and B-splines.

For the case when the condition of oscillation prevention does not provide the given accuracy of the modeling, it is necessary to impose more strict additional requirements to the law of change of geometrical characteristics along the formed contour. This can be a monotonous change in values of the curvature, rounding, radii of spheres in contact. Methods of piece-smooth approximations, which make possible to control a nature of a change of indicated geometric characteristics along the interpolating curve and give possibility to determine a number of output points required to provide a given accuracy of interpolation, are unknown.

The considered methods of piece-smooth approximation assume provision of the quality of joints of curve sections at the output points (presence of a joint point of a total tangential, static circle, etc.). The solution of the problem requires imposition of additional requirements for curves of lines, which form sections of the contour. The quality of joining sections of the contour is an important condition at solution of a number of practical problems (designing surfaces of working bodies of agricultural machines, blades of turbines, channels of internal combustion engines) [10].

For an unknown curve, it is possible to determine the area of a possible location based on the predicted properties of the curve only. Such properties can be the regularity and the direction of monotonic growth along sections of the characteristic curve: curvature, rounding, radii of spheres in contact, presence of special points. Considered methods for estimation of interpolation error do not define a limited area of space, within definitely contained curve line, on which the output point series is set. The implementation of the approach to the estimation of accuracy of interpolation requires development of methods for analyzing a point series on the possibility of forming contours according to given geometric conditions.

It is possible to solve the problem using approaches based on the methods of differential discrete geometric modeling [11–13]:

– discrete presentation of output data and modeling result;
– locality of formation;
– prevention of oscillation;
– step-by-step control and correction of the solution.

3. The aim and objectives of the study

The aim of present study is to develop a method for the formation of a flat one-dimensional contour with provision of the control of the maximum absolute error of interpolation.

The following tasks must be solved to achieve the objective:

– development of a method for determination of the area of possible location of a monotone curve line that interpolates a given point series based on its characteristics: absence of oscillations, a monotonous change in values of curvature along a curve;
– development of a method for the formation of one-dimensional contours, which interpolate a discrete set of points and belong to a given area of a possible location of a monotonous curve line.

4. Materials and methods to study the interpolation of a discrete set of points

The formation of a model based on a discrete set of points based on the given absolute interpolation error requires determination of boundaries of possible location of linear elements of a model. The forming curve is represented by an ordered set of points, which belong to it, and by geometric characteristics of a curve. The characteristics must be provided in the modeling process. We will call such a curve a discretely represented curve. The curve is formed on the basis of any point series in sections along which it is possible to provide a monotonous change in values of the curvature. The monotonous sections link up with the given order of smoothness.

Condensation, which involves determination for the output point series of intermediate points, forms the curve. Condensation points are assigned inside the section of possible location of a monotone curve. In the process of condensations of the point series, the section of the curve is sequentially localized. After reaching the given accuracy, nodes of the formed point series join each other with chords. The final solution has the form of an accompanying broken line, which consists of any numbers of chords, the distance from which to the curve of the line with given geometric characteristics does not exceed a predetermined any small value.

We build the geometric model on the assumption: if there is a curve line without special points that interpolates the point series, then there are no special points in the output object. Such points include points of inflection, changes in the direction of growth along the curve of values of the curvature, the rounding, etc.

We divide the output point series into areas, which may interpolate with the curve line along which the value of geometric characteristics monotonically increases or decreases. We define the area of space, where all curve lines with given geometric properties are located. We determine the maximum absolute error of interpolation on the base of a size of the area of a possible solution.

4. 1. Methodology for determining an error of presentation of the output curve line

The output flat curve is an ordered set of points, which belong to it. We determine the maximum absolute error, with which the curve interpolating the point series represents the output curve line, in the first approximation based on the absence of oscillation curves.

If every three successive output points are arranged in such a way that they are bypassed clockwise, then we assume that the point series belongs to the convex curve line. The output point series is divided into convex and concave sections and interpolated separately along these sections.

Any convex curve line that interpolates a point series, is located within a chain of basic triangles bounded by a chord, which connects the adjacent output points and tangent to the curve (t_i) at these points (Fig. 2).

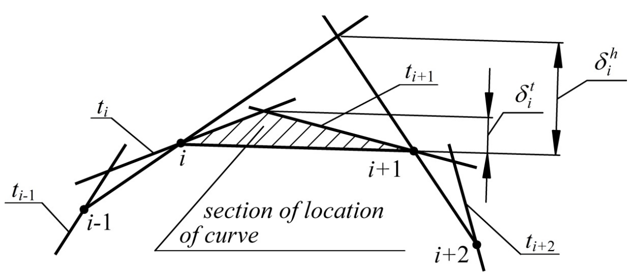

Fig. 2. Location region of the convex curve

The maximum deviation of the interpolating curve from the output one cannot exceed a height of the corresponding base triangle (δ_i^t) at each section. In the case when the forming contour is given only by the output points, the interpolation error cannot exceed the value of δ_i^h. This is a height of the triangle, sides of which belong to straight lines passing through three pairs of successive output points.

