Отримано систему рівнянь асинхронного двигуна в трифазній системі координат при нерухомому роторі, отримано аналітичне рішення рівнянь асинхронного двигуна для даного режиму роботи. Уточнена теоретична математична модель пускового струму статора асинхронного двигуна. Встановлено, що характер коренів характеристичного рівняння асинхронного двигуна і вид його перехідної характеристики залежать від початкового кутового положення ротора

Ключові слова: асинхронний двигун, трифазна система координат, метод простору станів, характеристичне рівняння, матриця передавальних функцій

Получена система уравнений асинхронного двигателя в трехфазной системе координат при неподвижном роторе, получено аналитическое решение уравнений асинхронного двигателя для данного режима работы. Уточнена теоретическая математическая модель пускового тока статора асинхронного двигателя. Установлено, что характер корней характеристического уравнения асинхронного двигателя и вид его переходной характеристики зависят от начального углового положения ротора

Ключевые слова: асинхронный двигатель, трехфазная система координат, метод пространства состояний, характеристическое уравнение, матрица передаточных функций

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ANALYTICAL STUDY OF STARTING CURRENT OF THE INDUCTION MOTOR STATOR

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1. Introduction

A theoretical mathematical model of starting current in the induction motor's stator is used in various studies that focus on diagnosing technical condition, identification of parameters for equivalent circuit, synthesis of control systems for the induction motor (IM). The analysis of induction motor based on its T-shaped equivalent circuit needs to be refined. To study dynamic modes of IM, a mathematical model of the induction motor in a three-phase coordinate system is widely employed.

The mathematical model of a three-phase IM is described by a nonlinear system of differential equations of eighth order with periodic coefficients. In a general case, it is impossible to solve such a system of equations analytically.

Therefore, the development of methods to solve analytically the equations of IM in a particular case of the starting mode will make it possible to refine existing understanding of characteristics of the stator starting current. Analytical solution to the equations of induction motor for a particular case of the starting mode is of independent scientific value with a rather wide scope of practical application. The results to be obtained could be used to refine methods for the identification of induction motor parameters based on data about starting mode.

2. Literature review and problem statement

Resolving a wide range of practical tasks related to the analysis of current technical condition of an induction motor, identification of its parameters, control over an asynchronous electric drive, ultimately relies on analysis of the starting currents of IM stator. Authors of [1], in order to determine asymmetry of the rotor in induction machines, suggest monitoring the evolution of the right lateral range (RSC) in the stator starting current.

In [2], to analyze technical condition of the stator winding, it is proposed to employ Motor Stator Current Envelope Analysis (MSCEA). Authors of [3] propose a method for the identification of active resistance of IM stator that utilizes a permanent integral component from starting current of the stator. In [4], authors consider a method for the analysis of technical condition of traction IM of electric locomotive transport, based on the analysis of stator winding current shape when switched on by direct voltage.

Existing understanding of the theoretical mathematical model for the formation of IM short-circuit currents is based on the analysis of a T-shaped equivalent circuit of IM [5]; it is significantly simplified and could be refined when solving the IM equations analytically for the mode of a stationary rotor.

A range of practical application of IM is extremely wide. Therefore, issues on mathematical modeling of IM are addressed in the sources of various subjects. Authors of [6] examine IM models for the analysis of static properties of IM. In [7], authors considered the use of mathematical models of IM in the composition of an automated electric drive. Authors of [8] outline an updated approach to the analysis of basic types of coordinate systems applied in the modeling of IM. Study [9] tackles various aspects of IM control. In [10], modeling of IM is used for the synthesis of IM scalar control system. Authors of [11] addressed the problem of vector control over IM with fuzzy regulators. In [12], authors defined variants of IM models in a three-phase coordinate system. Study [13] uses a mathematical model of IM to identify active resistances of IM stator and rotor.

A variety of mathematical models of the IM is predetermined by the following features:

– various techniques of power supply to an induction motor;

– various features of design of an induction motor;

– application of different coordinate systems for the mathematical description of an induction motor.

The single-phase models demonstrate more or less accurate representation of IM static properties, present very simplified reflection of dynamic properties, and lack information about instantaneous values of phase signals.

The two-phase models differ from each other by the rotation speed of coordinate system and its orientation, as well as by the presence of different signals in them. Such models are created using the decomposition of spatial (generalized) vectors of phase magnitude onto orthogonal axes of respective coordinate systems. Two-phase models, compared to the three-phase models, are of less order and contain simpler dependences for the calculation of currents and moments. They, however, cannot be applied to study asymmetrical modes.

