Досліджено різноманітні конструкції варіаторів з двох точок зору: аналіз власне конструкції і аналіз методів та прийомів, застосованих на шляху створення варіатору. Для розв'язання завдання пошуку нових конструкцій механізмів розроблений метод трансформації графів. Він включає наступні евристичні прийоми. Зменшення кількості вузлів графу при спрощення конструкції або для відсікання несуттєвих елементів. Зміна розташування ребр графу і зіставлення нового розташування ребр і необхідних змін в конструкції. Пошук всіх можливих варіантів розташування деталей, що відповідають одному і тому ж графу. Введення в граф вузлів відповідних властивості, для реалізації якої в конструкцію може бути додана деталь (вузол) або навпаки, деталь може бути видалена з конструкції, а властивість, яка вона реалізовувала, буде реалізовано іншими деталями. Розроблено оригінальну конструкцію варіатору, в якому навантаження передається зубчатим зачепленням без використання фрикційних дисків і гнучких ланок. Плавне регулювання передатного відношення забезпечується зубчастим колесом зі змінним кроком. Колесо складається з трьох зубчастих секторів два з яких беруть участь в зачепленні. Третій ненавантажений сектор здійснює поворот з кутовою швидкістю більшою, ніж швидкість обертання вихідного валу, поки не займе положення перед входом в зачеплення. Варіатор позбавлений такого недоліку, як коливання передатного відношення. На основі аналізу бондграфів визначено ККД, який в залежності від швидкості вихідного валу коливається в межах 95...97 %. Недоліком розробленої конструкції є недостатньо широкий діапазон регулювання R=1,5...1,6, який обмежений коефіцієнтом перекриття ε_α

Ключові слова; графова модель, колесо зі змінним кроком, варіатор зачепленням, метод трансформації графів

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DEVELOPMENT OF THE POSITIVE ENGAGEMENT CONTINUOUSLY VARIABLE TRANSMISSION DESIGN WITH THE APPLICATION OF GRAPH THEORY

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1. Introduction

There are several areas where application of continuously variable transmission (CVT) ensures higher technical performance of machines. In the first place, it is automobile engineering. Such car manufacturers as Honda, Toyota, Ford, and Nissan have been working on the improvement of CVTs which successfully compete with automatic gearboxes. CVTs ensure optimum transmission ratio at any moment, which improves the vehicle's dynamics, increases its efficiency and reduces harmful emissions. A CVT is also a promising technology for using in transmission of hybrid cars. Another relatively new area of using CVTs is wind turbines. Their drive parameters need to be adapted to wind velocity. There is a whole range of areas where CVTs have long been widely used: these are machines of food industry, agricultural machinery, as well as machine tools building.

Mechanical CVTs are much smaller in dimensions and weight than electric and hydraulic drives and provide a smooth change of angular velocity. That is why mechanical CVTs are used in vehicles. Mechanical CVTs are subdivided into frictional CVTs and those that transmit loads by gearing. The most widely used CVTs are frictional ones, mainly V-belt, bevel and toroidal. Mechanical CVTs compared with gears have large dimensions and weight, lower efficiency and rigidity of transmission ratio. There are only two types of CVTs with toothed gears: plate CVTs and those with sectors that are deformed. In this regard, the synthesis of the design of a CVT using positive engagement, which by its main indicators would be close to gear train with constant transmission ratio, is relevant.

No area of the mechanical engineering contains such a number of original ideas and a variety of design solutions, including patented. The search for new designs of CVTs is a rich soil for studying the methods of design that are traditional for the mechanical engineering and the new ones, adopted from other scientific disciplines. That is why designing the CVT is a perfect proving ground for application of heuristic methods, used in the system analysis, DSS, project management and the mechanical engineering itself. As a rule, the result of the search is patented designs or the designs, for which preliminary calculations were carried out and sketch layout was performed. However, the way an inventor passed in the search for an idea for a new CVT, then design implementation of this idea usually remain unknown. The inventor himself is later unable to clearly understand what stages of working on the design were passed, what ideas, methods and techniques were used explicitly or not fully realized. Most often, the experience of the search for a new design – a pattern of the designer's search can be more useful than the obtained result, implemented in the form of a patent.

2. Literature review and problem statement

Due to complexity and variety of designs of CVTs, any publication on this issue can be considered from two points of view: analysis of the design of a CVT itself and analysis of methods and techniques used in search of the design. We will follow such logic in the analysis of publications. V-belt and chain CVTs found their wide application in drives of cars. Despite the fact that the operation principle of these CVTs is different, they are similar in their layout. The simplified models were used to research the dynamics of the variators [1]. The belt of the V-belt variator is presented as a set of springs connected by hinge joints. The chain is presented in the form of sections (bars), connected by elastic elements. In each of such elements the power flow is divided into two flow; in one of the flow elasticity is modeled by a spring, and in the other – with the use of a hydraulic cylinder [1].

