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DEVELOPMENT OF THE METHOD FOR CALCULATION OF CANTILEVER CONSTRUCTION'S OSCILLATIONS TAKING INTO ACCOUNT OWN WEIGHT

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Запропоновано метод розрахунку на згинні коливання вертикальних консольних конструкцій з урахуванням власної ваги. Метод ґрунтується на точному розв'язку відповідного диференціального рівняння коливань в частинних похідних зі змінними коефіцієнтами. В аналітичному вигляді за допомогою безрозмірних фундаментальних функцій виписані формули для динамічних параметрів – переміщення, кута повороту, згинального моменту та поперечної сили, які повністю характеризують стан стрижня.

У загальному вигляді виписано частотне рівняння та визначено метод пошуку його коренів. Показано, що задача визначення частот власних коливань зводиться до знаходження із частотних рівнянь відповідних безрозмірних коефіцієнтів. Знайдені формули, якими визначаються власні форми коливань. Виписано алгоритм, який дозволяє визначати частоти та форми власних коливань консольних конструкцій з довільною наперед заданою точністю.

Алгоритм реалізовано на прикладі наскрізної стрижневої башти. Встановлено, що числові значення, отримані авторським методом, співпадають з результатами, отриманими за допомогою програмного комплексу, який реалізує метод скінченних елементів.

Порівняно з наближеними методами, даний метод дозволяє отримати достовірнішу картину коливань консольних конструкцій, оскільки саме точний розв'язок несе в собі інформацію якісного характеру та формує найбільш повну картину фізичного явища, яке розглядається. Завдяки використанню явних аналітичних формул, підвищується точність розрахунку на згинні коливання.

Запропонований метод не потребує дискретизації конструкції та являється реальною альтернативою застосуванню наближених методів при розв'язанні даного класу задач механіки деформівного твердого тіла

Ключові слова: консольна конструкція, власна вага, згинні коливання, частоти коливань, форми коливань

1. Introduction

When designing vertical structures, it is necessary to consider all possible impacts they may be exposed during construction and operation. The majority of these impacts have dynamic nature and lead to energy accumulation. The amplitude of structural oscillations, as well as the intensity of inertial forces gradually increases. This phenomenon is especially dangerous for high-rise buildings, which are widely used in modern construction.

Evaluation of dynamic parameters of the structure can be performed on the basis of analysis of such characteristics as natural frequencies and mode shapes. When investigating the structure in an unloaded state, the values of natural frequencies and mode shapes allow concluding about the rigidity and operability of the structure, evaluating the correctness of design solutions. Therefore, the determination of these characteristics is an actual scientific and practical problem.

2. Literature review and problem statement

A large number of modern publications have been devoted to the study of bending oscillations of various structures with allowance for the longitudinal force. Thus, in [1] bending oscillations of uniform beams under various boundary conditions and axial loads have been studied. The research is based on the energy method. Here the empirical formulas that allow finding oscillation frequencies without using the finite element method have been obtained. The paper [2] deals with the case when the axial force takes different constant values in different parts of the beam. Each of these parts of the beam corresponds to its own differential oscillation equation with constant coefficients. The general integrals of these equations are the basis for the solution of the problem. Here, the first three oscillation frequencies for various combinations of boundary conditions have been calculated. In [3], based on the Euler-Bernoulli hypothesis, free oscillations of beams with a set of lumped elements subjected to axial load have been investigated by the finite element method. The authors of [4] study oscillations of frame structures taking into account the influence of axial load. The load is calculated based on the total weight and number of floors of the building. Natural frequencies of frame structures are obtained by solving the differential equation written for an equivalent rod whose rigidity and weight must be uniformly distributed along the length.

In [5], the influence of the longitudinal force on free oscillations of a multi-span Tymoshenko beam with the mass-spring system has been studied. Here the solution of the problem is based on the general solution of the partial differential equation with constant coefficients. The publication [6] is devoted to an experimental study of the effect of axial force on the oscillation frequency of the rod. This paper is an attempt to find a connection between theoretical models and the phenomenon, which takes place in the real world.