Three-phase models of IM in the natural phase stator and rotor coordinates are characterized by a higher order of the system of differential equations. The equations of IM models include harmonic coefficients, which depend on the angular position of the rotor. To determine the stator and rotor currents, it is necessary to additionally solve a system of linear algebraic equations. These disadvantages are compensated for by the possibility to study asymmetrical modes of IM operation.

In order to synthesize and examine the systems of scalar and vector frequency control, typically used are the two-phase models of IM in the coordinate systems that rotate at different speed. The studies that address modelling of IM consider advanced models of IM, which make it possible to account for such nonlinear phenomena as the saturation of a magnetic system [14] and displacement of currents in the rotor circuits [15]. Given the complexity and non-linearity of IM models, their study employs numerical solutions. Development of computational tools led to a reduction in the number of scholarly works that dealt with analytical analysis of research problems, specifically papers related to the operation of electric machines of alternating current.

In the general case, numerical solutions significantly narrow the generality of obtained results, complicate studying parametric sensitivity of the solutions received. Therefore, analytical solution to the equations of IM has a separate scientific value.

3. The aim and objectives of the study

The aim of present study is to solve analytically the IM equations in three-phase coordinates for the starting mode, which would make it possible to refine existing theoretical mathematical models of the stator starting currents.

To accomplish the aim, the following tasks have been set:

– to construct IM equations taking into consideration the peculiarities of the process of acceleration;

– to examine solutions to the equations of IM under a starting mode.

4. Equations of IM in a three-phase coordinate system when the rotor is stationary

A three-phase induction motor has almost a century of improvements and has been actively studied. The most universal method for studying an induction motor is research into electromagnetic fields inside the electric machine based on the application of Maxwell's equations. Such an approach has no prospects for the analytical solution due to the complex structure of initial equations, complicated geometric configuration of IM.

Most popular for the analysis of IM as an object of control are the models based on the theory of electric circuits. A system of IM equations in a natural three-phase coordinate system contains periodic coefficients that depend on the angular position of the rotor. This system of equations cannot be represented in the Cauchy form, making it impossible to solve it analytically in the general case. However, at the stationary rotor of IM, it is possible to substantially simplify IM equations and there emerges a possibility to solve them analytically.

When IM is described mathematically, electromagnetic processes in the stator windings are described in a fixed coordinate system of the stator *ABC,* and electromagnetic processes in the rotor windings – in the rotor rotating coordinate system *abc*. The system of differential equations of electrical equilibrium of IM windings in a 3-phase coordinate system in the matrix form takes the following form [12]:

$$\begin{cases} \mathbf{U}_s = \mathbf{R}_s \mathbf{I}_s + \dfrac{d\mathbf{\Psi}_s}{dt}, \\ \mathbf{U}_r = \mathbf{R}_r \mathbf{I}_r + \dfrac{d\mathbf{\Psi}_r}{dt}, \end{cases} \quad (1)$$

where $\mathbf{U}_s$, $\mathbf{U}_r$ are the vectors-columns of feed voltage of the stator and rotor; $\mathbf{I}_s$, $\mathbf{I}_r$ are the vectors-columns of currents of separate windings of the stator and rotor; $\mathbf{R}_s$, $\mathbf{R}_r$ are the diagonal matrices of active resistances of windings of the stator and rotor; $\mathbf{\Psi}_s$, $\mathbf{\Psi}_r$ are the vectors-columns of linkage of the stator and rotor.

When constructing equations, we shall use the following assumptions associated with the notion of idealized engine:

– steel of the motor is not saturated;

– phase stator windings are symmetrical and evenly shifted in space;

– magnetomotive force (MMF) of windings and magnetic fields propagate sinusoidally along the circumference of the air gap;

– the rotor is electrically and magnetically symmetrical;

– really distributed windings of IM are replaced with lumped, and MMF shall be taken equal to the real winding.