The use of simplified schemes instead of actual designs is a common way of constructing a mathematical model of a mechanism. But construction, comparison and analysis of simplified schemes can be the impetus to search for new designs. In this case, similarity of dynamic models could give a boost to use a chain instead of a belt: V-chain CVTs: or location on the belt of metal plates that contact disks – metal V-belt [1].

A change of transmission ratio in a V-belt CVT is provided by axial motion of pulleys [2]. Designing a mechanism for motion of pulleys is quite a challenge and this mechanism contains more parts than V-belt transmission, which provides a load transmission and a continuously variable transmission ratio. A CVT typically must contain three units: the transmission with continuously variable transmission ratio, the mechanism that connects this transmission to the output shaft, and the mechanism of variator control.

The mechanism that connects transmission to the output shaft is compulsory in pulse variators. The advantage of such mechanisms is refusal to use friction to transfer the load [3]. The design of the hinge four bar mechanism with variable length of bar as a means of transmission with variable transmission ratio is quite simple. Along with this, simplicity of a single unit requires complicated mechanisms that control the change in length of bar and connect the bar with the output shaft at a time when its angular speed corresponds to an assigned value [3].

Another type of CVTs, which are widely used, including in cars, is toroidal. In particular, these are the CVTs of the company Ultimate Transmissions that include four disks with toroidal surfaces and two rows of the intermediate rollers (Double Roller Full Toroidal Variator). Toroidal CVTs are the most compact of frictional CVTs due to the use of the power split principle – location from two to four rollers in each row [4]. The principle of mutual compensation of forces arising in contact of rollers and disks was implemented. In addition, the function combination principle is used, as control of the two rows of rollers is carried out by a single mechanism. For toroidal CVTs, the problem of kinematic and geometric parameters optimization to achieve minimum weight and maximum efficiency was solved. The designed mechanism has efficiency of 86.7 % for a wide range of options, which is a good result for CVTs, but this value is much worse compared with gear transmission [4].

The spherical CVTs has less loading capacity that the toroidal, but makes it possible constantly to change the position of the axis of rotation of the output disks. This feature allowed using it in the drive of robots. In the spherical CVTs, mutual compensation of forces and power split are also used [5].

Despite the widespread use, frictional CVTs have a number of disadvantages: low efficiency, wear of parts, non-sustainability of transmission ratio, which is why the greatest number of patents were awarded to positive engagement CVTs [6]. The only CVT using gearing, manufactured by industry, is the chain CVTs, which has a number of original design implementations. They include the following CVTs: ICVT Varibox, Fixed-Pitch CVT and CVT Anderson, which have a sprocket with a variable shape instead of a chain that changes its shape depending on the location of the pulleys [6]. In ICVT Varibox, it is a set of parallel bars that move from the center of the sprocket. In Fixed-Pitch CVT, it is an assemblage of sprockets, located on a variable radius, which form a single sprocket. In Anderson CVT, sprockets are the cones, in which «floating» teeth are established. The teeth can change their position by height and be shifted around the perimeter within the gap. Thus, a sprocket is a gear with a variable pitch. If we manage to construct a gear, so that it can mesh with another gear, then the chain will not be required.

A cognitive map of search stages was used to develop a positive engagement CVT without a chain transmission [6]. Along with the cognitive maps, the mechanism can be represented as a graph. A variant of description of the mechanism in the form of a graph was proposed in article [7] and was based on the design method with the use of SFG (signal flow graphs). In this method, the nodes are variables, and the edges are functional dependences. The SFG method is based on the four following principles. Direction of flow between nodes is designated by arrows. To obtain the output signal, the input signal is multiplied by a weight factor of a given edge. The value of a signal in a node is the sum of all input signals. The value of a signal in a node is transmitted by all the edges that extend from it [7].

Bond-graphs offer a great opportunity for analysis of mechanisms and electromechanical systems. The bond-graph for a metal V-belt CVT, in which edges of the graph are in conformity with angular velocities and moments, and nodes are mechanical parts (nodes) and electrical components, was developed [8]. A bond-graph allows us to set the rules of direct causality between forces (moment, stress, pressure) and flows (velocity, flow, pressure), which made it possible to obtain the differential model that describes mechanical, electrical and thermal effects in the drive of a hybrid car [8]. It was noted that a bond-graph is a unique visual way of graphic analysis of power flow, the latter is important, not only for analysis, but also to search for new design solutions.

3. The aim and objectives of the study

The aim of present research is to develop a design of the CVT, which should have minimal dimensions and weight compared to known designs.

To accomplish the aim, the following tasks have been set:

– to eliminate the use of friction disks and flexible links in the design of a variator;

– to develop the design of gear transmission with a variable gear pitch;

– creation of a pattern of performed research, which could be used by other researchers, as well as summarizing the obtained experience in the use of the graph theory in the search of new designs of mechanisms.

