Запропоновано метод для класифікації процесів і об'єктів в транспортно-логістичних системах. Шляхом багатофакторного аналізу і використання системного методу аналізу ієрархій формується комплексний аналітичний показник. На основі логістичного аналізу здійснюється класифікація за ступенем впливу чинників. Проведено класифікацію країн-партнерів України у зовнішній торгівлі на основі аналізу товарообігу окремих груп вантажів за конкретними напрямами

Ключові слова: класифікація транспортно-логістичних процесів, багатофакторний аналіз, вантажопотоки, митна статистика, логістичний аналіз

Предложен метод для классификации процессов и объектов в транспортно-логистических системах. Путем многофакторного анализа и использования системного метода анализа иерархий формируется комплексный аналитический показатель. На основе логистического анализа осуществляется классификация по степени влияния факторов. Проведена классификация стран-партнеров Украины во внешней торговле на основе анализа товарооборота отдельных групп грузов по конкретным направлениям

Ключевые слова: классификация транспортно-логистических процессов, многофакторный анализ, грузопотоки, таможенная статистика, логистический анализ

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DEVELOPMENT OF METHOD OF MULTIFACTOR CLASSIFICATION OF TRANSPORT AND LOGISTIC PROCESSES

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1. Introduction

The European Union expansion has led to significant changes in the political and economic conditions for the further broadening of Ukraine's cooperation with the EU countries and created favorable conditions for gradual integration into the world economic system. The international transport corridors that pass through the territory of Ukraine, the national transport system as one of leading sectors of the national economy make a considerable contribution to the success of these processes [1, 2]. The plans for development of relations between the EU and Ukraine have enabled transition from cooperation to gradual economic integration and deepening of political cooperation. Integration of Ukraine's transport system into European structures becomes of particular relevance and implies further improvement of transport technologies and coordination with production, trade, warehouse and customs technologies [3].

Effective functioning of the transport sector is a necessary condition for stabilization, structural transformation of the national economy, development of foreign economic activities, satisfaction of the needs of population and social production in transportation and protection of Ukrainian economic interests. The main functions of the transport system invariably include ensuring unity of national goods markets, interconnection of regions, mobility of citizens, meeting the transportation needs of foreign trade [3].

A number of important tasks arise in the field of integration of the domestic transport system into European and international transport systems. It is necessary to reasonably reduce time of control procedures of checkpoints at the state borders and harmonize the checkpoint procedures with European norms. Also, the system of using customs statistics for monitoring and forecasting freight flows, etc. shall be improved [3].

For an effective solution of topical issues of the transport industry, it is necessary to apply complex methods of multifactor analysis of the problems of transport and logistics systems on the basis of systems approach and mathematical modeling.

2. Literature review and problem statement

Analysis of the main lines of the Ukrainian transport strategy indicates a number of problems related to the technical condition, level of technological and informational support of the sector, customer service, use of geopolitical advantages of the country, etc. Effective measures should be taken to ensure economic and transport and technological security of foreign economic activity to increase efficiency of international transportation. This will allow Ukraine to successfully integrate into European economic space and grow volumes of export-import cargo traffic [4].

For successful development of the Ukrainian transport system, it is necessary to apply logistic approaches in solving the sector problems. Multimodal, intermodal and combined transportation systems represent modern logistics concepts aimed at improvement of the package of services provided in the process of cargo movement from supplier to consumer. The active use of new concepts in logistics will improve the overall efficiency of the transport and logistics processes. Development of current concepts of traffic management in transport and logistics systems is considered in [5–10]. However, despite a significant number of scientific works devoted to studying the problems of improving the transportation process, elucidation of this problematics cannot be considered finally completed.

With the advent and active development of information technologies and computer systems, development of scientific approaches to modeling the processes of management of transportation and logistics systems is based on a multifactor analysis of technical, technological and logistics indicators.

Modeling of Ukrainian foreign economic activity to determine main routes of international cargo traffic and ensure further development based on the factor analysis of cargo flows is presented in [11]. The main factors influencing formation of the indicator of efficiency of container traffic were determined in [6]. However, it is necessary to develop a systems approach to the analysis of problems of transport and logistics systems [12]. Theoretical and methodological fundamentals of factor analysis of competitiveness of international motor freight are generalized and scientists' views concerning classification of the factors influencing development of international motor freight are analyzed in [13]. Improvement of methodological bases for assessing the state of financial and economic security of haulers taking into account the sector specifics on the basis of its modeling by factor analysis was set forth in [14].

In spite of the examples of successful application of the factor analysis methods in solving concrete problems in the study of complex multifactor systems, one should note the fundamental difficulty arising in application of this analysis. With the use of a multidimensional correlation matrix taking into account statistical relationships between factors, the factor analysis makes it possible to reduce the problem with a large number of influencing factors to analysis of simpler models with fewer factors which are linear combinations of source factors. The problem consists in a correct and adequate identification of the obtained synthesized factors. It cannot be solved successfully each time. Besides, correct application of the methods of factor analysis in the given problem is practically impossible for the following reason. The peculiarity of this task is that the statistical material used for analysis is a set of deterministic costs of the factors of the transport and logistic process which excludes the possibility of correct construction of a correlation matrix. Data accumulated for this purpose in several years do not save the situation because the fundamental principle of the probability theory, that is integrity and invariability of the mechanism and conditions for formation of the observed quantities is violated under conditions of the present-day market economy.

Other approaches to investigation of road freight and other multifactor processes and systems are known, in particular, the hierarchy analysis method. Application of systems analysis based on the hierarchy analysis method in the theory of decision-making, application of the hierarchy analysis method for solution of multifactor problems is given in the papers [5, 15–17]. The method well-known in the decision-making theory is very effective in making decisions under conditions of a large number of influential factors. Also, this method of systems analysis can be successfully applied in cases where a decision maker is forced to turn to the expert opinion because of lack of competence or for other reasons [15]. The procedure for determining effective means of cargo transportation of any type that meet requisite requirements using the hierarchy analysis method and paired comparisons, is proposed in [16]. A complex analytical indicator characterizing dynamics of international goods turnover for certain groups of goods was formed in [5] using the hierarchy analysis method based on the results of factor analysis of statistical data. The possibility of using the hierarchy analysis method and its simplified version for solving multifactor problems is proposed in [17]. However, this approach does not take into account modern methods of logistic analysis.

A wide range of works are devoted to theoretical aspects of the problem of classification, in particular, objects of transport and logistics systems. Classification methods enable division of the set of objects or observations into *a priori* groups, so-called classes. Inside each group, objects are considered similar to each other and have approximately the same properties and attributes. In this case, solution is based on the analysis of the costs of attributes.

Classification is one of the most important among the problems of data mining and covers a variety of applications including logistics, marketing, image recognition, medical diagnostics, and more. The classification methods are used in transportation and logistics systems when assessing the state of infrastructure objects, transport and logistics services, in analysis of cargo flows, etc.

Theoretical foundations and classification algorithms are considered in [18–20]. The use of information technology and intelligent models in the problems of classification in various fields of people's activity is described in [21–24]. Analysis of neural network methods for solving the classification problem is presented in [25–29]. Mathematical models of the classification problem presented in the form of discrete, non-smooth and multi-extreme optimization problems are considered in [18] for solving individual problems of recognition and analysis of data.

Various models of the algorithms intended for solving classification problems according to categorial features are considered in [19]. Both the simplest generalizations of classical algorithms and fundamentally new generalizations are considered: coding for the use of random forests and generalization of proximity-based algorithms.

Structure of the Statistica application package, methods of cluster, factor and discriminant analyses, implementation of the described methods of multifactor statistical analysis in this package are given in [20].

Various classifications of reverse flows of goods by spheres of circulation, production and consumption are given in [21]. Classification of conditions of reliability of the railroad tracks by allowable deviations during laying and maintenance of the track was made in [22]. Necessity of normalizing technical conditions of the ballast cross section and the ballast bed for the developed classification was established. The method of classification of railway sections and directions of operational activity for the formation of a system of estimation of capabilities and efficiency improvement in management of distribution of the railway infrastructure capacity is proposed in [23].

To solve the classification problem, many neural network architecture classifiers were created. They are widely used at present in various fields. Analysis of neural network methods for solving the classification problem is presented in [25–29] and clustering algorithms in data mining are given in [30]. The essential shortcomings of the classification methods are as follows. First, in classification problems, as a rule, the number of classes is predefined and the boundary costs of the attributes for each particular class are determined. Second, with an increase in the number of factors considered, the initial set of classification objects is divided into an increasing number of classes with a low level of representation of objects in each of them which naturally reduces the level of trust to the study results.

At the same time, a more substantiated theoretically approach is associated with the use of a multidimensional discriminant analysis.

The task of the multidimensional discriminant analysis in classifying objects in the simplest two-dimensional special case consists in assigning each particular object, w, to one of two sets, W_1 or W_2. based on the observation p of the controlled parameters, $x_1, x_2, \ldots, x_p$. The standard classification procedure consists in the following [31, 32]. It is assumed that the vector of observations, X, has a normal distribution with parameters (μ_1, Σ_1) if it belongs to the set W_1 and has a normal distribution with (μ_2, Σ_2) parameters if this vector belongs to the set W_2. Here, the vectors μ_1 and μ_2 also set the mathematical expectations of the components X under given assumptions, and the matrices Σ_1 and Σ_2 are variance matrices, moreover $\Sigma_1=\Sigma_2$.

Next, the so-called discriminant function is introduced

$$z = \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_p x_p. \tag{1}$$

The object W is attributed to W_1 if

$$z > C, \tag{2}$$

and is attributed to W_2 if otherwise (C is a certain constant).

In this case, parameters α_j, $j = 1, 2, \ldots, p$, and C are sought in the following way.