Detailed equations of linkage for the stator phases:

$$\psi_A = L_1 \cdot I_A + M \cdot \iota_a \cdot \cos(\gamma) + M \cdot \iota_b \cdot \cos\left(\gamma + \frac{2\pi}{3}\right) + M \cdot \iota_c \cdot \cos\left(\gamma - \frac{2\pi}{3}\right),$$

$$\psi_B = L_1 \cdot I_B + M \cdot \iota_a \cdot \cos\left(\gamma - \frac{2\pi}{3}\right) + M \cdot \iota_b \cdot \cos(\gamma) + M \cdot \iota_c \cdot \cos\left(\gamma + \frac{2\pi}{3}\right),$$

$$\psi_C = L_1 \cdot I_C + M \cdot \iota_a \cdot \cos\left(\gamma + \frac{2\pi}{3}\right) + M \cdot \iota_b \cdot \cos\left(\gamma - \frac{2\pi}{3}\right) + M \cdot \iota_c \cdot \cos(\gamma), \quad (2)$$

where γ is the angle of rotation of the IM rotor.

A complete system of IM equations, taking into consideration the equation of rotor motion, is a system of ordinary nonlinear differential equations of eighth order.

The main source of the non-linearity of IM equations is the presence in the equations of electrical equilibrium of harmonic coefficients that depend on the angular position of the rotor.

Another important feature of system (1) is that it cannot be represented in a canonical form, and numerical solution to system (1) is related to solving an intermediate system of linear algebraic equations of sixth order.

First of all, we shall transform system (1) to decrease its order. It is obvious that the system of equations has explicit redundancy for the case of IM operation without a zero wire.

According to the Kirchhoff's first law, in this case, the sum of phase stator and rotor currents at any point in time is equal to zero; these relations hold in the presence of asymmetry of the windings.

We shall rewrite the equation of linkage, excluding from them a current of phase C using the Kirchhoff's first law. After performing simple trigonometric transforms, we shall obtain the following reduced equations of linkage of the stator and rotor, with excluded currents of phases C of the stator and c of the rotor.

$$\begin{cases} \psi_A = L_1 \cdot I_A - M \cdot \sqrt{3} \cdot \iota_a \cdot \sin\left(\gamma - \frac{\pi}{3}\right) - M \cdot \sqrt{3} \cdot \iota_b \cdot \sin(\gamma), \\ \psi_B = L_1 \cdot I_B + M \cdot \sqrt{3} \cdot \iota_a \cdot \sin(\gamma) + M \cdot \sqrt{3} \cdot \iota_b \cdot \sin\left(\gamma + \frac{\pi}{3}\right), \\ \psi_a = L_2 \cdot i_a + M \cdot \sqrt{3} \cdot I_A \cdot \sin\left(\gamma + \frac{\pi}{3}\right) + M \cdot \sqrt{3} \cdot L_B \cdot \sin(\gamma), \\ \psi_b = L_2 \cdot i_b - M \cdot \sqrt{3} \cdot I_A \cdot \sin(\gamma) - M \cdot \sqrt{3} \cdot I_B \cdot \sin\left(\gamma - \frac{\pi}{3}\right). \end{cases} \quad (3)$$

Further simplification of equations (3) is possible only by limiting the considered operating modes of IM. We shall consider IM operation under a starting mode. At the initial stage of the start, driving moment T_e, generated by the electric motor, remains less than the moment of resistance of the working machine (WM) T_L. Because the moment of resistance of WM is typically created by the forces of friction, the moment of resistance T_L can be considered Coulomb while the rotor of the started engine remains stationary. The issues of formation of the WM moment of resistance T_L for different types of WM with complicated starting conditions are described in detail in [9]. It is shown that the moment of resistance T_L when starting an IM can substantially exceed its value in the established operation mode.

Thus, in the process of starting, IM operated under a short circuit mode, its angular speed is zero, and the angle of rotor rotation does not change and remains equal to its initial value γ_0.

This assumption considerably simplifies the structure of equations, both for a drive motor and for the equation of motion in general, thereby creating preconditions for obtaining elements of analytical solution to these equations.

We shall simplify IM equations in phase coordinates for the starting mode, assuming that the angular speed of IM remains zero.

We shall introduce the following notation:

$$\begin{cases} \frac{dI_A}{dt} = x; & \frac{di_a}{dt} = m; \\ \frac{dI_B}{dt} = y; & \frac{di_b}{dt} = n; \\ \alpha = M \cdot \sqrt{3} \cdot \sin(\gamma - \pi/3); & \beta = M \cdot \sqrt{3} \cdot \sin(\gamma); \end{cases} \quad (4)$$