4. Materials and methods of research into design of the variator that uses gearing

The methods of research are heuristic methods, based on the use of cognitive and graph models of representation of designs. These methods will be consistently used to search for an original scheme of a CVT, development and evaluation of its design. The following design methods were applied for development of a rational design: decomposition, split power flow, uniform strength, and rational layout. Analysis of efficiency of the designed gearbox was performed with the help of bond-graphs, and analysis of specific features of transmission geometry was carried out using the theory of gearing.

4. 1. Search for designs of a gear wheel with a variable pitch

It is advisable to start the search for an original idea with creating a cognitive model for the analysis of the place of the positive engagement CVT among other types of CVTs [9]. In this case the two super-classes of features are transmission and a CVT, the subclasses are kinds of transmissions and CVTs (Table 1).

Table 1

Cognitive model of relations of the superclass «transmission – variator»

Transmission	Relations	Variator
Friction transmission	Corresponds	Frictional
V-belt transmission	Corresponds	V-belt
Chain transmission	Corresponds	Chain
Gear transmission	Does not correspond	*Search for design*

The superset and a subset are related by AKO (a kind of). That is, every element of a subset is included in a superset and properties of a subset are inherited from a superset. The types of transmission and CVTs are connected by attributive relations. A friction, a chain, and a V-belt CVTs correspond to friction, chain, and V-belt transmissions. The class of existing technical solutions corresponds to these relations. Gear transmission exists, but there is no positive engagement CVT. Thus, there is a technical solution that can be implemented, but currently is not implemented in practice.

To implement a positive engagement CVT, a gear wheel with the number of teeth, which is continuously variable, is required. This is possible at a continuously variable pitch between teeth. To search for this design, it is necessary to use the graph «Interaction network», which displays the gear wheel with a continuously variable pitch. The graph «Interaction network» includes nodes A_1 and A_2 of the pinion and gear axis; three pairs of teeth – Z_2 and Z_2' are the teeth that are in mesh; Z_1 and Z_1' are the teeth that should engage into mesh; Z_3 and Z_3' are the teeth that come out of mesh. The nodes that correspond to teeth are connected by edges with the nodes that correspond to the axes of the gears (Fig. 1).

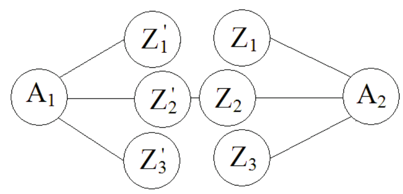


Fig. 1. Graph «Interaction network» of a gear wheel

The heuristic nature of the problem being solved requires another, more general approach to plotting the graph of the mechanism. Experience of using the graph «Interaction network» shows that it should not turn into a specification of all parts and elements of parts but rather should reflect only those nodes and parts that are important for analysis [10]. We will simplify the model, as we are interested not in transmission, but in the gear wheel with a continuously variable pitch 2 (Table 2). Gear mesh pitch is a property of teeth to be located at a certain distance. This property is provided by a set of parallel bar in ICVT Varibox or a set of sprockets in Fixed-Pitch CVT. Even not having a particular design solution, we will introduce into the model new nodes t that to implement the property – the distance between the teeth is equal to the pitch. We will join nodes Z_1, Z_2, Z_3 and nodes t with the help of edges. Functioning of gear mesh is provided by the tooth that is meshed and the tooth that is coming into mesh. The tooth that came out mesh had performed its function and its location makes no importance until it enters the mesh zone next time. Changing the value of pitch t, we will obtain a gear wheel with a continuously variable pitch and a variable number of teeth, including fractional. Mutual location of the tooth, which is coming into mesh and the tooth that is in mesh, can be assigned using a cog. Thus, the property, for implementation of which a part or a node can be added to the design, may be introduced to the graph of a mechanism in the form of a node. A new part of the cog, diameter of which determines the value of pitch t, corresponds to property t. A cog has a conical shape, which makes it possible to continuously variable a pitch in mesh.

We will transform the graph once again. Let us introduce to the graph node C, correspondent to the property to provide regulation of pitch t. As the location of the tooth that came out of mesh does not play any role, while the sector, composed of two teeth, is in mesh with the wheel, the third tooth rotates until the tooth takes the position before engagement into mesh, that is, until the cog stops in the tooth that is in mesh.

The graph of a gear with a variable number of teeth includes node J, connected with the teeth and the axis of the gear. Structurally, node J is a mechanism that connects the teeth under load with the output shaft and makes unloaded teeth rotate with angular velocity that is greater than the velocity of rotation of the output shaft (Table 3).

