Досліджено вплив модифікації топології гетерогенного строю безпілотних апаратів (БПА) на площу, охоплювану таким строєм. Запропонований підхід, за яким до опису поведінки строю БПА застосовується метод моделювання структури складних технічних систем. Розроблений метод визначення площі функціонування БПА із залученням теорії графів. Докладно розглянуто формування навантажених графів, що відповідають різним топологіям строю, проаналізовані матриця суміжності та матриця навантаженості для топології «ієрархічна зірка»

Ключові слова: безпілотний апарат, гетерогенний стрій, навантажений орграф, площа функціонування строю

Исследовано влияние модификации топологии гетерогенного строя беспилотных аппаратов (БПА) на площадь, охватываемую таким строем. Предложен подход, согласно которому к описанию поведения строя БПА применяется метод моделирования структуры сложных технических систем. Разработан метод определения площади функционирования БПА с привлечением теории графов. Подробно рассмотрено формирование нагруженных графов, соответствующих различным топологиям строя, проанализированы матрица смежности и матрица нагруженности для топологии «иерархическая звезда»

Ключевые слова: беспилотный аппарат, гетерогенный строй, нагруженный орграф, площадь функционирования строя

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DEVELOPMENT OF A METHOD FOR DETERMINING THE AREA OF OPERATION OF UNMANNED VEHICLES FORMATION BY USING THE GRAPH THEORY

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1. Introduction

People have always tried to transfer the phenomenon of a well-coordinated action of a flock of birds, a swarm of bees and other insects, fish and herd animals to the area of their research and to use it to achieve their own goals. Today this principle is used in the construction of heterogeneous networks for military, industrial and research purposes engaging unmanned vehicles designed for air, ground-based, and underwater operations [1, 2].

The actions of "formations" of unmanned vehicles (in other words, "drones") are based on "collective intelligence". It enables the rearrangement within a formation, action coordination under the general control of one operator, based on predetermined algorithms [3]. As a result of rearrangement within a formation, the area of the territory, at which unmanned vehicles can perform its functions, is determined both by technical characteristics of drones themselves and the order (patterns) of unmanned vehicles arrangement in a formation.

Control may be executed by giving episodic or continuous orders – in the latter case, unmanned vehicles are referred to as remotely controlled vehicles. Otherwise, drones can make a formation by exchanging navigation information with each other by means of telecommunication radio or optical channels established between unmanned vehicles [4, 5]. In both cases, a problem on the construction of a system of these vehicles can be solved using a mathematical model of the complex technical system with a dynamic structure [6].

When designing, it is necessary to represent in the structural scheme not only the composition of the system with a dynamic structure, but also functional relationships. One of the essential features of the system with a dynamic structure is its hierarchical organization, which is determined by the fact that separate elements of the system, technical specifications of which are given in detail, are combined in a totality of interrelated subsystems based on the hierarchical attribute.

A change in the topology of unmanned vehicles' formation as a system with a dynamic structure significantly changes the area, covered by the formation. As a result, it is necessary either to introduce additional unmanned vehicles in a formation or to review a subsequent route of the unmanned vehicles' motion. However, there are objective difficulties of these additional re-calculation, related

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to a limited capacity of unmanned vehicle's calculator. Moreover, an additional amount of computation results in a faster discharge of the accumulation battery aboard unmanned vehicle, thereby reducing the operation time of a formation.

Therefore, plotting of loaded graphs that correspond to different topologies of a formation will allow a substantiated approach to determining the area, covered by a unmanned vehicles' formation, in order to obtain certain effects from the applying of unmanned vehicles in various fields. In particular, environmental pollution can be reduced when spraying chemicals over farmland as the chemicals are sprayed from unmanned vehicles at lower altitudes than from airplanes and over a clearly determined area of a territory.

2. Literature review and problem statement

Based on an analysis of existing options of unmanned vehicles' design [7–9], the graphical designations of different unmanned vehicles' types were generalized, as shown in Fig. 1.

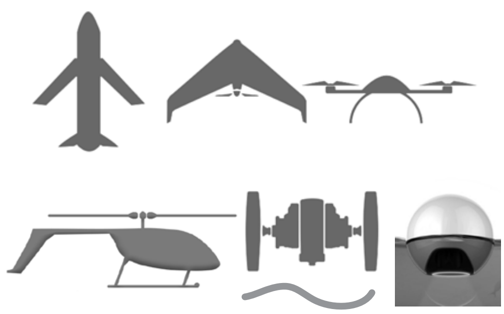


Fig. 1. Options of unmanned vehicles' design

Given the fact that unmanned aerial vehicles (UAV) are most common, we shall further consider them in the analysis and research in the form combined in the so-called UAV flock.