If observation $X \in W_1$, then z belongs to the normal distribution $N(\zeta_1, \sigma_z^2)$ and if, however $X \in W_2$, then z belongs to the distribution $N(\zeta_2, \sigma_z^2)$ where

$$\zeta_1 = \sum_{j=1}^{p} \alpha_j \mu_{1j}, \quad \zeta_2 = \sum_{j=1}^{p} \alpha_j \mu_{2j}, \quad \sigma_z^2 = \sum_{i=1}^{p} \sum_{j=1}^{p} \alpha_i \alpha_j \sigma_{ij}^2.$$

The set α_j, $j = 1, 2, \ldots, p$ is chosen so that the distance of Mahalanobis [32] between W_1 and W_2 calculated from the formula

$$\Delta^2 = \frac{(\zeta_1 - \zeta_2)^2}{\sigma_z^2}$$

was maximal. It was shown in [33] that the sought set is determined by solution of the system of linear equations.

$$\begin{aligned} &\alpha_1\sigma_{11} + \alpha_2\sigma_{12} + \ldots + \alpha_p\sigma_{1p} = \mu_{11} - \mu_{21}, \\ &\alpha_1\sigma_{21} + \alpha_2\sigma_{22} + \ldots + \alpha_p\sigma_{2p} = \mu_{12} - \mu_{22}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\alpha_1\sigma_{p1} + \alpha_2\sigma_{p2} + \ldots + \alpha_p\sigma_{pp} = \mu_1 p - \mu_2 p, \end{aligned} \tag{3}$$

and the constant C is calculated by formula

$$C = \frac{1}{2}(\zeta_1 + \zeta_2). \tag{4}$$

Next, the following are entered:

$$P\left(H_1 / X \in W_2 \right)$$

is the probability that according to the results of calculations using formulas (1), (2), a hypothesis H_1 is accepted about belonging of X to the set W_1, when in fact the object belongs to W_2,

$$P\left(H_2 / X \in W_1 \right)$$

is the probability that according to the results of calculations using formulas (1), (2), hypothesis H_2 is accepted about belonging of X to the set W_2 when in fact the object belongs to W_1.

The choice of C in accordance with (4) provides the minimum cost of the total probability of mix-up.

The difficulties in implementing the standard technology described herein are determined by the following circumstances. The practically insufficient number of observations leads to unpredictable errors in sample estimates of the mean costs and variances of controlled parameters that may be unpredictably large. In addition, the very hypothesis about normality of the random costs of these parameters cannot be reasonably accepted or rejected. Finally, the use of hyperplanes (1) as discriminating functions leads to errors in the classification of objects. The best results are ensured by the use of cluster analysis.

Cluster analysis occupies one of the central places among the data analysis methods and is a collection of methods and algorithms intended to find a certain partition of the investigated set of objects into subsets of objects similar to each other. At the same time, the following requirements are usually presented to the results of clustering:

– each cluster must contain objects with close of properties or attributes;

– the set of all clusters must be exhaustive, that is contain all objects of the set under study;

– no object from their set must not belong simultaneously to different clusters.

To solve the problem of clustering, many methods have been developed, the one-type concept of construction of which is displayed in the following way [34].

Let there be some set of objects that should be distributed among m clusters. At the same time, each object is characterized by a set of parameters $F=(F_1, F_2, \ldots, F_l)$ with their numerical costs for each object have been measured. Then, in the I-dimensional space of parameters, a point will correspond to each object and if a certain metric is given, then for each pair (j_1, j_2) of objects the distance between them can be calculated, for example, by formula

$$r_{j_1, j_2} = \left[\sum_{k=1}^{l} \left| F_{kj_1} - F_{kj_2} \right|^p \right]^{\frac{1}{p}}, \quad p \geq 1.$$

In many cases, coordinates of the objects are not entered, and immediately the matrix $R=(r_{j_1,j_2})$ of distances is set. At the same time, the problem of clustering is formulated in the following way. Enter the indicator

$$x_{ij} = \begin{cases} 1, & \text{if the j-th object was} \\ & \text{placed to the i-th cluster,} \\ 0, & \text{if otherwise.} \end{cases}$$

Let the number of clusters, m, be given. Then the formal statement of the problem of clustering takes the form: to find a set $X=(x_{ij})$ by maximizing

$$L(X) = \frac{\dfrac{\sum_{i_1=1}^{m} \sum_{i_2 \neq i_1} \sum_{j_1=1}^{n} \sum_{j_2=1}^{n} r_{j_1 j_2} x_{i_1 j_1} x_{i_2 j_2}}{\sum_{i_1=1}^{m} \sum_{i_2 \neq i_1} \sum_{j_1=1}^{n} \sum_{j_2=1}^{n} x_{i_1 j_1} x_{i_2 j_2}}}{\max\limits_{i} \left\{ \dfrac{\sum_{j_1=1}^{n} \sum_{j_2=1}^{n} r_{j_1 j_2} x_{ij_1} x_{ij_2}}{\sum_{j_1=1}^{n} \sum_{j_2=1}^{n} x_{ij_1} x_{ij_2}} \right\}}. \tag{5}$$

and satisfying the constraints

$$\sum_{i=1}^{m} x_{ij} = 1, \quad j = 1, 2, \ldots, n, \tag{6}$$

$$\sum_{j=1}^{n} x_{ij} \geq 1, \quad i = 1, 2, \ldots, m. \tag{7}$$

The meaning of the optimized criterion is understandable. The numerator (5) has the mean distance between the clusters; it is desirable to maximize it. The denominator (5) represents the mean distance between the objects of the most noncompact, "loose" cluster; it is desirable to minimize it. Constraints (6), (7) determine allowable distribution of the objects among clusters. The resulting problem is a fractional-quadratic Boolean problem of mathematical programming. Exact solution of this problem is possible by exhaustive search which is not realistic in the problems of practical dimensionality. In connection with this, to solve the problem, a number of approximate heuristic algorithms were developed which are used to ensure rapid and qualitative division of objects among clusters. Thus, the problem of clusterization in the formulation (5)–(7) does not cause fundamental difficulties.

Solving the clustering problems at a microlevel [35–38] and forming clusters at a macrolevel [39–41] of the enterprise contributes to the growth of efficiency of the supply processes.

In particular, this method is used in work [35] for analysis of Ukrainian industry sectors by sources of innovation financing, in the study of development of metallurgical enterprises on the basis of production, financial-economic and logistic indicators [36]. Efficiency of clustering transport and logistics enterprises was substantiated in [37]. In the above-mentioned works, clustering was carried out at a micro-level of enterprises.

Clustering of urban areas taking into account logistics characteristics was carried out in [38]. The results of analysis of regional motor transport network by the hierarchical clustering method are given in [39]. An analysis of preconditions for formation of near-border transport and logistics clusters for improvement of international rail freight traffic are presented in [40].

Comparison at the macrolevel on the basis of statistical data of international service trade was made in [41]. The paper estimates competitive position of Turkey among the 148 member states of the World Trade Organization based on cluster analysis.

An important disadvantage of the object grouping technologies using cluster analysis is the lack of informativeness of the results obtained in clustering. The belonging of an object to a cluster is weakly related to the real costs of its characteristics. They can vary significantly depending on whether this object is located in the center of the cluster or near its boundaries.

Another approach to classification of objects is significantly more informative. It provides search for an analytical relationship connecting the numerical costs of the sets of factors that determine main characteristics of the object with the cost of the resulting system indicator chosen in some way. For many reasons, such a correlation, usually called the response function, is conveniently chosen in the form of so-called Kolmogorov-Gabor polynomial [42]:

$$\begin{aligned} y(X) &= a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_n x_n + \\ &+ a_{12} x_1 x_2 + \ldots + a_{n-1,n} x_{n-1} x_n, \end{aligned} \tag{8}$$

where x_j is cost of the j-th factor, j=1, 2,..., n; a_j is weight factor, importance of the j-th indicator, j=1, 2,..., n, y is the resultant indicator.

Here, the maximum accountable degree of interaction of the factors is equal to two.

If the results of N experiments are used to estimate the polynomial (8) parameters, then the best vector in the sense of least squares is $A^T=(a_0\ a_1\ a_2 \ldots a_n\ a_{12} \ldots a_{n-1,n})$ vector determined by formula

$$\mathrm{A} = \left(\mathrm{H^T H}\right)^{-1} \mathrm{H^T Y}, \tag{9}$$

$$\mathrm{H} = \begin{pmatrix} 1 & x_{11} & x_{12} & \ldots & x_{1n} & x_{11}x_{12} & \ldots & x_{1,n-1}x_{1n} \\ 1 & x_{21} & x_{22} & \ldots & x_{2n} & x_{21}x_{22} & \ldots & x_{2,n-1}x_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & x_{N1} & x_{N2} & \ldots & x_{Nn} & x_{N1}x_{N2} & \ldots & x_{N,n-1}x_{Nn} \end{pmatrix},$$

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_N \end{pmatrix}.$$

Here x_{ij} is the cost of the i-th factor in the i-th experiment; y_i is the cost of the resulting indicator in the i-th experiment.

The above conventional scheme is realized when there is a sufficient amount of initial experimental data. However, organization and conduction of an appropriate number of experiments are not feasible in many practical situations. Theoretical problems of an adequate estimation of parameters of the regression equation (8) require studies and solutions.

Such a wide range of studies shows the interest of scientists in the problems of transport and logistics systems and complexity of the problems. However, for a more accurate and objective reflection of processes, development of an integrated approach to presentation, analysis and evaluation of efficiency of transport and logistics systems requires further development.

For efficient management of international transport-logistics delivery systems, systems analysis shall be applied based on integration of various methods. It is necessary to combine statistical analysis of data on transport and freight flows to determine factor influence, apply modern methods of logistic analysis, etc. Improvement of the methods of solving classification problems will ensure obtaining of a more accurate solution of multifactor problems occurring in activities of transport and logistics companies and will enable formulation of appropriate strategies during planning and control of goods supply.

3. The aim and objectives of the study

This study objective was to formulate a method for classification of objects and processes in transport and logistics systems on the basis of multifactor analysis.

To achieve this objective, the following tasks had to be solved:

– to formalize description of the classification problem in solving transport and logistics problems and the theory of organization of cargo traffic on the basis of multifactor analysis;

– to propose a model and an algorithm for solving problems of classification of transport and logistics processes on the basis of factor and logistic analysis;

– to solve the problem of classification of Ukraine's partner countries in terms of international trade turnover in various directions.

4. Formation of the method and the model for solving multifactor problems of classification of the transport and logistics processes

Formalized description of the classification problem for the transport and logistics systems on the basis of multifactor analysis was conducted in several stages. At the first stage, a regression model of the system was formed based on the systemic hierarchy analysis method. At the second stage, an analytical indicator is formed that characterizes influence of factors on the objects under study. At the third stage, ideas of the method of logistic ABC analysis are used for classification of transport and logistics processes and objects by the degree of manifestation of the factor influence. Let us make a formal description of these stages.