The estimation of the accuracy of interpolation through determination of a height of the base triangles is possible at formation of contours by methods that provide the control of the occurrence of oscillations. This is a method of curves of the second order [5], Bezier curves [6], B-splines [7] and methods of discrete interpolation [11–13].

The next condition, which reduces the area of a possible location of the curve and improves the accuracy of interpolation, is the condition of a monotonous change in values of the curvature along a curve. We will call such a curve line monotonous.

We draw a circle after every three successive starting points and call such circle adjacent (AC_i). If radii of adjacent circles increase or decrease along a point series monotonically, such a point series may interpolate with a curve line with a monotonous decrease or growth of curvature, respectively [12].

We divide the output point series into sections with a monotonous change in radii of adjacent circles and interpolate separately in these sections by monotonous discretely represented curves.

Any monotonous curve line, which interpolates the output point series, is located within a section bounded by successive adjacent circles [12].

We can estimate the absolute error of interpolation by the length of a segment δ_i^m (Fig. 3).

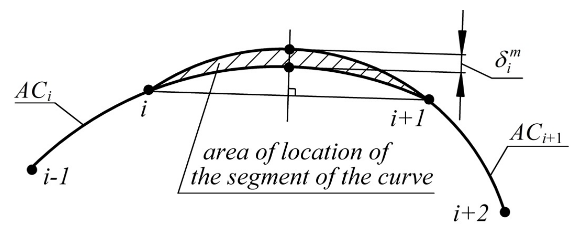

Fig. 3. Section of location of monotonous curve

The segment belongs to a line, which is perpendicular to the chord [i, i+1] and passes through the middle of the chord. Points, which limit the segment δ_i^m, belong to adjacent circles, which limit the area of the possible location of the curve on the section.

4. 2. Formation of a monotonous discretely represented curve, which interpolates the output point series

We determine location of points of condensation of the interpolating discretely represented curve on the basis of the conservation of regularity of the change in radii of adjacent circles along the point series, which is obtained as a result of successive condensations. The fulfillment of this condition ensures:

– correspondence of predicted geometric properties of the output curve of the line and the intended properties of the interpolating curve;

– the maximum error of interpolation, the value of which does not exceed a size of the area of possible location of the output discretely represented curve.

Condensation points are assigned to perpendiculars (n_i) of corresponding chords passing through the middle of the accompanying broken line. For the section (i...i+1), the range of possible location of the condensation point (Δ_i) is the intersection of the segments [A, C] and [B, D], where

– A is a point of intersection of the perpendicular n_i and the adjacent circle passing through the points i–1, i, i+1: $A \equiv n_i \times AC(i-1, i, i+1)$;

– $B \equiv n_i \times AC(i, i+1, i+2)$;

– $C \equiv n_i \times AC(i-2, i-1, i)$;

– $D \equiv n_i \times AC(i+1, i+2, i+3)$.

The purpose of the point of condensation on the section of the discretely represented curve (i, $i+1$) leads to formation of three new adjacent circles and localization of the region of possible location of the curve in the section (i–1...i+2).

Condensation points are consistently assigned within maximum ranges Δ_i. As a result of successive condensations, we obtain a curve interpolating the output point series with a regular monotonous change in values of the curvature.

4. 3. Methodology for determining maximum absolute error of formation of the interpolating curve

We obtain a new monotonous discretely represented curve interpolating the output point series as a result of the appointment of each point of condensation. In this case, we localize the area of possible location of the curve and it remains within the range of location of the output curve.

The presence of a location area, which is consistently localized, is a prerequisite for formation of a point series, which, with an arbitrarily small absolute error, represents a monotonous curve line. The localization of the location of the interpolating curve does not make possible to state that the accuracy, with which it represents the initial curve, increases.

We can reduce the error of interpolation by reducing the area of possible location of the curve due to increasing a fixation order of the forming contour. All monotonous curves that interpolate a sequence of nodes in which fixed positions of tangents (t_i) and values of radii of curvature (R_i) in the section ($i...i$+1) are located inside a region bounded by two box lines of circles [13]. Each of boundaries consists of two arcs of circles, one of which is a stretching circle of the monotonous curve at the point that restricts the area, and the other is a circle tangent to the monotonous curve at another point that restricts the area. Fig. 4 shows radii of the adjacent circles, which are marked as R_i and R_{i+1}.