However, it should be noted that all obtained results can be also used in respect to a UAV formation of any structural design.

There is a classification of UAV that takes into consideration both the weight and the type of control [10]. Based on weight, UAV are divided into 4 classes:

a) *class 0* (very small) – up to 250 g;
b) *class 1* (small) – from 250 g to 20 kg;
c) *class 2* (medium) – from 20 kg to 150 kg;
d) *class 3* (large) – over 150 kg.

UAV classification by the control type:

a) *class A* – manual visual;
b) *class B* – manual instrumental;
c) *class C* – autonomous.

It is implied in the research that UAV of class 1 (small), constructed by different aerodynamic schemes, are used within one flock (Fig. 1). It should be noted that the use of UAV, constructed by different aerodynamic schemes (of aircraft type, helicopter type, hybrid type, quadro copter, etc.), imposes some limitations on the use of a UAV flock in general and on the macro-task performance quality. The study of such limitations is beyond the scope of this research and is the subject of further work of Authors.

The following vehicles were taken as an example to compare different UAV models (Table 1):

– Parrot DISCO FPV [11];
– Walkera QR X350 Pro [12];
– DJI Phantom 4 PRO [13];
– Blade Chroma 4K [14].

These models are designed for monitoring tasks, professional aerial survey. They have everything necessary for this: a stable structure, a set of flight functions, a high-quality camera. However, it is necessary to take into account the limitations of the hardware platform of each model in the process of forming the bandwidth of the composition of signals from several UAV of the flock [15, 16].

Table 1

Comparative characteristics of UAV

Model	Battery capacity, min	Dimensions, mm	Communication standards
Parrot DISCO FPV	45	1,150×580×120	Wi-Fi
Walkera QR X350 Pro	30	289×289×200	Wi-Fi
DJI Phantom 4 Pro	25	289.5×289.5×196	Wi-Fi, radio channel
Blade Chroma 4K	25	332×332×242	radio channel

A heterogeneous flock of the above UAV (or similar) can be considered as a complex technical system, for description of functioning of which it is expedient to apply mathematical modeling. Paper [17] shows approaches to modeling the structure of complex technical systems with the use of the basic provisions of the graph theory. However, peculiarities of complex technical systems with a dynamic structure are not fully taken into account in [17]. In addition, the dynamic character of the structure leads to a significant increase in capacity of the graph, associated with it, which causes the need to search for the ways of simplifying the model without losing its adequacy.

The concept of the structure of a model is introduced to determine the composition, location and links between subsystems and system elements. The structure of the system is partial ordering of the elements and relations between them, i. e. construction of a certain hierarchy of relationships in the system. This hierarchy can be constructed because the total function of the system is always divided into sub-functions, to which separate subsystems correspond [18]. According to the tested approaches, it is appropriate to perform decomposition of a system into subsystems so that they would represent independent functional parts of a system (functional complexes that solve certain agreed-upon problems in the interest of best performance of general tasks facing the system). However, structural and functional properties are closely related. That is why considering them as an integrity, it is possible to construct swarms/flocks of drones, combining robotics, wireless sensors and executing networks [19].

The most widely tested apparatus for modeling hierarchical systems is provided by the graph theory [20]. The topology of a heterogeneous network is logically reduced to an directed graph, the nodes of which correspond to UAV, and the branches – to the shortest way of radio signal transmission. In addition, to consider a UAV flock as a complex system, except for the considered method, it is possible to use simulation and analytical modeling of a complex dynamic system, probabilistic-statistical methods, etc. [21–23].

The main thing that characterizes the structure of a system with a dynamic structure is composition, existence of a reliable signal from the elements and their location. Therefore, the structure changes with a change of one of these elements. Formation of the structure and variants of its stage-by-stage change is a part of the solution of the general problem of system construction, the use of directed graphs allows using the discrete optimization methods. A mathematical model of the structure of a system is characterized by a random number of vertices, which, accordingly, take into account the probable impact of internal and external factors. In this regard, it is advisable to conduct subsequent studies aimed at searching for the methods of optimization of construction of complex technical systems with a dynamic structure.

At the present stage of development of geo-information systems and technologies, the tools of the graph theory are mainly used in the field of transport navigation services. A detailed analysis of the properties of the graph, separated into several tree branches, was performed in [24], but peculiarities of the trees with vertices that have different properties were not considered in this research.