4. 1. Formal description of the problem using a regression model

In the conditions of a small sample of initial data, it is difficult to realize the standard technology of the method of least squares leading to (9).

Alternative technologies for assessing importance of concrete indicators (characteristics) of the objects are based on processing of the results of expert judgements. When doing this, experts, along with independent estimation of relative importance of partial indicators rank them after which the sum of ranks determines final estimate of importance of each indicator. The obtained weight coefficients are used further for estimation of resultant indicators of the objects.

It is worth paying attention to the general design shortcoming of the conventional methods which is related to the insufficient adequacy of the procedure for calculating weight coefficients. Essentially, the weight coefficient calculated according to the above scheme is uniquely determined only by the place taken by a corresponding indicator in the table of ranks. However, actual importance of the two indicators that occupy adjacent positions in this table may differ much more significantly than is determined by their position. This shortcoming is of a general nature. In a similar situation, when direct estimation of weights of the parameters of the compared objects is difficult and the use of solely ranking can lead to an inaccurate choice. Recently, the method based on the paired preferences given some parameters over others is finding an ever-growing application. Expert estimation of such preferences is certainly a simpler task than the task of direct estimation of importance of parameters, and it is decided by experts much more confidently. The method proposed and substantiated in [43] was called the hierarchy analysis method and is implemented as follows [44–47].

Let each object of the set be characterized by n parameters. At the first stage, a matrix A of pairwise comparisons of significance (weight) of parameters is formed:

$$A = \begin{pmatrix} 1 & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & 1 & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & 1 \end{pmatrix},$$

where a_{ij} is the number determining the preference level of the i parameter over the j parameter,

$$a_{ii} = 1, \;\; a_{ji} = (a_{ij})^{-1}, \;\; i = 1,2,\dots,n; \;\; j = 1,2,\dots,n.$$

Further, the eigen cost problem is solved for this matrix. In this case, the characteristic equation is formed

$$\det\begin{pmatrix} 1-\lambda & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & 1-\lambda & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & 1-\lambda \end{pmatrix} = 0.$$

Its solution gives a set $\lambda_1, \lambda_2, \dots, \lambda_n$ of eigen quantities of the matrix A. After this, the normalized (that is, one for which the sum of components is equal to one) eigenvector W of this matrix corresponding to the maximum eigen quantity $\lambda_{\max} = \max(\lambda_1, \lambda_2, ..., \lambda_n)$ is found. The matrix A is positive in its construction. As is known, the maximal eigen cost and the corresponding eigenvector are positive as well for such matrices. Let an eigen vector $W = (w_1, w_2, ..., w_n)$ be obtained as a result of solution. This vector components have meaning of the weight coefficients which characterize the relative importance of the object parameters. Let now the compared objects (alternatives) be characterized by a set of parameters $(F_1, F_2, ..., F_n)$ and the vector $F_k = (F_{k1}, F_{k2}, ..., F_{kn})$ corresponding to the k-th object. Then the weighted average characteristic of the preference level of the k-th object is determined by formula

$$V_k = \sum_{j=1}^{n} W_j F_{kj}, \quad k = 1, 2, ..., m. \tag{10}$$

The correlation connecting the maximal eigenvector of the matrix A with a set of weight coefficients $\mathrm{W} = (\mathrm{w_j})$ can be easily obtained if the matrix A is consistent, that is,

$$a_{ik} = a_{ij} a_{jk}, \quad i = 1, 2, ..., n, \quad j = 1, 2, ..., n, \quad k = 1, 2, ..., n.$$

Sum up the left and right members of this equality by j. Wherein

$$\sum_{j=1}^{n} a_{ik} = \sum_{j=1}^{n} a_{ij} a_{jk}, \quad i = \overline{1,n}, \quad k = \overline{1,n},$$

$$n \cdot a_{ik} = \sum_{j=1}^{n} a_{ij} a_{jk}$$

and

$$a_{ik} = \frac{1}{n} \sum_{j=1}^{n} a_{ij} a_{jk}, \quad i = \overline{1,n}, \quad k = \overline{1,n}. \tag{11}$$

The matrix analogue of this correlation takes the form:

$$\frac{1}{n}\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{pmatrix} =$$

$$= \frac{1}{n}\begin{pmatrix} \sum_{k=1}^{n} a_{1k}a_{k1} & \sum_{k=1}^{n} a_{1k}a_{k2} & \cdots & \sum_{k=1}^{n} a_{1k}a_{kn} \\ \sum_{k=1}^{n} a_{2k}a_{k1} & \sum_{k=1}^{n} a_{2k}a_{k2} & \cdots & \sum_{k=1}^{n} a_{2k}a_{kn} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{k=1}^{n} a_{ik}a_{k1} & \sum_{k=1}^{n} a_{ik}a_{k2} & \cdots & \sum_{k=1}^{n} a_{ik}a_{kn} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{k=1}^{n} a_{nk}a_{k1} & \sum_{k=1}^{n} a_{nk}a_{k2} & \cdots & \sum_{k=1}^{n} a_{nk}a_{kn} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix},$$

hence, it follows

$$\frac{1}{n} AA = A. \tag{12}$$

Suppose that the weight coefficients $w_1, w_2, \ldots, w_n$ defining significance (importance, value) of the parameters are known. Then it is expedient to estimate significance of the i-th parameter in comparison with the j-th parameter by formula

$$a_{ij} = \frac{w_i}{w_j}, \quad \mathrm{i}, j = \overline{1,n}. \tag{13}$$

Naturally, in this case,

$$a_{ij} a_{jk} = \frac{w_i}{w_j} \frac{w_j}{w_k} = \frac{w_i}{w_k} = a_{ik}, \quad a_{ji} = \frac{w_j}{w_i} = \frac{1}{\frac{w_i}{w_j}} = \frac{1}{a_{ij}}.$$

It follows from (13) that

$$a_{ij} \frac{w_j}{w_i} = 1, \quad \mathrm{i}, j = \overline{1,n}$$

and, consequently,

$$\sum_{j=1}^{n} a_{ij} w_j \frac{1}{w_i} = \frac{1}{w_i} \sum_{j=1}^{n} a_{ij} w_j = n, \quad \mathrm{i} = \overline{1,n}, \quad \sum_{j=1}^{n} a_{ij} w_j = n w_i,$$

which corresponds to the matrix equation $Aw = nw$.

It follows that for an reverse-symmetric positive consistent matrix, A, there is an eigen cost equal to n and a positive, corresponding to this number eigenvector, w, with its components being weights of the elements. Thus, the resulting correlation establishes a connection between the matrix of pairwise comparison of parameter significance and the set of weight coefficients. Thus, if a matrix A is given, then an unknown vector w can be obtained by calculating this matrix eigenvector corresponding to an eigen quantity equal to n. At the same time, this vector w can be obtained in a simpler way [48].

In accordance with (13), the matrix A takes the form

$$A = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{pmatrix}.$$

Calculate the sums of the elements for each of the rows of the matrix A. For an arbitrary i-th line, the following is obtained:

$$\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} \frac{w_i}{w_j} = w_i \sum_{j=1}^{n} \frac{1}{w_j} = C w_i, \quad i = \overline{1,n}. \tag{14}$$

It follows from (14) that the eigenvector, w, can be calculated to a constant directly from the elements of the matrix A. Define the constant C, proceeding from the natural requirement to normalization of the vector, w, in accordance with which the following condition shall be met:

$$\sum_{i=1}^{n} w_i = 1. \tag{15}$$

Sum up the left and right members of correlation (14) by i. Taking into account (15), the following is obtained:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} = \sum_{i=1}^{n} Cw_i = C\sum_{i=1}^{n} w_i = C,$$

$$w_i = \frac{1}{C}\sum_{j=1}^{n} a_{ij} = \frac{\sum_{j=1}^{n} a_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}}, \quad i=\overline{1,n}. \tag{16}$$

It is easy to verify whether the vector

$$w^T = (w_1 \quad w_2 \quad \ldots \quad w_n)$$

obtained in accordance with (16) is the matrix A eigenvector corresponding to an eigen quantity equal to n. Actually, calculate

$$Aw = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \ldots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \ldots & \frac{w_2}{w_n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \ldots & \frac{w_n}{w_n} \end{pmatrix} \cdot \begin{pmatrix} \sum_{j=1}^{n}\frac{w_1}{w_j} \\ \sum_{j=1}^{n}\frac{w_2}{w_j} \\ \ldots \\ \sum_{j=1}^{n}\frac{w_n}{w_j} \end{pmatrix} \cdot \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{w_i}{w_j}} =$$

$$= \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{w_i}{w_j}} \cdot \begin{pmatrix} \sum_{j=1}^{n}\left(\frac{w_1}{w_j}\sum_{i=1}^{n}\frac{w_i}{w_j}\right) \\ \sum_{j=1}^{n}\left(\frac{w_2}{w_j}\sum_{i=1}^{n}\frac{w_i}{w_j}\right) \\ \ldots \\ \sum_{j=1}^{n}\left(\frac{w_n}{w_j}\sum_{i=1}^{n}\frac{w_i}{w_j}\right) \end{pmatrix} =$$

$$= \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{w_i}{w_j}} \cdot \begin{pmatrix} \frac{w_1}{w_1}\cdot w_1\sum_{j=1}^{n}\frac{1}{w_j} + \frac{w_1}{w_2}\cdot w_2\sum_{j=1}^{n}\frac{1}{w_j} + \ldots + \frac{w_1}{w_n}\cdot w_n\sum_{j=1}^{n}\frac{1}{w_j} \\ \frac{w_2}{w_1}\cdot w_1\sum_{j=1}^{n}\frac{1}{w_j} + \frac{w_2}{w_2}\cdot w_2\sum_{j=1}^{n}\frac{1}{w_j} + \ldots + \frac{w_2}{w_n}\cdot w_n\sum_{j=1}^{n}\frac{1}{w_j} \\ \ldots\ldots\ldots \\ \frac{w_n}{w_1}\cdot w_1\sum_{j=1}^{n}\frac{1}{w_j} + \frac{w_n}{w_2}\cdot w_2\sum_{j=1}^{n}\frac{1}{w_j} + \ldots + \frac{w_n}{w_n}\cdot w_n\sum_{j=1}^{n}\frac{1}{w_j} \end{pmatrix} =$$

$$= \frac{n}{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{w_i}{w_j}} \cdot \begin{pmatrix} \sum_{j=1}^{n}\frac{w_1}{w_j} \\ \sum_{j=1}^{n}\frac{w_2}{w_j} \\ \ldots \\ \sum_{j=1}^{n}\frac{w_2}{w_j} \end{pmatrix} = nw,$$

as required.