The authors of [7] have calculated natural frequencies of the restrained rod with a mass at the end, applied with an eccentricity. Herewith, a linearly variable axial force representing the own weight of the rod has been considered. The method of solution is based on the Hamilton's principle. In [8], oscillations of the cantilever column under the action of compressive load have been studied. The boundary-value problem is formulated on the basis of the Hamilton's principle and Tymoshenko beam theory. A comparison of the numerical calculations obtained with the help of the Tymoshenko beam theory and the Euler-Bernoulli model has been carried out. The paper [9] deals with the study of bending oscillations of structures taking into account the own weight. Here the corresponding partial differential equation of oscillations with variable coefficients has been written out. To solve the problem, an approximate shape of the deflection curve of the cantilever has been adopted.

It is common for these publications that the exact solutions of the corresponding differential oscillation equations with variable coefficients are not given anywhere. In all such cases, approximate methods are used.

In real structures, longitudinal forces in different sections take different values. Examples of such structures are multi-storey buildings, columns of frame buildings, industrial structures (chimneys, water towers, through lattice towers), drill strings, wind generator supports, antennas, etc.

One of the most common computing schemes for studying bending oscillations of these structures is a vertical cantilever rod. An example is a rod with uniform cross-section, which is under the influence of the variable longitudinal force, represented by its own weight. The mathematical model of such a physical phenomenon is a partial differential equation with variable coefficients [10–12]. Studies of bending oscillations of cantilever structures with allowance for their own weight, which would be based on the exact solution of the corresponding differential equation, have not been found. Probably, this is directly related to the mathematical problem consisting in the lack of a universal method of direct integration of differential equations with variable coefficients.

It is quite clear that the exact solution carries information of a qualitative nature and forms the most complete picture of the physical phenomenon under consideration. However, the variability of the coefficients of the equation introduces significant mathematical difficulties in the procedure of constructing the exact solution, which, owing to the above, has not been known until recently. For example, the monograph [12] explicitly states that it is very difficult to obtain an exact solution because of the presence of variable parameters. This circumstance was one of the main reasons forcing researchers to resort to approximate methods.

Therefore, development of a method for calculating bending oscillations of structures with allowance for their own weight based on the exact solution of the corresponding differential equation is promising. This became possible after the general integral of the differential equation of rod oscillations has been found in [13] and, as a consequence, all necessary formulas for the state parameters have been written out.

3. The aim and objectives of the study

The aim of the work is to develop a method for calculating bending oscillations of vertical cantilever structures with allowance for their own weight based on the exact solution of the oscillation equation.

To achieve the aim, the following objectives were accomplished:

– to determine a general view of the frequency equation;

– to obtain analytically the formulas determining natural frequencies and mode shapes of structures;

– to describe the algorithm for the numerical determination of natural frequencies and mode shapes of structures;

– to determine natural frequencies and mode shapes of the through lattice tower.

4. Fundamental functions and analytical representation of the oscillation frequency

Fig. 1 shows the general scheme of oscillations of the vertical cantilever rod, and Fig. 2 shows the external and internal forces acting on its element.

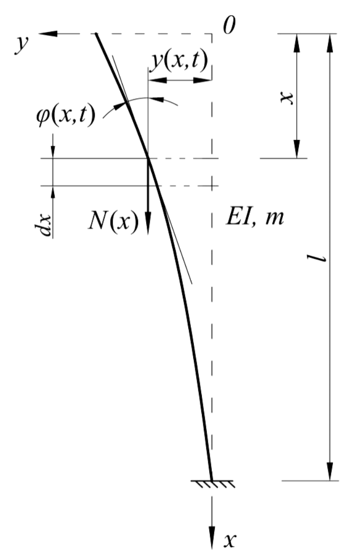

Fig. 1. Free transverse oscillations of the rod

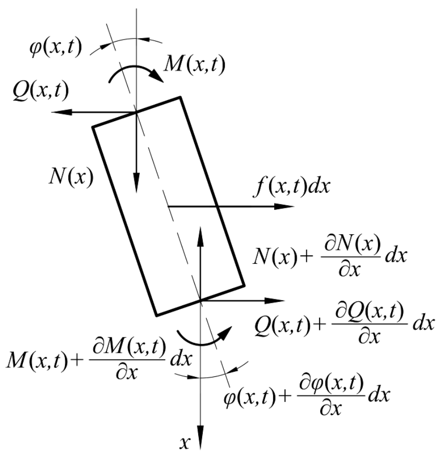

Fig. 2. Internal and external forces acting on the rod element

Here the following notations are used:

