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UDC 004.942:624.073.12
DOI: 10.15587/1729-4061.2018.126141

MINIMIZING THE MASS OF A FLAT BOTTOM OF CYLINDRICAL APPARATUS

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Отримано загальний розв'язок задач згину круглих пластин, товщина яких змінюється за експоненціальним законом із застосуванням вироджених гіпергеометричних функцій Куммера. Розв'язано задачу контакту циліндричної оболонки з круговою пластиною змінної товщини в загальному вигляді. Запропоновано методику мінімізації маси пластинчастих елементів конструкцій кругової форми. Розроблена конструкція зони переходу від днища до стінки, міцність якої перевірена методом скінчених елементів у реальному проектуванні

Ключові слова: днище змінної товщини, гіпергеометрична функція Куммера, контакт оболонки та кругової пластини

Получено общее решение задач изгиба круглых пластин, толщина которых меняется по экспоненциальному закону с применением вырожденных гипергеометрических функций Куммера. Решена задача контакта цилиндрической оболочки с круговой пластиной переменной толщины в общем виде. Предложена методика минимизации массы пластинчатых элементов конструкций круговой формы. Разработана конструкция зоны перехода от днища к стенке, прочность которой проверена методом конечных элементов в реальном проектировании

Ключевые слова: днище переменной толщины, гипергеометрическая функция Куммера, контакт оболочки и круговой пластины

1. Introduction

The main tasks in the design of machinery and vehicles imply achieving the highest technical-economical and operational indicators: performance, operational reliability, and cost. One of the most important factors when solving these tasks is the minimization of mass of structures. The greatest attention to the problem of reducing the mass is paid in transport engineering. Thus, the thickness of passenger cars made by different manufacturers is 1...2 mm; the thickness of the unloaded sections of modern automotive bodies (wing, trunk, etc.) does not exceed 0.7 mm, in some makes – 0.3 mm. The trend of decreasing thickness is limited mainly by the possibilities of manufacturing high-quality thin-walled rolled metal.

In transport engineering, one of the main criteria of quality is the ratio g_1 of payload mass to the mass of the structure. For marine transport, g_1=2.3...7.0; for railroad transport, g_1=2.0...2.5; for automobile transport, g_1=0.5...1.5, for aviation, g_1=0.3...0.75. The larger magnitudes here correspond to the largest size of vehicles. Similar characteristics are calculated for stationary machines and apparatuses. These data explain the steady trend of growth in the dimensions of machinery and technological equipment.

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The largest decrease in the mass of parts can be achieved by providing them with full equal strength. The ideal are the structures over whose entire volume such similar stresses occur that are equal to those permissible. Such results are possible for a very limited number of structures with a variable area of cross section with at the one-dimensional stressed state (rods, when stretched or compressed; disks, loaded with centrifugal forces). When bending, under torsion and complex loaded states, the stresses at the cross section are distributed unevenly: they have the highest magnitude at the extreme points of cross section and may drop to zero at other points. The effective reduction of mass of the one-dimensional parts, loaded by bending and torsion, is achieved by reducing the areas of cross sections in sections that experience insignificant internal forces.

When designing two- and three-dimensional parts, a minimum of mass is often not achieved, because of the growing complexity in estimation schemes; in addition, solution to such problems may prove ambiguous. When designing equipment, engineers resolve the issue of reducing the mass mostly intuitively, guided by the existing experience or known prototypes. Therefore, the development of analytical methods to minimize the mass of parts in structures of complex shapes is always an important task.

2. Literature review and problem statement

One of the main factors that determine the level of optimal structural design is the criterion of a mass minimum, because the cost of metal in the overall cost structure accounts for 60...80 % [1, 2]. For round plates, at bending, the stressed state depends on the character of a transverse load, geometrical dimensions (radius R and thickness) and fastening techniques. A minimum of mass of the round plate will be reached under uniform distribution of maximum stresses and the fulfillment, at all surface points, of the strength condition, $\sigma_{\bar{5}}(r)=[\sigma]=\text{const.}$ Here, $0\leq r\leq R$ is the radial coordinate, $[\sigma]$ and σ_e is the stress that is permissible and equivalent (for example, in line with the von Mises theory). Such parts are called equally strong. The condition of being equally strong is satisfied only when a round plate is stretched in its plane by centrifugal forces (the flat-stressed state). It is known that for the disks of turbo-machines the condition of being equally strong is met at a change in thickness in line with the exponential law [3]. The vast majority of plate parts of machines and vehicles may only come close to being equally strong [4].

The minimization of mass of round plates at bending can be achieved in different ways.

The first group, implying a decrease in area or load, or rational arrangement of supports, is apparent and is used at the first stage of designing.

To further minimize the mass, it is possible to apply the second group of techniques – the rationalization of a shape or a material: in regions with small internal efforts, the thickness decreases accordingly, or the material is replaced with that less strong and less expensive. The latter technique is more challenging to implement and it is hardly used in practice. Thus, designing round plates with a thickness that rationally changes in the radial direction may be considered the dominant minimization technique [5].