It is clear that correlation (16) will allow us to make an accurate estimate of the weights of the parameters being compared only if the matrix A is consistent. However, in practice, a matrix A containing the results of pairwise comparisons of significance of the signs formed by experts is definitely not consistent. Therefore, the vector determined in accordance with (16) estimates the weight coefficients with an error the greater the more the real matrix A differs from the consistent matrix. The resulting problem can be solved in one of two ways. First, it is clear that the required transitive matrix of pairwise comparisons can be calculated if a set of results of comparison of importance of any indicator with respect to all the others is known. Suppose, for example, that a row $\left(a_{kj}\right)$ is given. Because

$$a_{kj} = \frac{w_k}{w_j},$$

then

$$\frac{a_{kj}}{a_{ki}} = \frac{w_k}{w_j}\bigg/\frac{w_k}{w_i} = \frac{w_k w_i}{w_j w_k} = \frac{w_i}{w_j} = a_{ij}.$$

Thus, the i-th element of the j-th row of the matrix A can be reconstructed through the elements of the k-th row. Introduce

$$\frac{a_{kj}}{a_{ki}} = \frac{\overline{a}_{kj} + \zeta_{kj}}{\overline{a}_{ki} + \zeta_{ki}},$$

$$\overline{a}_{kj} = \frac{1}{m}\sum_{s=1}^{m} a_{kj}^{(s)},$$

$$\zeta_{kj} = a_{kj} - \overline{a}_{ki},$$

where $a_{kj}^{(s)}$ is the result of comparison of importance of indicators k and j obtained by the s-th expert, $s=\overline{1,m}$.

Calculate

$$\frac{\overline{a}_{kj} + \zeta_{kj}}{\overline{a}_{ki} + \zeta_{ki}} - \left(\frac{\overline{a}_{kj}}{\overline{a}_{ki}} + \frac{\zeta_{kj}}{\zeta_{ki}}\right) = \frac{\overline{a}_{kj} + \zeta_{kj}}{\overline{a}_{ki} + \zeta_{ki}} - \frac{\overline{a}_{kj}\zeta_{ki} + \overline{a}_{ki}\zeta_{kj}}{\overline{a}_{ki}\zeta_{ki}} =$$

$$= \frac{-\overline{a}_{ki}\overline{a}_{ki}\zeta_{kj} - \overline{a}_{kj}\zeta_{kj}\zeta_{ki}}{\left(\overline{a}_{ki} + \zeta_{ki}\right)\left(\overline{a}_{ki}\zeta_{ki}\right)} =$$

$$= -\frac{\zeta_{kj}\left(\overline{a}_{ki}^{2} + \overline{a}_{kj}\zeta_{ki}\right)}{\zeta_{ki}\left(\overline{a}_{ki}^{2} + \overline{a}_{ki}\zeta_{ki}\right)} \approx -\frac{\zeta_{kj}}{\zeta_{ki}}. \tag{17}$$

The approximate equality obtained follows from

$$M\left[\frac{\overline{a}_{kj} + \zeta_{kj}}{\overline{a}_{ki} + \zeta_{ki}}\right] = \frac{\overline{a}_{kj}}{\overline{a}_{ki}}.$$

Thus, it turns out that uncertainty in estimation of the calculated element a_{ij} of the matrix A is determined by uncertainty of a_{kj} and a_{ki} estimates. Let the random deviations ζ_{kj} and ζ_{ki} of these estimates from the corresponding mean costs be distributed normally with a zero average and variances σ_j^2 and σ_i^2, respectively. Then, as is known, density of distribution of a random quantity

$$Z = \frac{\zeta_{kj}}{\zeta_{ki}}$$

is determined by the correlation

$$f(z)=\frac{\frac{\sigma_j}{\sigma_i}}{\pi\left[z^2+\left(\frac{\sigma_j}{\sigma_i}\right)^2\right]} \text{ (the Cauchy distribution law).}$$

For a random quantity distributed in accordance with the Cauchy law, it is impossible to determine estimates of the mathematical expectation and variance because of divergence of the integral

$$\int_{-\infty}^{\infty}\frac{|z|^{\alpha}\frac{\sigma_j}{\sigma_i}dz}{\pi\left[z^2+\left(\frac{\sigma_j}{\sigma_i}\right)^2\right]} \text{ for any } \alpha\geq 1.$$

In this connection, implementation of the described variant of reconstruction of the matrix of pairwise comparisons can lead to gross errors in calculating coefficients of the regression equation and obtaining of an inadequate study result in general.

The second approach to solving the problem consists in finding the consistent matrix, *X,* minimally in the sense of least squares differing from the given matrix, *A*.

Let

$A=(a_{ij}),\quad i=\overline{1,n}, j=\overline{1,n}$ is the initial matrix of pairwise comparisons.

$X=(x_{ij}),\quad i=\overline{1,n}, j=\overline{1,n}$ is the sought consistent matrix of pairwise comparisons.

Formally, the problem can be formulated as follows: find a matrix *X* that minimizes

$$F(x)=\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij}-a_{ij})^2$$

and satisfies constraints

$$\frac{1}{n}\sum_{k=1}^{n}x_{ik}x_{kj}=x_{ij},\quad i=\overline{1,n}, j=\overline{1,n}.$$

Solve this problem by the method of undetermined Lagrange multipliers. Form the Lagrange function:

$$\Phi(x,\lambda)=\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{ij}-a_{ij})^2+\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{ij}\left(\frac{1}{n}\sum_{k=1}^{n}x_{ik}x_{kj}-x_{ij}\right). \quad (18)$$

Take partial derivatives of the problem variables from (18) and equate them to zero to obtain a system of equations

$$\frac{\partial\Phi(x,\lambda)}{\partial x_{ij}}=2(x_{ij}-a_{ij})+$$
$$+\lambda_{ij}\left[\frac{1}{n}\left(\sum_{k=1}^{n}x_{ik}+\sum_{k=1}^{n}x_{kj}\right)-1\right]=0,\quad i=\overline{1,n}, j=\overline{1,n}, \quad (19)$$

$$\frac{\partial\Phi(x)}{\partial\lambda_{ij}}=\frac{1}{n}\sum_{k=1}^{n}x_{ik}x_{kj}-x_{ij}=0,\quad i=\overline{1,n}, j=\overline{1,n}. \quad (20)$$

Rewrite equations of the system (19) in a more convenient form:

$$2(1+\frac{\lambda_{11}}{n})x_{11}+\frac{\lambda_{11}}{n}\left(\sum_{k\neq 1}^{n}x_{1k}+\sum_{k\neq 1}^{n}x_{k1}\right)=2a_{11}+\lambda_{11},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$2(1+\frac{\lambda_{ij}}{n})x_{ij}+\frac{\lambda_{ij}}{n}\left(\sum_{k\neq j}^{n}x_{ik}+\sum_{k\neq i}^{n}x_{kj}\right)=2a_{ij}+\lambda_{ij}, \quad (21)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$2(1+\frac{\lambda_{nn}}{n})x_{nn}+\frac{\lambda_{nn}}{n}\left(\sum_{k\neq j}^{n}x_{nk}+\sum_{k\neq i}^{n}x_{kn}\right)=2a_{nn}+\lambda_{nn}.$$

A compact representation of the system of equations (21) is obtained using matrix relations.

Introduce matrices

$$A_1=\begin{pmatrix} 2\left(1+\frac{\lambda_{11}}{n}\right) & \frac{\lambda_{11}}{n} & \ldots & \frac{\lambda_{11}}{n} \\ \frac{\lambda_{12}}{n} & 2\left(1+\frac{\lambda_{12}}{n}\right) & \ldots & \frac{\lambda_{12}}{n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\lambda_{1n}}{n} & \frac{\lambda_{1n}}{n} & \ldots & 2\left(1+\frac{\lambda_{1n}}{n}\right) \end{pmatrix},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$A_n=\begin{pmatrix} 2\left(1+\frac{\lambda_{n1}}{n}\right) & \frac{\lambda_{n1}}{n} & \ldots & \frac{\lambda_{n1}}{n} \\ \frac{\lambda_{n2}}{n} & 2\left(1+\frac{\lambda_{n2}}{n}\right) & \ldots & \frac{\lambda_{n2}}{n} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\lambda_{nn}}{n} & \frac{\lambda_{nn}}{n} & \ldots & 2\left(1+\frac{\lambda_{nn}}{n}\right) \end{pmatrix}.$$

$$B=\begin{pmatrix} \frac{\lambda_{11}}{n} & 0 & 0 & 0 \\ 0 & \frac{\lambda_{12}}{n} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{\lambda_{1n}}{n} \end{pmatrix},\ldots,B_s=$$

$$=\begin{pmatrix} \frac{\lambda_{s1}}{n} & 0 & 0 & 0 \\ 0 & \frac{\lambda_{s2}}{n} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{\lambda_{sn}}{n} \end{pmatrix},\ldots,B_n=$$

$$=\begin{pmatrix} \frac{\lambda_{n}}{n} & 0 & 0 & 0 \\ 0 & \frac{\lambda_{n2}}{n} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{\lambda_{nn}}{n} \end{pmatrix}$$

and vectors

$$X_1=\begin{pmatrix}x_{11} & x_{12} & \ldots & x_{1n}\end{pmatrix}\ldots,\ X_s=\begin{pmatrix}x_{s1} & x_{s2} & \ldots & x_{sn}\end{pmatrix}\ldots,$$
$$X_n=\begin{pmatrix}x_{n1} & x_{n2} & \ldots & x_{nn}\end{pmatrix},$$

$$P_1=\begin{pmatrix}2a_{11}+\lambda_{11}\\2a_{12}+\lambda_{12}\\\dots\\2a_{1n}+\lambda_{1n}\end{pmatrix},\dots,P_s=\begin{pmatrix}2a_{s1}+\lambda_{s1}\\2a_{s2}+\lambda_{s2}\\\dots\\2a_{sn}+\lambda_{sn}\end{pmatrix},\dots,P_n=\begin{pmatrix}2a_{n1}+\lambda_{n1}\\2a_{n2}+\lambda_{n2}\\\dots\\2a_{nn}+\lambda_{nn}\end{pmatrix}.$$

Write the system (21) using the notation introduced above:

$$\begin{aligned}&A_1X_1^{\mathrm{T}}+B_1X_2^{\mathrm{T}}+B_1X_3^{\mathrm{T}}+\ldots+B_1X_n^{\mathrm{T}}=P_1,\\&B_2X_1^{\mathrm{T}}+A_2X_2^{\mathrm{T}}+B_2X_3^{\mathrm{T}}+\ldots+B_2X_n^{\mathrm{T}}=P_2,\\&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\&B_nX_1^{T}+B_nX_2^{T}+B_nX_3^{T}+\ldots+A_nX_n^{T}=P_n.\end{aligned}\qquad(22)$$

The resulting system of linear algebraic equations can be solved numerically by any known method (Gauss, Jordan-Gauss, etc.). At the same time, the specific structure of the system (22) makes it possible to obtain a solution in an explicit form.