Table 2

Graph transformation

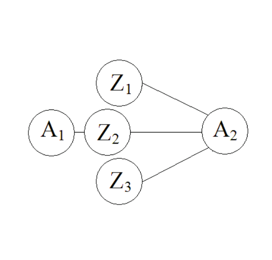
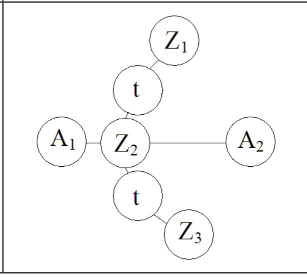
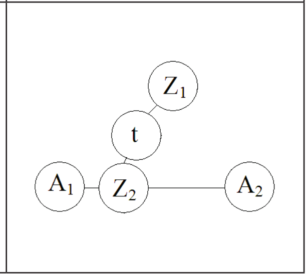
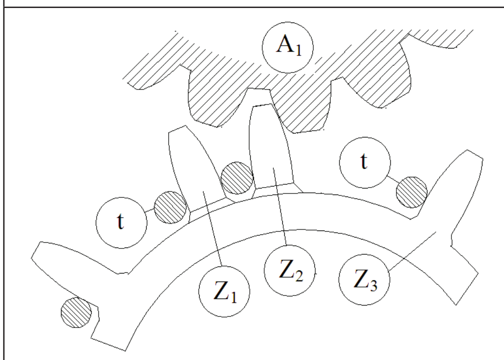
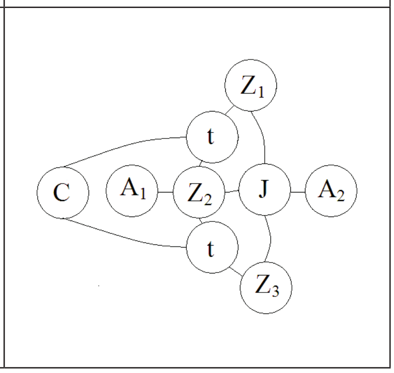
Simplified graph of transmission	Graph of transmission with nodes *t*	Graph of gear sector, replacing the gear
		

Table 3

Gear wheel with variable pitch and its graph

Scheme of a gear wheel	Graph of a gear wheel
	

Based on the resulting graph, we designed a gear wheel with a variable pitch, mutual location of the tooth that is coming into mesh and the tooth that is in mesh can be assigned by using the conical cogs (Table 3). Since the pitch of a gear is variable, the gear with which it is meshed must be bevel. The cogs move in parallel to the axis of the gear with a variable step, it is a cylindrical spur gear. Transmission that consists of the bevel and spur gears is cylindrical-bevel.

If we summarize the techniques, used to find a gear with a variable pitch, we will obtain, along with the original design, the method of the search for new designs of mechanisms and call it the method of graphs transformation. This method improves the method «Interaction network» and includes four heuristic techniques:

– a decrease in the number of graph nodes at simplification of a design or to remove non-essential elements;

– a change in location of graph edges and comparison of a new location of edges and necessary changes in design;

– a search for all possible options for location of the parts that correspond to the same graph;

– introduction to graph of the nodes, corresponding to the property, for implementation of which a part (a node) can be added to a design or, vice versa, can be removed from a design, and the property it implemented will be implemented by other parts.

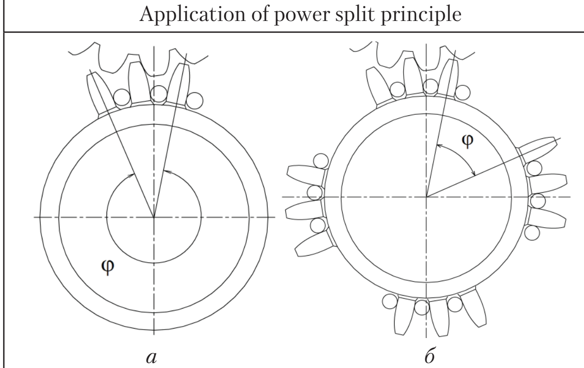
Coaxial structures of the CVT are required, thus transmission or transmissions that connect the input shaft with the bevel gear, node *A*, are required. The main node *V* is a gear wheel with a variable pitch. Node *C*, which provides regulation of pitch *t*, is also necessary. This node controls the motion by cogs and eventually by gear ratio to the CVT. Node *J* is the mechanism that connects the teeth under load with the input shaft and makes unloaded teeth rotate with angular velocity that is greater than the rotation rate of the output shaft. Thus, the decomposition principle was used, due to which the task of designing the CVT is split into four subtasks – designing nodes *A, V, C, J*.

The tooth, connected to the sleeve with a supporting bar, will be called a sector. The problem of this design is a high rotation velocity of the sectors that come out of mesh. A tooth that is in mesh, turns by a few degrees, and the tooth that came out of mesh, is to turn almost by 360 degrees within this time.

At a four-tooth sector (*b*) compared to a single-tooth sector (*a*), the angle, by which a tooth that came out of mesh is to turn until it stops in the cog, decreases by four times (Table 4). When designing node A, we will apply the power split principle. At a four-tooth sector, the number of driving bevel pinions may be increased up to two or even four.

