STUDY OF THE LAMB WAVES PROPAGATION ALONG A PLATE IN CONTACT WITH A RANDOMLY INHOMOGENEOUS HETEROGENEOUS MEDIUM

Представлені результати дослідження впливу характеристик неоднорідною гетерогенного середовища на параметри процесу поширення хвиль Лемба по пластині, що контактує з цим середовищем. Розглянуто вплив параметрів досліджуваного середовища на величину загасання хвиль Лемба. Встановлено, що в'язкість рідини і швидкість потоку не впливають на процес поширення хвиль Лемба. Отримані результати можуть бути використані при розробці методів і засобів контролю параметрів газовмісних суспензій

Ключові слова: газовмісна суспензія, хвилі Лемба, згасання ультразвуку, в'язкість рідини

Представлены результаты исследования влияния характеристик неоднородной гетерогенной среды на параметры процесса распространения волн Лэмба по контактирующей с данной средой пластине. Рассмотрено влияние параметров исследуемой среды на величину затухания волн Лэмба. Установлено, что вязкость жидкости и скорость потока не влияют на процесс распространения волн Лэмба. Полученные результаты могут быть использованы при разработке методов и средств контроля параметров газосодержащих суспензий

Ключевые слова: газосодержащая суспензия, волны Лэмба, затухание ультразвука, вязкость жидкости

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1. Introduction

Ultrasonic surface waves have been widely applied in the practice of control over characteristics of technological media [1–9]. The reason for this is the following features of ultrasonic waves. The first feature is the relatively large concentration of energy in a wave due to the small magnitude of the layer of its location. The second feature is the possibility for obtaining an ultrasonic signal from any point of the surface (including curvilinear), along which it propagates [2].

The most studied and utilized of all currently known ultrasonic waves are the Rayleigh surface waves [1–3]. The waves of a given type propagate along the boundary of a solid space. A Rayleigh wave consists of two flat inhomogeneous waves – longitudinal and transverse, which, similar to the Rayleigh wave composed of them, are characterized by vertical polarization. The Rayleigh waves possess the greatest concentration of energy along the surface of a solid body. However, the characteristics of the propagation process substantially depend on the state of the surface of propagation.

It is possible to observe reflection, scattering, caused even by the microdefects of this surface.

Similar by their nature to the Rayleigh waves, but with horizontal polarization, are the Love waves [4, 5]. The Love waves, as the surface waves, exist due to the addition of a solid layer to the semi-space, the former acting as a load for the semi-space. The Love waves are distinguished by a strong dependence on the state of inhomogeneity of the surface layer, due to which they do exist, which makes measuring surfaces along which they propagate rather vulnerable and thereby "unstable".

The third basic type of ultrasonic surface waves are the waves on the boundary of two semi-spaces – the Stoneley waves [1]. A Stoneley wave is characterized by the elliptical polarization oriented perpendicularly to the boundary of semi-spaces. The Stoneley waves propagate in both liquid and solid semi-spaces. That is why their component, which propagates in a liquid semi-space, is exposed to the same disturbing factors that are experienced by ordinary volumetric ultrasonic oscillations. For example, one should expect a

strong dependence of the magnitude of their attenuation on the content of gas bubbles in industrial suspensions.

Surface waves also include waves in plates: normal waves with horizontal polarization (transverse normal waves) and normal waves with vertical polarization – the Lamb waves [1, 6–9]. Given that the walls of technological containers and a number of industrial units are typically made of sheet metal, in the realization of ultrasonic control over parameters of media that come into contact with them, it is convenient to use the Lamb waves. These waves are characterized by a large enough concentration of energy and are exposed to disturbing factors to a lesser extent than the Rayleigh waves and the Love waves. However, when devising methods and tools for control over parameters of gas-containing suspensions, it is an important task to determine the degree of influence of parameters of the investigated medium on the magnitude of attenuation of the Lamb waves.

In many cases, when developing methods for ultrasonic control over characteristics of technological media, the ultrasonic Rayleigh, Love, Stoneley waves are employed. It should be noted that the use of these types of waves is appropriate under conditions of meeting a number of significant limitations. Such limitations are related to the state of the surface of wave propagation, as well as the presence of gas bubbles in the examined medium. Otherwise, when failing to meet the limitations, the measurement accuracy is compromised. It is possible to resolve these shortcomings by using the Lamb waves. This type of wave is much less sensitive to the state of a propagation surface, the existence of gas bubbles in the investigated media. Thus, the study of processes of the Lamb waves propagation along a plate in contact with randomly heterogeneous medium is a promising and important area of research.

2. Literature review and problem statement

Research into optimization process of enrichment of iron ore whose results are reported in papers [10, 11] led to the following conclusion. Efficiency of control over technological processes of ore enrichment depends on the accuracy of operative information on the status of technological processes.

The patterns of propagation of ultrasonic oscillations in a fluid under the cavitated mode were considered in paper [12]. Based on numerical methods, the authors calculated the energy dissipated by bubbles. They obtained a direct correlation between the energy lost by gas bubbles and the attenuation of ultrasonic oscillations, which leads to the formation of traveling waves. Based on the results described above [13], the authors estimated the magnitude of Bjerknes forces and predicted the structures of gas bubbles generated as a result of traveling waves.

Study into dissipation of acoustic oscillations in liquids in the presence of bubbles is described in [14]. The model formed makes it possible to predict nonlinear attenuation of ultrasonic waves. It is noted that the predicted values of damping are much higher than the numbers estimated by similar models.

A method for modeling propagation of ultrasonic waves under conditions of heterogeneous media was considered in papers [15, 16]. To generate controlled ultrasonic waves to control characteristics of ore pulp, it was proposed to use phased arrays.

Theoretical and experimental studies into ultrasound propagation in bubbly liquids were undertaken in paper [17]. An approach to modeling the process of ultrasound propagation was proposed, which implies consideration of a non-uniform pressure field away of bubbles. To quantify the instability of bubbles, the authors applied analytical methods.