A mathematical apparatus of the graph theory is very convenient for describing the infrastructure of spatial data. Measurements, which were taken with the equipment aboard UAV, are stored and transmitted with the use of computer components of a flock of drones, have a close relationship with the geographic information [25]. They allow overlapping the data coming from different UAV but are geometrically related to one territory. In addition, using the graph theory, it is possible to describe and determine location coordinates that connect mobile devices with spatial and temporal data. Such approach allows taking to the new level the process of combining principles of automatic control and autonomous behavior of a formation of moving objects of any design.

3. The aim and objectives of the study

The aim of present research is to study different variants of topology of the unmanned vehicles' formation, to construct graphs and to create the method for finding the area of territory coverage with an unmanned vehicles' formation, based on the ranges, within which a sustainable connection between the drones of different models is ensured.

To accomplish the aim, the following tasks have been set:

– to consider directions and ranges of data transmission between unmanned vehicles in a heterogeneous formation for different topologies;

– to construct loaded graphs that correspond to the most used topology "hierarchical star" and to mixed topologies, to analyze adjacency matrices and loading matrices.

4. Directions and ranges of data transmission in a heterogenic UAV flock

Let us consider the basic topologies of UAV dispersion (as a subset of all unmanned vehicles' designs) in the two-dimensional space. To solve the problems of placing servers (operators) and clients in the space for information transmission, processing and storage, the following topologies are used:

– *Star* (Fig. 2).

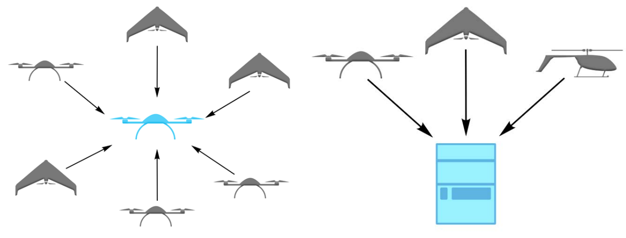


Fig. 2. Topology "star":
a — from UAV as a server; *b* — with a ground server

In the center of the topology, there is either the main drone or a ground operator, which serve for processing and storage of the collected data. Drones-clients that collect the necessary for a server information are attached to the central server unit.

– Ring (Fig. 3).

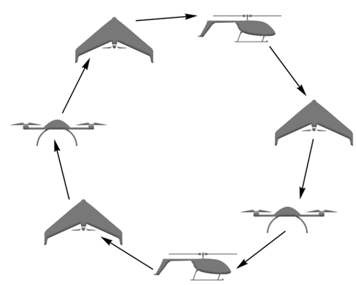


Fig. 3. Topology "ring"

The topology describes a circular relationship between drones that obtain certain data at the input to provide continuous information transmission around the closed circle between the objects of files collection and processing.

– *Bus* (Fig. 4).

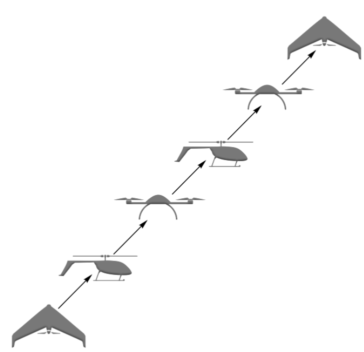


Fig. 4. Topology "bus"

Information transmission in this topology occurs sequentially from one extreme drone to the neighboring one by passing through the entire chain of intermediate clients. All drones, except for extreme ones, perform the role of data transfer points (repeaters).

– *Hierarchical star* (Fig. 5).

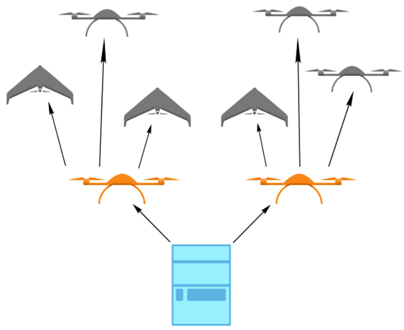


Fig. 5. Topology "hierarchical star"

The centerpiece of the hierarchy is a ground operator or a drone server, which are joined with the swarms connected to them. At the head of each swarm there is a leader that receives signals from child clients. Each swarm is an autonomous information collection unit, which is centrally buffered by the leader and sent for further processing and storage to ground operators or drones-servers.

– *Network-centric* (Fig. 6).

