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GEOMETRICAL MODELING OF THE SHAPE OF A MULTILINK ROD STRUCTURE IN WEIGHTLESSNESS UNDER THE INFLUENCE OF PULSES ON THE END POINTS OF ITS LINKS

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Досліджена геометрична модель нового способу розкриття в умовах невагомості багатоланкової стержневої конструкції, елементи якої з'єднані подібно багатоланковому маятнику. Розкриття ланок конструкції відбувається завдяки впливу імпульсів піротехнічних реактивних двигунів на їх кінцеві точки. Опис динаміки одержаного інерційного розкриття багатоланкової стержневої конструкції виконано за допомогою рівняння Лагранжа другого роду. Результати призначено для використання при проектуванні систем розкриття великогабаритних конструкцій в умовах невагомості, наприклад, силових каркасів для сонячних дзеркал чи космічних антен

Ключові слова: стержнева конструкція, процес розкриття у космосі, багатоланкова стержнева конструкція, рівняння Лагранжа другого роду

Исследована геометрическая модель нового способа раскрытия в условиях невесомости многозвенной стержневой конструкции, элементы которой соединены подобно многозвенному маятнику. Раскрытие звеньев конструкции происходит благодаря воздействию импульсов пиротехнических реактивных двигателей на конечные точки их звеньев. Описание динамики полученного инерционного раскрытия многозвенной стержневой конструкции выполнено с помощью уравнения Лагранжа второго рода. Результаты предназначены для использования при проектировании систем раскрытия крупногабаритных конструкций в условиях невесомости, например, силовых каркасов для солнечных зеркал или космических антенн

Ключевые слова: стержневая конструкция, процесс раскрытия в космосе, многозвенная стержневая конструкция, уравнение Лагранжа второго рода

1. Introduction

Development of space technologies in the leading countries of the world necessitates the construction of such large-size structures as the power frames for space antennas, mirrors, and other orbital infrastructures [1]. In this case, the prepared rod structures are mounted with reflective surfaces, which are manufactured from either a light-reflecting fabric – for space mirrors, or metal wires (a metal woven wire cloth) – for space antennas. These objects are unique by the Earth scale. The dimensions of space antennas for ultralong waves can be measured by the hundreds of meters while the areas of mirrors for lighting the settlements from space at night by reflected sunlight may amount to square kilometers. The area of the so-called solar sail, which is considered as a possible engine for space travel in future, will also be very large. The necessity to construct these, as well as other, space objects [2–4] calls for the development of technologies for constructing large-size space objects. Among them, an important role belongs to power frames of rod structures. Upon delivering into orbit, they would take the form of a combination of straight rods, coupled as the elements of a multilink rod structure. Existing technologies imply that large-size structures after having been delivered into orbit must unfold to acquire the designed shape. It is relatively easy to implement for a rod structure in the form of a multilink rod structure.

Control over unfolding of large-size structures in weightlessness is a complex scientific and technical challenge of mechanics, which has no analogues in the ground-based machinery. Creation of large-sized structures that would be transformed in space is related to solving several problems in engineering and mechanics predetermined by the uniqueness of objects. Their characteristic feature is a combination of conflicting requirements regarding a significant increase in the dimensions and ensuring sufficient rigidity at a rather limited mass of the power frame [5].

It is appropriate, for a structure in the form of a multilink rod system, to investigate mechanics of the unfolding process based on a variational principle of the Lagrangian dynamics. In this case, one should apply the concepts of kinetic and potential energy of a mechanical system. To calculate a geometrical form in the successive phases of unfolding «pendulum-like» rod structures, one should employ in the first place research results on the dynamics of multilink pendulums [6–8]. In practice, this implies constructing and solving the second-kind Lagrangian equations for the motion of a mechanical system relative to the generalized coordinates of a multilink pendulum and graphical interpretation of the solutions obtained.

The result is a question on adapting to weightlessness the process of fluctuations of a multilink pendulum as the basis of the geometrical model for unfolding a multilink rod structure of the orbital object. The answer can be found in papers that address the possibility of applying the second-kind Lagrangian equations for mechanical systems in weightlessness [9]. Formally, it is considered that calculations regarding the transformation of mechanical rod structures in weightlessness over time can be performed using only the notions of their kinetic energy. That is, when constructing the Lagrangian equations of the second kind, potential energy of the mechanical system can be considered as «zero». Such a formal approach is applied in further discussion in the development and study of geometrical models of unfolding space rod structures in the form of varieties of multilink pendulums as conservative systems. That is, it is accepted that after the initiation of fluctuations, the magnitude of kinetic energy is constant (for small periods of time).

Hence, it is expedient, for practical implementation, to examine a technique for the unfolding in weightlessness of large-size rod structures whose elements will be combined similar to a multilink pendulum. The sets of rods would be delivered into orbit in a folded form (a cartridge), after which it would be necessary to perform the operation of its unfolding to provide for a working shape of the entire spatial rod structure. Calculation of rod structures of such a class is proposed to be carried out based on the Lagrangian dynamics of multilink pendulums as a conservative system. This will make it possible to derive geometrical models for the successive phases of unfolding the rod structures with respect to dynamic properties. The application of such models at the design stage would help during subsequent studies to calculate functioning parameters of structures in general.

The relevance of the chosen subject is emphasized by the need to select and examine a possible engine for the process of unfolding a rod structure of pendulum type. It is proposed to use the pulse pyrotechnic jet engines mounted on the endpoints of links of the rod structure. They are much lighter and cheaper as compared, for example, with electric motors or spring devices. This is especially important when the process of unfolding a structure in orbit is planned to be performed only once. Most often, engines for the process of unfolding rod structures in orbit are used only once.

2. Literature review and problem statement

In order to substantiate the choice of design parameters for the elements of an unfolding system and to confirm the reliability of a given process, it is required to carry out a detailed mathematical modeling using an effective geometrical model. Paper [10] considered a procedure for modeling the dynamic process of unfolding a rod frame of space-based reflector of the antenna that is transformed. In this case, the authors employ a software complex based on the modeling using a finite element method. A special feature of the task is the possibility to take into consideration deformations of all elements of the structure at a radical change in configuration. In other words, it is possible to consider that an engine for the process of unfolding is the elastic properties of elements whose geometrical shape is difficult to handle in the case of large size of the structure. Paper [11] describes mathematical models of unfolding mechanisms, rope synchronization, braking and fixing the panels. The authors determined integrated dynamic characteristics and loading characteristics of elements of the solar cell. These models of unfolding mechanisms, however, do not apply the inertial technique to transform large-size solar batteries while preference is given to the rope synchronization, which is difficult to implement for large-size structures.

When studying the unfolding of structures of the multilink pendulum type, it appears interesting to consider a technique for enabling the required resultant arrangement of its links by the limited momentum applied to the first link. Authors in paper [12] constructed a law to control a multilink pendulum on a plane in the vicinity of the assigned position of equilibrium in the form of feedback. This makes it possible to drive a pendulum over finite time into equilibrium by the limited momentum applied to the first link. The efficiency of the derived control law is demonstrated using the example of computer simulation of the dynamics of a three-link rod structure. The authors substantiated the applicability of the derived control law for a nonlinear multilink pendulum. Such studies, however, are focused mainly on controlling the equilibrium of an inverse pendulum on a cart in the field of earthly gravitation.

In practice, more common are the frame rope unfolding systems. Papers [13, 14] describe mathematical models for the process of unfolding a multilink frame structure of the solar battery with a rope synchronization system. Fig. 1 shows a diagram of the unfolding of structure [13] in which the synchronization of change in the magnitudes of angles between adjacent links is performed by using electric motors and ropes.

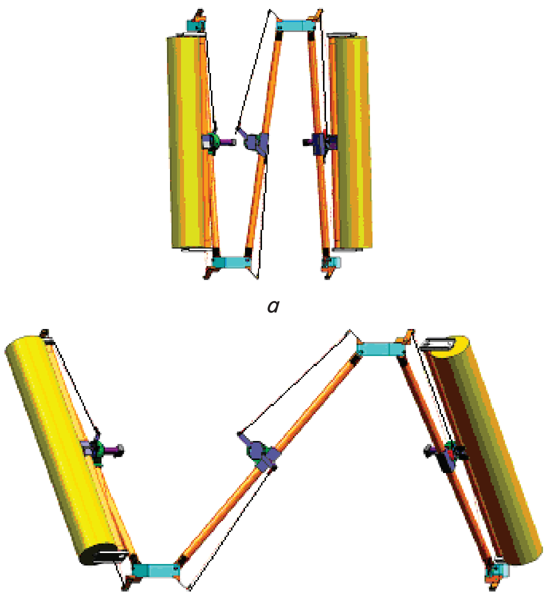


Fig. 1. Schematic of the structure with a rope unfolding system [13]: *a* – folded; *b* – unfolded

In this case, based on an analysis of the kinematic scheme of unfolding system, it is necessary to choose the size of rollers radii and a transfer ratio of the two types of gear mechanisms that enable the assigned sequence in the fixing of links. The second kind Lagrangian equation is used to study the process of unfolding a solar panel [14]. The application of a rope unfolding system is limited in practice by the dimensions of a structure and by the need to synchronize the action of electric motors, which is a separate task at a large number of links.

Capabilities of modern software packages to simulate the dynamics of mechanical systems are employed in order to analyze numerically the process of unfolding the transforming structures. Paper [15] addresses the method for calculating large-sized unfolding structures applying programming complexes MSC.Software. Paper [16] gives an example of calculating the unfolding process by using the software package for automated dynamic analysis of multicomponent mechanical systems EULER. The specified software, however, cannot realize, without appropriate add-ins, the inertial technique for unfolding multilink structures. Other variants of unfolding systems are outlined in the review of the scientific literature [17]. However, it lacks any information about the inertial technique for unfolding multilink rod structures, the preference being given to rope systems whose shortcomings were described above.