Numerical approach was applied to describe the process of ultrasound propagation in bubble liquids in paper [18]. This model is based on the methods of finite volume and finite differences. Such an approach makes it possible to solve a differential system formed by the wave equation and the Rayleigh-Plesset equation, which relates a sound pressure field with bubbles oscillations. The results obtained make it possible to observe the physical effects caused by the presence of bubbles in a liquid: nonlinearity, dispersion, attenuation.

The results of research into nonlinear propagation processes of ultrasonic waves in water at a non-uniform distribution of bubbles are presented in paper [19]. The mathematical model was synthesized using a set of differential equations that describes the connection between an acoustic field and bubble vibration. It is expected that the attenuation and nonlinear effects are due exclusively to the existence of bubbles. It should be noted that the heterogeneity in the distribution of bubbles is presented in the form of clusters of bubbles that can act as acoustic screens, and affect the behavior of ultrasonic waves.

The necessity of applying means of nondestructive testing in the process of enrichment of mineral-technological varieties of ore raw materials is substantiated in works [20–22]. Specifically, the means of ultrasonic testing will make it possible to obtain necessary information for the operational control over technological processes.

As a means of nondestructive testing, authors of paper [23] used the multimode Lamb waves. By measuring the characteristics of different modes in experimental dispersion curves of the Lamb waves and matching them against theoretical curves, the authors derived estimates for some physical parameters of the investigated medium. It is noted that the Lamb waves dispersion curves depend only on the plate parameters while the frequency and phase speed can be set relative to the speed of a shear wave and thickness of the layer of the investigated medium.

The use of controlled ultrasonic influence on the ore raw material in the enrichment process, in order to improve the efficiency of a given process, was studied in papers [24–25]. The authors presented a mathematical description of cavitation processes in a heterogeneous medium and generalized model of the dynamics of gas bubbles. The technique for determining optimal parameters for the source of ultrasonic oscillations was proposed.

The advantage of a Lamb wave in the field of nondestructive testing, as noted in [26], among the multitude of ultrasonic waves is that they can scan a large area with a minimum number of receivers. Since the Lamb waves are dispersing, it is recommended that the sinusoidal emission signal should be used. Simulation of the Lamb waves was performed using the software ATILA.

In paper [27], it was also noted that the Lamb waves are the most widely used ultrasonic waves to control various media. However, a theoretical analysis of the propagation of a controlled wave is a complex task. The authors considered a method for modeling the local interaction during propagation of waves in metallic structures. It should be noted that

the application of the proposed method is complicated by the coexistence of at least two highly dispersive modes at any given frequency.

A method of control over parameters of liquid media using the ultrasonic Lamb waves was presented in paper [28]. It is shown, that a change in the characteristics of waves can be used as a function that depends on the level of the liquid. It is noted that in order to determine the optimal conditions for measuring the parameters of a liquid medium using the Lamb waves, further research is needed.

Development of approaches to the use of means of ultrasonic testing in order to optimize control over technological enrichment processes was described in papers [29–31]. The results obtained suggest the appropriateness of employing ultrasonic control over characteristics of ore material in the technological flows at the enrichment line.

An analysis of the scientific literature revealed that in most cases, when developing methods of ultrasonic control over characteristics of heterogeneous media, certain types of waves are utilized. The most commonly used for these purposes are the Love, Stoneley, Rayleigh and Lamb surface waves. Selecting a specific type of waves for solving the set tasks requires taking into consideration a number of strict requirements and limitations imposed both on the characteristics of a propagation surface and properties of the controlled medium. To define characteristics of gas-containing suspensions, it appears promising to use the Lamb waves. However, still unresolved is the task on assessing the extent of influence of perturbing factors on the results of measuring the parameters of propagation of these waves.

3. The aim and objectives of the study

The aim of present research is to refine the patterns in the propagation of Lamb waves along a plate in contact with randomly heterogeneous medium, and to estimate the degree of influence of viscosity and motion speed of a gas-containing suspension on the attenuation magnitude of the Lamb waves.

To accomplish the aim, the following tasks have been set:
– to form a mathematical description of the Lamb wave propagation process in a metallic plate;
– to investigate the dependence of Lamb waves attenuation on the characteristics of a fluid in contact with the propagation medium.

4. Mathematical description and modeling of the Lamb wave propagation process in a metallic plate

Consider a flat harmonic Lamb wave that propagates in a plate with a thickness of $2d$ in the positive direction of the X axis. Introduce for the area occupied by the plate the scalar φ and vector ψ potentials of displacements that describe, respectively, the longitudinal and transverse waves. The values of φ and ψ can be represented in the following form [1, 2]

$$\varphi = A_s chqze^{ikx} + B_a shqze^{ikx}; \qquad (1)$$

$$\psi = D_s shsze^{ikx} + C_a chsze^{ikx}, \qquad (2)$$

where A_s, B_a, C_a, D_s are the arbitrary constants; k is the wavenumber of the Lamb waves;

$$q = \sqrt{k^2 - k_l^2}; \quad s = \sqrt{k^2 - k_t^2}.$$

Values of wavenumber k are derived from characteristic equations

$$\left(k^2 + s^2\right)^2 ch \cdot qh \cdot sh \cdot sd - 4k^2 qs \cdot sh \cdot qd \cdot ch \cdot sd = 0; \qquad (3)$$

$$\left(k^2 + s^2\right)^2 sh \cdot qd \cdot ch \cdot sd - 4k^2 qs \cdot ch \cdot qd \cdot sh \cdot sd = 0. \qquad (4)$$