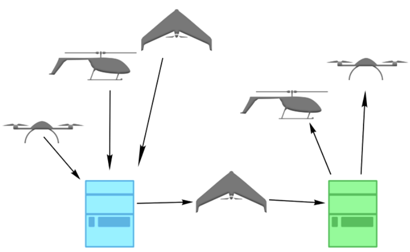


Fig. 6. Topology "network-centric"

The essence of the network-centric topology is that drones-clients that collect information transmit data to the nearest servers – both to ground-based operators and drones. Information exchange between servers is performed using transfer drones – the ones that relay the information that arrived from the neighboring node to another node of a wireless network.

Each topology can be presented in the form of a loaded graph. The weight of the edges of these graphs will be determined as the maximum range, within which connection between its nodes can be performed.

5. Results of research into the dynamic structure of UAV flock, constructed based on the topology "hierarchical star"

By the structure of a system with a dynamic structure, we will imply the interconnected totality of the subsystems (grouped UAV) and elements (drones-clients), united by the purpose of interaction with impact objects (ground operator or drone-server). The mentioned structure is characterized not only by relations of subsystems and elements (topology), but also by their location in time and space. The main thing that characterizes the structure of the system with a dynamic structure is composition, existence of a reliable signal from the elements and their location. Therefore, the change of the structure occurs during the change of one of the specified elements. Formation of the structure, variants of its staged change is a part of the solution of the general problem of construction of a system, which does not fully reveal it in general.

For example, it is possible to take the topology "*hierarchical star*" (Fig. 5) and to plot the loaded graph that correspond to this topology (Fig. 7).

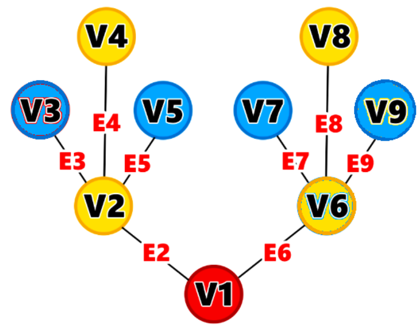


Fig. 7. Loaded graph of topology "hierarchical star"

In the graph in Fig. 7, V_n is the vertices of the graph, E_n is the weights of the edges. The color of the vertex designates a certain function, which can be performed by the hardware unit: red – the main drone or a ground operator (v_1), yellow – executors with the function of repeaters (v_2, v_6, v_4, v_8), blue – performers (v_3, v_5, v_7, v_9). It should be noted that in case of a loss of repeaters v_2 and/or v_6 during critical application, drones v_4 and/or v_8 can serve as repeaters relative to nodes $\{v_3, v_5\}$ and $\{v_7, v_9\}$. To have the possibility to perform these functions, appropriate computer components and downloaded software must be found aboard UAV {v4, v8} in cold reserve. In case of giving or losing an internal control signal in relation to the function of relaying, such components, accordingly, will start or stop consuming power from the electric power source aboard UAV.

The matrix of graph loading by the topology "hierarchical star" is shown in Table 2. Given that drones can receive commands for themselves and perform their own tasks, addressing "themselves" in the matrix is designated as "0".

For UAV models, shown in Table 1, $E_{n\ max}$=5 km. Using these calculations, it is possible to find the minimum route from one drone to another.

Subsystems of a system with a dynamic structure Pd_i can be the following:

$$Pd_i = \{D_c,\ B_m,\ L_s,\ I_p\}, \tag{1}$$

where D_c is the subsystem of detection of impact objects at every i-th moment. Subsystem D_c contains a set of relations with impact objects and is assigned by the list according to subordination relation $D_c \subset V$:

$$D_c = \begin{pmatrix} v_1 : v_2, v_3; \\ v_2 : v_3, v_4, v_5; \\ v_3 : \varnothing; \\ v_4 : \varnothing; \\ v_5 : \varnothing; \\ v_6 : v_7, v_8, v_9; \\ v_7 : \varnothing; \\ v_8 : \varnothing; \\ v_9 : \varnothing. \end{pmatrix}; \tag{2}$$

Table 2

Matrix of graph loading by topology "hierarchical star"