Rewrite equations of the system as follows:

$$\begin{aligned}&\left(A_1-B_1\right)X_1^{\mathrm{T}}+B_1X_1^{\mathrm{T}}+B_1X_2^{\mathrm{T}}+B_1X_3^{\mathrm{T}}+\ldots+B_1X_n^{\mathrm{T}}=P_1,\\&\left(A_2-B_2\right)X_2^{\mathrm{T}}+B_2X_1^{\mathrm{T}}+B_2X_2^{\mathrm{T}}+B_2X_3^{\mathrm{T}}+\ldots+B_2X_n^{\mathrm{T}}=P_2,\\&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\&\left(A_n-B_n\right)X_n^{\mathrm{T}}+B_nX_1^{\mathrm{T}}+B_nX_2^{\mathrm{T}}+B_1X_3^{\mathrm{T}}+\ldots+B_1X_n^{\mathrm{T}}=P_n\end{aligned}$$

or

$$\left(A_1-B_1\right)X_1^{T}+B_1\sum_{j=1}^{n}X_j=P_1\ \dots,$$

$$\left(A_n-B_n\right)X_n^{\mathrm{T}}+B_n\sum_{j=1}^{n}X_j=P_n.$$

Then

$$B_1^{-1}(A_1-B_1)X_1^{T}-B_1^{-1}P_1=-\sum_{J=1}^{n}X_j^{\mathrm{T}}\ \dots,$$

$$B_n^{-1}(A_n-B_n)X_n^{\mathrm{T}}-B_n^{-1}P_n=-\sum_{J=1}^{n}X_j^{\mathrm{T}}.$$

As a result,

$$\left(B_1^{-1}A_1-I\right)X_1^{T}-B_1^{-1}P=\ldots=\left(B_n^{-1}A_n-I\right)X_n^{T}-B_n^{-1}P_n.\qquad(23)$$

Correlation (23) allows us to express $X_j^{\mathrm{T}},\ j=\overline{2,n},$ through X_1^{T}. The following is obtained:

$$\begin{aligned}&X_j^{\mathrm{T}}=\left(B_j^{-1}A_j-I\right)^{-1}\left[\left(B_1^{-1}A_1-I\right)X_1^{\mathrm{T}}-B_1^{-1}P_1+B_j^{-1}P_j\right]=\\&=\left(B_j^{-1}A_j-I\right)^{-1}\left(B_1^{-1}A_1-I\right)\times\\&\times X_1^{\mathrm{T}}+\left(B_j^{-1}A_j-I\right)^{-1}\left(B_j^{-1}P_j-B_1^{-1}P_1\right)=\\&=C_{1j}X_1^{\mathrm{T}}+D_{1j},\ j=\overline{2,n}\ .\end{aligned}\qquad(24)$$

Substituting (23) into the first equation of system (22), the following is obtained:

$$\left(A_1+\sum_{j=2}^{n}C_{1j}\right)X_1^{\mathrm{T}}+\sum_{J=2}^{n}D_{1j}=P_1,$$

whence

$$X_1^{\mathrm{T}}=\left(A_1+\sum_{j=2}^{n}C_{1j}\right)^{-1}\left(P_1-\sum_{j=2}^{n}D_{1j}\right),\qquad(25)$$

and then, calculate $X_2^{\mathrm{T}},\dots,X_n^{\mathrm{T}}$ using (24). Now a system of equations is obtained for finding values of the uncertain Lagrange multipliers which provides solution of the problem. Unfortunately, the resulting nonlinear system of equations can only be solved numerically. The complexity of solution of this problem increases rapidly (quadratically) with an increase in dimension of the problem. It should be mentioned that a simpler, iterative procedure for obtaining a consistent matrix was proposed in [49]. This procedure provides an approximate solution of the problem but accuracy of approximation improves at each step. The procedure is implemented as follows.

The fact is used that in the consistent matrix A, for all i, j pairs, the following equalities are feasible

$$a_{ij}=a_{ik}a_{kj},\quad na_{ij}=\sum_{k=1}^{n}a_{ik}a_{kj},$$

$$a_{ij}=\frac{1}{n}\sum_{k=1}^{n}a_{ik}a_{kj},\quad i=\overline{1,n},\quad j=\overline{1,n},\quad \mathrm{k}=\overline{1,n}.\qquad(26)$$

This correlation, together with the set of equalities

$$a_{ij}=\left(a_{ji}\right)^{-1},\quad i=\overline{1,n},\quad j=\overline{1,n},\qquad(27)$$

determines the consistent matrix. Since the real matrix of pairwise comparisons possesses only the property (27) but does not satisfy (26), the procedure of correction of the real matrix was proposed. It approximates this matrix to the consistent one. First of all, let us see that for any matrix A with its elements satisfying (27), the diagonal elements of the matrix

$$\frac{1}{n}AA=A_1=\left\{a_{ij}^{(1)}\right\}$$

are equal to one. Indeed, in accordance with (26),

$$a_{ii}^{(1)}=\frac{1}{n}\sum_{k=1}^{n}a_{ik}\cdot a_{ki}=\frac{n}{n}=1,\quad i=\overline{1,n}.\qquad(28)$$

Next, the correction procedure is introduced as follows. The computational scheme is iterative. At each iteration, three steps are performed. Let l correction iterations be done resulting in obtaining of the matrix A_l.

At the next (l+1)-th iteration, the following calculations are performed.

Step 1.

Calculate

$$\frac{1}{n}A_lA_l=\hat{A}_{l+1}=\left\{\hat{a}_{ij}^{(l+1)}\right\}.$$

Step 2.

Calculate

$$a_{ij}^{(l+1)}=\frac{\hat{a}_{ij}^{(l+1)}}{\left(\hat{a}_{ij}^{(l+1)}\cdot\hat{a}_{ji}^{(l+1)}\right)^{\frac{1}{2}}},\quad a_{ji}^{(l+1)}=\frac{\hat{a}_{ji}^{(l+1)}}{\left(\hat{a}_{ij}^{(l+1)}\cdot\hat{a}_{ji}^{(l+1)}\right)^{\frac{1}{2}}}\ .\qquad(29)$$

It is clear that as a result of transformation of (29), the matrix A_{l+1} will satisfy (27) which in accordance with (28), ensures equality of diagonal elements of the matrix A_{l+2} to one. The matrix will be obtained in the next step. Since the original matrix possessed the property of (27), obviously all subsequent matrices $A_1, A_2, \ldots, A_l, \ldots$, with (28) taken into account, will have the same property.

Step 3.

Calculate

$$\eta_l = \max_{i,j} \left| a_{ij}^{(l)} - a_{ij}^{(l+1)} \right|.$$

If the value obtained $\eta_l < \varepsilon$ where ε is some sufficiently small preassigned number (for example, $\varepsilon = 10^{-3}$), then the procedure is considered completed. Otherwise, proceed to the next iteration.

Convergence of the correction procedure has been verified experimentally [50].

Thus, the proposed procedure for constructing the regression equation ensures restoration of an unknown relationship between a set of influencing factors and the selected resultant indicator under conditions of a small initial data sample based on the hierarchy analysis method.

Rapid determination of this dependence can be performed using the ABC classification. The ABC analysis is an effective management tool in logistics systems. It makes it possible to allocate certain groups of material resources depending on cost concentration. In general, the ABC analysis is a method by which the degree of distribution of a concrete characteristic (cost) between individual elements of a set is determined. The method of logistics ABC analysis is development of the Pareto principle in logistics problems. According to the Pareto principle (20/80 rule), a fifth (20 %) of the total number of objects usually give approximately 80 % of the result cost. Accordingly, contribution of the remaining 80 % of the objects yields only 20 %. In logistics, the ABC method offers a deeper division: into three parts [40]. This ensures rational allocation of resources and their effective management to achieve the goal.

According to the method of logistics ABC analysis, normalization and control of the objects under study consists in distribution of the set elements into three non-uniform subsets A, B, C based on a certain formal algorithm.

Let us apply the following algorithm for the classification based on the ABC analysis:

1) for each of the set elements of the objects of classification, take their value on the basis of the complex analytical indicators (indices) obtained in the second stage;

2) arrange the set elements in order of decreasing value;

3) determine share of each element of the set in the total amount of contribution;

4) build a cumulative curve: display the cumulative percentage in the number of objects on the abscissa axis and the cumulative percentage by contribution of each of the set elements to the overall estimate by the adopted cost indicator on the ordinate axis;

5) divide the set elements into classes corresponding to their shares in the total indicator.

Depending on the value of individual objects, the set is divided into three groups. The following classification is most typical [40]. The group "A" includes the most valuable objects which account for about 75–80 % of the total cost of resources, but they account for only 10–20 % of the total sum. Group "B" includes objects of an average cost with a share in the total cost about 10–15 % but it is 30–40 % in the quantitative terms. Group "C" contains the cheapest objects, 5–10 % of the total cost and 40–50 % of the total quantity.

However, the distribution is not necessarily made in three groups. The number of groups and limits can be chosen arbitrarily. Experts' opinions and their experience can be used in this stage.