Upon performing mathematical transformations, one can obtain, in order to calculate the desired potentials, the following expressions

$$\phi = A_s ch \cdot q_s ze^{ik_s x} + B_a sh \cdot q_a ze^{ik_a x}; \qquad (5)$$

$$\psi = \frac{2ik_s q_s sh \cdot q_s d}{\left(k_s^2 + s_s^2\right) shs_s d} A_s sh \cdot s_s ze^{ik_s x} +$$
$$+ \frac{2ik_a q_a ch_a \cdot q_a d}{\left(k_a^2 + s_a^2\right) ch \cdot s_a d} B_a chs_a ze^{ik_a x}, \qquad (6)$$

where

$$q_{s,a} = \sqrt{k_{s,a}^2 - k_l^2}; \ S_{s,a} = \sqrt{k_{s,a}^2 - k_t^2}.$$

Expressions (5) and (6) allow us to calculate displacement components U and W

$$U = U_S + U_a; \qquad (7)$$

$$W = W_S + W_a; \qquad (8)$$

$$U_S = Ak_s \left(\frac{ch \cdot q_s z}{sh \cdot q_s d} - \frac{2q_s s_s}{k_s^2 + s_s^2} \frac{sh \cdot s_a z}{ch \cdot s_a d}\right) \times e^{i\left(k_s x - \omega t - \frac{\pi}{2}\right)}; \qquad (9)$$

$$U_a = Bk_a \left(\frac{sh \cdot q_a z}{ch \cdot q_a d} - \frac{2q_a s_a}{k_a^2 + s_a^2} \frac{sh \cdot s_a z}{ch \cdot s_a d}\right) \times e^{i\left(k_a x - \omega t - \frac{\pi}{2}\right)}; \qquad (10)$$

$$W_S = -Aq_s \left(\frac{sh \cdot q_s z}{sh \cdot q_s d} - \frac{2k_s^2}{k_s^2 + s_s^2} \frac{sh \cdot s_s z}{sh \cdot s_s d}\right) \times e^{i\left(k_s x - \omega t - \frac{\pi}{2}\right)}; \qquad (11)$$

$$W_a = -Bq_a \left(\frac{ch \cdot q_a z}{ch \cdot q_a d} - \frac{2k_a^2}{k_a^2 + s_a^2} \frac{ch \cdot s_a z}{ch \cdot s_a d}\right) \times e^{i\left(k_a x - \omega t - \frac{\pi}{2}\right)}, \qquad (12)$$

where A and B are constants.

Analyzing expressions (5)–(12), one can notice that in the first group of waves, denoted with symbol s, the motion occurs symmetrically to the plane Z=0. That is, at the upper and bottom parts of the plate displacement U has the same signs, while displacement W – opposite. In the second group of waves, denoted with index a, the motion occurs anti-symmetrically relative to Z=0. That is, at the upper and bottom parts of the plate, displacement U has opposite signs, and displacement W – similar. The waves of the first group are the symmetrical Lamb waves while waves in the second group – anti-symmetrical.

In the plate with a thickness of $2d$ at frequency ω there can be a certain finite number of symmetric and antisymmetric Lamb waves. The mentioned waves are different one

from another in phase and group velocities, as well as distribution of displacements and stresses along the thickness of the plate.

Fig. 1–4 show simulation results of the Lamb wave propagation process in an aluminum plate using the software package LAMSS [34].
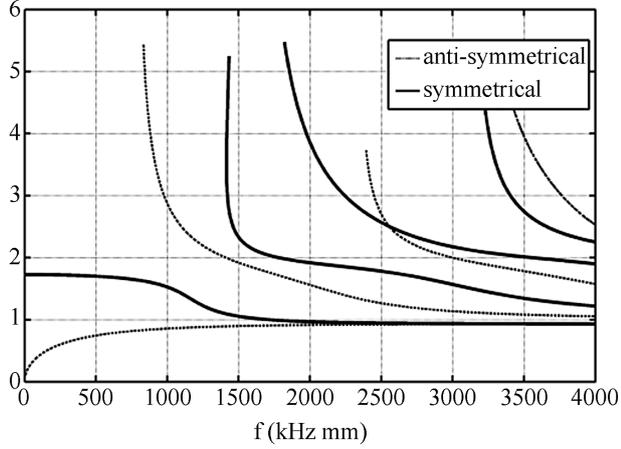

Fig. 1. Lamb wave phase speed in aluminum 2024-T3
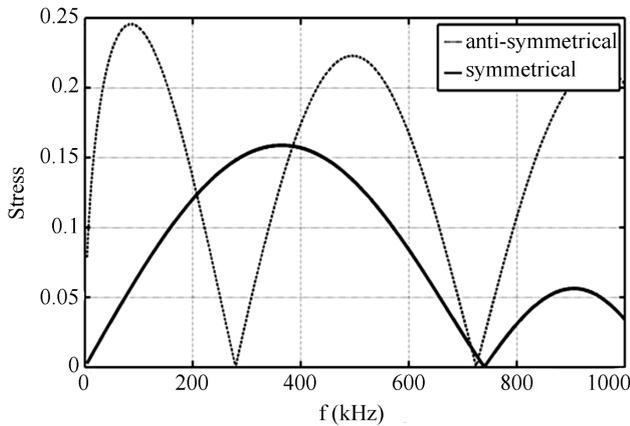

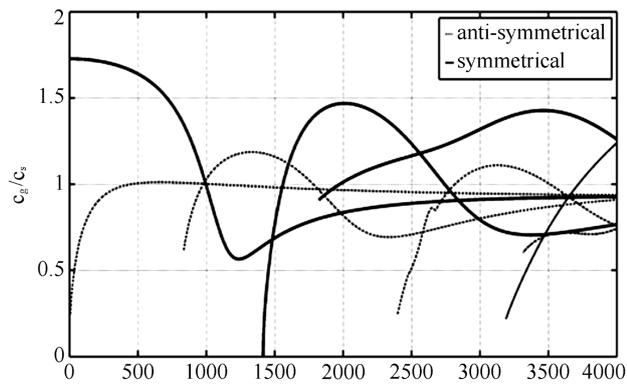Fig. 2. Stresses along the thickness of plate made of aluminum 2024-T3