No of node	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
v_1	0	e_2	e_2+e_3	e_2+e_4	e_2+e_5	e_6	e_6+e_7	e_6+e_8	e_6+e_9
v_2	e_2	0	e_3	e_4	e_5	e_2+e_6	$e_2+e_6+e_7$	$e_2+e_6+e_8$	$e_2+e_6+e_9$
v_3	e_2+e_3	e_3	0	e_3+e_4	e_3+e_5	$e_3+e_2+e_6$	$e_3+e_2+e_6+e_7$	$e_3+e_2+e_6+e_8$	$e_3+e_2+e_6+e_9$
v_4	e_2+e_4	e_4	e_3+e_4	0	e_4+e_5	$e_4+e_2+e_6$	$e_4+e_2+e_6+e_7$	$e_4+e_2+e_6+e_8$	$e_4+e_2+e_6+e_9$
v_5	e_2+e_5	e_5	e_3+e_5	e_4+e_5	0	$e_5+e_2+e_6$	$e_5+e_2+e_6+e_7$	$e_5+e_2+e_6+e_8$	$e_5+e_2+e_6+e_9$
v_6	e_6	e_2+e_6	$e_3+e_2+e_6$	$e_4+e_2+e_6$	$e_5+e_2+e_6$	0	e_7	e_8	e_9
v_7	e_6+e_7	$e_2+e_6+e_7$	$e_3+e_2+e_7$	$e_4+e_2+e_6+e_7$	$e_5+e_2+e_6+e_7$	e_7	0	e_7+e_8	e_7+e_9
v_8	e_6+e_8	$e_2+e_6+e_8$	$e_3+e_2+e_8$	$e_4+e_2+e_6+e_8$	$e_5+e_2+e_6+e_8$	e_8	e_7+e_8	0	e_8+e_9
v_9	e_6+e_9	$e_2+e_6+e_9$	$e_3+e_2+e_9$	$e_4+e_2+e_6+e_9$	$e_5+e_2+e_6+e_9$	e_9	e_7+e_9	e_8+e_9	0

B_m is the subsystem of interaction with impact objects that contains a set of executive functions, $B_m \subset V$. Subsystem B_m is described be the compliance matrix (3), where the lines are vertices and columns are functions:

$$\begin{array}{cc} & \begin{array}{cccccccc} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 \end{array} \\ B_m = & \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ & & & \dots & & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}. \end{array} \quad (3)$$

Description of function f_i is presented in Table 3.

Table 3

Functions of interaction system

Check of object existence in time:	
– signal transmission (1)	– response reception (2)
Getting orders to be fulfilled: – transition (3) – aerial survey/spraying (4) – results transmission/ reception (5)	Transmission orders for impact objects: – transition (6) – aerial survey/spraying (7) – results transmission/ reception (8)

L_s is the communication subsystem, which contains a set of communication tools, $L_s \subset V$, L_s={Wi-Fi, radio channel}; I_p is the distributed information subsystem with a set of elements, $I_p \subset V$.

Given the fact that the mentioned set Pd_i is finite and orderly, the dynamic change in a set of vertices W can be represented in the vector form:

$$W = \{w_1, w_2, w_3, \dots, w_9\}, \quad (4)$$

where w_i is the elements of set W, indicators of the state of the elements of the structure indicate existence of a specified element at the i-th moment. In the case of a transmitter leaving the range, the object has a sub-program of returning towards the root vertex or towards the moment of signal returning.

We will assume that existence of a relationship between two elements of set W depends only on existence of the specified elements, which does not contradict the principles of construction and operation of a system with a dynamic structure.

The existence of relationship d_{ij} between elements v_i and v_j is designated respectively:

$$d_{ij} = M * w_i * w_j, \quad (5)$$

where w_i and w_j are the indicators of the state of structure elements; M is the adjacency matrix of graph G_α that has the form:

$$M = \begin{pmatrix} 0 & L & R \\ L & 0 & 0 \\ R & 0 & 0 \end{pmatrix}, \quad (6)$$

where L, R are the matrices of loading E_n of the left (L) and the right (R) branches of graph G_α in Fig. 7. The specified matrices (7) and (8) indicate subordination objects and are used for calculation of coverage area of a certain territory by a flock of the accepted topology:

$$L = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{pmatrix} 0 & e_2 & 0 & 0 & 0 \\ e_2 & 0 & e_3 & e_4 & e_5 \\ 0 & e_3 & 0 & 0 & 0 \\ 0 & e_4 & 0 & 0 & 0 \\ 0 & e_5 & 0 & 0 & 0 \end{pmatrix}, \quad (7)$$

$$R = \begin{array}{c} v_1 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} \begin{pmatrix} 0 & e_6 & 0 & 0 & 0 \\ e_2 & 0 & e_7 & e_8 & e_9 \\ 0 & e_7 & 0 & 0 & 0 \\ 0 & e_8 & 0 & 0 & 0 \\ 0 & e_9 & 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

UAV of various models with different communication range can be used as the nodes in Fig. 7, different by functions (Table 1). An increase in the range of consistent communication on a single selected model can be achieved if we activate FCC mode on the communication modules of UAV, set the JP region and activate 33 dBi on the transmitter (Table 4). After updating the module, the maximum transmission range again will be 100 m.