A prototype for the technique of unfolding a multilink rod structure described in the work is the rope system. An analysis of the scientific literature that we conducted revealed that the existing schemes of the unfolding of rope systems are too complex to implement for the case of large-size links (of the order of tens of meters). This conclusion is based on the necessity to synchronize and switch electric motors to adjust the magnitudes of angles in the nodes of a structure in order to render a multilink structure the calculated geometrical shape. This is a separate task, solving which is expedient from the standpoint of the dynamics of rod structures [18, 19]. In addition, the idea of applying multilink rod structures is promising for development given the newest technologies SpiderFab for manufacturing, directly in space, the links of rod structures as elements of large-size structures [20].

Papers [21, 22] initiated a geometrical model for the unfolding of the rod structure as a multilink pendulum in the imaginary plane in weightlessness. It was believed that the unfolding was enabled by pulsed pyrotechnic jet engines mounted at the endpoints of links. To actuate the sensors of fixation locks of the position of adjacent links of the rod structure in the unfolded state, it is proposed to use lateral movements of nodes (a tremor) at the final phases of the unfolding a rod structure. The conducted test calculations demonstrated a possibility of employing multilink rod structures with a common point of attachment. Still studied insufficiently are the issues related to the impact of a possible error in the magnitude of pulse on the realization of the unfolding in line with the calculated shape, as well as detailed description of the types of rod structures in order to obtain acceptable schemes of unfolding. Papers [23, 24] report a technique for constructing in weightlessness a metal woven wire cloth designed for antennas of ultralong waves. Stitches in a metal woven wire cloth are obtained using circular trajectories of the final load while opening a double rod structure. It is expedient, in this direction, to explore the selection of parameters for rod structures in order to obtain the required forms of circular trajectories of endpoints. Paper [25] gives illustrations to some of the provisions that would facilitate understanding the results of this paper.

To compare results obtained in this paper, we present [26, 27] some schemes for the unfolding of rod structures. Known structures and technologies for their unfolding are much more complex compared to the ones considered in this paper. There is a video at the website [28], which demonstrates a technique for initiating the oscillations of a rod structure in weightlessness.

The result of analysis of the scientific literature [10–28] is the issues that we identified as not having been investigated by other authors, which allowed us to formulate the following research problem. In order to implement the idea of unfolding multilink large-size structures in weightlessness, it is necessary to explore the inertial system of unfolding using the examples of particular rod structures where the initiation of their unfolding in space is performed by the pulses of pyrotechnic jet engines that affect certain nodal elements of the structure. It is necessary to analyze manifestations of possible errors in the magnitudes of pulses on the geometrical shape of the arrangement of links of the rod structure, acquired as a result of its unfolding. It is also required to consider the issue of slowing down and fixation of the elements of a multilink structure in the unfolded state, calculated in advance. In this case, the presence of extended links in the structure should not fundamentally affect the universal implementation of the inertial technique of unfolding.

3. The aim and objectives of the study

The aim of present study is to develop a geometrical model of the unfolding of a multilink rod structure under conditions of weightlessness. A rod structure is identified as a multilink pendulum that moves in the imaginary plane. To initiate the motion of a structure, it is proposed to use pulse pyrotechnic jet engines mounted at the end points of links of the structure.

To accomplish the aim, the following tasks have been set:

– to solve a system of the second-kind Lagrangian differential equations in order to describe phases in the motion of elements of particular varieties of multilink rod structures in weightlessness;

– in order to simulate the action of pulse jet engines, it is required to develop a scheme for the initiation of motion of a multilink rod structure by the influence of engine pulses on the end points of links of the structure;

– to explore a permissible error in the magnitude of the pulse for initiating the unfolding of a multilink rod structure with the inertial unfolding system under condition of obtaining the required arrangement of its links;

– by using computer animation, to predict in time mutual arrangement of the elements of a multilink rod structure and to determine, based on it, the required moment for the fixation of a mutual position of rods by employing a special «stop code»;

– to define parameters and initial conditions for the initiation of oscillations of a two-link rod structure in order to obtain a cyclic trajectory of the end point of the second link;

– to present test examples of the unfolding of certain variants of multilink rod structures in weightlessness.

4. Construction of a geometrical model of the unfolding in weightlessness of rod structures as multilink rod structures

4. 1. Explanation of the general scheme of the unfolding process of rod structures

Among possible configurations of multilink rod structures with a stationary attachment point, we shall consider their varieties of two types: linear – double and four-link structures, and non-linear two-link – Magdeburg and Thomson-Tait. Stationarity of the attachment point in a rod structure is provided by its attachment to a spacecraft whose mass is orders of magnitude larger than the total mass of load in nodes. Because the mass of a spacecraft is much greater than the mass of a two-link rod structure, its orientation is stabilized.

The term «Magdeburg» denotes a structure that resembles a pendulum exhibited at the Museum «Century Tower « in the German city of Magdeburg [29] in order to demonstrate chaotic oscillations. The term «Thomson-Tait» denotes a pendulum structure described in paper [30] and intended to explain relations between a symmetry of the Lagrangian function and the laws of conservation.

It is believed, for linear rod structures, that the first link is attached by its beginning to a fixed point and the beginning of the second link is attached to the end point. The following links of the rod structure are attached with its beginning to the end points of the preceding links. The difference between nonlinear rod structures is in the possibility for the existence of «child» structures with initial points at nodes of the «parent» rod structure.

In what follows we assume that cylindrical joints in the nodes of rod structures enable their unfolding only within the limits of an abstract plane, which passes through a motionless point of the structure. In practice, the implementation of the considered scheme of unfolding a rod structure will result in two key technological problems – the choice of techniques for the activation and termination of unfolding process. That is, the choice of driving forces as a means to initiate the unfolding of rod structures in weightlessness. As well as the technique for fixing the angles between the links of the mechanisms of cylindrical hinges when attaining the spatial arrangement implied by the designer. That is, the selection of time for giving a signal to slow down the unfolding by specialized devices mounted into cylindrical joints.

We can consider, as possible variants of techniques for initiating the unfolding of a rod structure, various units (devices) that are located in the nodal cylindrical hinges. First, these are the stepper electric motors, by using which one can monitor a change in the angles between links of the rod structure and fix their position. Second, this is the application of various elastic elements and metals with «thermal memory», programmed for a specific sequence of actions. Third, this is the use of centrifugal forces to change the spatial arrangement of links of the rod structure while it rotates around a fixed attachment point.

Theoretically, such methods are capable of implementing the unfolding of space objects of the multilink rod structures type. The choice of units for unfolding and braking, however, should depend on meeting certain conditions that would enable a failure-free functioning. Among the main ones are – the capacity of the specified units to counteract overloads in the process of being delivered into orbit. When using stepper electric motors, it is necessary to ensure the supply of current and switching in their work. To perform a one-time unfolding of the rod structure (as the most probable), such a technology would prove too expensive. In addition, prior to «launching» the unfolding, it is necessary to perform certain activities in order to initiate and test the structure.

All the aforementioned prompted us to choose another variant for selecting driving forces to initiate the unfolding of «pendulum-like» rod structures. Specifically, the use of pulse jet engines (the type of a squib), mounted at the endpoints of links of a rod structure. Note that pyrotechnic devices have been already applied in space technologies. For example, NASA employs pyrotechnic cartridges to «split» the screws when it is required to instantaneously disconnect orbital objects [31].

Each pulse pyrotechnic jet engine must provide for the magnitude of the pulse calculated for it. In addition, jet engines must be attached so that the action is directed along normal to the respective link. We shall explain this idea using a trivial example of a dual-rod structure. For this purpose, we use an imaginary plane with Cartesian coordinates Oxy in which under conditions of weightlessness a dual rod structure would move. It would consist of two weightless non-stretchable rods of lengths L_1 and L_2, coupled by a nodal cylindrical hinge. The ends of the rods hold fixed loads with masses m_1 and m_2 (Fig. 2).

Friction-free motion in the cylindrical hinges is enabled by the displacement of loads within the chosen plane. The beginning of the first link of the rod structure coincides with the coordinate origin O. Count direction is accepted to be the Oy axis. To determine a mutual position of links of the rod structure, we choose generalized coordinates – angles $u_1(t)$ and $u_2(t)$, formed in the plane by respective links along a count direction.

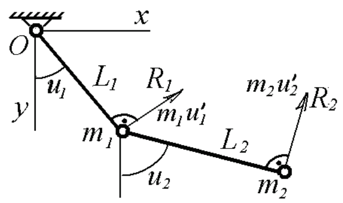


Fig. 2. Schematic of a two-link rod structure

The initiation of oscillations of the rod structure in weightlessness is carried out by selecting the magnitudes of pulses assigned to each deviation angle. For example, $\mathbf{U}'=\{u_1'(0), u_2'(0)\}$ means that load No. 1 with mass m_1 is given the pulse of magnitude $m_1u_1'(0)$, with load No. 2 with mass m_2 is given the pulse of magnitude $m_2u_2'(0)$. That is, the unfolding angles $u_1(0)$ and $u_2(0)$ are given initial velocities $u_1'(0)$ and $u_2'(0)$, respectively. Vectors $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ of the velocity setting direction coincide with the direction of action of pulse jet engines and are located perpendicular to the corresponding link in the rod structure at the end points (Fig. 1). With respect to the instantaneous velocities $u_1'(0)$ and $u_2'(0)$ given by the jet engines, the rod structure must unfold by inertia. The aforementioned explains the term of «inertial unfolding system». It is possible to apply as a pulse jet engine any device capable of providing the pre-calculated magnitude of the pulse (the type of a squib).

When the arrangement of links in a rod structure reaches the required spatial position, implied by design, then the question arises related to slowing down the process of unfolding. For this purpose, it is necessary to develop means to fix (lock) the angle between links at the calculated moment using a specialized device built into a cylindrical hinge of the rod structure. For example, it is possible to predict electromagnetic locks in cylindrical hinges, which would be operated by signals when reaching the estimated unfolding angle. These signals should be given by a computer program in the form of a «stop-code» at the moment of the unfolding, which would ensure the estimated arrangement of all links in the rod structure. Thus, a «stop-code» will refer to the sequence of values for the magnitudes of angles between links of the rod structure, pre-calculated by a designer, reaching which would initiate a command for the simultaneous fixation (locking) of links positions.