Fig. 3. Lamb wave group speed

Formulas (7)–(12), obtained above, for the mathematical description of the Lamb wave propagation process allow us to proceed to the exploration of factors influencing the degree of attenuation of a given type of waves.
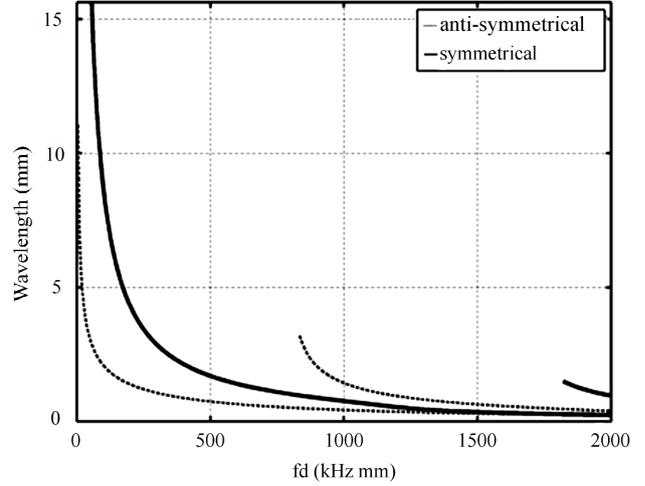

Fig. 4. Lamb wave length

5. Investigation of dependence of the Lamb wave attenuation on characteristics of the plate in contact with a liquid propagation medium

If a plate in which the Lamb wave propagates borders the fluid, and the speed of sound in liquid C_l is less than speed C of the wave in the plate, the Lamb wave will attenuate, radiating energy into liquid. Attenuation coefficient of the Lamb waves per unit length is determined from formula [1, 15]

$$k_2 = -i\frac{\rho_l}{\rho} k_1 \cdot A_{s,a}, \tag{13}$$

where ρ_l is the density of the fluid that borders the surface of the plate; ρ is the density of a material of the plate;

$$A_{s,a} = -\frac{ik_t^4 h S_{s,a} \cdot d}{8k_{s,a}^2 \cdot S_{s,a}\sqrt{k_c^2 - k_{s,a}^2}}\left[1 + \frac{k_{s,a}^2}{2S_{s,a}^2} + \frac{k_{s,a}^2}{2q_{s,a}^2} - \right.$$
$$-\frac{4k_{s,a}^2}{k_{s,a}^2 + S_{s,a}^2} + \frac{k_{s,a}^2 \cdot d}{2S_{s,a}^2}\left(thS_{s,a}d - cthS_{s,a}d\right) -$$
$$\left. -\frac{k_{s,a}^2 \cdot d}{2q_{s,a}}\left(thq_{s,a}d - cthq_{s,a}d\right)\right]^{-1}, \tag{14}$$

where $k_{s,a}$ is the wavenumber of the symmetric and antisymmetric Lamb waves;

$$q_{s,a} = \sqrt{k_{s,a}^2 - k_l^2};$$

$$S_{s,a} = \sqrt{k_{s,a}^2 - k_t^2};$$

k_l and k_t are the wavenumbers of longitudinal and transverse waves of the plate's material.

It should be noted that coefficient k_2 of the Lamb waves attenuation monotonically grows with an increase $\rho_l \cdot \rho^{-1}$, which means that k_2 can be represented as

$$k_2 = \frac{\rho_l}{\rho} C_v, \tag{15}$$

where C_v is the magnitude that almost does not depend on the fluid density.

Fig. 5 shows dependence of the attenuation coefficient per a Lamb wavelength on parameters of the medium: 1 – $C_t \cdot C_l^{-1} = 1,5$; 2 – $C_t \cdot C_l^{-1} = 2$; 3 – $C_t \cdot C_l^{-1} = 3$. Poisson's ratio of the plate's material is equal to 0.3.
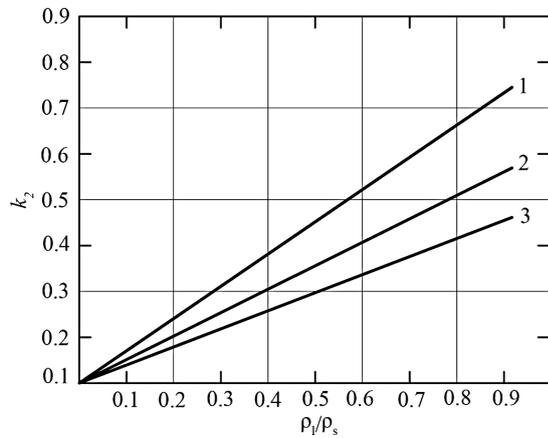

Fig. 5. Dependence of attenuation coefficient per a Lamb wavelength on parameters of the medium
$1 - C_t \cdot C_l^{-1} = 1,5;\ 2 - C_t \cdot C_l^{-1} = 2;\ 3 - C_t \cdot C_l^{-1} = 3$

Fig. 6 shows dependence of the magnitude of relative change in the Lamb wave speed on parameters of the medium under the same conditions.
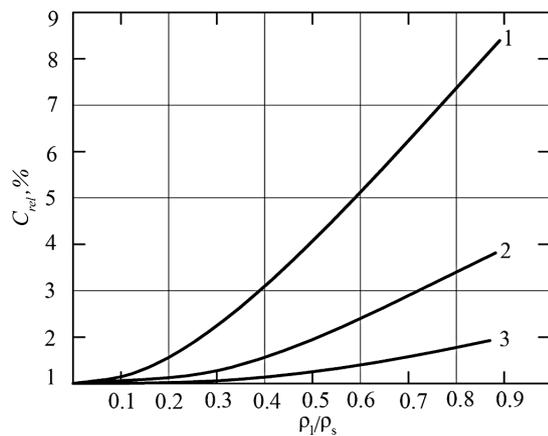

Fig. 6. Dependence of the magnitude of relative change in the Lamb wave speed on parameters of the medium
$1 - C_t \cdot C_l^{-1} = 1,5;\ 2 - C_t \cdot C_l^{-1} = 2;\ 3 - C_t \cdot C_l^{-1} = 3$

Consider the case where a plate is in contact with water, which contains solid phase particles of various size (suspension) and gas bubbles, that is a randomly heterogeneous medium. An example of such a medium is the iron ore pulp. As the gas phase of pulp has almost no effect on its density, gas bubbles do not affect the weakening of Lamb waves. In this case, the density of suspension ρ_l is determined by the volumetric fraction of particles of solid phase W, their average density ρ_s, and the density of water ρ_w

$$\rho_l = (1 - W)\rho_w + W\rho_s. \tag{16}$$

Attenuation coefficient k_2 of the Lamb waves in this case can be represented as

$$k_2 = \left[(1 - W)\frac{\rho_w}{\rho} + W\frac{\rho_s}{\rho} \right]C_v. \tag{17}$$