The most interesting in terms of mathematical modeling is the shape of a round plate whose thickness gradually changes in the radial direction, in line with the linear, power, exponential, or other laws.

The simplest shape of variable thickness is linear (as a particular case of the power thickness). Calculations of such structures are performed often enough for the elements of building structures, for plates with local loads along a circle and in the center [6, 7]. Note that the linear dependence of the plate thickness on radial coordinate r is rational if radial bending moment $M_r(r)$ decreases from the center to the periphery, and thus the thickness could decrease in this direction in line with dependence $h(r)=h_0-k_1r$. Such a dependence is applicable for plates, freely leaned against external contour R. If moment $M_r(r)$ increases with increasing coordinate r, one should accept $k_1<0$. Such a problem has a simple analytical solution if thickness at the center of the plate $h_0=0$ (and it is typically applied), but such a form is acceptable only for annular plates.

The exponential law of change in thickness is employed in the form of Gaussian function:

$$h(r)=h_0\exp\left(\frac{nr^2}{6R^2}\right), \quad (1)$$

where h_0 is the thickness in the center of the plate, n is a parameter.

Parameter n defines the character of change in thickness: at $n>0$, thickness from the center to the periphery grows, at $n<0$ – decreases. A special case $n=0$ defines the plate with a constant thickness. The form of mathematical modeling of variable thickness (1) is applicable for both solid plates and plates annular in plan, thereby it is more universal than the linear form.

The problem on bending a round plate with variable thickness (1) was stated in the form of a differential equation of second order with variable coefficients by O. Pichler. An unknown function was considered be the turning angle φ of normal to the middle surface of the plate, which forms when bending the plate. The general solution to this equation was obtained as the sum of two power series with coefficients of quite a complex structure. Solutions to some problems on bending the plates with thicknesses that change according to power or exponential laws were obtained in hypergeometric functions by A. D. Kovalenko. For round plates with the shape (1) of radial cross sections, a general solution to the differential equation of bending is determined using the confluent hypergeometric functions by Kummer's. Note that the Kummer's functions theory is well developed and these functions are effectively applied in such diverse fields as statistics [8], physics [9], soil mechanics [10], metrology [11], biology [12], etc.

Real plates can have very diverse shapes of contours and cross sections, complex laws of transverse load, and non-uniform structure. Thus, lenses and mirrors of optical apparatuses, lithospheric plates of the Earth, are calculated, in particular, for the action of thermal load [13, 14]. Some parts of modern structures utilize specialized materials, which are called functionally-gradient materials (FGM). Plates made of FGM have variable mechanical characteristics (density, modulus of elasticity, etc.). The equations that describe the stressed-strained state of such plates and the plates with a variable thickness, as well as their solutions, are similar [15, 16]. For plates with a complex configuration, it is appropriate, instead of analytical methods of calculation, to apply MFE [17].

Much simpler are the problems on bending the round plates of constant thickness, symmetrical relative to the center (axisymmetric bend), in this case, a general solution to the differential equation of bending is obtained in elementary functions:

$$w_0(r)=a_1+a_2r^2+a_3\ln r+a_4r^2\ln r,$$

where $w_0(r)$ is the displacement of the median surface due to transverse load; a_i is the constants of integration.

Due to the simplicity of a given solution, an estimated model is selected for a plate with equivalent stepwise-constant thickness, or plates with such a shape are constructed [18, 19]. Analytical calculation of such plates can be performed applying the method of initial parameters. It should be noted that at bending of plates with a stepwise-constant thickness, zones of change in thickness are exposed to the concentration of stresses. Such zones require special methods, such as the method of finite elements (MFE).

When considering the above scientific sources, it should be noted that their authors, when calculating round plates of variable thickness, solve and analyze problems only for simple boundary conditions (rigid fastening, free leaning, or a free edge). Actual structures are often under complex conditions when a round plate is in contact with another elastic structure: an annular rim (pulleys, toothed wheels, railroad wheels) or a shell (cylindrical, conical, or another). Therefore, stating and solving the problems on contact for plates of variable thickness with simultaneous determining of the shape of a radial cross section, which is achieved with a minimum mass, is an important theoretical and practical task.

3. The aim and objectives of the study

The aim of present study is to minimize the mass of circular plate parts with regard to the conditions of contact with other elements of the structure by enabling their variable thickness. In this case, the strength and initial dimensions of a machine or an apparatus remain unchanged.

To accomplish the aim, the following tasks have been set:

– to solve the problem on bending a bottom whose variable thickness is assigned by the exponential Gaussian function, and to determine the constant of integration from conditions of contact with the wall of a cylindrical apparatus;

– to develop a method for the optimization of shape of a radial cross-section of the bottom, which implies passing from its shape as a plate with constant thickness to the plate of variable thickness and minimal volume, through a rational use of the specified material;

– to work out a rational design of the circular zone of connection between the designed bottom and a body.