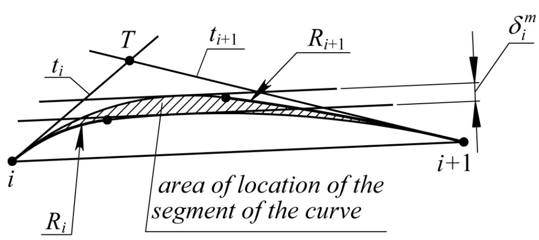

Fig. 4. Area of location of the segment of a contour of the second fixation order

The deviation of the contour of the second fixation order with the monotonous curvature change from the output curve in the segment ($i...i$+1) cannot exceed the maximum width of the area of possible location of the curve – δ_i^m.

5. Results of the study on the absolute error of interpolation

As a test case, we selected a point series, which belongs to the parabola branch. Table 1 shows the value of the absolute error of the representation of the output curve.

Table 1

Estimation of the absolute error of representation of the output curve

Number of section, $i...i$+1	Chord length h_i=|i;i+1|, mm	Absolute error, mm	
		δ_i^t	δ_i^m
1...2	34.73	8.9638	3.68
2...3	48.2689	5.0317	2.445
3...4	53.3633	1.6393	0.549
4...5	51.3769	0.7093	0.1557
5...6	34.5599	0.2498	0.0243

The values of the absolute error of interpolation of the convex curve determined on the basis of basis triangles (δ_i^t) exceed values determined on the basis of adjacent circles (δ_i^m) for a monotonous curve in 2–3 times.

The error determined on the basis of the output point series (δ_i^t, δ_i^m) is equal to the maximum possible deviation of a non-oscillating generated curve formed of the output curve. We can consider such error can as a sampling error. And we can reduce such error by increasing the number of output nodes. Consolidation of the point series in 2 times, characteristics of which are in Table 1, leads to a decrease in the error δ_i^t in 2–3 times, and δ_i^m – in 5–8 times (Table 2).

Table 2

Estimation of the absolute error of presentation of the output curve based on the point series, which consists of a double number of nodes

Number of section, $i...i$+1	Chord length h_i=|i;i+1|, mm	Absolute error, mm	
		δ_i^t	δ_i^m
1...2	16.9426	3.235737	0.6642
2...3	18.6058	3.377989	0.6934
3...4	23.747	1.760375	0.4277
4...5	24.5925	0.469214	0.114
5...6	25.6318	0.523739	0.0877
6...7	27,7316	0,210213	0,0352
7...8	24,4898	0,238711	0,0262
8...9	26.8903	0.101133	0.0111
9...10	17.5891	0.168795	0.00821
10...11	16.9712	0.105677	0.00514

Table 3 shows the value of the absolute error of interpolation for the segments of the contour of the second fixation order.

Table 3

Estimation of the absolute error of interpolation by the second-order fixation contour

Number of section, $i...i$+1	Chord length h_i=|i;i+1|, mm	Absolute error, δ_i^m, mm
1...2	34.73	0.352423
2...3	48.2689	0.28445
3...4	53.3633	0.055597
4...5	51.3769	0.016804
5...6	34.5599	0.002738

We took a point series considered in the previous example (Table 1) as the output data. The position of the tangent to the contour at the i-th point is defined as an average of positions of the tangents to the adjacent circle passing through the points i–1, i and i+1 (AC(i–1, i, i+1)), and the closest tangent by location to AC(i–2, i–1, i) or AC(i, i+1, i+2). The value of the radius of the contour at the i-th point (R_i) is based on the condition of the location of the point series, in the nodes of which the value of radii of the curvature of the contour is less than R_i inside the i-th adjacent circle. The fulfillment of the mentioned conditions permits to form a second-order fixation contour with a monotonous curvature change [13].

Increasing the order of fixing of the contour to the second one at constant number of output nodes reduced the absolute error of the interpolation of the sections of the monotonous curve by an average of 9–10 times.

The estimation of the accuracy of interpolation based on the condition of growth or decrease of values of the curvature along a smooth curve is possible in the modeling of contours by methods of formation of monotonous curves. Among the well-known methods are methods developed within a framework of differential discrete geometric modeling [11–13].

6. Discussion of results of study of the error in discrete interpolation

Linear interpolation with a given absolute error requires determination of the area of possible location of the discretely represented curve line. Known methods of geometric modeling do not give possibility to determine such an area.

The conducted studies showed that we can determine the area based on predicted properties of the output curve and their corresponding properties of the interpolating curve.

This study presents the solution to the problem for a flat smooth curve based on the condition of absence of oscillations and a condition of a monotonous change of the curvature. The chosen conditions are universal, because any curve line can be divided into areas, along which values of the curvature monotonically increase or decrease.

One can use research results with approximate calculations, graphs construction, modeling of surfaces.

The proposed method was first developed for formation of surface frameworks, function of which is interaction with the environment. In the process of study, we established that the best solution for the task is based on the area of possible location of the curve. One can use such an area to solve any tasks that require estimation of the accuracy of interpolation.