Thus, the intensity of Lamb waves at a distance l from the source of waves can be derived from formula

$$I_{l,v} = I_{0,v} \exp\{-k_2 l\} =$$
$$= I_{0,v} \exp\left\{ -\left[(1 - W)\frac{\rho_w}{\rho} + W\frac{\rho_T}{\rho} \right]C_v l \right\}. \tag{18}$$

If we accept in formula (18) that $W=0$, we obtain the expression that will determine the intensity of Lamb waves in contact between the plate and clean water.

$$I_{l,v}^* = I_{0,v} \exp\left\{ -\frac{\rho_w}{\rho}C_v l \right\}. \tag{19}$$

Consider the influence of viscosity of the investigated medium and the speed of its motion on the magnitude of Lamb waves attenuation.

At the border between liquid and solid media there arise the outflowing waves propagating along the border and continuously re-emitting energy into the fluid. This event leads to the Lamb waves attenuation.

As the fluid motion can affect the Lamb waves parameters only by internal forces of friction (viscosity), studying this factor gives an idea not only of the degree of influence of fluid flow velocity, but also its viscosity.

In terms of theoretical analysis of the specified situation, it is appropriate to consider an outflowing wave of the Rayleigh type at the border of solid and liquid semi-spaces. From a mathematical point of view, this problem is much easier, and from a physical standpoint is similar, which is why quantitative ratios of such a problem will properly reflect the influence of the factors under consideration.

Let the semi-space of the solid medium occupy region $Z>0$ and fluid $Z<0$. As it is known, in a general case, the equation of motion of elastic medium is written in the following form

$$\rho\frac{\partial^2 \vec{U}_l}{\partial t^2} - \mu\Delta\vec{U}_t + (\lambda + 2\mu)\operatorname{grad}\vec{U}, \tag{20}$$

where $\vec{U}$ is the vector of medium particles displacement; ρ is the density; λ and μ are the elastic constants of the medium (Lame parameters); Δ is the Laplace operator.

Represent displacement vector in the form

$$\vec{U} = \vec{U}_l + \vec{U}_t,$$

where $\vec{U}_l = \operatorname{grad}\phi$; $\vec{U}_t = \operatorname{rot}\psi$; ϕ and $\vec{\psi}$ are the scalar and vector potentials.

Then equation (20) can be reduced to two independent equations

$$\rho\frac{\partial^2 \vec{U}_l}{\partial t^2} - (\lambda + 2\mu)\Delta\vec{U}_l = 0; \tag{21}$$

$$\rho\frac{\partial^2 \vec{U}_t}{\partial t^2} - \mu\Delta\vec{U}_t = 0. \tag{22}$$

The first one describes the propagation of longitudinal waves, the second – transverse waves.

Consider a flat Rayleigh wave propagating in the positive direction of the X axis along the boundary between the semi-space and liquid.

In this case, the movement does not depend on coordinate Y. Then vector's potential $\vec{\psi}$ will be different from zero only by the component that is denoted ψ. Proceed from $\vec{U}_l$ and $\vec{U}_t$, to the scalar and vector potentials ϕ and ψ. For a flat harmonic wave the equations of motion (21) and (22) will hold if ϕ and ψ are the solutions to wave equations of the form

$$\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Z^2} + k_l \phi = 0; \tag{23}$$

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Z^2} + k_t^2 \psi = 0 \tag{24}$$

for a solid semi-space and

$$\frac{\partial^2 \phi^l}{\partial X^2} + \frac{\partial^2 \phi^l}{\partial Z^2} + k_l^2 \phi^l = 0, \tag{25}$$

for a liquid semi-space.

Here

$$k_l = \omega \sqrt{\rho/(\lambda + 2\mu)}, \quad k_t = \sqrt{\rho/\mu}$$

are the wavenumbers, respectively, of the longitudinal and transverse waves of a solid semi-space;

$$k_c = \sqrt{\rho/\lambda_l}$$

is the wavenumber of longitudinal waves propagating in a liquid half-space.

Solutions to equations (23), (24) and (25) will take the form:

$$\phi = A \exp[-qZ] \exp[i(kx - \omega t)];$$

$$\psi = B \exp[-SZ] \exp[i(kx - \omega t)]; \tag{26}$$

$$\phi^l = C \exp[q_l Z] \exp[i(kx - \omega t)],$$

where $q^2 = k^2 - k_l^2$; $S^2 = k^2 - k_l^2$; $q_l = k^2 - k_l^2$; A, B and C are the arbitrary constants.

Particle displacement components in a wave along the X and Z axes are expressed through potentials ϕ, ψ, ϕ^l as follows

$$U_x^{(1)} = \frac{\partial \phi}{\partial X} - \frac{\partial \psi}{\partial Z};$$

$$U_x^{(2)} = \frac{\partial \phi^l}{\partial X};$$

$$U_z^{(1)} = \frac{\partial \phi}{\partial Z} + \frac{\partial \psi}{\partial X}; \tag{27}$$

$$U_z^{(2)} = \frac{\partial \phi^l}{\partial Z}.$$

Here indexes (1) and (2) apply to the medium with solid and liquid semi-spaces, respectively.

The components T_{xx}, T_{zz}, T_{xz}, of the stress tensor can be expressed via ϕ, ψ, ϕ^l

$$T_{xx} = \lambda \left(\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Z^2} \right) + 2\mu \left(\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \psi}{\partial X \partial Z} \right),$$

$$T_{zz} = \lambda \left(\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Z^2} \right) + 2\mu \left(\frac{\partial^2 \phi}{\partial Z^2} + \frac{\partial^2 \psi}{\partial X \partial Z} \right),$$

$$T_{xz} = \lambda \left(2 \frac{\partial^2 \phi}{\partial X \partial Z} + \frac{\partial^2 \psi}{\partial X^2} - \frac{\partial^2 \psi}{\partial Z^2} \right). \tag{28}$$

Expressions (26)–(28) are initial for determining the arbitrary constants A, B и C and finding the wavenumber k. Boundary conditions are used for this purpose at $Z=0$. In this case, we assume the equality of displacement components along the Z axis of particles of solid and liquid semi-spaces. That is

$$U_z^{(1)}(Z=0) = U_z^{(2)}(Z=0),$$

and when $Z=0$ component T_{xz} and T_{zz} of stress tensors of both media are the same.