Proceeding from this classification, different levels of detail are laid down for each group of goods during planning and control.

Comparative characteristics of the three variants (empirical, differential and tangential) of the ABC analysis and analysis of the advantages and disadvantages of the used modifications are given in [51] by the example of drug consumption

5. Classification of Ukraine's partner countries based on a multi-factor analysis

Let us classify Ukraine's partner countries by identifying the level of connections based on analysis of goods turnover in the areas of export and import of certain goods groups (GG) according to the Ukrainian Foreign Economic Activity Classifier (UFEAC) [52].

Initial data are formed based on the volumes and cost of goods of each group based on official customs statistics for 2016 [53]. Simulations were carried out in the Maple-7 environment using the Symbolic Computation package [54] and in the Microsoft EXCEL environment. For the classification based on the logistic ABC analysis, the advanced procedure of the graphical method [55] was used.

5. 1. Modeling on the basis of volumes and costs of international flows of individual goods groups

Let us consider the problem of classification of Ukraine's trading partner countries in terms of volumes and cost of goods flows in "export" and "import" modes. Initial data were formed based on Ukrainian customs statistics for 2016 [53]. For the study, take individual groups of goods that can be transported in universal containers according to UFEAC. These are food and industrial goods of wide consumption, products of industrial-engineering use, agricultural products and household belongings, etc. Perishable stuffs, nonpacked loose cargoes, explosives, flammable, caustic, stinking and toxic substances, etc. are unsuitable for transportation in universal containers. Goods that contaminate container walls and floor are excluded from the list as well. Also, goods that cannot be loaded into a container or unloaded without the use of cargo handling equipment are not included in the list of goods for container shipment.

For analysis, the weight and cost indicators for thirteen goods groups (GG) according to the UFEAC were taken. These are goods flows of goods groups under following UFEAC codes: 43, 51, 52, 53, 57, 59, 60, 61, 62, 64, 65, 95 and 96 with the corresponding numbering of factors i=1...3.

The aggregate weight and cost characteristics of the goods groups taken for analysis of the goods shipped in export and imports modes are presented in Table 1. For convenience of further use of this information, numeration of Ukraine's partner countries was made by indicating the country ID in the set.

Table 1

Aggregate weight and cost characteristics

Export				Import			
Country	ID	Cost, USD 1,000	Weight, 1,000 t	Country	ID	Cost, USD 1,000	Weight, 1,000 t
Australia	1	975	242	Austria	1	975	242
Austria	2	3,694	786	Azerbaijan	2	3,694	786
Bangladesh	3	47,261	4,700	Albania	3	4,7261	4,700
Belgium	4	6,688	2,089	Afghanistan	4	6,688	2,089
Belarus	5	26,181	4,537	Belgium	5	26,181	4,537
Bulgaria	6	1,475	111	Belarus	6	1,475	111
Great Britain	7	978	24	Bulgaria	7	978	24
Vietnam	8	36,012	2,366	Great Britain	8	36,012	2,366
Denmark	9	2,553	429	Vietnam	9	2,553	429
Egypt	10	72	29	Hong Kong	10	72	29
India	11	11,896	3,555	Georgia	11	11,896	3,555
Indonesia	12	10,195	647	Denmark	12	10,195	647
Ireland	13	1	0	Estonia	13	1	0
Spain	14	2	0	Egypt	14	2	0
Italy	15	61,722	5,751	India	15	61,722	5,751
Kazakhstan	16	149	17	Spain	17	149	17
Cambodia	17	2,923	141	Italy	18	2,923	141
Canada	18	10	0	Kazakhstan	19	10	0
China	19	504,052	64,402	Canada	20	504,052	64,402
Korea	20	14	6	Kyrgyzstan	21	14	6
Latvia	21	840	64	China	22	840	64
Malaysia	22	390	112	Congo	23	390	112
Moldova	23	1,178	727	Latvia	24	1,178	727
Nepal	24	11	0	Lithuania	25	11	0
Netherlands	25	9,065	2,118	Moldova	26	9,065	2,118
Germany	26	63,956	6,879	Nonidentified countries	27	63,956	6,879
Pakistan	27	134	15	Netherlands	28	134	15
Poland	28	93,320	20,243	Germany	29	93,320	20,243
Russia	29	37,521	9,247	Panama	30	37,521	9,247
Rumania	30	215	219	Poland	31	215	219
Syria	31	11	4	Portugal	32	11	4
Slovakia	32	899	164	Russia	33	899	164
Slovenia	33	225	10	Rumania	34	225	10
USA	34	2,113	328	Slovakia	35	2,113	328
Thailand	35	604	66	USA	36	604	66
Taiwan	36	729	56	Turkey	37	729	56
Turkey	37	80,243	13,466	Turkmenistan	38	80,243	13,466
Turkmenistan	38	2,627	1,235	Hungary	39	2,627	1,235
Hungary	39	12,815	3,759	Uzbekistan	40	12,815	3,759
Uzbekistan	40	5,974	2,985	France	41	5,974	2,985
Uruguay	41	40	12	Croatia	42	40	12
Philippines	42	32	21	Czech Republic	43	32	21
Finland	43	51	2	Switzerland	44	51	2
France	44	8,743	387	Sweden	45	8,743	387
Czech Republic	45	6,736	1,186	–	–	–	–
Sri Lanka	46	200	102	–	–	–	–
Japan	47	6,491	237	–	–	–	–

The results of classification on the basis of logistics ABC analysis are shown in Fig. 1. Cumulative percentages by quantity represent the share of the total number of countries. The cumulated percentages by contribution represent the share by the estimated criterion of the cost of international goods flows moving in the "import" mode.

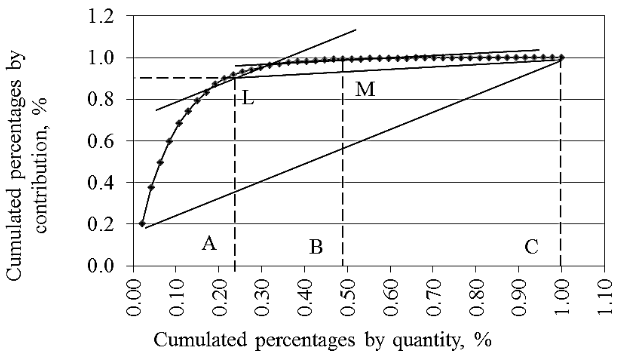

Fig. 1. Classification of Ukraine's partner countries

The tangent to the cumulative curve indicates the boundaries of classes (points of contact L, M) which are shown by a dashed line in Fig. 1. Class A includes 23 % of the total number of countries and the share by cost is 92 %. Class B includes 30 % of the total number of countries and the share by the cost indicator is 7.5 %. Class C includes 47 % of countries with their cost contribution is only 0.6 %.

Similarly, modeling was carried out for classification of countries by the criterion of the weight of goods flows in the "import" shipment mode as well as the criteria of cost and weight in the "export" shipment mode.

In the analysis of the main components of the data by weight of imported goods with a reliability of 0.9, five equations with a share of total variance were obtained, respectively, 53 %, 14 %, 10 %, 8 % and 7 %.

The analysis of data by the cost of exported goods with a reliability of 0.81 has allowed us to obtain seven equations of the main components with a share of the total variance for each, 26 %, 20 %, 14 %, 8 %, 7 %, 5 % and 3 %, respectively.

Based on the weight of exported goods, seven equations of the main components with reliability of 0.86 for the variance shares of 28 %, 18 %, 13 %, 9 %, 8 %, 6 %, and 5 % were obtained, respectively.

At the second stage, based on hierarchy analysis method, analytical indicators (indices) were determined. They are presented in summary Tables 2, 3 and arranged in an order of their reduction. Countries and their identification numbers (ID) as well as the classification results are given in these tables.

Table 2

Classification of Ukraine's partner countries by the direction of movement of goods flows in the "import" mode

By the cost indicator				By the weight indicator			
ID	Country	Index	Class	ID	Country	Index	Class
1	2	3	4	5	6	7	8
19	China	0.52774	A	19	China	0.32138	A
37	Turkey	0.43955	A	15	Italy	0.31791	A
26	Germany	0.31205	A	3	Bangladesh	0.30005	A
28	Poland	0.25974	A	28	Poland	0.22661	A
15	Italy	0.22448	A	37	Turkey	0.21142	A
4	Belgium	0.15206	A	5	Belarus	0.18163	A
25	Netherlands	0.12779	A	29	Russia	0.15160	A
29	Russia	0.10325	A	26	Germany	0.13820	A
5	Belarus	0.10162	A	23	Moldova	0.08875	A
44	France	0.06456	A	38	Turkmenistan	0.07687	A
11	India	0.05185	A	4	Belgium	0.06084	A
40	Uzbekistan	0.03133	B	11	India	0.05848	A
3	Bangladesh	0.02948	B	44	France	0.05500	A
7	Great Britain	0.02934	B	1	Australia	0.05148	A
39	Hungary	0.02487	B	25	Netherlands	0.04784	A
23	Moldova	0.02220	B	30	Romania	0.04659	B
38	Turkmenistan	0.01462	B	40	Uzbekistan	0.03004	B
8	Vietnam	0.00982	B	45	Czech Republic	0.02855	B
47	Japan	0.00843	B	39	Hungary	0.02219	B
45	Czech Republic	0.00829	B	46	Sri-Lanka	0.01346	B
1	Australia	0.00752	B	9	Denmark	0.01053	B
21	Latvia	0.00346	B	21	Latvia	0.00354	B
35	Thailand	0.00292	B	8	Vietnam	0.00332	B
17	Cambodia	0.00263	C	41	Uruguay	0.00255	B
12	Indonesia	0.00252	C	10	Egypt	0.00206	C
43	Finland	0.00206	C	47	Japan	0.00116	C
2	Austria	0.00198	C	32	Slovakia	0.00097	C
36	Taiwan	0.00196	C	2	Austria	0.00071	C
34	USA	0.00184	C	36	Taiwan	0.00070	C