The interpolation error, which is determined from the condition of the convexity of the curve, is maximal and is the output error. The imposition of more strict conditions: a monotonous change in values of the curvature along the curve and the purpose of fixed characteristics at the output points localize an area of a possible solution.

The main advantage of the proposed method is possibility of interpolation of a point series consisting of any number of nodes. This ensures regular and appropriate change in values of the curvature along the contour. The interpolation curve forms in the form of a condensed point series in sections that may interpolate the curve with a monotonous curvature change. This makes it possible to interpolate a point series of binary configuration. Condensation points are assigned based on the condition of existence of a possible location of the curve with given characteristics. The area of a possible solution is localized as a result of successive condensations of the point series.

An increase in the number of output points reduces the absolute error, with which the interpolating curve represents the output curve, but this does not result in accumulation of calculation errors or uncontrolled occurrence of special points.

The proposed method involves increase in the accuracy of interpolation at an unchanged number of output points due to the build-up of conditions imposed on the curve. For flat interpolation, the next step may be to control the rate of growth of the curvature along the curve.

The formed point series may represent a curve line with given geometric characteristics with an arbitrarily small absolute error. The value of the maximum absolute error of the representation of the output curve makes it possible to control the required amount of output information, which provides necessary accuracy of the problem solution. When solving practical problems, acceptable values of the error can be based on capabilities of a measuring equipment, requirements of product layout, permissible deviations of a mass and dimensions of the object being modeled.

The method gives possibility to take into account an arbitrary number of additional conditions in the process of modeling. This may be the accuracy that can be achieved by the processing equipment or special requirements for a quality of the surface of a product. For example, operation surfaces of products, functional purposes of which are interactions with the environment, should ensure laminar nature of their flow. The permissible error of interpolation in formation of linear elements of the geometric model of such a surface depends on a rate of contact of the surface with the flow of the environment and can be measured by micron particles. In the general case, the permissible error of further interpolation can be on orders of magnitude smaller than the error of presentation of the output curve line. The proposed method provides the accuracy required for the solution of such problems.

Formation of an interpolating curve with given geometric characteristics on the basis of a possible location makes possible to reduce the problem of ensuring the accuracy of geometric modeling to the solution of technical issues to ensure the necessary accuracy of measurements and calculations.

The main disadvantage of the developed method is that the formed discretely presented curve is unambiguous. Providing the required accuracy involves formation of a point series consisting of a large number of nodes at presentation of the modeling result in the form of an accompanying broken line. In addition, an algorithm for determination of ranges of the location of points of condensations requires implementation of sequential algebraic actions, each of which is performed with some degree of error. As a result of consecutive condensations of a point series, when a width of the area of possible location of a monotonous discretely represented curve is measured by values of 10^{-5} mm or less, these errors do not give possibility to control the given geometric properties of the contour. The mentioned problems include a large amount of required calculations and increased requirements for the accuracy of calculations.

Another problem caused by the discrete representation of geometric images is the complexity of solution of a number of positional geometric modeling tasks. This are a definition of a length of the discretely represented curve, points of intersection of the curve with given geometric images, and others. The mentioned problems require a solution when forming based on line frameworks of discretely presented surfaces.

The direction of further development of the method of interpolation based on the area of a possible location of curve line may be formation of spatial one-dimensional contours. Interpolation with given accuracy of spatial point series requires a regular change along a contour of the curvature, the rounding and the radii of spatial spheres.

7. Conclusions

1. We developed a method for the determination of the area of a possible location of a monotonous curve line, which interpolates a given point series. The method is based on

determination of segments of a point series, which may be interpolated by a monotonous curve line, a curve along which a value of the curvature increases or decreases monotonically. This makes possible to determine the output curve of the line as a curve with a minimum number of special points - points of inflection and points of change in the direction of growth of curvature. The method gives possibility to estimate the maximum absolute error of interpolation by the magnitude of the area of a possible location of monotonous curve lines. The estimation of the accuracy of discrete interpolation on the basis of the monotonous curvature change makes possible to reduce the absolute error by 2–3 times with respect to the estimation of accuracy based on the condition for prevention of oscillations.

2. We developed the method of formation of a one-dimensional contour, which represents a monotonous curve line with given accuracy. The method makes it possible to:

– form a monotonous discretely represented curve, which interpolates sections of the output point series, the area of a possible location of which is arbitrarily small;

– create a contour in the form of an accompanying broken line, which represents a monotonous curve with given accuracy.

Only capabilities of equipment limit the accuracy of representation of a monotonous curve. The calculation of test cases showed that the developed algorithms work steadily in the formation of a discretely presented monotonous curve located inside the area, a width of which is up to 10^{-5} mm.

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