Expression (26), when recording a scalar potential ϕ^l, does not reflect the fact of the liquid motion. Therefore, before proceeding to the boundary conditions to obtain the characteristic equation for k, let us find out how the magnitude ϕ^l changes when taking into account the motion of liquid.

Let the liquid medium flows along the X axis from right to left (Fig. 7) at speed V.
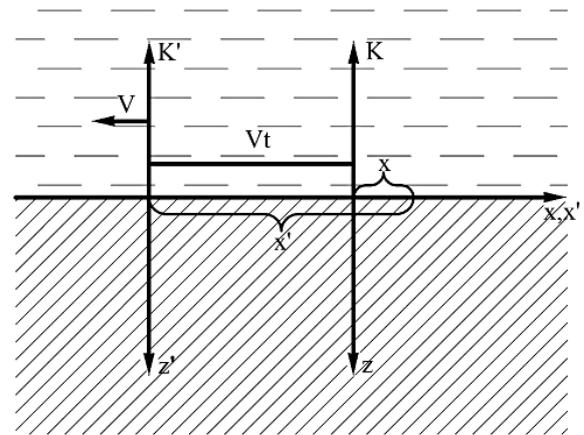

Fig. 7. Geometrical interpretation of the simulated space

Proceed to the k', coordinate system in which fluid is resting, it means that the system k' moves along the negative X axis direction at speed V. If the Rayleigh wave with frequency ω propagates along the positive direction of the X axis at velocity c, the frequency and speed of this wave in the k system will be equal to $\omega' = \omega(1 + V/c)$ according to the Doppler effect, and $c' = c + V$, according to the classical law of addition of velocities. Wave numbers k' and q_l' in the system k' do not change because

$$k' = \frac{\omega'}{c'} = \frac{\omega(1 + V/c)}{c(1 + V/c)} = \frac{\omega}{c} = k.$$

The same applies to q_l', so the scalar potential ϕ^l will take the form

$$\phi^l = C \exp(q_l z) \exp\left[i(kx' - \omega' t) \right]. \tag{29}$$

The presence of internal friction (viscosity) of liquid leads to the occurrence of tangential forces that can be described by assigning the stress tensor

$$T_{xy}^{(2)} = -\eta \frac{\partial V_x^l}{\partial z} = -\eta \frac{\partial V_x'^l}{\partial z} = -\eta \frac{\partial^2 V_x'^l}{\partial t \cdot \partial t}, \tag{30}$$

where η is the viscosity coefficient;

$$U_x^l = \frac{\partial \phi^l}{\partial x^l} ik\phi^l.$$

From the condition of equality of components of the stress tensor of a solid medium and fluid with respect to coordinate transformations of Galileo $X' = X + Vt$, we obtain equation

$$2ikq_1 A + \left(k^2 + S_1^2\right)B - \frac{\eta}{\lambda_1} q_c \omega' k C = 0. \tag{31}$$

Thereafter, the magnitude η/λ_1 will be denoted via β. The second condition of equality

$$T_{zz}^{(1)}\left(X, Z = 0\right) = T_{zz}^l\left(X' = X + Vt, Z = 0\right)$$

leads to equation:

$$\left(k^2 + S_1^2\right)A + 2iS_1 kB - k_{t_1}^2 \frac{\rho_l}{\rho_1} C = 0. \tag{32}$$

And, finally, the condition of equality of components of the Z displacement

$$U_z^{(1)}\left(X, Z\right) = 0 = U_z^l\left(X' = X + Vt, Z = 0\right)$$

produces equation

$$q_1 A - ikB + q_l C = 0. \tag{33}$$

Thus, we obtain a system of linear homogeneous equations relative to arbitrary constants A, B and C.

$$2ikq_1 A + \left(k^2 + S_1^2\right)B - \beta q_l \omega' k C = 0;$$

$$\left(k^2 + S_1^2\right)A + 2iS_1 kB - k_{t_1}^2 \frac{\rho_l}{\rho_1} C = 0; \tag{34}$$

$$q_1 A - ikB + q_l C = 0.$$

The condition for the existence of a nontrivial solution to this system is the equality of its determinant to zero

$$\begin{vmatrix} 2ikq_1 & k^2 + S_1^2 & \beta q_l \omega' k \\ k^2 + S_1^2 & 2iS_1 k & -k_{t_1}^2 \\ q_1 & -ik & q_l \end{vmatrix} = 0. \tag{35}$$

This produces the following characteristic equation to find the wavenumber k

$$2k^2 q_1 S_1 + \left(k^2 + S_1^2\right) =$$

$$= \frac{\rho_l k_{t_1}^4 q_1}{\rho_1 q_l} + iBk^2 \omega' k \left[\left(k^2 + S_1^2\right) + 2q_1 S_1\right]. \tag{36}$$

It can be shown that under condition $c > c_R$, which holds for almost all actual media, equation (36) has a complex root $k = k_R$, corresponding to the system of three waves. In this case, for a given wave, q_l in equation (36) should be understood as the next branch of the root

$$q_l = \sqrt{k^2 - k_l^2} = -i\sqrt{k_l^2 - k^2}.$$

The complexity of k has a simple physical sense: a surface wave in this case continuously radiates energy into a liquid, forming in it a heterogeneous wave diverging from the boundary (an outflowing wave).