EI – bending rigidity of the rod;

E – elastic modulus of the material;

I – moment of inertia of the cross-section;

m – intensity of the distributed mass (mass per unit length) of the rod;

$N(x)=qx$ – variable longitudinal (compressive) force, where q is the mass per unit length of the rod;

$y(x, t)$ – cross motion of the axis point of the rod with the coordinate x at time t (dynamic deflection);

$\varphi(x,t)$ – dynamic angle of rotation;

$M(x, t)$ – dynamic bending moment;

$Q(x, t)$ – dynamic shear force;

$f(x, t)$ – intensity of the inertia forces that arise during oscillation (the d'Alembert's force).

The differential oscillation equation of the rod for this case has the form [10–12]

$$EI\frac{\partial^4 y}{\partial x^4}+q\frac{\partial}{\partial x}\left(x\frac{\partial y}{\partial x}\right)+m\frac{\partial^2 y}{\partial t^2}=0. \tag{1}$$

Further calculations are based on the exact solution of (1). According to the results of [13], for the dynamic state parameters of the rod, the following representations take place:

$$y(x,t)=v(x)T(t);\ \ \varphi(x,t)=\varphi(x)T(t); \tag{2}$$

$$M(x,t)=M(x)T(t);\ \ Q(x,t)=Q(x)T(t), \tag{3}$$

where $v(x)$, $\varphi(x)$, $M(x)$, $Q(x)$ are the amplitude functions depending only on the variable x.

These functions are expressed through the dimensionless fundamental functions $X_n(x)$, $\tilde{X}_n(x)$, $\overset{\frown}{X}_n(x)$, $\hat{X}_n(x)$ (n=1, 2, 3, 4) by means of the formulas [13]:

$$v(x)=v(0)X_1(x)+\varphi(0)lX_2(x)-$$
$$-M(0)\frac{l^2}{EI}X_3(x)-Q(0)\frac{l^3}{EI}X_4(x); \tag{4}$$

$$\varphi(x)=v(0)\frac{1}{l}\tilde{X}_1(x)+\varphi(0)\tilde{X}_2(x)-$$
$$-M(0)\frac{l}{EI}\tilde{X}_3(x)-Q(0)\frac{l^2}{EI}\tilde{X}_4(x); \tag{5}$$

$$M(x)=-v(0)\frac{EI}{l^2}\overset{\frown}{X}_1(x)-$$
$$-\varphi(0)\frac{EI}{l}\overset{\frown}{X}_2(x)+M(0)\overset{\frown}{X}_3(x)+Q(0)l\overset{\frown}{X}_4(x); \tag{6}$$

$$Q(x)=-v(0)\frac{EI}{l^3}\left(\hat{X}_1(x)+\alpha\frac{x}{l}\tilde{X}_1(x)\right)-$$
$$-\varphi(0)\frac{EI}{l^2}\left(\hat{X}_2(x)+\alpha\frac{x}{l}\tilde{X}_2(x)\right)+$$
$$+M(0)\frac{1}{l}\left(\hat{X}_3(x)+\alpha\frac{x}{l}\tilde{X}_3(x)\right)+$$
$$+Q(0)\left(\hat{X}_4(x)+\alpha\frac{x}{l}\tilde{X}_4(x)\right). \tag{7}$$