Table 4

Reliable communication distance for nodes with different functions

Functions of nodes	Node number	Reliable communication, m
Master	v_1	2,000
Slave	v_3, v_5, v_7, v_9	470
Repeater	v_2, v_6, v_4, v_8	833

For computations, we will also need a matrix of distances between vertices according to topology. Columns *in* and *out* indicate maximum capacity of transmitters in the conjunction "Master – Slave".

Taking into account the ranges, specified in Table 4, the loading matrix, shown in Table 2, takes the form of Table 5. The specified matrix is symmetric, the service auxiliary information is highlighted in color.

In addition to the mathematical model of a dynamic structure, it is also necessary to control the available area and the area that is already treated by UAV in different time intervals (Fig. 8).

The coordinates of the nodes of the graph, shown in Fig. 8 and attributed to a specific area in different moments, are summarized in Table 6.

Table 6

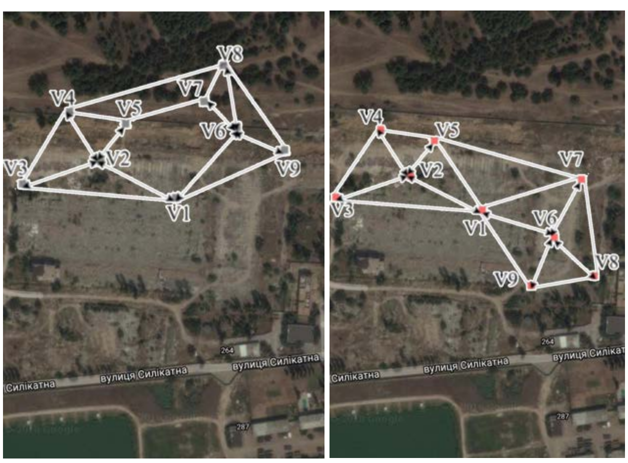
Coordinates of UAV of topology "hierarchical star"

v_i	Coordinates at the i-th moment		Coordinates at the j-th moment	
1	47.009337	31.904993	47.009176	31.904653
2	47.009758	31.903245	47.009598	31.903982
3	47.009487	31.902429	47.009064	31.902880
4	47.010352	31.902580	47.009916	31.903481
5	47.010095	31.903610	47.009796	31.904332
6	47.009905	31.905783	47.008879	31.905836
7	47.010300	31.905340	47.009460	31.906530
8	47.010615	31.906016	47.008441	31.906638
9	47.009743	31.906729	47.008354	31.905549

Table 5

Matrix of loading graph G_α

Node number	in	out	E	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
			max	2,000	2,000	830	830	830	2,000	830	830	830
v_1	0	2,000	2,000	0	2,000	2,830	2,830	2,830	2,000	2,830	2,830	2,830
v_2	2,000	830	2,000	2,000	0	830	830	830	4,000	4,830	4,830	4,830
v_3	830	470	830	830	2,000	0	1,660	1,660	4,830	5,660	5,660	5,660
v_4	830	830	830	830	2,830	1,660	0	1,660	4,830	5,660	5,660	5,660
v_5	830	470	830	830	2,830	1,660	1,660	0	4,830	5,660	5,660	5,660
v_6	2,000	830	2,000	2,000	2,000	4,830	4,830	4,830	0	830	830	830
v_7	830	470	830	830	2,830	5,660	5,660	5,660	830	0	1,660	1,660
v_8	830	830	830	830	2,830	5,660	5,660	5,660	830	1,660	0	1,660
v_9	830	470	830	830	2,830	5,660	5,660	5,660	830	1,660	1,660	0



a *b*

Fig. 8. Area coverage according to graph of topology "hierarchical star" and range of transmitters at different moments: *a* – at *i*-th moment; *b* – at *j*-th moment

We will calculate the surface area, covered by UAV as the area of a polygon – a closed broken line without self-intersections (Fig. 9).

The order of passing vertices P for calculation of the area in Fig. 9 takes the form:

$$P=\{v_1, v_3, v_4, v_5, v_7, v_8, v_9, v_1\}. \qquad (9)$$

Area S, covered by UAV flock:

$$S=\left|\begin{array}{l}\dfrac{y_0+y_3}{2}(x_0-x_3)+\\ +\dfrac{y_3+y_4}{2}(x_3-x_4)+\\ +\dfrac{y_4+y_5}{2}(x_4-x_5)+\ldots+\\ +\dfrac{y_4+y_5}{2}(x_4-x_5)+\\ +\dfrac{y_9+y_0}{2}(x_9-x_0)\end{array}\right|. \qquad (10)$$

The results of calculations of the operation area of UAV flock, obtained by applying the developed method of determining the area of UAV operation, using the graph theory, are shown in Table 7.