4. Statement and solution to the problem on bending the bottom of a cylindrical apparatus under the action of internal pressure q_0

In apparatuses with flat bottoms, a cylinder wall typically has a constant thickness, which is determined from the condition of strength of the shell at the moment-free stressed state. The thickness of flat bottoms is also given as constant. The reason for such structural solutions is explained by the ease of making these elements in apparatuses, although in the forged bottoms, the thickness in a zone of transition to the wall always increases in order to reduce moment stresses [20].

We shall accept that the wall of a cylindrical apparatus has constant thickness h_1, and thickness of the bottom h changes in line with law (1).

Place the origin of the O cylindrical coordinate system at the center of the plate. We shall denote: z is the axial coordinate, r is the radial coordinate. The problem is axisymmetric, so the stresses and displacements do not depend on the circular coordinate t. We shall conditionally separate a wall of the apparatus from the bottom and accept the line that crosses middle surfaces as a connecting line (Fig. 1). We shall formulate compatibility conditions of deformations of the wall of the vessel and the bottom at the connecting line, at $r=R$ and $z=0$: $\theta_c=-\theta_b$; $w_c=0$ (the deformation of stretching the bottom is disregarded), where θ_c and θ_b are the wall and the bottom turning angles, respectively, w_c are the radial displacements of the wall.

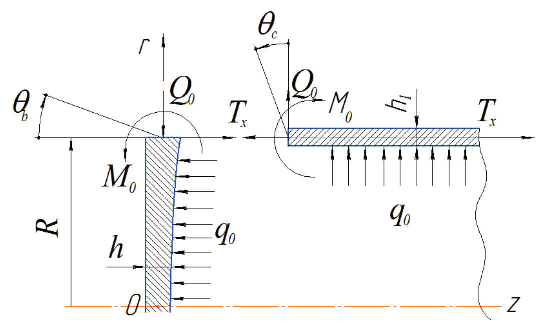


Fig. 1. Calculation scheme of a connection node between the wall and the bottom of cylindrical vessel

Turning angle of the edge of the wall of the apparatus:

$$\theta_c=-\frac{M_0}{D_1\beta}-\frac{Q_0}{2D_1\beta^2}, \tag{2}$$

where M_0 and Q_0 are the initial parameters (bending moment and a lateral force, arising in the wall at $z=0$).

Cylindrical rigidity of the wall and the parameter that determines attenuation speed of the moment state with the growth of coordinate z are determined, respectively, by formulae:

$$D_1(r)=\frac{Eh_1^3}{12(1-\mu^2)},\quad \beta=\sqrt[4]{\frac{3\left(1-\mu^2\right)}{R^2h_1^2}}. \tag{3}$$

Radial displacement of the vessel's wall edge in line with a boundary condition produces an equation with two unknown initial parameters:

$$w_c=\frac{M_0}{2D_1\beta^2}+\frac{Q_0}{2D_1\beta^3}+\left(1-\frac{\mu}{2}\right)\frac{pr^2}{Eh_1}=0, \tag{4}$$

where E and μ are the modulus of elasticity and Poisson's coefficient of the apparatus's material.

Differential equation of bending, formulated for the turning angle of normal $\varphi(r)$ to the median surface of the bottom as a round plate, and upon replacing $r=Rx$ takes the form [2]:

$$\frac{d^2\varphi}{dx^2}+\left(\frac{1}{x}+nx\right)\frac{d\varphi}{dx}-\left(\frac{1}{x^2}-\mu n\right)\varphi=$$
$$=-\frac{1}{r}\int_0^r q(\rho)\rho d\rho=-\bar{p}x\exp\left(-\frac{nx^2}{2}\right), \quad (5)$$

where a dimensionless multiplier is

$$\bar{p}=6(1-\mu^2)\frac{q_0}{E}\cdot\frac{R^3}{h_0^3}. \quad (6)$$

Solution to equation (5)

$$\varphi(x)=\frac{nx}{2}\exp\left(-\frac{nx^2}{2}\right)\times$$
$$\times\left[C_1 M\left(\frac{3-\mu}{2},2;\frac{nx^2}{2}\right)+C_2 U\left(\frac{3-\mu}{2},2;\frac{nx^2}{2}\right)\right]+$$
$$+\frac{\bar{p}x}{(3-\mu)n}\exp\left(-\frac{nx^2}{2}\right), \quad (7)$$

where $M(a, b; z)$ and $U(a, b; z)$ are the Kummer's functions of the first and second kind with argument $z=0.5nx^2$ [21, 22].