Linkage equation (3), taking into consideration notation (4), will take the form

$$\begin{cases} \psi_A = L_1 \cdot I_A - \iota_a \cdot \alpha - \iota_b \cdot \beta, \\ \psi_B = L_1 \cdot I_B + \iota_a \cdot \beta + \iota_b \cdot \alpha, \\ \psi_a = L_2 \cdot i_a + I_A \cdot \alpha + I_B \cdot \beta, \\ \psi_b = L_2 \cdot i_b - I_A \cdot \beta - I_B \cdot \alpha, \end{cases} \quad (5)$$

and the equation of electrical equilibrium of the reduced system of IM equations can be written as:

$$\begin{cases} L_1 \cdot x - \alpha \cdot m - \beta \cdot n = U_A - I_A \cdot R_1, \\ L_1 \cdot y + \beta \cdot m + \alpha \cdot n = U_B - I_B \cdot R_1, \\ \alpha \cdot x + \beta \cdot y + L_2 \cdot m = -i_a \cdot R_2, \\ -(\beta \cdot x + \alpha \cdot y) + L_2 \cdot n = -i_b \cdot R_2. \end{cases} \quad (6)$$

Assuming the constancy of IM rotor rotation angle α, numerical coefficients α and β become constants, and a system of equations (6) acquires a linear character.

To solve a given system of differential equations, it is necessary to represent it in a canonical form.

Applying the procedures of consistent exclusion of variables to system (6), after the reduction of similar and inverse substitution of variables from (4), we shall obtain

$$\begin{cases} \dfrac{dI_A}{dt}=\left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{R_1}{L_1}\cdot I_A-\dfrac{\alpha R_2}{\sigma\cdot L_1}\cdot i_a- \\ -\dfrac{\beta R_2}{\sigma\cdot L_1}\cdot i_b-\left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{1}{L_1}\cdot U_A, \\ \dfrac{dI_B}{dt}=\left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{R_1}{L_1}\cdot I_B+\dfrac{\alpha R_2}{\sigma\cdot L_1}\cdot i_a+ \\ +\dfrac{\beta R_2}{\sigma\cdot L_1}\cdot i_b-\left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{1}{L_1}\cdot U_B, \\ \dfrac{di_a}{dt}=\dfrac{\alpha\cdot R_1}{\sigma\cdot L_1}\cdot I_A+\dfrac{\beta\cdot R_1}{\sigma\cdot L_1}\cdot I_B-\dfrac{R_2}{\sigma}\cdot i_a-\dfrac{\alpha}{\sigma\cdot L_1}\cdot U_A-\dfrac{\beta}{\sigma\cdot L_1}\cdot U_B, \\ \dfrac{di_b}{dt}=-\dfrac{\beta\cdot R_1}{\sigma\cdot L_1}\cdot I_A-\dfrac{\alpha\cdot R_1}{\sigma\cdot L_1}\cdot I_B-\dfrac{R_2}{\sigma}\cdot i_b+\dfrac{\beta}{\sigma\cdot L_1}\cdot U_A+\dfrac{\alpha}{\sigma\cdot L_1}\cdot U_B, \end{cases} \quad (7)$$

where the following notation is accepted

$$\sigma=\frac{\alpha^2-\beta^2}{L_1}+L_2.$$

The system of IM equations for a starting mode (7) is given in a canonical form and can be solved analytically.

5. Solution to IM equations when the rotor is stationary

To solve the IM equations when the rotor is stationary (7), we shall represent this system of equations in the classical form of a state space, [16]

$$\frac{d\mathbf{x}}{dt}=\mathbf{A}\cdot\mathbf{x}+\mathbf{B}\cdot\mathbf{u},$$

$$\mathbf{y}=\mathbf{C}\cdot\mathbf{x}+\mathbf{D}\cdot\mathbf{u}, \quad (8)$$

where **A** is the functional matrix of the system state with a dimensionality of $n\times n$; **B** is the functional control matrix (input) with a dimensionality of $n\times r$; **C** is the functional matrix of output by state with a dimensionality of $m\times n$; **D** is the functional matrix of output by control with a dimensionality of $m\times r$.

IM state vector includes the currents of stator and rotor:

$$x=\left[I_A(t)\quad I_B(\mathrm{t})\quad i_a(t)\quad i_b(\mathrm{t})\right]^{\mathrm{T}}. \quad (9)$$