Table 4

Designing node *V*

Application of power split principle	Solid model of the sector
 *a* – single-tooth sector; *b* – four-tooth sector	

For structural implementation of the obtained model of a gear wheel with a variable pitch, selection of pitch diameter of sectors *d* should be based on the condition of an integer of gear teeth at cutting. Taking into account that there can be some sectors of a tooth wheel, it is necessary to meet the condition when selecting a pitch diameter:

$$\frac{d}{mq} = n \in N,$$

where m is the module of teeth of sectors, q is the number of sectors in a gear with a variable pitch, n is the integer, N is the set of integers.

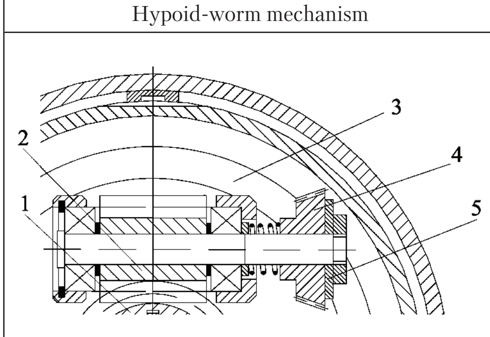
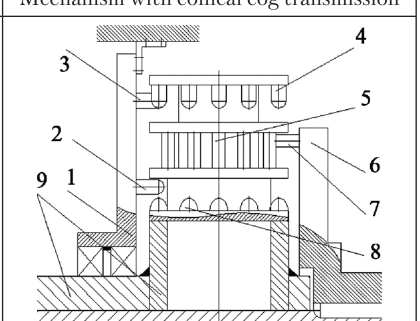
4. 2. Search for a design of the mechanism that connects the gear with a variable pitch with the input shaft

As a result of development of the design of control node *J*, two alternative designs were created: the hypoid-worm mechanism and the mechanism with conical cog transmission. In the hypoid-worm mechanism, each of the sectors has a supporting bar, in the bearings of which rotates the worm 2 that comes into engagement with the worm wheel 1, rigidly connected with a sleeve. Hypoid pinion 4 that comes into mesh with the hypoid wheel 3 rotates on the worm axis (Table 4). Hypoid pinion 4 transmits rotation to the shafts of worms 2 through the friction coupling 5. The tooth of the sector transmits loads through the worm to the worm wheel. At this, the worm does not rotate but operates as an element of the gear coupling. Hypoid pinion, rolling on a stationary hypoid wheel, cannot turn the worm, since the moment, at which the friction coupling slips, is less than the moment that is necessary to rotate a worm in loaded state. After the tooth of the sector comes out of mesh, its worm, being unloaded, starts to rotate under the influence of a force in pinion mesh. In this case, the sector will rapidly rotate until its tooth catches up with the tooth of the sector, which through the conical cog stops in the tooth the next sector. At this, the friction coupling of the sector operates in sliding mode (Table 5).

Control mechanism with bevel cog transmission includes fixed disk 1, on which external 3 and internal 2 rows of cogs, forming bevel cog wheels, are located. The sprocket is connected to supporting arm 9 through a sleeve. The three-ring cog pinion is mounted in the supporting bar with the possibility of rotation and axial motion: upper ring 4, middle ring 5, and lower ring 8. Middle ring 5, connected to the bevel cog wheel, in the form of disk 6, is connected with the input shaft and cogs 7. Depending on the ratio of friction forces in cog transmission mesh, the pinion takes the upper or the lower position. In this case, either pinion 4 and wheel 3 or pinion 8 and wheel 2 are in mesh, thus angular velocity of the loaded sector is equal to angular velocity of the output shaft, and the unloaded sector rotates with angular velocity that is greater than angular velocity of the output shaft (Table 5).

Table 5

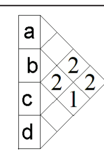
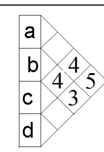
Designing node *J*

Hypoid-worm mechanism	Mechanism with conical cog transmission
	

Two variants of node *J* were developed at draft designing of nodes *A*, *V*, *C*, *J*; we will compare these two designs using the method of estimation of the options for a design solution [10]. The casing, the supporting arm, the sectors and the output shaft are considered as unchanged elements. Combined elements were separated: transmission that connects the sectors with the output shaft – *a*; transmission that ensures rotation of sectors with angular velocity greater than angular velocity of the shaft – *b*; the physical principle that provides mechanism operation in the mode of power transmission from the sectors to the output shaft – *c*; the physical principle that provides the mechanism operation in the mode of rotation of the sectors with angular velocity greater than angular velocity of the shaft – *d*. Numerical values were assigned to each of the combinations of the elements. The total of the estimates is the degree, to which the design corresponds to the task (Table 6).

Table 6

Comparative analysis of the options of node *J*

Parameters of combined elements	Points	Score
cog conical transmission	2	
cog conical transmission	2	
force transmission by pressing	2	
sliding off teeth of cog transmission	1	conical cog mechanism
worm transmission	4	
hypoid transmission	4	
self-braking in worm transmission	5	
slippage in mufti-friction coupling	3	hypoid-worm mechanism

As a result of application of the method of estimation of design solution options, it was found that the hypoid-worm mechanism is better; its score is 16, whereas the mechanism with the conical cog transmission has the score of 7.