It is known that Kummer's function of the second kind $U(a,b;z)|_{z=0}=\infty$. The turning angle of normal to the curved median surface of the round plate $\varphi(0)=0$, which is why in (7) we assign $C_2=0$. As a result, equation (7) takes the form:

$$\varphi(x)=$$
$$=\left[C_1\frac{n}{2}F_{K1}(x)+\frac{\bar{p}}{(3-\mu)n}\right]x\exp\left(-\frac{nx^2}{2}\right), \quad (8)$$

where for the Kummer's function of the first kind the following designation is accepted:

$$M\left(\frac{3-\mu}{2},2;\frac{nx^2}{2}\right)=F_{K1}(x). \quad (9)$$

Boundary condition for the turning angles, given expressions (2) and (8), produces the second equation for initial parameters M_0 and Q_0 that also includes an unknown constant of integration C_1:

$$\frac{M_0}{D_1\beta}+\frac{Q_0}{2D_1\beta^2}-$$
$$-C_1\frac{n}{2}F_{KM}(1)\exp\left(-\frac{n}{2}\right)=\frac{\bar{p}}{(3-\mu)}\exp\left(-\frac{n}{2}\right). \quad (10)$$

At transverse load, the plate is exposed to the radial and circular bending moments:

$$M_r=\frac{D(x)}{R}\left(\frac{d\varphi(x)}{dx}+\frac{\mu}{x}\varphi(x)\right), \quad (11)$$

$$M_t=\frac{D(x)}{R}\left(\frac{\varphi(x)}{x}+\mu\frac{d\varphi(x)}{dx}\right), \quad (12)$$

which define estimated stresses

$$\sigma_t=\frac{6M_t(x)}{h^2(x)},$$
$$\sigma_r=\frac{6M_r(x)}{h^2(x)}. \quad (13)$$

Given (8), we obtain formula

$$M_r(x)=$$
$$=\frac{D_0}{R}\left[C_1 n\frac{1+\mu}{2}F_{K2}(x)+\frac{\bar{p}}{(3-\mu)}\cdot\frac{1+\mu-nx^2}{n}\right], \quad (14)$$

where

$$D_0=\frac{Eh_0^3}{12(1-\mu^2)},\quad F_{K2}(x)=M\left(\frac{1-\mu}{2},2;\frac{nx^2}{2}\right).$$

The condition of equality of bending moments M_0 on the edges of the wall and the bottom, in which for bottom $M_0=M_r(x)|_{x=1}$, Fig. 1, produces equation:

$$M_0=\frac{D_0}{R}\left[C_1 n\frac{1+\mu}{2}F_{K2}(1)+\frac{p}{(3-\mu)}\cdot\frac{1+\mu-n}{n}\right]. \quad (15)$$

Solution to the system of three equations (4), (10) and (15) takes the form:

$$Q_0=\frac{q_0\beta^2R^2D_1}{(3-\mu)nEh_1}\times$$
$$\times\frac{(1+\mu)nF_{K2}\left[Eh_1R\cdot\exp(-0.5n)+2D_0\beta(2-\mu)(3-\mu)\right]+F_{K1}R\cdot\exp(-0.5n)\left[(n-\mu-1)Eh_1-2(2-\mu)(3-\mu)\beta^2nD_1\right]}{(1+\mu)F_{K2}D_0-2F_{K1}\beta RD_1\exp(-0.5n)},$$

$$C_1=\frac{2q_0R^3}{(3-\mu)n^2D_0Eh_1}\frac{\beta^2(2-\mu)(3-\mu)nD_0D_1+Eh_1\left[\beta Rn\exp(-0.5n)D_1+0.5(n-\mu-1)D_0\right]}{(1+\mu)F_{K2}D_0-2F_{K1}\beta RD_1\exp(-0.5n)},$$

$$M_0=\frac{q_0\beta R^2D_1}{(3-\mu)nEh_1}\frac{(1+\mu)nF_{K2}\left[Eh_1R\cdot\exp(-0.5n)+D_0\beta(2-\mu)(3-\mu)\right]+(n-\mu-1)F_{K1}REh_1\cdot\exp(-0.5n)}{(1+\mu)F_{K2}D_0-2F_{K1}\beta RD_1\exp(-0.5n)}.$$

These three quantities contain all the geometrical and mechanical characteristics of the body, which are assigned during design, except for parameter n. The procedure of determining the magnitude n is demonstrated by the example in the next chapter.

5. Optimization of the shape of a radial cross-section of the bottom

The set task is solved for the following data: radius of the inner surface of the wall $R_i=1$ m, $q_0=5$ MPa, $[\sigma]=180$ MPa. Material of machine is steel 16GS. The design of the bottom, shown in Fig. 1, is simplified; we consider it flat-concave, leaving dependence (1) for the variable thickness. Such a shape somewhat reduces the concentration of stresses in the connection between the wall and the bottom.

Estimated thickness of the wall (Fig. 2) [20]:

$$h_1=\frac{q_0 R_i}{\varphi[\sigma]-0.5q_0}=\frac{0.5\cdot 5\cdot 2000}{0.95\cdot 180-0.5\cdot 5}=29.67 \ \text{ mm.}$$

We accept $h_1=30$ mm.