IM control vector includes phase voltages of stator and rotor:

$$u=\left[U_A\quad U_B\right]^{\mathrm{T}}. \quad (10)$$

IM output vector by state determines the phase currents of stator:

$$y=\left[I_A\quad I_B\right]^{\mathrm{T}}. \quad (11)$$

Given the system of equations (7), matrix coefficients **A, B, C** of the vector-matrix model of IM can be computed from expressions

$$\mathrm{A}=\begin{bmatrix} \left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{R_1}{L_1} & 0 & -\dfrac{\alpha R_2}{\sigma\cdot L_1} & -\dfrac{\beta R_2}{\sigma\cdot L_1} \\ 0 & \left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{R_1}{L_1} & \dfrac{\beta R_2}{\sigma\cdot L_1} & \dfrac{\alpha R_2}{\sigma\cdot L_1} \\ \dfrac{\alpha\cdot R_1}{\sigma\cdot L_1} & \dfrac{\beta\cdot R_1}{\sigma\cdot L_1} & -\dfrac{R_2}{\sigma} & 0 \\ -\dfrac{\beta\cdot R_1}{\sigma\cdot L_1} & -\dfrac{\alpha\cdot R_1}{\sigma\cdot L_1} & 0 & -\dfrac{R_2}{\sigma} \end{bmatrix},$$

$$\mathrm{A}=\begin{bmatrix} a & 0 & c & d \\ 0 & a & -d & -c \\ e & f & b & 0 \\ -f & -e & 0 & b \end{bmatrix},$$

$$\mathrm{B}=\begin{bmatrix} -\left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{1}{L_1} & 0 \\ 0 & -\left(\dfrac{(\alpha^2-\beta^2)}{\sigma\cdot L_1}-1\right)\cdot\dfrac{1}{L_1} \\ -\dfrac{\alpha}{\sigma\cdot L_1} & -\dfrac{\beta}{\sigma\cdot L_1} \\ \dfrac{\beta}{\sigma\cdot L_1} & \dfrac{\alpha}{\sigma\cdot L_1} \end{bmatrix}=\begin{bmatrix} b_{11} & 0 \\ 0 & b_{11} \\ b_{31} & b_{32} \\ -b_{32} & -b_{31} \end{bmatrix},$$

$$\mathrm{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (12)$$

Matrix **D** in (8) is zero. Characteristic equation of the system will be defined by state matrix **A**, (12), from the following expression, [10]:

$$\varphi(s)=\det(sI-A), \quad (13)$$

where **I** is the square unit matrix whose dimensionality coincides with the dimensionality of the matrix of system state **A**.

Substituting in (19) expressions (18) for the matrix of *system state* **A** and the identity matrix **I**, we obtain the characteristic equation of IM under a starting mode

$$\varphi(s)=\det\left(\begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix}-\begin{bmatrix} a & 0 & c & d \\ 0 & a & -d & -c \\ e & f & b & 0 \\ -f & -e & 0 & b \end{bmatrix}\right),$$

$$\varphi(s)=\det\begin{bmatrix} s-a & 0 & -c & -d \\ 0 & s-a & d & c \\ -e & -f & s-b & 0 \\ f & e & 0 & s-b \end{bmatrix}=0. \quad (14)$$

After computing the determinant, we obtain

$$\varphi(s)=\left((a-s)(b-s)-(c+d)(e-f)\right)\times \times\left((a-s)(b-s)-(c-d)(e+f)\right). \quad (15)$$

The roots of the characteristic equation can be calculated as the eigenvalues of the matrix of system state **A**. By using the online-service wolframalpha.com for analytical computations, we shall obtain the following expressions for the eigenvalues of the matrix of system state **A**.

$$s_1=\frac{1}{2}\left(-\sqrt{(a-b)^2+4(ce-cf+de-df)}+a+b\right),$$

$$s_2=\frac{1}{2}\left(\sqrt{(a-b)^2+4(ce-cf+de-df)}+a+b\right),$$

$$s_3=\frac{1}{2}\left(-\sqrt{(a-b)^2+4(ce+cf-de-df)}+a+b\right),$$

$$s_4=\frac{1}{2}\left(\sqrt{(a-b)^2+4(ce+cf-de-df)}+a+b\right). \qquad (16)$$

By employing formulae (12), it is easy to show that the following identities hold: $s_1 \equiv s_3$; $s_2 \equiv s_4$.

We shall define equivalent matrix of transfer functions, which connects state vectors and control vectors by the expression given in [14]:

$$H(s)=\frac{1}{\varphi(s)}\cdot adj(s\cdot \mathbf{I}-\mathbf{A})\cdot \mathbf{B}. \qquad (17)$$

Matrix (s**I**–**A**) and characteristic equation φ(s) can be determined from (14).