Continuation of Table 2

1	2	3	4	5	6	7	8
32	Slovakia	0.00174	C	7	Great Britain	0.00068	C
30	Rumania	0.00166	C	12	Indonesia	0.00063	C
9	Denmark	0.00060	C	34	USA	0.00058	C
33	Slovenia	0.00044	C	35	Thailand	0.00057	C
6	Bulgaria	0.00041	C	17	Cambodia	0.00045	C
18	Canada	0.00040	C	43	Finland	0.00026	C
24	Nepal	0.00033	C	22	Malaysia	0.00019	C
41	Uruguay	0.00031	C	42	Philippines	0.00012	C
22	Malaysia	0.00025	C	6	Bulgaria	0.00008	C
10	Egypt	0.00008	C	16	Kazakhstan	0.00007	C
42	Philippines	0.00006	C	27	Pakistan	0.00007	C
16	Kazakhstan	0.00006	C	33	Slovenia	0.00006	C
27	Pakistan	0.00005	C	20	Korea	0.00004	C
20	Korea	0.00003	C	31	Syria	0.00002	C
31	Syria	0.000021	C	13	Ireland	0.00000	C
14	Spain	0.00002	C	14	Spain	0.00000	C
13	Ireland	0,00001	C	18	Canada	0.00000	C
46	Sri-Lanka	0.00000	C	24	Nepal	0.00000	C

Table 3

Classification of Ukraine's partner countries by the direction of movement of goods flows in the "export" mode

By the cost indicator				By the weight indicator			
ID	Country	Index	Class	ID	Country	Index	Class
1	2	3	4	5	6	7	8
32	Russia	0.41097	A	32	Russia	0.28325	A
30	Poland	0.35561	A	30	Poland	0.26791	A
28	Germany	0.21595	A	17	Italy	0.21491	A
33	Romania	0.20503	A	33	Romania	0.21430	A
17	Italy	0.18355	A	1	Austria	0.17135	A
18	Kazakhstan	0.17797	A	21	China	0.16806	A
12	Denmark	0.13874	A	5	Belgium	0.16138	A
39	Uzbekistan	0.13701	A	28	Germany	0.13800	A
37	Turkmenistan	0.11862	A	12	Denmark	0.08721	A
6	Belarus	0.10133	A	24	Lithuania	0.07000	A
5	Belgium	0.04304	A	6	Belarus	0.06776	A
38	Hungary	0.04227	A	38	Hungary	0.04180	B
24	Lithuania	0.03480	B	29	Panama	0.02183	B
9	Vietnam	0.03205	B	25	Moldova	0.02126	B
25	Moldova	0.03175	B	4	Afghanistan	0.01894	B
42	Czech Republic	0.02949	B	27	Netherlands	0.01826	B
27	Netherlands	0.02089	B	3	Albania	0.01775	B
3	Albania	0.01548	B	42	Czech Republic	0.01723	B
40	France	0.01459	B	19	Canada	0.00830	B
29	Panama	0.01420	B	35	USA	0.00800	B
13	Estonia	0.00946	B	40	France	0.00736	B
2	Azerbaijan	0.00704	B	18	Kazakhstan	0.00726	B
8	Great Britain	0.00524	C	9	Vietnam	0.00697	B
31	Portugal	0.00454	C	13	Estonia	0.00348	B
35	USA	0.00412	C	31	Portugal	0.00234	C
11	Georgia	0.00405	C	2	Azerbaijan	0.00186	C
4	Afghanistan	0.00333	C	20	Kyrgyzstan	0.00160	C
20	Kyrgyzstan	0.00278	C	11	Georgia	0.00133	C
43	Switzerland	0.00122	C	34	Slovakia	0.00127	C
23	Latvia	0.00117	C	8	Great Britain	0.00122	C
34	Slovakia	0.00091	C	23	Latvia	0.00118	C

Continuation of Table 2

1	2	3	4	5	6	7	8
26	Nonidentified countries	0.00088	C	43	Switzerland	0.00066	C
1	Austria	0.00056	C	26	Nonidentified countries	0.00055	C
19	Canada	0.00048	C	10	Hong Kong	0.00009	C
41	Croatia	0.00035	C	22	Congo	0.00005	C
36	Turkey	0.00030	C	7	Bulgaria	0.00003	C
7	Bulgaria	0.00020	C	14	Egypt	0.00001	C
10	Hong Kong	0.00013	C	15	India	0.00001	C
16	Spain	0.00009	C	44	Sweden	0.00001	C
15	India	0.00007	C	16	Spain	0.00000	C
44	Sweden	0.00005	C	36	Turkey	0.00000	C
22	Congo	0.00005	C	37	Turkmenistan	0.00000	C
14	Egypt	0.00001	C	39	Uzbekistan	0.00000	C
21	China	0.00000	C	41	Croatia	0.00000	C

The analysis presented in Tables 2, 3 shows that classification of a set of Ukraine's trading partners has resulted in three subsets (classes) which is well correlated with the classic ABC analysis.

5. 2. Classification of the main Ukraine's partner countries in foreign trade based on an analysis of the specific value of the goods flows

Goods structure of goods flows between countries affects the indicator of specific value which is formed on the basis of data on the cost and weight of goods flows of individual goods groups. The flows between Ukraine and the trading partner countries, in particular, the modes of transportation (export, import), are also taken into account.

For the study, countries of the first twenty partners of Ukraine were selected based on the analysis of official statistics for 2016 based on the analysis of export and import flows. For selected countries, specific freight traffic costs were determined by analyzing weight and cost characteristics [53] for each of the flows of goods of individual groups according to UFEAC [52] to each of the selected countries in terms of export and import.

By the ratio of the goods cost to the goods weight for each of the traffic flows indicators were determined that characterize the value of goods in international goods turnover. Specific value indicators for each of twenty one groups of goods under the UFEAC (numbered as I, II,..., XXI) moving in the export and import modes according to the specified directions between Ukraine and the trading partner countries were calculated. These indicators were accepted as criteria for classification of countries.

An example of initial data for modeling of individual goods groups (I–V) and several partner countries are given in Table 4.

Factor analysis of Ukraine's goods turnover in terms of exports of goods with major partner countries revealed with a reliability of 0.6 five equations of the main components that characterize influence of each of the factors: 19 % of the total variance are described by the first component, 13 % by the second component, 11 %, by the third component, 9 % by the fourth component and 8 % by the fifth component.

Table 4

Indicators of the specific cost of export and import goods flows

Country	Movement direction	ID	I	II	III	IV	V	...	XXI
Russia	export	1	0.624	0.352	1.115	0.890	0.031	...	0
	import	1	0.235	0.607	0.404	1.552	0.122	...	0.830
Poland	export	3	1.606	0.156	0.741	0.277	0.047	...	1.402
	import	5	0.811	0.915	1.113	1.922	0.157	...	0
Turkey	export	4	1.491	0.370	1.085	0.140	0.103	...	1.075
	import	9	6.395	0.971	0	1.699	0.102	...	0
Italy	export	5	0	0.558	0	0	0.039	...	2.344
	import	8	0	1.708	4.079	1.275	0	...	0
India	export	6	0	0.297	0.765	0	0	...	2.329
	import	18	0	0.949	2.804	4.745	0.597	...	0
...	...	...	...	...	...	...	...	...	...
Georgia	export	20	1.395	0.535	0	0,91	0.13	...	0
	import	20	0	2.941	0	0.948	0.178	...	0

The factor analysis of Ukraine's goods turnover in terms of import of goods with major partner countries revealed with a reliability of 0.6 six equations of the main components that characterize influence of each of the factors: 15 % of the total variance are describes by the first component, 12 % by the second component, 10 % by the third component, 9 % by the fourth component, 8 % by the fifth component and 7 % by the sixth component.

The classification results are shown in Fig. 2, 3 and summarized in Table 5. Classification of international goods flows by the criterion of specific value of the goods flows moved in the "export" mode is shown in Fig. 5 and analogous data for the "import" mode are given in Fig. 3. Table 5 shows in a decreasing order the analytical indices determined at the second stage by the of hierarchy analysis method, countries with their identification numbers (ID) as well as the results of classification.

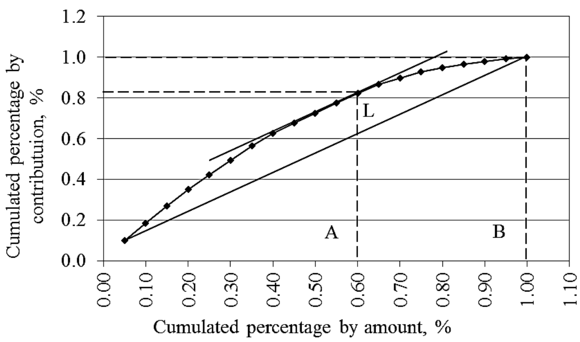

Fig. 2. Classification of Ukraine's partner countries in the "export" mode

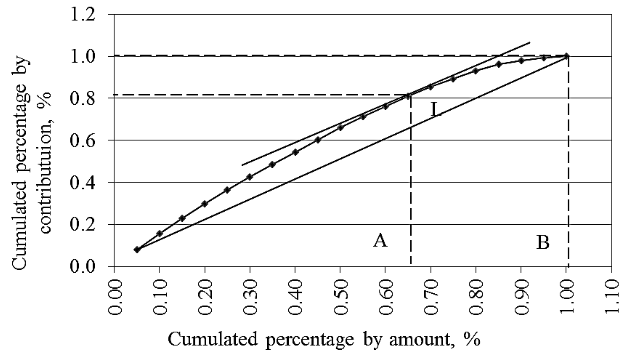

Fig. 3. Classification of Ukraine's partner countries in the "import" mode

The classification results are summarized in Table 6. For each of the analyzed variants of initial data, indicators are presented that characterize classification of the set of Ukraine's partner countries in the international goods turnover according to the corresponding criteria.

Analysis of data given in Table 5 shows that according to the criterion of specific value of goods flows, a set of studied countries can be divided into two classes. This is due to the fact that only twenty countries with the largest turnover were selected previously.

From the data given in Table 6, it appears that the boundaries of classes are dynamic and depend on the specific formulation of the problem and the criteria taken into consideration.