The bottom's thickness at $h=\text{const}$ [13]:

$$h_d=0.45K_0 D\sqrt{\frac{q_0}{[\sigma]}}=0.45\cdot 1\cdot 2000\sqrt{\frac{5}{180}}=150 \ \text{ mm.}$$

The bottom's thickness, variable in the direction of dimensionless radial coordinate x,

$$h(x)=h_0\exp\left(\frac{nx^2}{6}\right). \qquad (16)$$

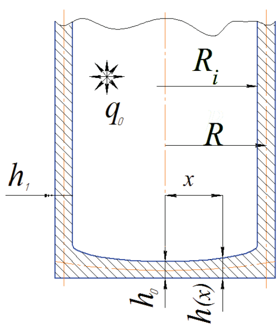


Fig. 2. Schematic of a body with the bottom of variable thickness

Dimensionless multiplier (6) in this case accepts values:

$$p=6(1-\mu^2)\frac{q_0}{E}\cdot\frac{R^3}{h_0^3}=$$

$$=6\cdot(1-0.3^2)\cdot\frac{5}{2\cdot 10^5}\cdot\frac{1000^3}{150^3}=0.04\cdot 10^{-3},$$

and the cylindrical rigidity of the wall:

$$D_1=\frac{Eh_1^3}{12\left(1-\mu^2\right)}=\frac{2\cdot 10^5\cdot 30^3}{12\left(1-0.3^2\right)}=4.945\cdot 10^8 \ \text{ Nmm.}$$

The cylindrical rigidity in the center of the bottom and parameter of the wall:

$$D_0=\frac{Eh_0^3}{12\left(1-\mu^2\right)}=\frac{2\cdot 10^5\cdot 150^3}{12\left(1-0.3^2\right)}=618.13\cdot 10^8 \ \text{ Nmm.}$$

$$\beta=\sqrt[4]{\frac{3(1-\mu^2)}{R^2h_1^2}}=\sqrt[4]{\frac{3\cdot(1-0.3^2)}{1000^2\cdot 30^2}}=7.42\cdot 10^{-3} \ \text{ mm}^{-1}.$$

We shall determine the magnitudes of coefficients:

$$K_1=\frac{q_0\beta R^2 D_1}{(3-\mu)Eh_1}=$$

$$=\frac{5\cdot 7.42\cdot 10^{-3}\cdot 1000^2\cdot 618.13\cdot 10^8}{(3-0.3)\cdot 2\cdot 10^5\cdot 30}=$$

$$=1415.6\cdot 10^5 \ \text{Nmm},$$

$$K_2=D_0\beta(2-\mu)(3-\mu)=$$

$$=4.945\cdot 10^8\cdot 7.42\cdot 10^{-3}\cdot 1.7\cdot 2.7=168.4\cdot 10^5 \ \text{H},$$

$$K_3=Eh_1R=2\cdot 10^5\cdot 30\cdot 1000=6\cdot 10^9 \ \text{H},$$

$$Z_0(n)=(1+\mu)F_{K2}D_0-2F_{K1}\beta RD_1\exp(-0.5n).$$

Taking into consideration these coefficients, formulae for the initial parameters M_0, Q_1, C_1 of the problem take the form:

$$Q_0=\beta^2K_1\frac{(1+\mu)nF_{K2}\left[K_3\cdot\exp(-0.5n)+2K_2\right]+F_{K1}R\cdot\exp(-0.5n)\left[(n-\mu-1)Eh_1-2(2-\mu)(3-\mu)\beta^2nD_1\right]}{Z_0(n)}, \qquad (17)$$

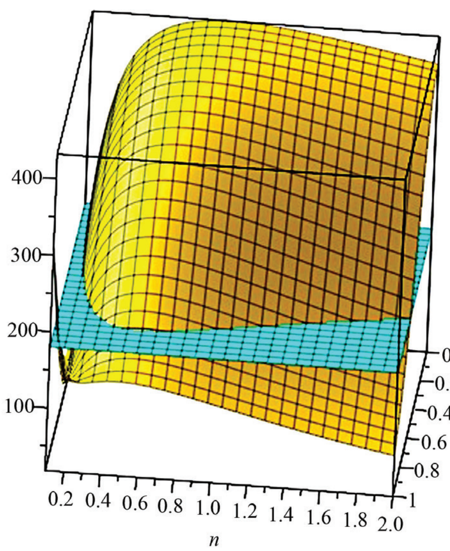
$$C_1=K_1\frac{2R}{\beta n^2D_0D_1}\frac{K_2\beta D_1+Eh_1\left[\beta Rn\exp(-0.5n)D_1+0.5(n-\mu-1)D_0\right]}{Z_0(n)}, \qquad (18)$$

$$M_0=K_1\frac{(1+\mu)nF_{K2}\left[K_3\cdot\exp(-0.5n)+K_2\right]+(n-\mu-1)F_{K1}K_3\cdot\exp(-0.5n)}{Z_0(n)}. \qquad (19)$$