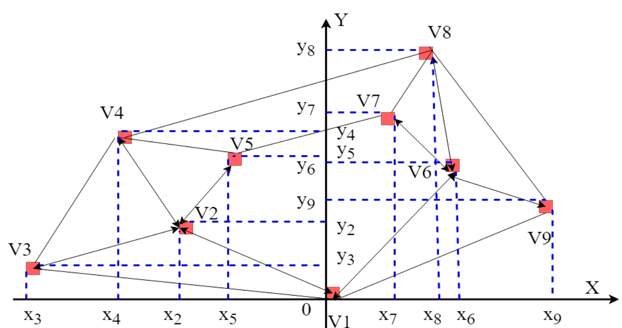


Fig. 9. Polygon from vertices of topology "hierarchical star"

Table 7

Areas according to UAV coordinates

Area at the i-th moment		Area at the i-th moment	
m^2	ha	m^2	ha
40,162.29	4.0	37,109.39	3.7

When generalizing the calculation of area S by the considered route, we can use formula (11), which is applied for calculating the areas of polygons, assigned by their vertices:

$$S=\frac{1}{2}\left|\sum_{i=0}^{n}(x_i-x_{i+1})(y_i+y_{i+1})\right|,\quad x_0=x_{n+1},y_0=y_{n+1}. \qquad (11)$$

For this formula, it is possible to apply the concept of interpolation for finding intermediate values of magnitudes by a discrete set of the known values. Let us assume that there is a system of non-coinciding points $x_i=(i\in 0,1,...,N)$ from the region of coordinate values and the value of functions in this points $y_i=f(x_i)$, $i=1,...,N$. The interpolation process will involve finding from this values of such function $F(x_i, y_i)$, $i=1,...,N$, which would correspond to the value of the sum of areas of trapezia, constructed on points (x_i, y_i).

However, *mixed topologies* of UAV flocks are most common, for example, "*hierarchical star with a bus*". Vertices v_1, v_2, v_3 and v_{10} form the topology "bus". The correspondent graph is shown in Fig. 10.

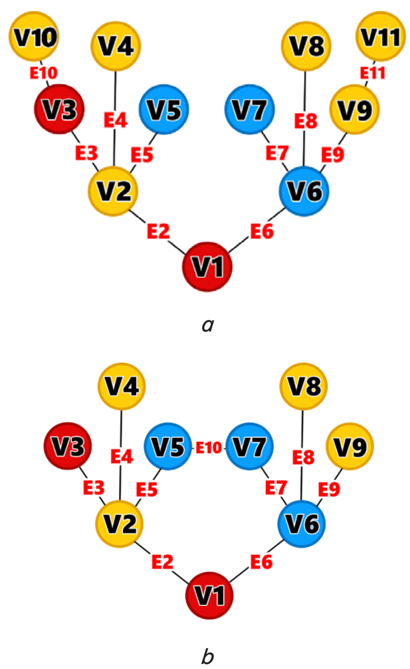


Fig. 10. Loaded graphs of mixed topologies:
a — "hierarchical star with a bus";
b — "hierarchical star with a ring"

The order of passing the graph, shown in Fig. 10, *a*, is determined as:

$$P=\{v_1,\ v_2,\ v_3,\ v_{10},\ v_4,\ v_5,\ v_7,\ v_8,\ v_{11},\ v_9,\ v_6,v_1\}. \qquad (12)$$

The matrix of loading of "hierarchical star with a bus" is composed in the same way, which was considered in detail for the topology "hierarchical star".

Another example is construction of the loaded graph for the topology "hierarchical star with a ring". In this case, vertices v_2, v_5, v_7, v_6 and v_1 form a "ring". The corresponding graph is shown in Fig. 10.

Existence of a "ring" gives variations for a search for the minimal route.

Thus, summarizing the above, it is possible to propose the method for determining the operation area of UAV flock, constructed separately for each of the considered topologies.

6. Discussion of results of study into determining the area, covered by UAV flock, by using a graph theory

In the present research, the method of modeling the structure of complex technical systems applying the graph theory is used to describe the behavior of an UAV flock. The proposed approach allows us to plan more carefully performance of the tasks that use unmanned vehicles of various designs (models).

It should be noted that a set of impact objects L and R should be included in the system with a dynamic structure, which will allow breaking the loading matrix into subordination units and will enable decreasing computational complexity. To determine a graph, it is common to apply adjacency matrices of its separate sub-graphs and loading matrices.