An equivalent matrix of transfer functions

$$H(s)=\begin{bmatrix} \frac{I_A(s)}{U_A(s)} & \frac{I_A(s)}{U_B(s)} \\ \frac{I_B(s)}{U_A(s)} & \frac{I_B(s)}{U_B(s)} \\ \frac{i_a(s)}{U_A(s)} & \frac{i_a(s)}{U_B(s)} \\ \frac{i_b(s)}{U_A(s)} & \frac{i_b(s)}{U_B(s)} \end{bmatrix}. \qquad (18)$$

Applying the rules of matrix multiplication, it is easy to obtain expressions for individual elements of the equivalent matrix of transfer functions. We give the expression for transfer function $H_{11}(s)$ along the channel "phase A voltage – phase A current". By employing a notation from (12), we obtain:

$$H_{11}(s)=\frac{\begin{pmatrix}(b-s)\left(-s^2+bs+ce-df+a(s-b)\right)b_{11}+ \\ +\left(e\left(c^2-d^2\right)-c(s-b)(s-a)\right)b_{31}+ \\ +\left(fc^2+ad(s-b)-d\left(s^2-bs+df\right)\right)b_{32}\end{pmatrix}}{\varphi(s)}. \qquad (19)$$

Thus, the IM transfer function along the channel "phase A voltage – phase A current" is a fractional rational function with the numerator of third order and the denominator of fourth order. Coefficients of this transfer function, as well as the roots of the characteristic equation, depend on the initial angular position of the rotor.

6. Verification of results and mathematical modeling

To test the adequacy of analytical transformations performed, we have carried out experimental study into the starting of IM based on the mathematical model. We used in the mathematical model the induction motor of type 4A100 4SY5 with a power of 15 kW with a rotation speed of 1,500 rpm. Parameters for the T-shaped equivalent circuit of the motor 4A100 4SY5: inductance of the stator winding scattering is 0.002645 H, reduced inductance rotor winding scattering is 0.004017 H, circuit inductance magnetization is 0.0546 H, active resistance of the stator is 0.462 Ohm, reduced active resistance of the rotor is 0.312 Ohm.

Realization of the IM mathematical model for the starting mode in a state space based on the above expressions (12) was implemented in the MATLAB/Simulink software; the developed structural diagram is shown in Fig. 1.

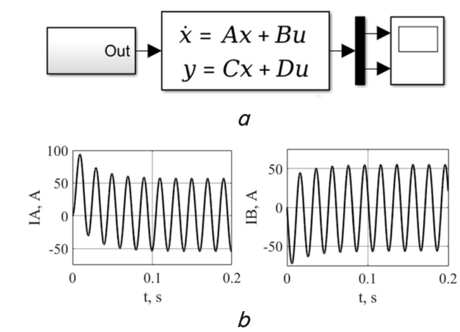

Fig. 1. Mathematical modeling of IM in a state space for the starting mode: *a* — block diagram of the vector-matrix model; *b* — charts of phase currents of the stator under a starting mode

The shape of starting currents coincides with a general theoretical understanding of the start of IM and the values – with specification data on the pilot IM, which confirms correctness of the conducted transforms and obtained equations.

The obtained expressions (14), (16), (19) allow us to analytically determine the values of roots of the IM characteristic equation and IM transfer function coefficients when the rotor is stationary. This makes it possible to build transitional characteristics of IM for different values of the initial angular position of the rotor.

Calculations for the induction motor 4A100 4SY5 were performed using the MATLAB software; the results obtained are shown in Fig. 2.

A change in the initial angular position of the rotor from 0 to π leads to a change in the character of roots of the characteristic equation. For the values of initial rotor rotation angle in the neighborhood of 0 and π, the roots of the IM characteristic equation are complex conjugated. At the initial rotor angle values in the neighborhood of π/2, the roots of the IM characteristic equation have a valid character.

The results obtained are valid for all structural designs of three-phase IM with stator connection to the power circuit without a zero wire: with a short-circuited and a phase rotor, for motors with a deep-seated rotor, etc.