To prevent over-stresses in the elements of a structure, resulting at slowing down the unfolding of the structure, rods should be fabricated from ultralight materials. For example, polymeric composite materials based on epoxy resin, reinforced by graphite or kevlar fibers. Graphite and kevlar fibers have a negative coefficient of thermal expansion, whereas epoxy resin – normal, and their combination forms a composite, almost not affected by thermal expansion or compression in a wide temperature range. At the same time, under conditions of weightlessness, strength of a material in orbit is secondary relative to rigidity. Papers [32] described elements of structures that on Earth would not withstand their own mass of 411 kg, however, in orbit they could withstand loads of up to 24.5 tons. In addition, by applying the phase trajectories of unfolding processes, obtained in the work, there is a possibility to determine the speed of elements at the time of fixing the rod structure. And, consequently, to define other physical parameters.

Finally, we shall consider a scheme for initiating and registering oscillations, generalized for the case of a linear n-link rod structure. It consists of n weightless non-stretchable rods of lengths L_i ($i=1..n$), coupled by hinges at the end nodal points, on which the loads with masses m_i ($i=1..n$) are fixed. In this case, we shall consider that the generalized coordinates are angles $u_i(t)$ ($i=1..n$), formed by respective links along the direction of axis Oy in the plane.

At moment t, mutual position of the links of a rod structure can be determined by using a sequence of numbers $\mathbf{U}\{u_1(t), u_2(t), u_3(t),...,u_n(t)\}$. To finalize the process of unfolding, one must specify the calculated time $t=t_0$ when the mutual position of the links in a rod structure, implied by design, is reached. In this paper, we propose conducting these calculations by applying a computer animation of the unfolding process. For the moment $t=t_0$ we shall calculate the desired sequence $\mathbf{U_{STOP}}\{u_1(t_0), u_2(t_0), u_3(t_0),..., u_n(t_0)\}$. This would enable the formation of the «stop code», employing which would ensure a simultaneous fixation of locks in all cylindrical hinges. In this case, links of the rod structure will attain the mutual position preset by the calculations.

4. 2. Geometrical modeling of the unfolding of two-link rod structures

We present calculations for the unfolding of a two-link structure. Assume the beginning of the first link of a rod structure coincides with the coordinate origin O. The generalized coordinates are considered to be angles $u_1(t)$ and $u_2(t)$, formed in the plane by respective links to the reference direction (Fig. 1). In the absence of dissipative forces, and taking into consideration the «null» potential energy, a description of the motion of a rod structure in the imaginary plane will be performed based on Lagrangian:

$$L=0.5[m_1\left(x_1'^2+y_1'^2\right)+m_2\left(x_2'^2+y_2'^2\right)], \qquad (1)$$

where

$$x_1(t)=L_1\sin\left(u_1(t)\right);$$

$$y_1(t)=L_1\cos\left(u_1(t)\right);$$

$$x_2(t)=x_1(t)+L_2\sin\left(u_2(t)\right);$$

$$y_2(t)=y_1(t)+L_2\cos\left(u_2(t)\right). \qquad (2)$$

In this case, a system of the Lagrange equations of second kind takes the form:

$$m_1L_1u_1''+m_2L_2u_2''\cos(u_1-u_2)+$$
$$+m_2L_2\left(u_1'\right)^2\sin(u_1-u_2)+m_2L_1u_1''=0;$$

$$L_1u_1''\cos(u_1-u_2)-$$
$$-L_1\left(u_1'\right)^2\sin(u_1-u_2)+L_2u_2''=0. \qquad (3)$$

Here

$$u_1'=\frac{d}{dt}u_1(t);\quad u_2'=\frac{d}{dt}u_2(t);$$

$$u_1''=\frac{d^2}{dt^2}u_1(t);\quad u_2''=\frac{d^2}{dt^2}u_2(t)$$

are the derivatives from the function of description of generalized coordinates.

When solving a system of equations (3), it is necessary to consider coordinates of the following vectors: the lengths of links in a rod structure: $\mathbf{L}=\{L_1, L_2,\}$; values of the mass of loads: $\mathbf{m}=\{m_1, m_2\}$; values of the initial deviation angles: $\mathbf{U}=\{u_1(0), u_2(0)\}$, as well as values of velocities given to the deviation angles: $\mathbf{U}'=\{u_1'(0), u_2'(0)\}$.

By meeting the initial conditions, a system of the Lagrange equations of second kind (3) was approximately solved by the Runge-Kutta method in the environment of the mathematical package maple; the solutions obtained are denoted by symbols $U_1(t)$, $U_2(t)$. This allows us, in the plane in the *Oxy* coordinate system, to determine coordinates of the end nodal point (x_2, y_2) of the second link in a rod structure at time t. To calculate these coordinates in expressions (2), one should replace lowercase letters u with the uppercase letters U. The approximated form of the displacement trajectory will be obtained by connecting the near points with straight sections.

For the case of unfolding a two-link rod structure, of significance are the non-chaotic cyclic displacements of the end point (x_2, y_2) of the rod structure. It is ensured by the proper choice of parameters for a rod structure. By using the software created by maple, it is possible, in addition to the displacement of the specified end point, to determine speed that makes it possible to build a respective phase trajectory. At the same time, we derive approximated values for the current magnitudes of angles $u_1(t)$, $u_2(t)$, as well as the representation of a non-chaotic trace left by the displacement of the second load. In addition, the software makes it possible to obtain a sequence of N frames of the animated images showing the process of unfolding a rod structure depending on time. Given this, one can monitor mutual position of links in a rod structure at arbitrary point in time.

Here are examples of determining the non-chaotic oscillation trajectories of the second load of the rod structure by solving a system of the Lagrange equations of second. Given that a two-link structure is to be delivered folded, the initial condition, common to all the examples, is $\mathbf{U}=\{0, \pi\}$. Figures that show examples will demonstrate a cyclic non-chaotic trajectory of end point of the second link in a rod structure, which has a «pointed» point of return. As well as one of the phases in the position of links in a rod structure in the process of unfolding implemented counterclockwise. The triangle denotes a fixed fastening nod of the rod structure. Values for all parameters are given in arbitrary units. Examples of the implementation of the unfolding of rod structures in weightlessness can be found, in the form of animations, at a website [25].

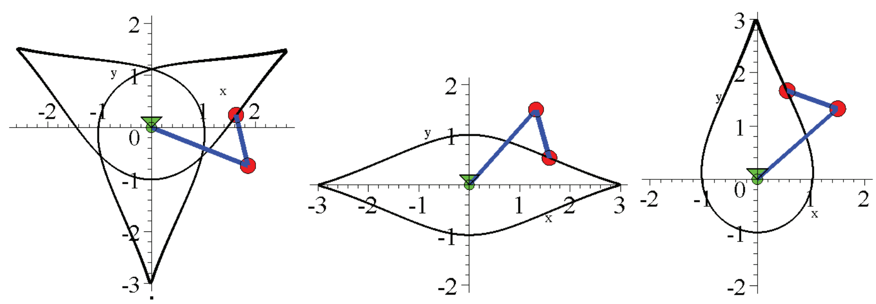
Example 1. $\mathbf{L}=\{2, 1\}$; $\mathbf{m}=\{1, 2\}$. Fig. 3 shows results of calculations depending on the coordinates of vector $\mathbf{U}'=\{u_1'(0), u_2'(0)\}$.

The curve, depicted in Fig. 3, *b*, is an example of a cyclic non-chaotic trajectory of the end point of a two-link rod structure (the cyclic trajectory with special points of return), which can be applied when manufacturing a metal woven wire cloth in weightlessness.

Example 2. $\mathbf{L}=\{2.5; 0.5\}$; $\mathbf{m}=\{1; 1{,}5\}$. Fig. 4 shows results of calculations depending on the coordinates of vector $\mathbf{U}'=\{u_1'(0), u_2'(0)\}$.

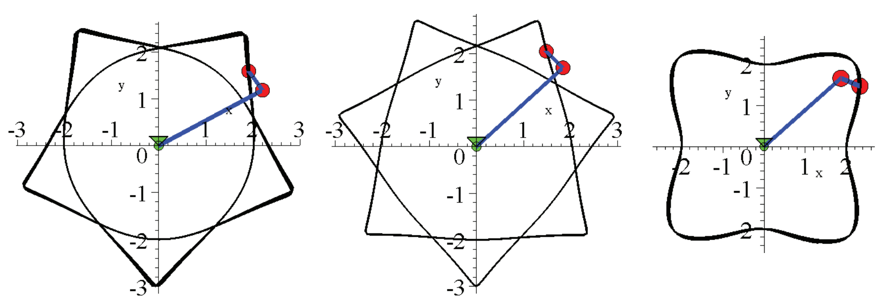
Example 3. $\mathbf{L}=\{0.5; 2.5\}$; $\mathbf{U}'=\{1; 1.5\}$. Fig. 5 shows results of calculations depending on the coordinates of vector $\mathbf{m}=\{m_1, m_2\}$.

We shall examine a question on the influence of error in the magnitude of the pulse on a geometrical shape of the result of unfolding. We accept as the criterion for the «quality» of unfolding the shape of a cyclic non-chaotic trajectory of the end point of the second link in a road structure depending on error in the magnitude of the pulse within the percentage of its estimated value.