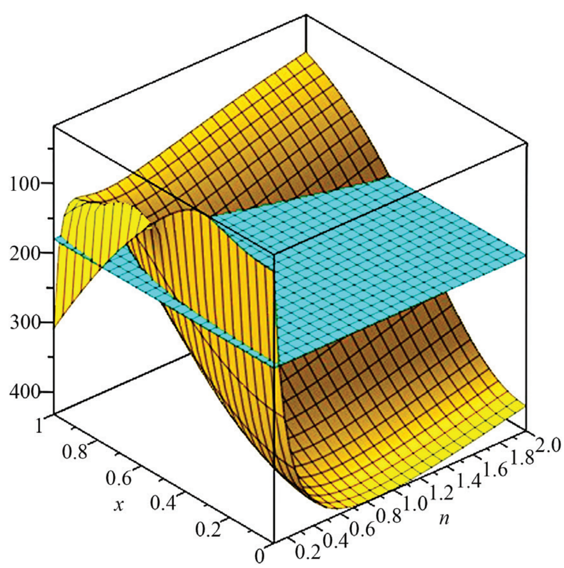
We assume the thickness of bottom in the center $h_0 = 120$ mm, next we build the surface of equivalent (in line with von Mises theory) stresses $\sigma_e(x, n)$, which are determined through main stresses (13):

$$\sigma_e^{(4)} = \sqrt{\sigma_r^2 - \sigma_r \sigma_t + \sigma_t^2}$$

and the plane of permissible stresses to be $[\sigma] = 180$ MPa; next, using a graphical technique, we determine the parameter of change in thickness n. This parameter must be positive, because the thickness of the bottom must increase when approaching its contour, that is, at $x \to 1$. To determine the character of change in the thickness of the bottom, it is necessary to find parameter n, responsible for a given change. Its magnitude must be such so that stresses in the bottom do not exceed permissible ones. To determine the range of values of n, at which condition $\sigma_e \leq [\sigma]$ is met, we build, in the first approximation, surface $\sigma_e(x, n)$ in the range $n = 0.1...2$ (Fig. 3).



a



b

Fig. 3. Surface of equivalent stresses σ_e and plane of permissible stresses $[\sigma] = 180$ MPa at $n = 0.1...2$: *a* — top view; *b* — bottom view

An analysis of the character of surface of stresses σ_e indicates that the condition of strength is satisfied under any values of argument x if the parameter is within $0.1 < n < 0.25$. Building the surface $\sigma_e(x, n)$ in this narrowed range made it possible to find the resulting magnitude of parameter $n = 0.1448$ at which maximum stresses do not exceed 192 MPa and are distributed almost evenly. We accept that the maximum stresses may exceed the permissible ones by 7 %, that is, be equal to 193 MPa. Thus, at $n = 0.1448$, the condition of strength:

$$\sigma_{e\max} = 192 < [\sigma] = 193 \text{ MPa}$$

is satisfied. Thickness of the bottom as a function of radius is determined from formula:

$$h(x) = 120 \cdot \exp(0.02413x).$$

6. Construction of the annular zone of connection between the bottom and the wall of the apparatus

Dimensions of the conical zone of transition from the bottom to the wall are determined based on the recommendations from reference book [20].

Conical transition is formed by two surfaces, radius with $r_k \geq h_b$ and conical with horizontal $h_g \geq 0.6h_b$ and vertical $h_v \geq 1.8h_b$ cathets (Fig. 4). In this case, when determining thickness $h_b = 150$ mm, we have $r_k = 150$ mm, $h_g \geq 0.6 \cdot 150 = 90$ mm, $h_v \geq 1.8 \cdot 150 = 270$ mm. Flange height is determined from inequality $h_2 \geq h_0$. Accept $r_k = 150$ mm, $h_v = 270$ mm, $h_b = 150$ mm.

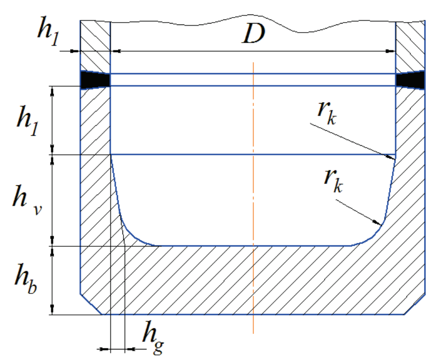


Fig. 4. Schematic of conical transition from the bottom to the wall of the apparatus

Calculation of stresses in the transition zone, which has a complex geometry, is impossible using analytical methods. Such objects are successfully treated with sampling methods.

In this case, we used the finite element method (MFE) in the software ANSYS. To simplify the procedure, boundary zones of the vessel are taken to be equal (two bottoms); the sampling of the structure is shown in Fig. 5, 6. Preliminary calculation was performed at larger FE and was refined at smaller FE.