5. Analysis of design of the variator that uses gearing

The developed variant of design of the CVT with the hypoid-worm mechanism is shown in the longitudinal (*a*) and transversal (*b*) cross-sections, as well as in the solid model (*c*) (Fig. 2). The CVT includes casing 3, lids 1 and 4, driving shaft 5, countershaft shafts with conical pinions 8 and 10, the wheel with variable pitch, rotating on shaft 6. The wheel with variable pitch is composed of three sectors 11, 12 and 13, which can be rotated relative to each other and to sleeve 9. Each of these sectors has supporting bar 17, in the bearings of which there rotates worm 18, which come into mesh with worm wheel 2, rigidly connected to sleeve 9. On the worm axis, there rotates hypoid pinion 20 that comes into mesh with hypoid wheel 21, fixed in cup 22. The hypoid pinion 20 transmits rotations to worm shafts 18 through friction sleeve 23. Sleeve 22 moves on the spline, cut on the casing, using the rack transmission and flywheel 19 and in this case moves sleeve 9 on the slots of driven shaft 6 through the ball bearing. Between the teeth of the sectors, there are conical cogs 14, fixed in rings 15 that freely rotate in bearing 16. The motion of driving shaft 5 through the conical tooth transmission is transmitted to countershaft shaft 7, from conical wheel 10, it is transmitted to the sectors and then, using worms 18, to worm wheel 2, sleeve 9 and through the slot connection to driven shaft 6. The tooth of sector 12 transmits the load by means of worm 18 to the worm wheel. After the tooth of sector 13 comes out of mesh, its worm begins to rotate under the influence of the force in pinion mesh 20. The sector rotates rapidly until its tooth catches up with the tooth of sector 11, which through conical cog 14 stops to the tooth of sector 12.

The diameter of the conical cogs was selected so that the tooth pitch of the driven wheel should be equal to the pitch of teeth of conical pinion 10. When the driven wheel moves in relation to this pinion, the diameter of the cogs and the pitch between the teeth change. Transmission is designed so that at meshing of the wheel with a variable pitch at the maximum radius of conical pinion 10, there are no relative movements of the sectors, as the number of teeth of the wheel with a variable pitch is equal to the number of teeth of three sectors.

Gear ratio range R may be limited by the transverse contact ratio ε_α at the larger radius of the bevel gear wheel, because the height of the teeth of the toothed cone is permanent, and the pitch increases from the smaller to the larger radius.

$$\varepsilon_\alpha = \frac{c_1\sqrt{R} + c_2 R + c_3}{\pi m_{\min} R \cos\alpha_w},$$