The existence, quantity, and weight of relationships, revealing the impact object, interaction with impact objects, and control of the state of a system are also of importance. However, there will be changes in the structure of the system during its operation. Such changes can be caused by the failure of the elements of the system due to the influence of external factors. The structure of the system can change as a result of the system restoration and extension. Regarding the proposed mathematical model of the system, these factors will affect, first of all, subset V of vertices of graph G_α. When calculating the area, covered by an UAV flock, by the considered route, the use of a large number of interpolation nodes leads to a dramatic increase in the degree of interpolation polynomials, which makes them inconvenient for calculations. It is possible to avoid this problem by splitting the interpolation segment into the right and the left parts with construction of the own polynomial on each of them. Thus, in the given problem it is advisable to consider separately the right and the left wing of the flock (left and right branch of a tree). The simplest, and at the same time most frequently used kind of the interpolation type is piece-linear interpolation. In this case, the assigned points of linear interpolation will be connected by rectilinear segments and the function will approximate to the broken line with vertices in the given points. In this case, it is necessary to calculate the area, covered by a flock.

Latitude and longitude coordinates on an UAV is provided by GPS-modules and the above ground altitude is provided by an altimeter (if altitude is more than 3 m, if lower – it is necessary to use data from an ultrasound altimeter-sonar). Based on pressure measurements by using of a barometer and their comparison with reference values, it is possible to calculate the height above sea level. Three-dimensional coordinates are not necessary for an aerial survey or fertilizer spraying. To find a projection onto the surface, coordinates of GPS-modules are enough. Coordinate rounding to an integer number of seconds gives accuracy of the order of 111 km/3,600≈30 m, which also decreases computational complexity.

If the above conditions are complied with, it is possible to plan a transfer of an UAV flock considering the area that is intended for each architecture of a mobile network, constructed according to different topologies. This approach at the basis of the proposed method for determining the area, covered by an UAV flock, will not allow switching off, for example, communication modules, as it is usually implied by the control systems of UAV to save battery charge. Partition of a loading matrix into subordination units allows decreasing computational complexity, thus reducing power consumption by UAV and prolonging the operation of a formation.

But, it is necessary to keep in mind that companies-manufacturers of UAV in many cases do not allow intervention in the control of navigation and computing processes during creation of proprietary firmware of drones. That is why the implementation of the proposed method can be limited for flight programming of some UAV models.

The proposed approach is capable to increase efficiency of the use of a heterogeneous network based on unmanned vehicles of any structures, promote development of technologies for monitoring various indicators, remote sensing of natural resources, photogrammetry etc. The research results can appear useful for agricultural enterprises to control the types, similarities and other characteristics of farmed crops and for differentiated spraying crops. Now it is difficult to anticipate the challenges of objective or subjective nature when attempting to develop the described research in these directions. Nevertheless, it is possible to say that no additional costs are required to use the proposed approach in specific production situations.

It is advisable to carry out subsequent research in the direction of searching for optimization methods for constructing complex technical systems with a dynamic structure, taking into account the features of the architecture of implemented structures, technical characteristics and functionality of devices aboard unmanned vehicle.

7. Conclusions

A mathematical model of the system with a dynamic structure was developed in the conducted research. The provisions of the graph theory and the methods for discrete optimization were used in order to analyze a change in the area, covered by a flock. As a result:

1. We analyzed the change of data transmission range between the UAV in a flock, the topology of which is dynamic and changes due to the influence of external factors and as a result of a change in the route of each drone. The use of the transfer nodes in case of exceeding the range of sustainable communication between a drone-executor and the server was foreseen. As far as hardware is concerned, such relay is provided by cold reservation of additional computer components aboard UAV. In order to save the limited capacity of the drone battery, it is foreseen to connect such additional components to the power supply only in the event of a critical situation, in which signal relaying is required.

2. Formalization of the structure of the system in the form of a loaded directed graph was proposed, adjacency matrices and loading matrices for the basic and hybrid topologies of UAV flock were analyzed. This approach makes it possible to identify a number of indicators and efficiency criteria for a system structure, as well to ensure its synthesis.

3. The proposed method for calculation of the area of UAV flock operation was proposed. This area was composed from the sum of areas of polygons, assigned by vertices with the coordinates, obtained from GPS modules of UAV.

All results, obtained for UAV, can be generalized for any unmanned vehicles' design.

The implementation of the proposed method for determining the operation area of a unmanned vehicles' formation by applying the graph theory will make it possible to plan more effectively the time to perform the task and the number of drones in a formation for covering the territory of the assigned size.

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