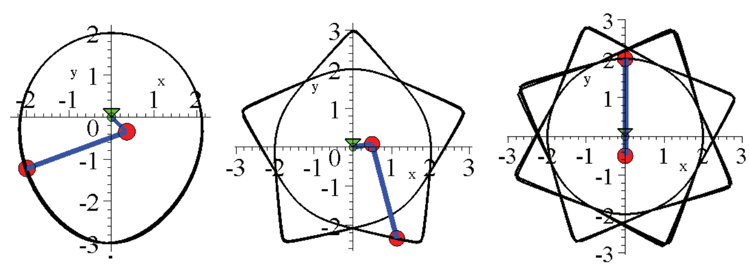
a *b* *c*

Fig. 3. Calculations for example 1 depending on the coordinates of vector $\mathbf{U}'$: *a* – $\mathbf{U}'=\{1; 0.65\}$; *b* – $\mathbf{U}'=\{1; 0.05\}$; *c* – $\mathbf{U}'=\{1; 0.947\}$



a *b* *c*

Fig. 4. Calculations for example 2 depending on the coordinates of vector $\mathbf{U}'$: *a* – $\mathbf{U}'=\{1; 0.28\}$; *b* – $\mathbf{U}'=\{1; -1.336\}$; *c* – $\mathbf{U}'=\{1; 2.66\}$



a *b* *c*

Fig. 5. Calculations for example 3 depending on the coordinates of vector $\mathbf{m}$: *a* – $\mathbf{m}=\{1; 0.244\}$; *b* – $\mathbf{m}=\{1; 8.543\}$; *c* – $\mathbf{m}=\{1; 13.48\}$

To this end, we build aligned patterns of trajectories that would match the following values for the magnitudes of pulses:

– $\{u_1'(0)-u_1'(0)/100,\ u_2'(0)-u_2'(0)/100\}$ – denoted by green color;

– $\{u_1'(0),\ u_2'(0)\}$ – denoted by black color (derived trajectory);

– $\{u_1'(0)+u_1'(0)/100,\ u_2'(0)+u_2'(0)/100\}$ – denoted by green color.

We repeat, with respect to these conditions, the construction of example 1. Fig. 6 shows aligned images depending on the coordinates of vector $\mathbf{U}'$.

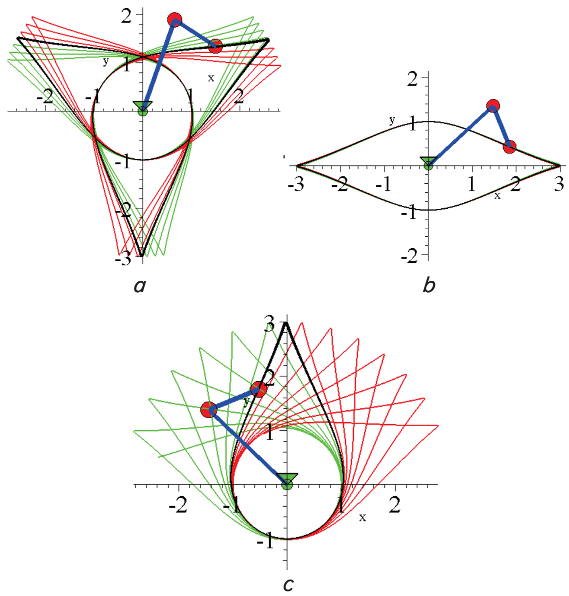


Fig. 6. Calculations depending on the coordinates of vector $\mathbf{U}'$: *a* – $\mathbf{U}'=\{1\pm0.01;\ 0.65\pm0.065\}$; *b* – $\mathbf{U}'=\{1\pm0.01;\ 0.05\pm0.005\}$; *c* – $\mathbf{U}'=\{1\pm0.01;\ 0.947\pm0.0947\}$

In this example, we have demonstrated that the error in the quantity «magnitude percentage» differently affect the result of the unfolding of different double rod structures. The most important result is the fact that for the rod structure with parameters $\mathbf{L}=\{2,1\}$; $\mathbf{m}=\{1,2\}$ $\mathbf{U}'=\{1, 0.05\}$ (Fig. 6, *b*) such an error almost does not affect the outcome of its unfolding. The specified curve is an example of a cyclic non-chaotic trajectory of the end point in a two-link rod structure. A given property makes it possible to recommend a pendulum with parameters $\mathbf{L}=\{2,1\}$; $\mathbf{m}=\{1,2\}$ $\mathbf{U}'=\{1, 0.05\}$ as a pretext for weaving a metal wire cloth where it is necessary to ensure the displacement of an end point of the second link in a rod structure along a direction of the platform.

Fig. 6 allows us to also explain a technique for finding cyclic non-chaotic trajectories (Fig. 3–5), which underlies the algorithm for determining such curves. Initially, we build a parametric family of curves whose parameter is C from expression $\mathbf{U}'=\{1, C\}$. Next, we derive a function of the number of pixels that make up an image (without a rod structure) in Fig. 6. The desired value for C, which would provide for a cyclic non-chaotic trajectory, matches the extreme (minimum) value of the function of the number of pixels. When there are several such extrema, we choose a global minimum.

Summing up, we note that periodic movements of the second load of a dual rod structure can be used when implementing certain operations in orbit. For example, in the mechanism for weaving a metal wire cloth for large-size structures [23, 24].

4. 3. Geometrical modeling of the unfolding of four-link rod structures

By building on the results of papers [21, 22], we shall give as the second example calculations of the unfolding of a four-link rod structure. We consider that conditions for all preceding assumptions are satisfied. The generalized coordinates are the angles $u_1(t)$, $u_2(t)$, $u_3(t)$ i $u_4(t)$, formed in the plane by respective links along the reference direction (Fig. 7).

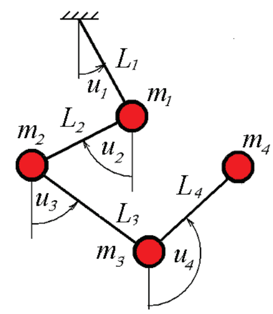


Fig. 7. Schematic of a four-link rod structure

Using the generalized coordinates, we compute coordinates of the nodal points in a rod structure:

$$x_1(t)=L_1\sin(u_1(t));$$

$$y_1(t)=L_1\cos(u_1(t));$$

$$x_2(t)=x_1(t)+L_2\sin(u_2(t));$$

$$y_2(t)=y_1(t)+L_2\cos(u_2(t));$$

$$x_3(t)=x_2(t)+L_3\sin(u_3(t));$$

$$y_3(t)=y_2(t)+L_3\cos(u_3(t));$$

$$x_4(t)=x_3(t)+L_4\sin(u_4(t));$$

$$y_4(t)=y_3(t)+L_4\cos(u_4(t)). \quad (4)$$

In the absence of dissipative forces, and with respect to the «null» potential energy, a description of the unfolding of a rod structure in the imaginary plane, will be performed based on Lagrangian:

$$L=0.5\begin{bmatrix} m_1\left(x_1'^2+y_1'^2\right)+m_2\left(x_2'^2+y_2'^2\right)+ \\ +m_3\left(x_3'^2+y_3'^2\right)+m_4\left(x_4'^2+y_4'^2\right) \end{bmatrix}. \quad (5)$$

A description of the motion of a four-link rod structure will be obtained in the form of a system of four differential Lagrange equations of second kind relative to functions $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ (not given here for the sake of brevity). When solving the obtained system, one should take into account coordinates of the following vectors: lengths of the links of a rod structure: $\mathbf{L}=\{L_1, L_2, L_3, L_4\}$; values of the masses of balls: $\mathbf{m}=\{m_1, m_2, m_3, m_4\}$; values of the initial deviation angles: $\mathbf{U}=\{u_1(0),\ u_2(0),\ u_3(0),\ u_4(0)\}$, as well values of the initial velocities, given to deviation angles $\mathbf{U}' = \{u_1'(0), u_2'(0), u_3'(0), u_4'(0)\}$. All values for parameters are given in arbitrary units.

Taking into consideration the respective initial conditions, a system of Lagrange equations of second kind is solved approximately using the Runge-Kutta method in the environment of the mathematical software maple; the solutions obtained are denoted by symbols $U_1(t)$, $U_2(t)$, $U_3(t)$ and $U_4(t)$. We define in the coordinate system *Oxy,* selected in the plane, by using the obtained solutions, coordinates of the nodal points at time t. For this purpose, we apply expressions (4) to calculate coordinates of the nodes in a rod structure using the generalized coordinates, replacing there lowercase letters u with uppercase letters U. By employing the software created by maple, it is possible, in addition to the displacement of nodal points, to determine velocities, which would make it possible to build respective phase trajectories of displacement.

A multi-link frame structure is to be delivered folded into orbit (visually, it reminds a household meter ruler when folded). That is, the initial position of the set of links in a rod structure takes a «folded» form; the vector of values of the initial deviation angles will always have coordinates $\mathbf{U} = \{\pi/2, -\pi/2, \pi/2, -\pi/2\}$. We emphasize the fact that, when folded, the initial position of a jet engine of the fourth link is located in the zone when a rod structure is attached to a spacecraft. This simplifies the assembly of the installation for transportation; moreover, it also makes it possible to compensate for the effect of impulse on the system through the symmetrical position relative to the device of a similar multilink rod structure.

Here are the test examples of geometrical modeling of the unfolding four-link frames by solving a system of Lagrange equations of second kind. We shall consider, as a promising variant, a rod structure composed of six four-link structures with a joint attachment node, the angles between them are 60 degrees. It is believed that the unfolding of rod structures is performed in closely-spaced parallel planes that do not hinder mutual displacements.

Following the execution of the program, we shall receive a sequence of frames of animated images, depending on the time of the unfolding of the structure. In the examples presented, we determined, by using computer animation, the points in time when there appeared the required form of rod structures. At the same time, we derive approximated values of the current magnitudes of angles $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ for the selected time t. These values will be used to form a «stop-code» of the process of unfolding. The examples are illustrated by axonometric images of the acquired final phases of links in a rod structure. Animated videos of unfolding processes can be viewed at website [25]. The phase trajectories built are intended to analyze the values of change in the angles at the final stage of unfolding a multilink structure.

Example 4. Length of the links: $\mathbf{L} = \{4, 4, 4, 4.5\}$. The pulses of magnitudes $\mathbf{U}' = \{0, 1, -1, 1\}$ are provided by the jet engines installed on the second, third, and fourth loads.

Fig. 8 shows a rod structure following the unfolding of six rod structures at time $t = 2.65$. Values for the coordinates of vector of the «stop-code» will be defined by the following numbers: $\mathbf{U_{STOP}} = \{0.2312, -0.9965, 2.115, 1.023\}$.

Fig. 9 shows phase trajectories of the generalized coordinates of unfolding a rod structure for example 1. Analysis shows that at the final stages of unfolding, speeds of the respective nodes will acquire values: $u_1'(2.65) = 0.5$; $u_2'(2.65) = -1.8$; $u_3'(2.65) = 1$; $u_4'(2.65) = 0.75$.