where coefficients are

$$c_1 = \sqrt{r_{a1\max}^2 - r_{f1\max}^2}\sqrt{r_{a1\min}^2 + r_{f1\min}^2},$$

$$c_2 = r_{1\min}\sin\alpha,$$

$$c_3 = \sqrt{r_{a2}^2 - r_{f2}^2} - r_2\sin\alpha_w.$$

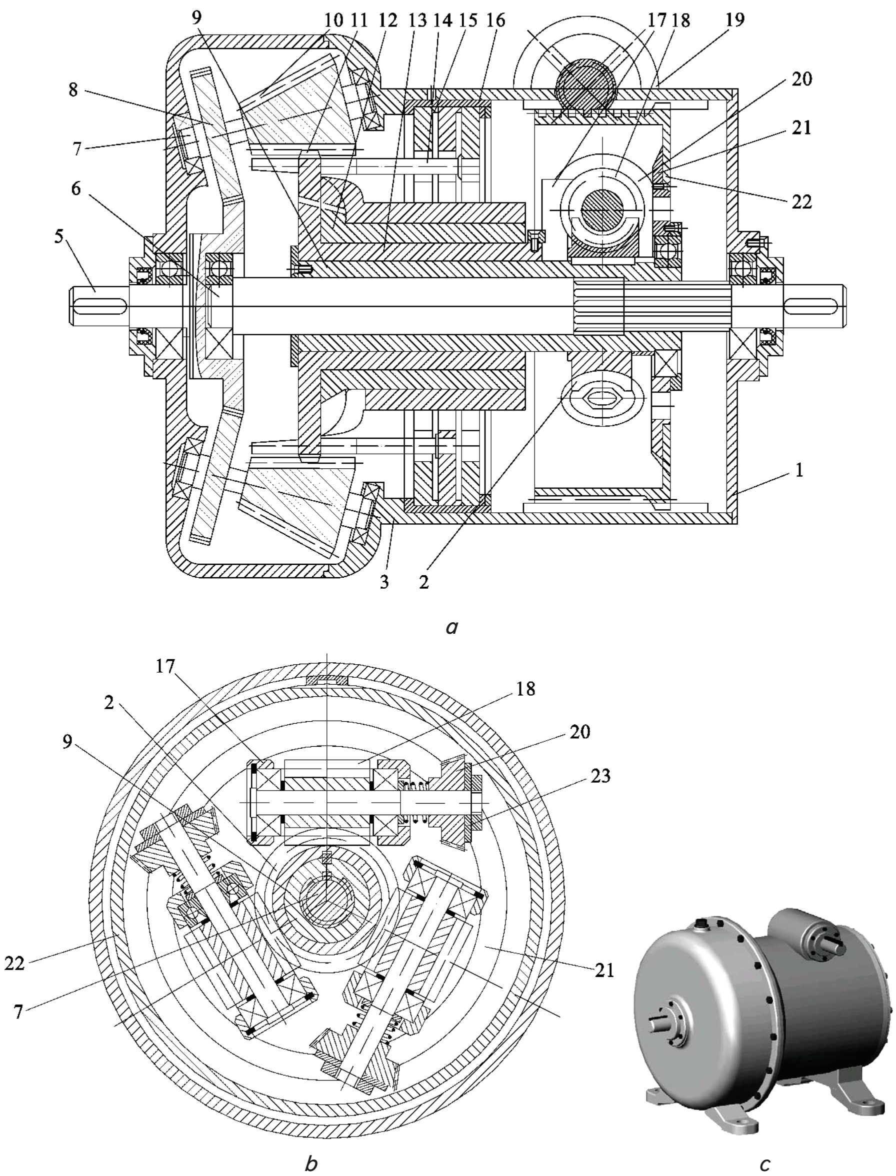


Fig. 2. Design of the CVT:
a — longitudinal cross-section; *b* — transverse cross-section; *c* — solid model

In these dependences, the radii of the circles of the gear wheel with a variable pitch and the maximum and minimum of the bevel pinion are: r_{a2}, $r_{a1\max}$, $r_{a1\min}$ are the radii of root; r_{f2}, $r_{f1\max}$, $r_{f1\min}$ are the radii of tip; α_w is the pressure angle. For the designed variator, the control range $R=1.52$, at the overlap factor on the larger radius of the conical wheel $\varepsilon_\alpha=1.07$.

The bond-graphs for the case when one sector transfers the load, while the other two are moving rapidly (*a*), and when the two sectors transfer the load, and one is moving rapidly (*b*) (Fig. 3), were constructed.

The power from the input shaft is transmitted through the bevel pinion (*b*) to the sector (*s*). Worm gearing is a node of type «1» – the speed of the worm wheel is the same for the main power flow and control flow. On the one hand, from the worm wheel, the motion is transmitted to the output shaft, on the other hand, through the worm to the hypoid pinion (*g*). The bond-graph allows determining efficiency of the CVT. In the case, when the two sectors are meshed, they move as a unit, but the load on them is different, because of uneven distribution of load between the teeth. That is, there is an opportunity to explore uneven distribution of load on efficiency. In the case when the two sectors are moving rapidly, efficiency is lower because more power is consumed in node J. That is efficiency of the CVT decreases at an increase in transmission ratio.

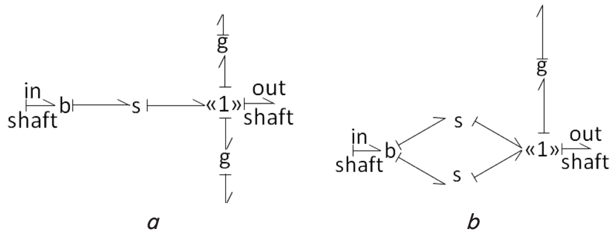


Fig. 3. Bond-graph of the CVT:
a — one tooth is meshed; *b* — two teeth are meshed

To determine efficiency, the moment that is transferred in node J was found, it is equal to the moment of triggering a friction couples. The moment of friction of the control mechanism, reduced to the worm shaft is:

$$T_f = (T_s + T_r)/i + T_w + T_g + T_b,$$

where moments of friction in the sleeve of the sector is T_s, of rings with cogs is T_r, in worm gearing – T_w and hypoid gearing – T_g, in worm bearings – T_b, and i is the transmission ratio of worm gearing.

Inertia moment, brought to the worm shaft is:

$$T_i = (J_s + J_c)\varepsilon / i + (J_w + J_g)\varepsilon,$$

where J_s is the inertia moment of the sector, J_c is the inertia moment of the cogs, J_w is the inertia moment of the worm, J_g is the inertia moment of hypoid gearing, ε is the acceleration of the worm. The friction couples must be set so that the moment of its triggering T should be more than the moment that is needed for the accelerated motion of the sector, but less than friction moment in loaded worm gearing T_0:

$$T_f + T_i \le T \le T_0.$$

The highest permissible inertia moment was found from this dependence. It is possible to achieve maximum values of acceleration of the sectors by decreasing inertia moment of the system, for this purpose, the hypoid gearing should be made of textolite and the worm – of light alloys. Efficiency of the designed CVT at maximum velocity of the output shaft is 97 %, which approximately corresponds to efficiency of the double-step cylindrical gearbox. At the minimum velocity of the output shaft, efficiency is equal to 95 %.