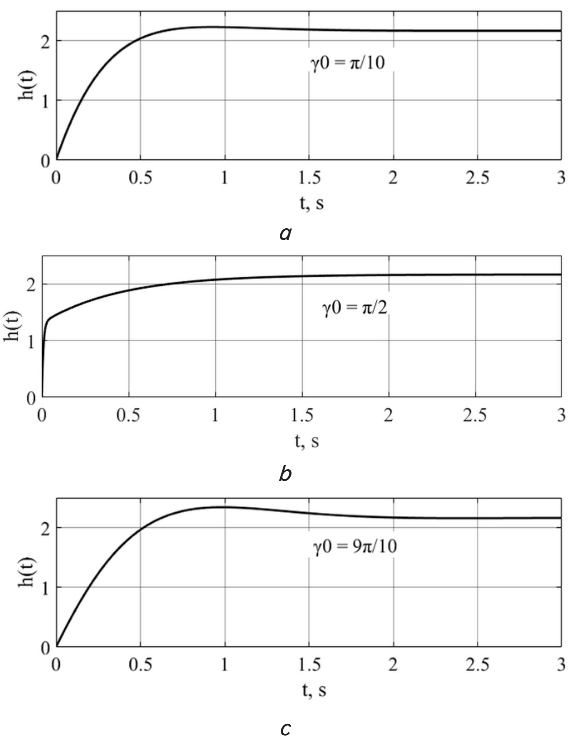

Fig. 2. Transient characteristics and roots of the characteristic equation of IM under a starting mode for different values of the initial angular position of the IM rotor: *a* – γ 0=π/10; *b* – γ 0=π/2; *c* – γ 0=9π/10

7. Discussion of results of research into a starting current of the stator of an induction motor

Operation of IM is considered under a mode of the stationary rotor at its permanent angular position. These patterns make it possible to further simplify the IM equations. Based on the analysis of the system of IM equations in a three-phase coordinate system, it was established that at the stationary rotor it is possible to analytically solve the IM equations because periodic coefficients in the IM equations are transformed into constant magnitudes, which transfers the system of IM equations to the class of linear dynamic systems. Because there is no motion of the rotor and it has no effect on the electromagnetic processes of IM, the equations of rotor motion are redundant for the problem being solved, and can be excluded from consideration. For the most common connection scheme of IM without a zero wire, the stator and rotor currents are linked via the first law by Kirchhoff. Therefore, determining one of the stator currents and one of the rotor currents can be conducted not through the differential equation of electrical equilibrium of the respective winding, but applying the algebraic equation that expresses the first Kirchhoff law. This makes it possible to exclude from the system of IM equations two equations of electric equilibrium of windings, for one stator and one rotor winding. The main advantages of the proposed approach are the reduction in the order of the system of IM equations from the eighth to fourth order and a linear character of the obtained system of equations, which makes it possible to solve it analytically. The assumption of the switching scheme of IM is the most significant constraint in the application of the proposed solutions.

Employing algebraic transforms, a system of IM equations was represented in a canonical form, and subsequently in the vector-matrix form of a state space. The main advantage of this approach is the existence of verified methods for assessing dynamic characteristics of control objects.

Using the methods of analysis of dynamic objects in a state space, we derived expressions for coefficients of the characteristic IM equation and its roots, as well as for the matrix of IM transfer functions when the rotor is stationary. This makes it possible to eventually obtain algebraic expressions for direct estimation of the influence of IM parameters on characteristics of its starting currents.

Correctness of the performed analytical transformations is confirmed by the results of examining the developed mathematical model of IM in the form of a state space using the Matlab/Simulink software.

An analysis of the mathematical model of the expressions obtained allowed us to derive values for the roots of the IM characteristic equation, to build its transitional functions for different values of the initial angular position of the IM rotor.

An analysis of expressions for the roots of the characteristic equation shows that the real or complex conjugate character of roots of the IM characteristic equation depends on the initial angular position of the rotor. This can be explained in the following way. A change in the initial angular position of the rotor results in a change in the value of mutual inductance between separate windings of IM, which affects the processes of energy transfer between stator and rotor windings.

With a complex-conjugate character of roots of the characteristic equation, the transitional characteristic of the rotor starting current will be oscillatory in nature.

The theoretical results obtained could be used to refine the results of research in the field of the identification of IM parameters based on the analysis of starting modes.

In the future, it will be possible to apply the proposed method for transforming the IM equations when the rotor is stationary in order to analyze the starting current of synchronous motor.

8. Conclusions

1. It was established that the system of IM equations in a three-phase coordinate system can be solved analytically under assumption of the stationary rotor. This is related to the fact that under such an assumption the periodic coefficients in the IM equations are transformed to the constant magnitudes. Further simplification of the system of IM equations implies the exclusion of motion equations, which is also related to the accepted assumption about the immobility of the rotor. For a circuit of IM connection without a zero wire, it is possible to apply the Kirchhoff's first law and exclude from the overall system two equations of electrical equilibrium of windings, for one stator and on rotor winding.