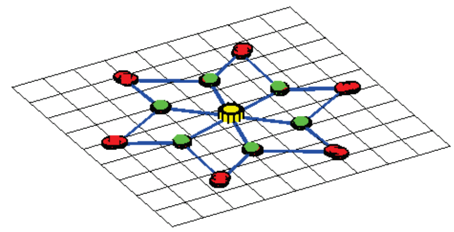


Fig. 8. A rod structure from example 4 at time $t = 2.65$

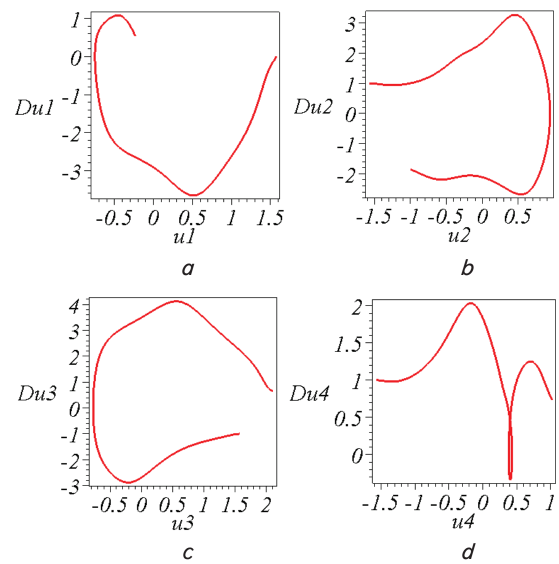


Fig. 9. Phase trajectories of the generalized coordinates for example 1: *a* – $u_1(t)$; *b* – $u_2(t)$; *c* – $u_3(t)$; *d* – $u_4(t)$

Next, we analyze the impact of the magnitude of pulses on the geometrical form of the result of unfolding. We shall demonstrate graphically a dependence of the magnitude of errors on the magnitude of pulse within the percentage of its estimated value. First, we consider the unfolding of one of six rod structures. We build aligned images of unfolding phases, which would correspond to the following values of the magnitudes of pulses:

– $\{u_1'(0) - u_1'(0)/100,\ u_2'(0) - u_2'(0)/100\}$ – denoted by green color;

– $\{u_1'(0), u_2'(0)\}$ – denoted by red color (found pattern);

– $\{u_1'(0) + u_1'(0)/100,\ u_2'(0) + u_2'(0)/100\}$ – denoted with blue color.

Fig. 10 shows aligned images of unfolding phases at time $t_0 = 2.65$. Yellow color marks a motionless point of the rod structure.

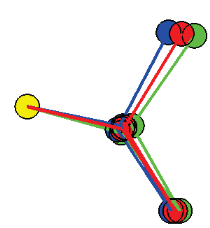


Fig. 10. Aligned images of the unfolding phases of a rod structure depending on the magnitudes of pulses at time $t_0 = 2.65$

Example 5. Length of the links: $\mathbf{L} = \{4.3, 5, 3.5, 6\}$. The pulses $\mathbf{U}' = \{1, 0, 0, 0.5\}$ are provided by the jet engines mounted on the first and fourth loads.

Fig. 11 shows the state of a rod structure after the unfolding of six rod structures at time t=1.76. Values for the coordinates of vector of the «stop-code» will be defined by the following numbers: $\mathbf{U_{STOP}}$={3.074, –2.276, 1.5, –0.6652}.

Fig. 12 shows phase trajectories of the generalized coordinates of unfolding a rod structure for example 2. Analysis shows that at the final stages of unfolding the speeds of nodes will have the following values: $u_1'(1.76)=1$; $u_2'(1.76)=-0.8$; $u_3'(1.76)=0$; $u_4'(1.76)=0.45$.

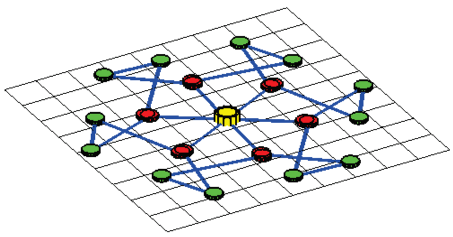


Fig. 11. A rod structure from example 2 at time $t=1.76$

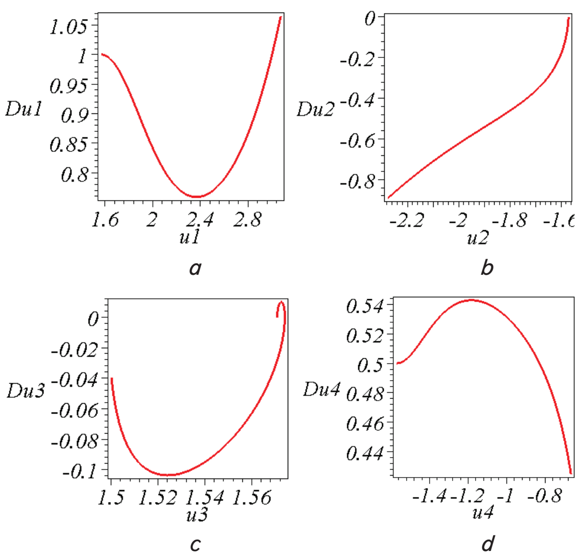


Fig. 12. Phase trajectories of the generalized coordinates for example 2: $a - u_1(t)$; $b - u_2(t)$; $c - u_3(t)$; $d - u_4(t)$

We shall analyze the impact of the magnitude of pulses on the geometrical form of the result of unfolding. Fig. 13 shows the unfolding of one of six rod structures from example 5. We have built the aligned images of unfolding phases based on data from the previous example. Yellow color marks a motionless point of the rod structure.

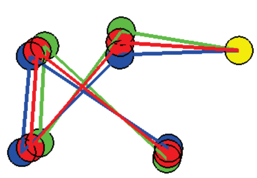


Fig. 13. Aligned images of unfolding phases of a rod structure from example 7 depending on the magnitudes of pulses at time $t_0=1.76$

Example 6. Length of the links of a rod structure $\mathbf{L}$={1, 3, 3, 5.4}. The initial speeds $\mathbf{U}'$={0, 5, 0, 0} are provided by the jet engine installed on the second load only.

Fig. 14 shows a rod structure after the unfolding of a six four-link rod structures at time t=1.03. Values for the coordinates of vector of the «stop-code» will be defined by the following numbers: $\mathbf{U_{STOP}}$={5.19, 0.2166, –1.132, 1.539}.

Fig. 15 shows phase trajectories of the generalized coordinates of unfolding a rod structure for example 3. Analysis shows that at the final stage of unfolding the speeds of nodes will acquire the following values: $u_1'(1.03)=10$; $u_2'(1.03)=-2$; $u_3'(1.03)=0$; $u_4'(1.03)=6$.

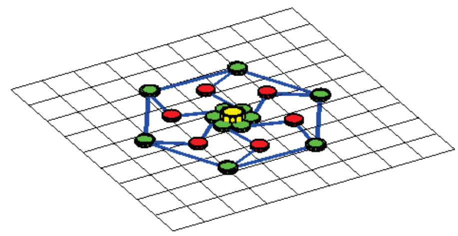


Fig. 14. A rod structure from example 3 at time $t=1.03$

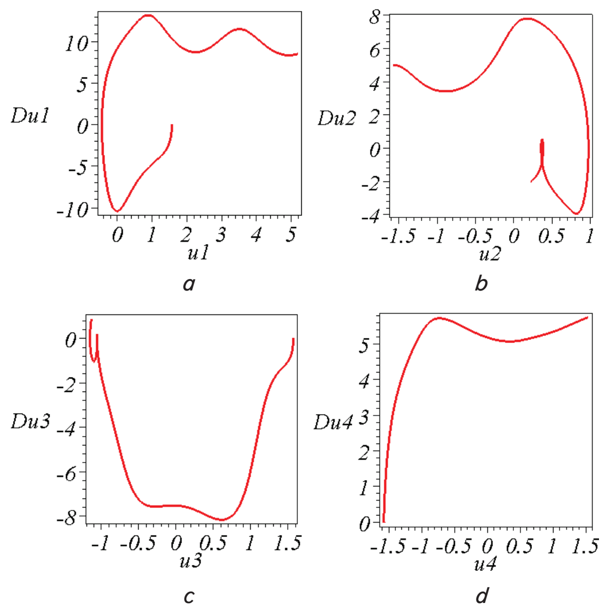


Fig. 15. Phase trajectories of the generalized coordinates for example 3: $a - u_1(t)$; $b - u_2(t)$; $c - u_3(t)$; $d - u_4(t)$

Fig. 16 shows aligned images of the unfolding of one of rod structures of the example (designations are similar to the previous examples). Yellow color marks a motionless point in the rod structure. Paper [25] contains animated images of the unfolding processes.

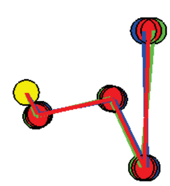


Fig. 16. Aligned images of the unfolding phases of a rod structure from example 7 depending on the magnitudes of pulses at time $t_0=1.03$

The obtained aligned images allow us to conclude at the qualitative level about the effect of error in the magnitude of pulse on the result of unfolding. Comparison of Fig. 10, 13, and 16 shows that the smallest impact of the error will be demonstrated by a variant of the rod structure with parameters **L**={1, 3, 3, 5}; **U′**={0, 5, 0, 0}. Such a conclusion at the qualitative level was drawn because the differences in the position of colored end points will be the smallest. Note that in this example of initiation, movements are enabled by only one jet engine installed on the site of the second load.

4. 4. Geometrical modeling of the unfolding of a rod structure of the Magdeburg pendulum type

As the third example, we present calculations for the unfolding of a non-linear rod structure in the form of a Magdeburg pendulum. The structure is composed of two weightless rods. The first rod is attached to the fixed point *O* by one of the internal points; the second rod is attached to the end point of the first link with its end. Fig. 17 shows designations for the elements of a rod structure, similar to designations in Fig. 2.