Table 5

Classification of countries by the indicator of value of goods flows

In the «export» mode				In the «import» mode			
ID	Country	Index	Index	ID	Country	Index	Index
8	Germany	0.386071	A	8	Italy	0.363417	A
9	Hungary	0.340034	A	7	France	0.354946	A
12	Belarus	0.330739	A	11	Hungary	0.340211	A
20	Georgia	0.319889	A	3	Germany	0.323278	A
1	Russia	0.286304	A	18	India	0.29856	A
6	India	0.282967	A	13	Czech Republic	0.283953	A
7	China	0.277638	A	2	China	0.278306	A
18	Moldova	0.239381	A	12	Great Britain	0.270627	A
3	Poland	0.199425	A	5	Poland	0.269522	A
13	Runania	0.19323	A	19	Austria	0.266977	A
16	Czech Republic	0.189345	A	6	USA	0.236012	A
2	Egypt	0.187958	A	14	Japan	0.232749	A
11	Netherlands	0.179817	B	10	Switzerland	0.213365	A
19	Slovakia	0.12254	B	1	Russia	0.209051	B
5	Italy	0.117688	B	15	Netherlands	0.185483	B
10	Spain	0.078537	B	9	Turkey	0.165933	B
4	Netherlands	0.068727	B	4	Belarus	0.15214	B
17	Israel	0.057136	B	17	Lithuania	0.074865	B
15	Saudi Arabia	0.050304	B	16	Spain	0.061853	B
14	Iran	0.030317	B	20	Georgia	0.031014	B

Table 6

Results of the logistic ABC analysis based on graphical method

Indicator	Value, export	Value, import	Cost, export	Cost, import	Weight, export	Weight, import
Limit of classes A/B by quantity, %	60	65	27	23	25	32
Limit of classes A/B by contribution, %	82	81	89.9	92	90	93
Limit of classes B/C by quantity, %	–	–	50	49	54.5	51
Limit of classes B/C by contribution, %	–	–	98.4	99.2	99.4	99.6
Number of objects in Class A, %	60	65	27	23	25	32
Number of objects in Class B, %	40	35	23	26	29.5	19
Number of objects in Class C, %	–	–	50	51	46.5	49
Ponderability of objects in Class A, %	82	81	89.9	92	90	93
Ponderability of objects in Class B, %	18	19	8.5	7.2	13.4	8.6
Ponderability of objects in Class C, %	–	–	1.6	0.8	0.6	0.4

6. Discussion of results obtained in application of the method of multifactor analysis for classification of transport logistics processes

For a more accurate representation of the processes taking place in transport and logistics systems, it is necessary to use the results of statistical analysis of data on freight flows in a comprehensive manner, use current methods of logistic analysis.

This work proposes a method for classifying a set of objects and/or processes in transport and logistics systems on the basis of multifactor analysis and provides a formalized description. The method is implemented by combining regression analysis, system hierarchy analysis method and logistic ABC analysis.

A model and algorithm of solution of multifactor classification problems in solving problems of transport and logistics and theory of organization of cargo transportation were proposed. Simulation was carried out in two stages. At the first stage, a regression analysis model was developed based on the system hierarchy analysis method. At this stage, an analytical indicator characterizing influence of factors on the objects under study was created. At the second stage, the ideas of the method of logistic ABC analysis for classification of transport and logistics processes and objects by the degree of manifestation of factor influence were used. Simulations were performed using symbolic composition package in the Maple-7 and the Microsoft Excel environments.

Classification of Ukraine's partner countries by the indicators of international turnover of certain groups of goods which can be transported in universal containers in various directions according to the Ukrainian foreign economic activity classifier (UFEAC) have shown the following:

– classification of countries with the help of an advanced procedure of the graphical method of ABC analysis [55] has allowed us to divide the set of Ukraine's partner countries in foreign trade into three classes;

– boundaries of the classes are dynamic and depend on the specific problem statement and the criteria taken into account.

When classifying countries by the criterion of cost of goods flow in the "import" mode, 23 % of the total number of countries whose share according to the cost indicator is 92 % were assigned to class A. 26 % of the total number of countries whose share according to the cost index is 7.2 % were assigned to class B. 51 % of the countries whose share according to the cost indicator is only 0.8 % were assigned to class C (Tables 2, 6). Class A includes the following countries: China, Turkey, Germany, Poland, Italy, Belgium, the Netherlands, Russia, Belarus, France, and India.

When classifying countries according to the criterion of weight of goods flows in the "import" mode, 32 % of the total number of countries with their share according to the cost indicator is 93 % were assigned to class A. Class B includes 19 % of the total number of the countries with their share according to the cost index is 8.6 %. Class C includes 49 % of the countries whose share according to the cost index is only 0.4 % (Tables 2, 6). Class A includes the following countries: China, Italy, Bangladesh, Poland, Turkey, Belarus, Russia, Germany, Moldova, Turkmenistan, Belgium, India, France, Australia, and the Netherlands.

When classifying countries according to the criterion of cost of goods flows in the "export" mode, 27 % of the total number of countries, the share according to the cost index is 89.9 % were assigned to class A. 23 % of the total number of countries with their share according to the cost indicator is 8.5 % were assigned to class B Class C includes 50 % of the countries with their according to the cost indicator is only 1.6 % (Table 3, 6). Class A includes the following countries: Russia, Poland, Germany, Romania, Italy, Kazakhstan, Denmark, Uzbekistan, Turkmenistan, Belarus, Belgium, and Hungary.

In classification of countries according to the criterion of weight of goods flows in "export" mode, 25 % of the total number of countries with their share of the cost indicator is 90 % were assigned to class A. Class B includes 29.5 % of the total number of countries with their share according to the cost indicator is 13.4 %. Class C includes 46.5 % of the countries with their share according to the cost indicator is only 0.6 % (Tables 2, 6). Class A includes the following countries: Russia, Poland, Italy, Romania, Austria, China, Belgium, Germany, Denmark, Lithuania, and Belarus.

Since classification was carried out for a set of all partner countries, the classification results are in line with the classic ABC analysis.

Classification of countries according to the criterion of specific value of goods flows has allowed us to divide the set of countries under study into two classes. This is explained by the fact that only twenty countries with the largest goods turnover were selected previously.

When classifying countries according to the criterion of the specific value of goods flows in the mode of "export" 60 % of the total number of countries with their share of contribution is 82 % were assigned to class A. Class B includes

40 % of the total number of countries with contribution share of 18 % (Tables 5, 6).

In classification of countries according to the criterion of specific value of goods flows in the “import” mode, 65 % of the total number of countries with their contribution share of 81 % were assigned to class A. Class B includes 35 % of the total number of countries with their contribution share of 19 % (Tables 5, 6). Class A includes the following countries: Italy, France, Hungary, Germany, India, Czech Republic, China, Great Britain, Poland, Austria, USA, Japan, and Switzerland.

This study is the development of the ideas set forth in papers [7, 8, 13, 56].

Using the hierarchy analysis method in an unifying model helps in structuring the problems of decision makers by building a hierarchy in accordance with the purpose, task and understanding of factors. Based on the use of statistical data, time series of dynamics are formed. Meaningful analysis of information makes it possible to create various sets of factors. This may be the characteristics by the goods groups, directions of the movement of goods flows, etc.

Application of the simplified hierarchy analysis method in the study facilitates formation of a versatility analytical indicator. A thorough statistical analysis adds objectivity to the estimates since it is based on actual data. However, processing of large statistical data bodies in preparation of initial data is a labor intensive procedure.

The use of the hierarchy analysis method in solving multifactor decision-making problems may involve experts in this process. This is a complicated procedure connected with formation of a group of experts, conduction of polls, analyzing questionnaires, and the like. However, when analyzing intricate problems, it can give more accurate estimates of the problem.

More generally, use of the ideas of the method of logistic ABC analysis enables distribution of objects of classification among various numbers of groups and their boundaries can be chosen arbitrarily. Experts’ opinions and experience may be useful at this stage.

The proposed classification method can be applied in transport, logistics, customs and brokerage enterprises, as it ensures planning and control of the goods supply, provide for varying degrees of detail and apply appropriate strategies.

Further development and improvement of the study is possible for using intelligent systems and algorithms in solving the problems of multifactor analysis of transport and logistics systems. In this case, the main goal may be development of a method of joint analysis of numerical characteristics of transport and logistics systems obtained over several years. It is clear that the problems arising from processing of a small sample of initial data seriously impede the possibility of using conventional probability-theoretic methods. A natural alternative may be the mathematical apparatus of the fuzzy [57] and inaccurate [58] mathematics using the methods proposed in [59, 60].

7. Conclusions

1. The method of classification of a set of objects and/or processes in the transport and logistics systems based on the multifactor analysis was proposed. The method was implemented by combining regression analysis, systems of hierarchy analysis method and the method of logistic ABC analysis. This allows one to form dynamic classes based on a comprehensive assessment of factor effects depending on the specific problem statement and the criteria to be taken into account. The use of the proposed classification method in activities of transport and logistics companies will enable formation of appropriate strategies for planning and control of goods supply.

2. Combination of methods of statistical and regression analysis, the systems hierarchy analysis method and modern methods of logistic analysis has allowed us to form a unifying model of classification. A three-stage algorithm was proposed for solving multifactor classification problems in the field of transport and logistics and the theory of organization of cargo transportation. At the first stage, the regression method investigates the factor influence based on the chosen system of criteria. At the second stage, an analytical indicator characterizing degree of factor influence on the objects under investigation and based on the systemic hierarchy analysis method was formed. The use of the hierarchy analysis method makes it possible both to use factual data based on a thorough statistical analysis and involve experts in solving multicriterial decision-making problems. The use of modern logistic analysis methods at the third stage enables substantiated division of classified objects in the transport and logistic systems into various numbers of dynamic classes. At this stage, it is also possible to involve experts in formation of boundaries between classes. The model was implemented using symbolic computation package in the Maple-7 and in the Microsoft Excel environments.

3. Classification of Ukraine’s trading partners according to the international turnover of certain groups of goods which can be transported in universal containers according to the Ukrainian foreign economic activity classifier (UFEAC) has been made. Through simulation, the set of partner countries in foreign trade was divided into three classes. The classification results are in good agreement with the classic ABC analysis. Classification of countries according to the criterion of specific value of goods flows has allowed us to divide the set of countries under study into two classes. This is explained by the fact that classification was carried out for a set of all partner countries in the first example and previous selection of twenty countries with the largest turnover was made in the second example.

Application of the proposed method of multifactor classification enables division of the classified objects, in general, among various numbers of groups. At the same time, boundaries of the classes are dynamic and depend on concrete formulation of the problem and the criteria taken into account.

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