Construction was executed in successive steps, at each of them, based on the results of analysis of the previous step, we improved the designed structure, in this case – in order to reduce stresses in the transition zone bottom-wall.

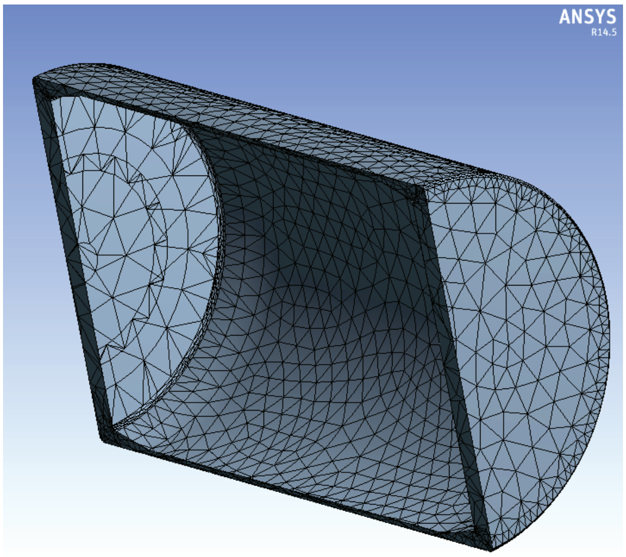


Fig. 5. Simulation of the body. Grid of large-size elements

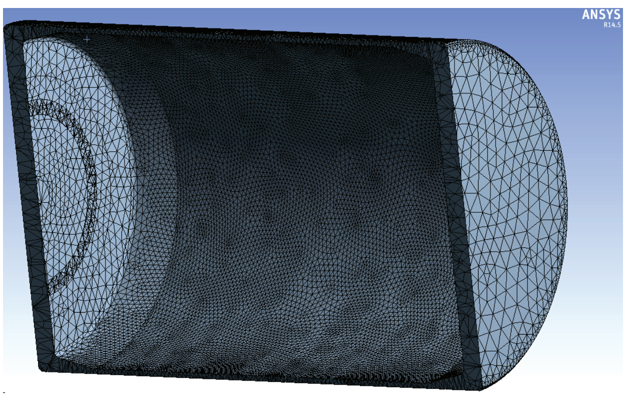


Fig. 6. Simulation of the body. Grid of smaller-size elements

At the first stage, the calculation was performed in the absence of rounding, $r_k=0$ for thickness at the center of the bottom $h_0=120$ mm. Calculation using MFE produced maximum stresses in the zone of transition – 245.2 MPa; they, however, exceed the permissible stresses $[\sigma]=193$ MPa (Fig. 7).The calculation showed that a decrease in thickness in the center (120 mm instead of 150 mm) did not cause an overload in the bottom itself as a result of assigning a variable thickness.

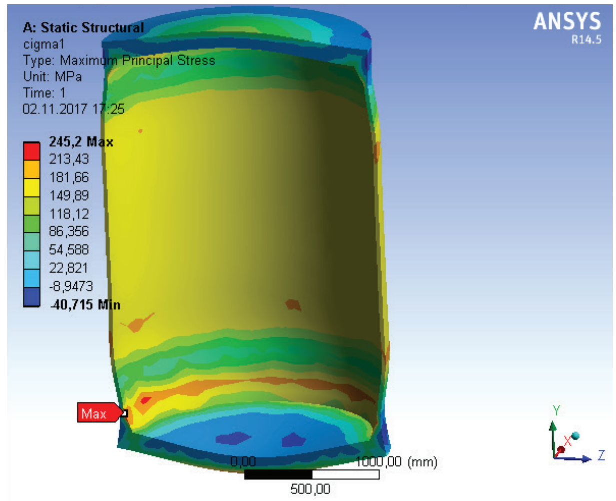
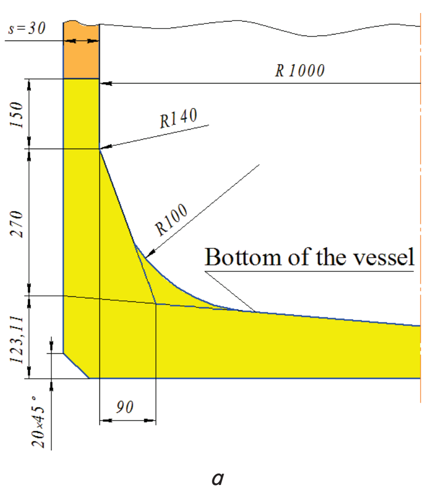


Fig. 7. Calculation of the transition zone (the first stage of constructing)

At the second stage, transition from the conical surface to the wall of the vessel was performed using the rounding by a radius, which, based on the recommendation from reference books [20, 23], is equal to the thickness of the wall, 30 mm. It was established that the overstress in a given zone of the conical transition amounted to 23 %.