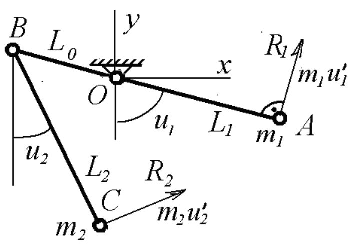


Fig. 17. Schematic of the Magdeburg rod structure

We shall consider that all conditions for preceding assumptions are satisfied. The generalized coordinates are the angles $u_1(t)$ and $u_2(t)$, formed in the plane by respective links with the *Oy* axis as the reference direction. By using the generalized coordinates, we shall define the Cartesian coordinates of the nodal points in a rod structure:

$$x_A(t) = L_1 \sin(u_1(t));$$

$$y_A(t) = L_1 \cos(u_1(t));$$

$$x_B(t) = -L_0 \sin(u_1(t));$$

$$y_B(t) = -L_0 \cos(u_1(t));$$

$$x_C(t) = x_B(t) + L_2 \sin(u_2(t));$$

$$y_C(t) = y_B(t) + L_2 \cos(u_2(t)). \qquad (6)$$

In the absence of dissipative forces, and with respect to the «null» potential energy, a description of the unfolding of a rod structure will be performed based on Lagrangian [29]:

$$L = 0.5 m_1 L_1^2 u_1'^2 - m_2 L_0 L_2 u' v' \cos(u_1 - u_2) + $$
$$+ 0.5 m_2 L_0^2 u_1'^2 + 0.5 m_2 L_2^2 u_2'^2 + 0.5 J u_2'^2. \qquad (7)$$

In this case, a system of the Lagrange equations of second kind will take the form:

$$m_1 L_1^2 u_1'' - m_2 L_0 L_2 u_2'' \cos(u_1 - u_2) -$$
$$- m_2 L_0 L_2 u_2'^2 \sin(u_1 - u_2) + m_2 L_0^2 u_1'' = 0; \qquad (8)$$

$$- m_2 L_0 L_2 u_1'' \cos(u_1 - u_2) + m_2 L_0 L_2 u_1'^2 \sin(u_1 - u_2) +$$
$$+ m_2 L_2^2 u_2'' + J u_2'' = 0.$$

Here

$$u_1' = \frac{d}{dt} u_1(t); \quad u_2' = \frac{d}{dt} u_2(t);$$

$$u_1'' = \frac{d^2}{dt^2} u_1(t); \quad u_2'' = \frac{d^2}{dt^2} u_2(t)$$

are the derivatives from the function of description of the generalized coordinates; $J=0.1$ is the inertia momentum of the second link.

When solving a system of equations (8), one should take into account the following coordinates of these vectors: lengths of the links in a rod structure: **L**=$\{L_0, L_1, L_2,\}$; values of the mass of loads: **m**=$\{m_1, m_2\}$; values of the initial deviation angles: **U**=$\{u_1(0), u_2(0)\}$, as well as values of speed given to the deviation angles: **U′**=$\{u_1'(0), u_2'(0)\}$. With respect to the initial conditions, the system of Lagrange equations of second kind (8) was solved approximately using the Runge-Kutta method in the environment of the mathematical software maple; the solutions obtained are denoted by symbols $U_1(t)$, $U_2(t)$. To calculate the Cartesian coordinates of point *C* on the second link of the rod structure, one should replace in expressions (6) the lowercase letters *u* with the uppercase letters *U*.

The result of execution of the program is a time-dependent sequence of frames that show the unfolding of the structure in the form of computer-animated images. By using the animation, we determined values for parameters of a rod structure when the trajectory of point *C* would take the form of a non-chaotic curve, interesting for consideration.

Here are examples of determining the result of unfolding rod structures of the Magdeburg pendulum type by solving a system of Lagrange equations of second kind (8). Since a two-link structure is to be delivered folded, the initial condition **U**={0, 0} will be common to all the examples. The examples will be illustrated by the images of cyclic non-chaotic trajectories of the end point of the second link in a rod structure. We shall also show one of the phases of arranging the links of a rod structure in the process of unfolding, implemented counter-clockwise. Values for all parameters are given in arbitrary units.

Example 7. **L**={1, 0.5, 1.5}; **m**={1, 1}. Fig. 18 shows trajectories of the displacement of a point that coincide in the case of three variants, namely, for the coordinates of vectors **U′**={1, –0.788}; **U′**={1, –0.5743}; **U′**={1, –0.8815} and differ in the directions of its displacement.

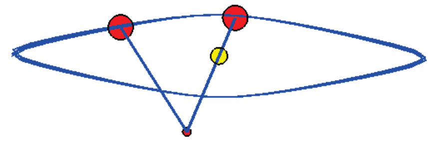


Fig. 18. Coinciding trajectories of the displacement motion

Curve in Fig. 18 is also an example of a cyclic non-chaotic trajectory of the end point in a two-link rod structure, namely, a cyclic trajectory with two special points of return. Such a property also makes it possible to use the rod structure with parameters from example 7 as a pretext for weaving a metal wire cloth. For this purpose, the displacement of the end point of the second link in a rod structure along the direction of a platform is ensured.

Two-link rod structures might also find application, with their cyclic trajectories acquiring a drop-like shape; they are examined in the next example.

Example 8. **L**={1, 0.5, 1.5}; **m**={1, 1}. Fig. 19 shows drop-shaped trajectories of the point depending on the coordinates of vector **U′**={u_1′(0), u_2′(0)}.

Example 9. **L**={1, 1.5, 1.5}; **m**={1, 1}. Fig. 20 shows drop-shaped trajectories of the point depending on the coordinates of vector **U′**={u_1′(0), u_2′(0)}.

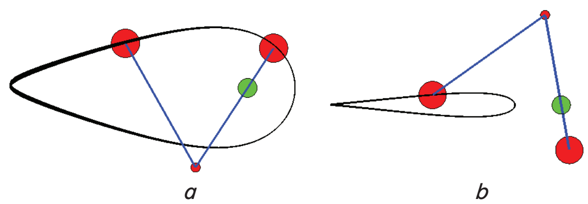


Fig. 19. Calculations for example 2 depending on the coordinates of vector **U′**: *a* –**U′**={1; –0.8365}; *b* – **U′**={1; –0.6569}

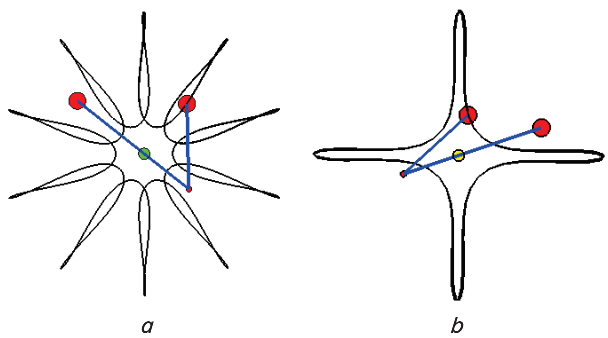


Fig. 20. Calculations for example 2 depending on the coordinates of vector **U′**: *a* – **U′**={1; –0.7098}; *b* – **U′**={1; –0.6569}

Summing up, we note that the periodic movements of the second load in a two-link rod structure can be employed when implementing certain operations in orbit. For example, in the mechanism for fabricating a metal woven wire cloth for large-size structures [23, 24]. It was found that an error within the percentage of the calculated value does not affect the result of the unfolding of various double rod structures. The most important is the fact that for the rod structures with parameters **L**={1, 0.5, 1.5}; **m**={1, 1}; **U′**={1, –0.788}; **U′**={1, –0.5743}; **U′**={1, –0.8815} trajectories of the motion of the end point coincide, differing only by the directions of its displacement.

4. 5. Geometrical modeling of the unfolding of rod structures of the Thomson-Tait type

In the fourth example, we shall consider the unfolding of a non-linear rod structure similar to the Thomson-Tait pendulum. The structure is composed of two rods. The first rod is attached to a fixed point O by its beginning; the second rod is attached to the end point of the first link at its middle [30]. Fig. 21 shows designations for the elements of a rod structure similar to designations in Fig. 1.

We shall consider that all conditions for the previous assumptions are satisfied. The generalized coordinates are the angles $u_1(t)$ and $u_2(t)$, formed in the plane by respective links with the *Oy* axis as the reference direction.

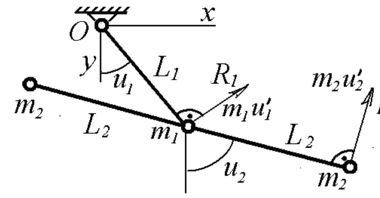


Fig. 21. Schematic of the Thomson-Tait rod structure

By using the generalized coordinates, we shall define the Cartesian coordinates of the nodal points in a rod structure:

$$x_1(t)=L_1\sin(u_1(t));$$

$$y_1(t)=L_1\cos(u_1(t));$$

$$x_2(t)=x_1(t)+L_2\sin(u_2(t));$$

$$y_2(t)=y_1(t)+L_2\cos(u_2(t));$$

$$x_3(t)=x_1(t)-L_2\sin(u_2(t));$$

$$y_3(t)=y_1(t)-L_2\cos(u_2(t)). \quad (9)$$

In the absence of dissipative forces, and taking into account the «null» potential energy, a description of the unfolding of a rod structure will be performed based on Lagrangian:

$$L=m_2\left(2L_1^2u_1'^2+L_2^2u_2'^2\right). \quad (10)$$

In this case, a system of the Lagrange equations of second kind will take the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial u_1'}\right)-\frac{\partial L}{\partial u_1}=m_2L_1^2u_1''=0; \quad (11)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial u_2'}\right)-\frac{\partial L}{\partial u_2}=m_2L_2^2u_2''=0.$$

Here

$$u_1'=\frac{d}{dt}u_1(t);\quad u_2'=\frac{d}{dt}u_2(t);$$

$$u_1''=\frac{d^2}{dt^2}u_1(t);\quad u_2''=\frac{d^2}{dt^2}u_2(t)$$

are the derivatives from the function of description of the generalized coordinates. We emphasize the fact that equation (8) does not include m_1. The explanation of a given phenomenon is in paper [26].