In characteristic equation (36) proceed to the dimensionless variable

$$\mu^2 = k^2 / k_{t_1}^2.$$

The result is

$$4\mu^2 = \sqrt{\mu^2 - \zeta^2}\sqrt{\mu^2 - 1} + \left(2\mu^2 - 1\right) =$$

$$= i\frac{\rho_c \sqrt{\mu^2 - \zeta^2}}{\rho_1 \sqrt{\nu^2 - \mu^2}} + i\beta\omega\mu^2\left(1 + \alpha\mu\right) \times$$

$$\times \left[\left(2\mu^2 - 1\right) + 2\sqrt{\mu^2 - \zeta^2}\sqrt{\mu^2 - 1}\right]. \tag{37}$$

Because k is the complex magnitude, represent it as

$$k = k_1 + ik_2,$$

where $k_2 \ll k_1$, since $k \approx k_R$.

Comprehensive part of the wave number is the attenuation coefficient of a surface wave of the Rayleigh type, so the main task is to determine this magnitude.

The method for its determining will imply the following: a dimensionless variable is represented in the form

$$\mu = \mu_1 + i\mu_2,$$

where $\mu_1 = k_1 / k_{t_1}$; $\mu_2 = k_2 / k_{t_1}$. In this case, $\mu_2 \ll \mu_1$, because $k_2 \ll k_1$. Next, decomposing in equation (37) all magnitudes containing μ, into a series, and, confining ourselves to the terms of the first-order smallness relative to μ_2, we obtain the equation that contains, separately, the integrated and the real parts

$$4\mu_1^2 = \sqrt{\mu_1^2 - \zeta^2}\sqrt{\mu_1^2 - 1} + \left(2\mu_1^2 - 1\right) + i\mu_2\left[8\mu_1\left(2\mu_1^2 - 1\right) + \right.$$

$$+ 4\mu_1\left(2\sqrt{\mu_1^2 - \zeta^2}\sqrt{\mu_1^2 - 1}\right) + \mu_1^2 \frac{\sqrt{\mu_1^2 - 1}}{\sqrt{\mu_1^2 - \zeta^2}} + \mu_1^2 \frac{\sqrt{\mu_1^2 - \zeta^2}}{\sqrt{\mu_1^2 - 1}}\right] =$$

$$= \beta\omega\mu_1\mu_2\left\{\left(2 + 3\mu_1^2\right)\left[\left(2\mu_1^2 - 1\right) + \sqrt{\mu_1^2 - 1} \cdot \sqrt{\mu_1^2 - \zeta^2}\right] + \right.$$

$$\left. + \mu_1^2\left(4 + \frac{\sqrt{\mu_1^2 - 1}}{\sqrt{\mu_1^2 - \zeta^2}} + \mu_1^2 \frac{\sqrt{\mu_1^2 - \zeta^2}}{\sqrt{\mu_1^2 - 1}}\right)\right\} -$$

$$- \frac{\rho_c}{\rho_1}\mu_1\mu_2 \times \left(\frac{1}{\sqrt{\mu_1^2 - \zeta^2}} + \mu_1^2 \frac{\sqrt{\mu_1^2 - \zeta^2}}{\nu^2 - \mu_1}\right) +$$

$$+ i\left\{\frac{\rho_c \sqrt{\mu_1^2 - \zeta^2}}{\rho_1 \sqrt{\nu^2 - \mu_1^2}} + \beta\omega\left[\left(2\mu_1^2 - 1\right) + \sqrt{\mu_1^2 - \zeta^2}\sqrt{\mu_1^2 - 1}\right]\mu_1\left(1 + \alpha\mu\right)\right\}. \tag{38}$$

Next, by equating imaginary and real expressions from the left and right side of ratio (38), we obtain two equations, solving which produces information about μ_1 and μ_2

$$\mu_2\left[8\mu_1\left(2\mu_1^2-1\right)+4\mu_1\left(2\sqrt{\mu_1^2-\zeta^2}\sqrt{\mu_1^2-1}\right)+\right.$$

$$\left.+\mu_1^2\frac{\sqrt{\mu_1^2-1}}{\sqrt{\mu_1^2-\zeta^2}}+\mu_1^2\frac{\sqrt{\mu_1^2-\zeta^2}}{\sqrt{\mu_1^2-1}}\right]=$$

$$=\frac{\rho_c\sqrt{\mu_1^2-\zeta^2}}{\rho_1\sqrt{v^2-\mu_1^2}}+\beta\omega\mu_1\left(1+\alpha\mu_1\right)\times$$

$$\times\left[\left(2\mu_1^2-1\right)+\sqrt{\mu_1^2-1}\cdot\sqrt{\mu_1^2-\zeta^2}\right]; \qquad (39)$$

$$4\mu_1^2\sqrt{\mu_1^2-\zeta^2}\sqrt{\mu_1^2-1}+\left(2\mu_1^2-1\right)^2=$$

$$=-\left[\frac{\rho_c}{\rho_1}\mu_1\left(\frac{1}{\sqrt{\mu_1^2-\zeta^2}}+\frac{\sqrt{\mu_1^2-\zeta^2}}{v^2-\mu_1^2}+\right.\right.$$

$$\left.\left.+\beta\omega\mu_1\phi_1\left(\mu_1\right)\left(1+\alpha\mu_1\right)\right]\mu_2, \qquad (40)$$

where

$$\phi_1\left(\mu_1\right)=\left\{\left(2+3\mu_1^2\right)\left[\left(2\mu_1^2-1\right)+\sqrt{\mu_1^2-1}\cdot\sqrt{\mu_1^2-\zeta^2}\right]+\right.$$

$$\left.+\mu_1^2\left(4+\frac{\sqrt{\mu_1^2-1}}{\sqrt{\mu_1^2-\zeta^2}}+\mu_1^2\frac{\sqrt{\mu_1^2-\zeta^2}}{\sqrt{\mu_1^2-1}}\right)\right\}. \qquad (41)$$