6. Discussion of research results and comparison of the found design to known designs of CVTs

The problem of all friction CVTs is fluctuation of transmission ratio depending on the load. This limits the possibilities of their using in the mechanisms that require kinematic accuracy, for example, in machine tools. In the chain CVT this phenomenon also occurs through deformation of plates of the chain, which leads to a change in its location on sprockets at the change of torsion moment. In a well-designed CVT, an increase in load leads to deformation of the sectors and worm gearing. But gear and worm transmissions have much greater rigidity, and most importantly, their deformation in no way affects transmission ratio. That is why we can assume that this is the only CVT, devoid of such shortage as non-sustainability of the selected transmission ratio.

A CVT using positive engagement has efficiency within 95–97 %, which is better than in chain CVT – 92–95 % and is much better than in friction CVT – 80–88 %.

The designed CVT is more reliable than friction variators and has no surfaces that wear out under load and the chain, in which plates that are in contact with sprockets wear out and get jammed. It is more compact, thus, at maximum moment of 160 Nm, its length is 260 mm and diameter is 180 mm, at approximately the same loading capacity, V-belt variator of VARMEC company has dimensions of 160×270×555, the most compact disk CVTs are: SITI series MK-MKF, made by manufacturer SITI S.p.A, has dimensions of 350×390×400; series VAR 30/1 of company-manufacture VARMEC has dimensions of 307×320×393. For clarity, Fig. 4 shows comparison of dimensions of the V-belt, friction disk and the designed CVT of approximately the same load capacity.

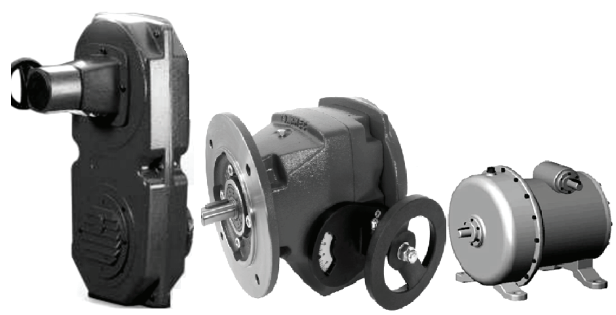


Fig. 4. Comparison of dimensions of CVTs

The drawback of the developed design is an insufficiently wide gear ratio range R=1.52. Mentioned above CVTs have the gear ratio range within 3.5...4. To increase the gear ratio range R, it is advisable to locate two mechanisms of the CVT sequentially in one box with the general control mechanism similarly to the design of the toroidal type – Double Roller Full Toroidal.

The main direction for further research is to increase the gear ratio range through the use of cycloidal meshing of a bevel pinion with sectors instead of the involute one, as well as development of the design with the double node of the gear wheel with a variable pitch. Together, these two innovations can increase the gear ratio range up to two.

The developed CVT can be used in cars, and in first place with large engine capacity. It is also possible to use it in machine tools, where rigidity of the kinematic chain is extremely important. In this case, a CVT can be mounted along with a gearbox. In this case, the common ratio of the preferred numbers of gearboxes must be smaller than the gear ratio range. In this case, a continuously variable transmission ratio within the scope of the product of gear ratio range of a gearbox and a CVT is achieved. In machine tools, transmission ratio of the kinematic chain is set before the beginning of machining a part, so the need to adjust both a CVT and a gearbox is not a disadvantage.

7. Conclusions

1. The original design of the CVT, in which load is transmitted by gear engagement without using friction disks and flexible sections, was developed. The CVT has minimum dimensions and weight compared with the known designs of both friction and chain CVTs. The disadvantage of the design is an insufficiently wide gear ratio range $R \leq 1.6$.

2. A continuously variable of transmission ratio is ensured by a gear wheel with a variable pitch, composed of three gear sectors, two of which are involved in meshing, and the third unloaded sector performs a turn with angular velocity that is more than the velocity of rotation of the output shaft until it takes the position before coming into mesh. Rotation velocity of the third sector limits the gear ratio range. Rational design of the positive engagement CVT must be split flow with the number of teeth of the sectors not less than four.

3. To solve the problem of finding new designs of mechanisms, we developed the graph transformation method, which includes four heuristic techniques:

– a decrease in the number of graph nodes at simplification of a design or to remove non-essential elements;

– a change in location of graph edges and comparison of a new location of edges and necessary changes in a design;

– a search for all possible options for location of the parts that correspond to the same graph;

– introduction to graph of the nodes, corresponding to the property, for implementation of which a part (a node) can be added to the design or, vice versa, can be removed from the design, and the property it implemented will be implemented by other parts.

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