When solving a system of equations (11), one should take into account coordinates of the following vectors: lengths of links in a rod structure: **L**={L_1, L_2,}; values of the masses of loads: **m**={m_1, m_2}; values of the initial deviation angles **U**={$u_1(0)$, $u_2(0)$}, as well as values of speed given to the deviation angles: **U′**={u_1′(0), u_2′(0)}. With respect to the initial conditions, a system of the Lagrange equations of second kind (10) was solved approximately by the Runge-Kutta method in the environment of the mathematical software maple; the solutions obtained are denoted by symbols

$U_1(t)$, $U_2(t)$. To calculate the Cartesian coordinates of the end points at the second link in a rod structure, one should replace in expressions (9) the lowercase letters u with the uppercase letters U.

Here are examples of determining a result of unfolding rod structures of the Thomson-Tait type by solving a system of the Lagrange equations of second kind. Since a two-link structure is to be delivered folded, the initial condition $\mathbf{U}=\{0, 0\}$ will be common to all the examples. Values of all parameters are given in arbitrary units.

Example 10. $\mathbf{L}=\{1.52, 0.91\}$; $\mathbf{m}=\{1, 1\}$ $\mathbf{U}'=\{1, -0.5\}$. Fig. 22 shows sequential frames of the process of unfolding a structure depending on time t. The «stop-code» at time $t=1.05$ is the numbers $\mathbf{U_{STOP}}\{1.05; -0.525\}$.

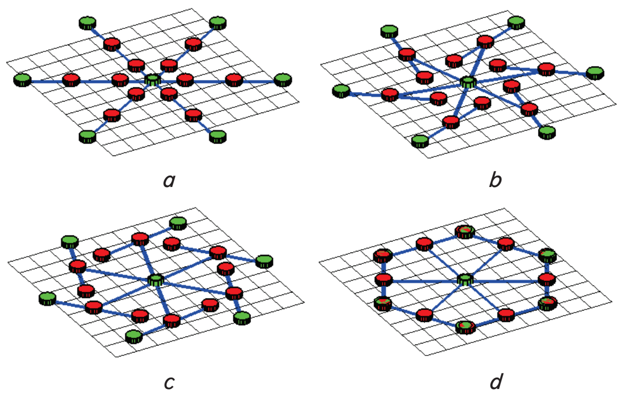


Fig. 22. Time-dependent unfolding of the Thomson-Tait rod structure: a — $t=0$; $u_1=0$; $u_2=0$; b — $t=0.315$; $u_1=0.315$; $u_2=-0.158$; c — $t=0.718$; $u_1=0.718$; $u_2=-0.359$; d — $t=1.05$; $u_1=1.05$; $u_2=-0.525$

By using thirty six unfolded rod structures of the Thomson-Tait type, it is possible to build a power rod structure for space objects (Fig. 23).

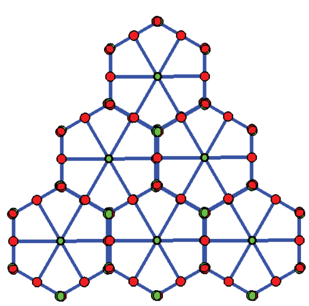


Fig. 23. Schematic of a power rod structure, erected out of 36 unfolded Thomson-Tait rod structures

Example 11. $\mathbf{L}=\{1, 1.7\}$; $\mathbf{m}=\{1, 1\}$ $\mathbf{U}'=\{1, 0.5\}$. Fig. 24 shows sequential frames of the process of unfolding a structure depending on time t. The «stop-code» at time $t=3.07$ is the numbers $\mathbf{U_{STOP}}\{3.07; 0.535\}$.

Using the result from example 11 it is possible to build a reinforced rod structure (Fig. 25) for space infrastructure (for example, space enterprises or settlements).

Summing up, it should be noted that the results of calculations verified the following properties of the Thomson-Tait rod structure, which would contribute to its application when designing systems for the unfolding of rod structures in weightlessness.

1. At time t, magnitudes of the unfolding angles are proportional to the respective initial speeds of change in angles $u_1'(0)$ and $u_2'(0)$.

2. In the process of unfolding, current speeds of change in the angles are constant and coincide with the initial speeds $u_1'(0)$ and $u_2'(0)$.

3. Unfolding of a rod structure of the Thomson-Tait type of rod structure does not depend on mass m_1.

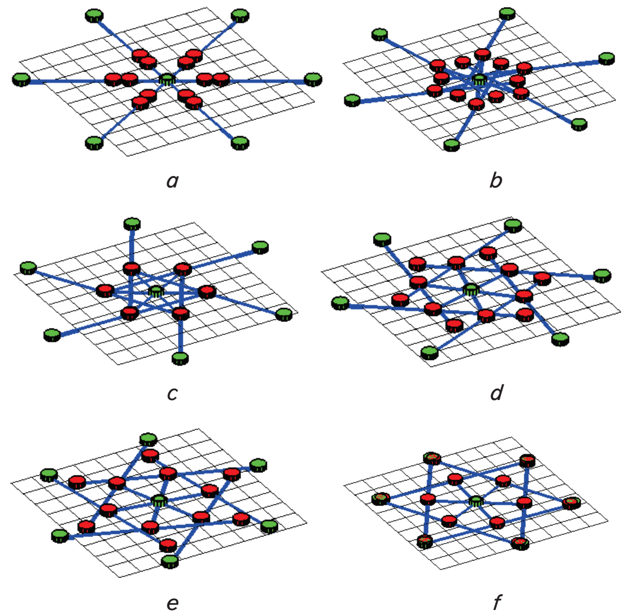


Fig. 24. Time-dependent unfolding of the Thomson-Tait rod structure: a — $t=0$; $u_1=0$; $u_2=0$; b — $t=0.46$; $u_1=0.46$; $u_2=0.23$; c — $t=1.023$; $u_1=1.023$; $u_2=0.512$; d — $t=1.84$; $u_1=1.84$; $u_2=0.92$; e — $t=2.46$; $u_1=2.46$; $u_2=1.23$; f — $t=3.07$; $u_1=3.07$; $u_2=0.535$

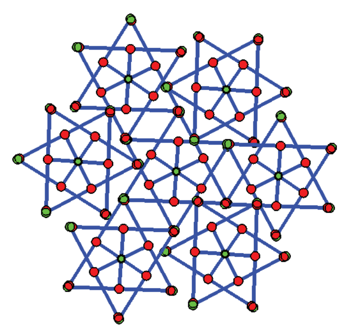


Fig. 25. Reinforced rod structure for space facilities

Other properties of a rod structure of the Thomson-Tait pendulum type are reviewed in paper [30].

5. Discussion of results of geometrical modeling of the oscillations of rod structures in weightlessness under the influence of pulses on the end points of links

The research undertaken here addressed the shape-formation of multilink rod structures in weightlessness. The coupled rods, upon delivery into orbit in a folded state, must acquire the planned spatial shape by using a mechanical operation of unfolding. In present study, the process of unfolding a multilink rod structure in weightlessness is similar to the process of oscillation of a multilink pendulum in the field of earthly gravitation. Their mathematical models may differ, in fact, by the values of constant g (free fall acceleration): $g\approx 0$ for the first case, and $g\approx 9.81$ for the second case.

Therefore, by applying the developed algorithm for the simulation of oscillation of a multilink pendulum in the field of gravity (at $g\approx 9.81$), and having checked the test results obtained for adequacy, one can employ this algorithm for calculating the unfolding of rod structures in weightlessness (at $g\approx 0$). Moreover, the adequacy of oscillations can be confirmed by a human eye analyzer when analyzing a computer model of the oscillation process.

This work investigated the issue of adaptation to weightlessness of the process of a geometrical model of unfolding a multilink rod structure. In this case, important is the notion of oscillation initialization. It has a physical analogy in the form of an impact (click) to the nodal element of a multilink rod structure. At the formal level, it is proposed to implement it using the pulses that act on the end points of links in a rod structure. Practical implementation of this approach could be achieved through a series of pulse pyrotechnic jet engines whose action is directed along a normal to the respective links in a rod structure at the end points. Importantly, following the action of pulses to the nodal elements of a rod structure, a system of unfolding is considered to be conservative (for small periods of time). That is why the magnitude of kinetic energy is assumed to be constant. Following the action of pulses to the end points of links, the unfolding of a rod structure should occur by inertia. Therefore, we use the term – inertial technique for unfolding multilink rod structures.

The results obtained can be explained by the possibility of applying a variational Lagrange principle to the calculation of mechanical structures with respect to the patterns described above. This allowed us to use the Lagrange equations of second kind to describe the motion of a pendulum system in weightlessness, regardless of the type of the rod structure.

We have obtained geometrical models of successive phases of the unfolding of rod «pendulum-like» structures when the engines of this process are pulse pyrotechnic jet engines installed at the end points of links in a rod structure. Such engines are much lighter and cheaper as compared, for example, with electric motors or string devices. This is especially important when the unfolding of a structure is performed only once, which is often the requirement. We analyzed manifestations of possible errors in the magnitudes of pulses on the geometrical shape of the arrangement of links in a rod structure, acquired as a result of its unfolding. We have also investigated the issue of braking the elements of a multilink structure in the pre-calculated unfolded state using a special «stop-code». Development of an illustrative geometrical model of the inertial unfolding of a multilink rod structure explains the use of conventional units for parameters in the test examples.

The not-yet-realized possibilities of research into the motion of a pendulum system in weightlessness include a time-dependent accounting for a variable mass of loads on the nodal elements of pendulum structures. Development of this area of research would imply the application of other variants of multilink rod structures that execute movements not in parallel planes only (unfolding schemes for 3D-structures).

The results obtained could help find the ways to solve the inverse problem of assembling a multilink rod structure. Specifically, based on the preset arrangement of elements of a given structure, it is necessary to define initial conditions for its motion (that is, the arrangement of links and the magnitudes of pulses from jet engines). In a certain sense, this is the ultimate goal of such a research.