From equation (40), μ_2 will be represented as

$$\mu_2=\frac{\rho_l}{\rho_1}f_1\left(\mu_1\right)+\beta\omega\left(1+\alpha\mu_1\right)f_2\left(\mu_1\right), \qquad (42)$$

where

$$f_1\left(\mu_1\right)=\sqrt{\mu_1^2-\zeta^2}\left/\sqrt{v^2-\mu_1^2}\left[8\mu_1\left(2\mu_1^2-1\right)+\right.\right.$$

$$\left.+4\mu_1\left(2\sqrt{\mu_1^2-\zeta^2}\sqrt{\mu_1^2-1}+\mu_1^2\frac{\sqrt{\mu_1^2-1}}{\sqrt{\mu_1^2-\zeta^2}}+\mu_1^2\frac{\sqrt{\mu_1^2-\zeta^2}}{\sqrt{\mu_1^2-1}}\right)\right];$$

$$f_1\left(\mu_1\right)=\left[\left(2\mu_1^2-1\right)+\sqrt{\mu_1^2-1}\cdot\sqrt{\mu_1^2-\zeta^2}\right]\sqrt{v^2-\mu_1^2}\cdot f_1\left(\mu_1\right)\left/\sqrt{\mu_1^2-\zeta^2}\right..$$

In turn, μ_1 is found from the solution to equation

$$4\mu_1^2\sqrt{\mu_1^2-\zeta^2}\sqrt{\mu_1^2-1}+\left(2\mu_1^2-1\right)^2=-\left(\frac{\rho_l}{\rho_1}\right)^2\mu_1\phi_2\left(\mu_1\right)f_1\left(\mu_1\right)-$$

$$-\frac{\rho_l}{\rho_1}\beta\omega\mu_1\left(1+\alpha\mu_1\right)\left[\phi_1\left(\mu_1\right)f_1\left(\mu_1\right)+\phi_2\left(\mu_1\right)f_2\left(\mu_1\right)\right], \qquad (43)$$

where

$$\phi_2\left(\mu_1\right)=\frac{1}{\sqrt{\mu_1^2-\zeta^2}}+\frac{\sqrt{\mu_1^2-\zeta^2}}{v^2-\mu_1^2}.$$

Since $\mu_1\approx\mu_R$ (an analysis reveals that the difference between these magnitudes does not exceed 1–2 %), with a good approximation μ_2 can be estimated from formula (42) assuming that in this case $\mu_1=\mu_R$.

As seen from expression (42), μ_2 is defined by two elements; the second one assesses the contribution of viscosity and movement of liquid on the surface wave attenuation factor. Multiplier $\left(1+\alpha\mu_1\right)$ shows effect of the fluid motion. However, the magnitude $\alpha\mu_R\ll1$ $\left(\alpha\mu_R\sim10^{-3}\right)$.

6. Discussion of results of research into the processes of Lamb wave propagation in a metallic plate

We have considered results of research into the Lamb wave propagation process along a plate in contact with randomly-heterogeneous medium. The importance of the results obtained is emphasized by that they could be used to develop improved methods and tools to control parameters of gas-containing suspensions.

There are objective difficulties for using ultrasonic methods of control over parameters of gas-containing suspensions related to the existence of microdefects at the surface of propagation and gas bubbles in the examined liquid. In this case, the results of the measurements of an ultrasonic field will contain significant error. Study of the process of Lamb wave propagation under similar conditions has shown that the use of this type of waves makes it possible to overcome these disadvantages, which is the benefit of the proposed approach. The value of measurement error in this case does not exceed permissible limits under different condition of the surface of propagation of ultrasonic waves and varying content of gas bubbles in the examined medium.

It was established that the attenuation is due to the density of the studied medium. Viscosity and speed of a gas-containing suspension, for example the iron ore pulp, have almost no effect on the magnitude of attenuation of the Lamb waves.

From a practical point of view, the results obtained could be used for calculation of parameters of measuring channels in the systems of ultrasonic control over characteristics of shredded materials in the flow of pulp. For example, at ore dressing plants, processed ore is categorized based on the main chemical-mineralogical and physical-mechanical characteristics. The resulting separation of ore into varieties is subsequently used in the planning and organization of ore extraction. At the same time, the task is to ensure constant characteristics of ore over a certain time interval. Such an approach makes it possible to better comply with technology regulations for the enrichment process and contributes to the improvement of quality of the finished product the concentrate. However, the mining system does not at present make it possible to extract the same type of ore to provide for a sufficient time for stable operation of the processing plants. Hence the relevance of operational control over characteristics of ore in technological processes at an enrichment plant and recognition of its mineralogical-technological varieties. In this case, the ultrasonic control methods solve the important task on ensuring that the management systems of ore-enrichment receive reliable operational information on the characteristics of the material. Thus, there appears an opportunity to improve the efficiency of technological processes in the mining and metallurgical industry, specifically, the enrichment of ore.

The disadvantages of the proposed approach include the lack of a possibility to control characteristics of the particles of the solid phase of the pulp that belong to a specific size class. That is, under present conditions, it is not possible to obtain information on the characteristics of solid phase particles of a specific size class. This information is important in the formation of control actions in the process of enrichment of ore.

The prospect of the development of present research is to study the possibilities of combining the tools for ultrasonic (using surface and volumetric waves) and nuclear physical

measurements. This would make it possible to control the characteristics of certain particle size classes that make up the solid phase of the pulp. It is necessary to investigate the effects of a simultaneous impact of various types of radiation on the ore pulp. Further research in this direction will require taking into consideration numerous disturbing factors that differently affect various kinds of energy impact.

frequency ω. The possibility is shown of the existence under these conditions of a specific finite number of symmetric and antisymmetric Lamb waves. The mentioned waves are different one from another in phase and group velocities, as well as the distribution of displacements and stresses along the plate's thickness.

2. We have investigated dependence of the Lamb wave attenuation on the characteristics of fluid in contact with the propagation medium. In the case when the Lamb wave propagation medium borders the fluid and the speed of sound in liquid C_l is less than speed C of the wave in a plate, the Lamb wave will attenuate radiating energy into liquid. It was established that the attenuation is due to the density of the studied medium. Viscosity and speed of a gas-containing suspension, for example iron ore pulp, have almost no effect on the magnitude of attenuation of the Lamb waves.

7. Conclusions

1. We have formed a mathematical description of the Lamb wave propagation process in a metallic plate. Analytical expressions were derived for determining the displacement potentials that describe, respectively, the longitudinal and transverse waves in a plate with a thickness of $2d$ at

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