The result of performed transformations is the simplified system of IM equations IM when the rotor is stationary, which, in contrast to the complete system, is a system of linear differential equations of the fourth order and is given in the Cauchy form.

2. Using the methods of analysis of dynamic objects in a state space, we obtained expressions for the coefficients of IM characteristic equation and its roots, as well as for the matrix

of IM transfer functions along the channel "power phase voltage – stator phase currents" when the rotor is stationary.

An analysis of expressions for the roots of the characteristic equation shows that the character of roots of the IM characteristic equation depends on the initial angular position of the IM rotor. This is explained by the fact that a change in the initial angular position of the rotor changes the magnitude of mutual inductance between separate windings of IM, which affects the processes of energy transfer between stator and rotor windings.

References

1. Complementary diagnosis of rotor asymmetries through the tracing of the Right Sideband Component in the stator startup current / Antonino-Daviu J., Jover P., Riera-Guasp M., Arkkio A., Pineda-Sanchez M. // 2008 18th International Conference on Electrical Machines. 2018. doi: 10.1109/icelmach.2008.4799988
2. Babu W. R., Ravichandran C. S. Diagnosis of stator fault of Medium Voltage Induction Motors using Motor Stator Current Envelope Analysis (MSCEA) // Jan 2016 in 2016 3rd International Conference on Advanced Computing and Communication Systems (ICACCS). 2016. doi: 10.1109/icaccs.2016.7586395
3. Identification of the active resistances of the stator of an induction motor with stator windings dissymmetry / Tytiuk V., Pozigun O., Chornyi O., Berdai A. // 2017 International Conference on Modern Electrical and Energy Systems (MEES). Kremenchuk, 2017. P. 48–51. doi: 10.1109/mees.2017.8248949
4. Identifikaciya elektricheskih parametrov tyagovyh asinhronnyh dvigateley elektrovozov / Sinchuk O. N., Zaharov V. Yu., Sinchuk I. O., Smenova L. V. // Elektrotekhnichni ta kompiuterni systemy. 2013. Issue 10. P. 50–59.
5. Monitoring parametrov elektricheskih dvigateley elektromekhanicheskih sistem / Cherniy A. P., Rod'kin D. I., Kalinov A. P., Vorobeychik O. S. Kremenchug: ChP Shcherbatyh A.V., 2008. 246 p.
6. Krause P. C. Analysis of Electric Machinery. New York: McGraw-Hill, 1994. 135 p.
7. Leonhard W. Control of Electrical Drives. 3rd ed. Springer-Verlag, 2001. 460 p.
8. Krause P. C., Wasynczuk O., Sudhoff S. D. Analysis of Electric Machinery and Drive Systems. Wiley-IEEE Press, 2002. 632 p. doi: 10.1109/9780470544167
9. Trzynadlowski A. M. Control of Induction Motors. Academic Press, 2001. 230 p.
10. Pena J. M., Diaz E. V. Implementation of V/f scalar control for speed regulation of a three-phase induction motor // 2016 IEEE ANDESCON. 2016. doi: 10.1109/andescon.2016.7836196
11. Zhou H., Long B., Cao B. Vector Control System of Induction Motor Based on Fuzzy Control Method // 2008 Workshop on Power Electronics and Intelligent Transportation System. 2008. doi: 10.1109/peits.2008.110
12. Mathematical models and specifics of numerical calculations of dynamic characteristics of electric drives with induction motors: monograph / Chornyi O., Tolochko O., Tytyuk V., Rodkin D., Chekavskiy G. Kremenchuk: PE Shcherbatykh O.V., 2016. 302 p.
13. Marino R., Peresada S., Tomei P. On-line stator and rotor resistance estimation for induction motors // IEEE Transactions on Control Systems Technology. 2000. Vol. 8, Issue 3. P. 570–579. doi: 10.1109/87.845888
14. Improved dynamic model of induction motor including the effects of saturation / Singh A. K., Dalal A., Roy R., Kumar P. // 2014 IEEE International Conference on Power Electronics, Drives and Energy Systems (PEDES). 2014. doi: 10.1109/pedes.2014.7042108
15. Maddi Z., Aouzellag D. Dynamic modelling of induction motor squirrel cage for different shapes of rotor deep bars with estimation of the skin effect // Progress In Electromagnetics Research M. 2017. Vol. 59. P. 147–160. doi: 10.2528/pierm17060508
16. Fairman F. W. Linear Control Theory: The State Space Approach. John Wiley & Sons, 1998. 315 p.