The difficulties related to the development of studies into this area include the construction of an adequate mathematical model and its formalization in the form of Lagrange equations of second kind. Important is the technique to solve these equations using computational equipment with the obligatory visualization of obtained results.

Thus, the results of present work could be used when designing the unfolding of large-size structures under conditions of weightlessness, for example, frames for space solar mirrors or antennas. The study conducted lays the basis for calculating spatial 3D-multilink rod structures in which links, in the process of unfolding, would reach beyond the limits of one plane.

Summing up, it should be noted that the result of the research conducted is the constructed idealized geometrical models of the successive phases of unfolding the multilink rod structures when the engines of this process are the pulse jet engines mounted at the end points of links in a rod structure.

6. Conclusions

1. The obtained solutions to a system of the Lagrange differential equations of second kind allowed us to describe motion in weightlessness of the following varieties of multilink rod structures: double, four-link, Magdeburg, and Thomson-Tait. This made it possible to specify geometrical models of the unfolding of rod structures and to observe them in the mode of computer animation.

2. To simulate the action of a pulse pyrotechnic jet engine, we developed a scheme for the initiation of oscillations through the influence of pulses to the end points of links in a rod structure, allowing the simulation of the dynamics of unfolding a particular multilink rod structure. This made it possible to demonstrate, in the form of geometrical models, the implementation of pulse jet engines as possible engines for the process of unfolding a multilink rod structure of the pendulum type.

3. We have investigated at the graphical level the magnitude of a possible error in the value of pulse for initiating the oscillations of a rod structure in order to obtain a proper arrangement of its links. The technique is based on the aligned images showing three variants of unfolding with close values of the initial conditions, acquired in the mode of computer animation. This allowed us to assert that those errors in the magnitudes of pulses would be permissible, which did not exceed the percentage of the calculated value.

4. By applying computer animation, we predicted time-dependent mutual arrangement of links in rod structures, obtained as a result of inertial unfolding of respective links using jet engines. The specified possibility to determine the current values of angles between links made it possible to

form a numeric «stop-code» for fixing the mutual position of links in the structure. The obtained phase trajectories of the process of unfolding allow us to estimate speed of the elements of structures at the moment of braking the unfolding.

5. We have determined parameters of two-link rod structures in order to derive cyclic trajectories of their end points, which are characterized by special points of self-return. This allowed us to consider the possibilities of their application in the space-based machinery. For example, as the controlling elements for making stitches when constructing metal woven wire cloth in orbit for large-size antennas.

6. The test examples are given for the unfolding of variants of particular rod structures as multilink power frames in weightlessness: double, four-link, Magdeburg, and Thomson-Tait. The examples confirm the possibility of geometrical modeling of objects in weightlessness based on the unfolding of multilink rod structures when the engines of the process are the pulse jet engines installed at the end points of links in a rod structure.

References

1. Alpatov A. P., Gorbulin V. P. Space platforms for orbital industrial complexes: problems and prospects // News of the National Academy of Sciences of Ukraine. 2013. Issue 12. P. 26–38.
2. Large reflecting surfaces in space. Antennas by the communication satellite / Alpatov A. P., Belonozhko P. A., Belonozhko P. P., Vitushkin A. A., Fokov A. A. // System technologies. 2007. Issue 3 (50). P. 73–87.
3. Large reflecting surfaces in space. Radotelescopes, solar concentrators, flat reflectors / Alpatov A. P., Belonozhko P. A., Belonozhko P. P., Vitushkin A. A. Fokov A. A. // System Technology. 2007. Issue 3 (50). P. 88–101.
4. Robert H. SpiderFab. Architecture for On.Orbit Manufacture of Large Aperture Space Systems. FISO Briefing, 2015. 33 p.
5. Alpatov A. P. Dynamics of perspective space vehicles // Visnik NAN Ukraine. 2013. Issue 7. P. 6–13.
6. Udwadia F. E., Koganti P. B. Dynamics and control of a multi-body planar pendulum // Nonlinear Dynamics. 2015. Vol. 81, Issue 1-2. P. 845–866. doi: 10.1007/s11071-015-2034-0
7. Lopes A. M., Tenreiro Machado J. A. Dynamics of the N-link pendulum: a fractional perspective // International Journal of Control. 2016. Vol. 90, Issue 6. P. 1192–1200. doi: 10.1080/00207179.2015.1126677
8. Fritzkowski P., Kaminski H. Dynamics of a rope as a rigid multibody system // Journal of Mechanics of Materials and Structures. 2008. Vol. 3, Issue 6. P. 1059–1075. doi: 10.2140/jomms.2008.3.1059
9. Szuminski W. Dynamics of multiple pendula without gravity // Chaotic Modeling and Simulation. 2014. Issue 1. P. 57–67. URL: http://www.cmsim.eu/papers_pdf/january_2014_papers/7_CMSIM_Journal_2014_Szuminski_1_57-67.pdf
10. Modeling of the dynamics of the opening of the truss frame of the transformed reflector of the space-based antenna by the finite element method / Gutovsky I. E., Zolin A. V., Kurkov S. V., Panteleev V. A., Khlebnikov V. A. // Modern machine building. Science and education. 2012. Issue 2. P. 276–285.
11. Simulation of the process of solar battery opening / Bakulin D. V., Borzykh S. V., Ososov N. S., Shchiblev Yu. N. // Matem. Modeling. 2004. Vol. 16, Issue 6. P. 88–92.
12. Anokhin N. V. The reduction of a pendulum pendulum to a position of equilibrium by means of a single control moment // Izv. RAN. Theory and control systems. 2013. Issue 5. P. 44–53.
13. Deployable Perimeter Truss with Blade Reel Deployment Mechanism. URL: https://www.techbriefs.com/component/content/article/tb/techbriefs/mechanics-and-machinery/24098
14. Bushuev A. Yu., Farafonov B. A. Mathematical modeling of the process of disclosure of a large-scale solar battery // Mathematical Modeling and Numerical Methods. 2014. Issue 2. P. 101–114.
15. Schessnyak S., Romanov A. Designing and calculating large-scale unfolding structures using software packages MSC.Software // CADmaster. 2009. Issue 2-3. P. 28–36.
16. Boykov V. G. Program complex of automated dynamic analysis of EULER multicomponent mechanical systems // CAD and graphics. 2009. Issue 9. P. 17–20.
17. Peculiarities of calculating the opening of large-sized transformable structures of various configurations / Zimin V. N., Krylov A. V., Meshkovsky V. E., Sdobnikov A. N., Faizullin F. R., Churilin S. A. // Science and Education. MGTU im. N.E. Bauman. 2014. Issue 10. P. 179–191.
18. Martinez-Alfaro H. Obtaining the dynamic equations, their simulation, and animation for N pendulums using Maple. URL: http://www2.esm.vt.edu/~anayfeh/conf10/Abstracts/martinez-alfaro.pdf
19. Kinematic analysis of the deployable truss structures for space applications / Yan X., Fu-ling G., Yao Z., Mengliang Z. // Journal of Aerospace Technology and Management. 2012. Vol. 4, Issue 4. P. 453–462. doi: 10.5028/jatm.2012.04044112
20. Hoyt R., Cushing J., Slostad J. SpiderFab: Process for On-Orbit Construction of KilometerScale Apertures. NASA Goddard Space Flight Center 8800 Greenbelt Road Greenbelt, MD 20771, 2013. 53 p.
21. Geometrical modeling of the inertial unfolding of a multi-link pendulum in weightlessness / Kutsenko L., Shoman O., Semkiv O., Zapolsky L., Adashevskay I., Danylenko V. et. al. // Eastern-European Journal of Enterprise Technologies. 2017. Vol. 6, Issue 7 (90). P. 42–50. doi: 10.15587/1729-4061.2017.114269

22. Kutsenko L. M. Illustrations for geometric modeling of inertial disclosure of a multi-faceted pendulum in weightlessness // 2017. URL: http://repositsc.nuczu.edu.ua/handle/123456789/4868
23. Geometrical modeling of the process of weaving a wire cloth in weightlessness using the inertial unfolding of a dual pendulum / Kutsenko L., Semkiv O., Zapolskiy L., Shoman O., Kalynovskyi A., Piksasov M. et. al. // Eastern-European Journal of Enterprise Technologies. 2018. Vol. 1, Issue 7 (91). P. 37–46. doi: 10.15587/1729-4061.2018.121022
24. Kutsenko L. M. Heometrychne modeliuvannia pletinnia sitkopolotna v nevahomosti za dopomohoiu inertsiynoho rozkryttia podviinoho maiatnyka // URL: http://repositsc.nuczu.edu.ua/handle/123456789/5143
25. Kutsenko L. M. Iliustratsiyi do statti heometrychne modeliuvannia protsesu rozkryttia sterzhnevykh konstruktsiyi u nevahomosti // URL: http://repositsc.nuczu.edu.ua/handle/123456789/6335
26. An umbrella-shaped deployable mechanism constructed by six Myard linkages a Two 5R Myard linkages, b 2-Myard mechanism by sharing one common sub-chain. URL: https://www.researchgate.net/figure/An-umbrella-shaped-deployable-mechanism-constructed-by-six-Myard-linkages-a-Two-5R-Myard_271570634
27. Self deployable truss. URL: https://www.youtube.com/watch?v=sH7NHZwPzMM
28. Wang Yaping shows pendulum motion in space. URL: https://www.youtube.com/watch?v=dqcVONfly8U
29. Gladkov S. V. Computer simulation of oscillations of the «Chaotic pendulum». URL: http://old.exponenta.ru/educat/referat/student8/index.asp
30. Ter Haar D. Fundamentals of Hamiltonian mechanics. Moscow: Nauka, 1974. 224 p.
31. Pyrotechnics Test Facility. URL: https://www.nasa.gov/centers/johnson/engineering/human_space_vehicle_systems/energy_systems_test_area/pyrotechnics/index.html
32. Prospects for the application of spatial structures from plastics in space technology. URL: http://stroi-archive.ru/polimery-v-stroitelstve/706-perspektivy-primeneniya-prostranstvennyh-konstrukciy-iz-plastmass-v-kosmicheskoy-tehnike.html