The final, the third, variant of the calculation was performed for the scheme shown in Fig. 8.

Construction with a consistent increase in the rounding radii of the conical transition at each new stage makes it possible to ultimately meet the criteria of strength at a minimal consumption of material.



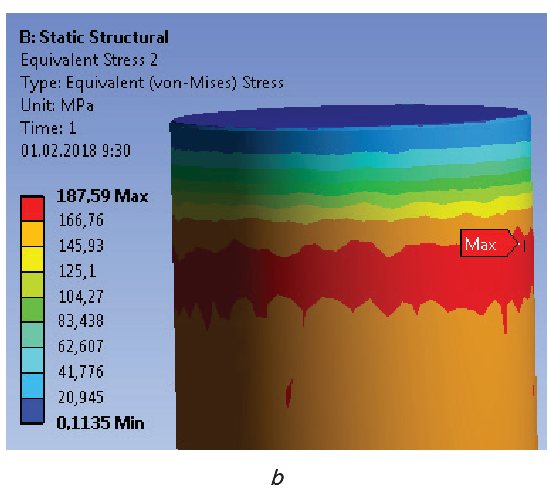


Fig. 8. The third stage in the construction of geometry of the transition zone from the bottom to the wall: *a* – construction of the transition zone; *b* – distribution of equivalent stresses defined using MFE

7. Results of the study into the minimization of mass of round plates

It was established that the calculation of the bottom as a component in the design of the body of an apparatus is implemented in a closed analytical form using the Kummer's functions. The result of present research is the confirmation of a possibility to reduce the mass of round flat bottoms, established for the round plates, applying simple techniques from papers [2, 23, 24]. The calculations show that a decrease in the mass of the bottoms of vessels in the transition from the fixed thickness to the variable thickness can range from 10...25 % (larger values from a given range are reached when the magnitudes of pressure q_0 and radius R increase).

The positive effect of present study is confirmed by the practical implementation of the applied methods of optimization in an actual CAD system, employed in the production of vessels that operate under pressure. For this purpose, we provided PAT «Berdychiv Machine-Building Plant Progress» (Ukraine) with the software «CAD system for flat bottoms of vessels loaded by internal pressure» with a block-hierarchical structure. Its elements include the following units: initial data, technical task, formalization of the object, mathematical model, and the optimization of design by the brute force method.

8. Discussion of results of examining the method for optimizing the shape of a bottom

Calculation of a round flat bottom of variable thickness as the element of a vessel's body, which operates under pressure, is based on the method of mathematical modelling of the bottom proposed in papers [1, 2, 25].

The advantage of the developed calculation method is the representation of solution in the analytical form. The obtained formulae make it possible to estimate the impact of separate characteristics of the structure (material, wall thickness, bottom thickness, dimensions) in the analytical or graphical fashion without repeatedly solving the problem.

The shortcoming of the proposed mathematical model of variable thickness $h(r)$ in the form of a Gaussian function (1) is the inability to create a perfect equally-stressed design of the bottom. This task can be resolved by optimizing the function $h(r)$ for separate annular zones of the bottom.

The proposed method of optimization of the shape of a radial cross-section of the bottom is demonstrated in present paper using as an example the calculation of the bottom of a vessel loaded by internal pressure, for which we achieved a substantial reduction in the mass of the bottom.

A variable shape of the cross section appears rational for many structures with round plate parts. The proposed procedure to minimize the mass is applicable in the presence or absence of reinforcements of contours (diaphragms of pipelines and vessels, screw shafts, etc.). It is recommended to use the method developed for solving the problem on minimizing the mass of flat parts with a round or an annular shape.

If appropriately improved, a given method could address problems on a cyclically symmetrical loading of round plate parts with variable thickness (flat lids of apparatuses, disks of toothed of rail wheels, disk locks of pipelines). We plan to solve the problems of this type in the future studies.

6. Conclusions

1. We have proposed a mathematical model for the bottom of a tank with variable thickness in the form of a Gaussian function (1). The problem on bending the bottom, connected to the wall of a cylindrical apparatus, was reduced to a system of three algebraic equations. The resulting analytical solution includes all the preset mechanical and geometrical characteristics of the structure.

2. We have developed a method of optimization of the shape of a radial cross section of the bottom of an apparatus for the criterion of a mass minimum, based on the analysis of a three-dimensional graphical interpretation of the condition of strength. The method makes it possible to significantly reduce the radial uneven distribution of equivalent (in line with von Mises theory) stresses in the bottom.

3. The most dangerous in an apparatus is the annular zone of connection between a flat bottom of variable thickness and a cylindrical wall. Momentous stresses in this region exceed the stresses in a cylindrical wall by 3...4 times. A reduction in stresses to the level of permissible ones is achieved by applying a conical transition with smooth rounding. The rational form of transition is constructed by sequential approximations using MFE at every stage. For the designed bottom with variable thickness, the reduction in mass is 22 % compared with the bottom of constant thickness.

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