In turn, $X_n(x)$, $\tilde{X}_n(x)$, $\overset{\frown}{X}_n(x)$, $\hat{X}_n(x)$ (n=1, 2, 3, 4) are represented by uniformly convergent series in powers of the unknown dimensionless parameter K:

$$X_n(x)=\beta_{n,0}(x)+K^2\beta_{n,1}(x)+$$
$$+K^4\beta_{n,2}(x)+K^6\beta_{n,3}(x)+\ldots\ ; \tag{8}$$

$$\tilde{X}_n(x)=\tilde{\beta}_{n,0}(x)+K^2\tilde{\beta}_{n,1}(x)+$$
$$+K^4\tilde{\beta}_{n,2}(x)+K^6\tilde{\beta}_{n,3}(x)+\ldots; \tag{9}$$

$$\overset{\frown}{X}_n(x)=\overset{\frown}{\beta}_{n,0}(x)+K^2\overset{\frown}{\beta}_{n,1}(x)+$$
$$+K^4\overset{\frown}{\beta}_{n,2}(x)+K^6\overset{\frown}{\beta}_{n,3}(x)+\ldots \tag{10}$$

$$\hat{X}_n(x)=\hat{\beta}_{n,0}(x)+K^2\hat{\beta}_{n,1}(x)+$$
$$+K^4\hat{\beta}_{n,2}(x)+K^6\hat{\beta}_{n,3}(x)+\ldots, \tag{11}$$

where

$$\beta_{1,0}(x)=1;$$

$$\beta_{n,0}(x)=\left(\frac{x}{l}\right)^{n-1}\left(c_{n,0,0}+\sum_{j=1}^{\infty}(-1)^j\alpha^j c_{n,0,j}\left(\frac{x}{l}\right)^{3j}\right)$$

$$(n=2,3,4); \tag{12}$$

$$\beta_{n,k}(x)=\left(\frac{x}{l}\right)^{n+4k-1}\left(c_{n,k,0}+\sum_{j=1}^{\infty}(-1)^j\alpha^j c_{n,k,j}\left(\frac{x}{l}\right)^{3j}\right)$$

$$(k=1,2,3,\ldots); \tag{13}$$

$$\alpha=\frac{ql^3}{EI}; \tag{14}$$

$$\tilde{\beta}_{n,k}(x)=l\beta'_{n,k}(x),\ \ \overset{\frown}{\beta}_{n,k}(x)=l\tilde{\beta}'_{n,k}(x),$$
$$\hat{\beta}_{n,k}(x)=l\overset{\frown}{\beta}'_{n,k}(x)\ \ (k=0,1,2,\ldots). \tag{15}$$

In order to determine the dimensionless coefficients of the series (12), (13), the following set of recurrence formulas is used:

$$c_{n,0,0}=\frac{1}{(n-1)!}; \tag{16}$$

$$c_{n,0,j}=\frac{(n-1)(n+2)...(n+3j-4)}{(n+3j-1)!}; \quad (17)$$

$$c_{n,k,0}=\frac{1}{(n+4k-1)!}; \quad (18)$$

$$c_{n,k,j}=\frac{1}{(f_{n,k,j}-2)(f_{n,k,j}-1)f_{n,k,j}}\times$$

$$\times\left(\frac{c_{n,k-1,j}}{f_{n,k,j-1}}+f_{n,k,j-1}c_{n,k,j-1}\right), \quad (19)$$

where

$$f_{n,k,j}=n+4k+3j-1 \quad (20)$$

$$(n=1,2,3,4)(k=1,2,3,...)(j=1,2,3,...).$$

The time function has the form

$$T(t)=T(0)\cos pt+\frac{\dot{T}(0)}{p}\sin pt, \quad (21)$$

where $T(0)$, $\dot{T}(0)$ are the parameters of the initial conditions of motion; p is the oscillation frequency of a weighty structure, for which the analytic representation is obtained [13]

$$p=\frac{K}{l^2}\sqrt{\frac{EI}{m}}. \quad (22)$$

The dimensionless parameter K will further be called the oscillation coefficient.

We note that the formulas (2)–(22) given in [13] are universal in the sense that they are suitable for calculating bending oscillations of the rod under any possible boundary conditions.

The implementation of the given boundary conditions in each particular case leads to the frequency equation with respect to the unknown parameter K. After finding the required number of the roots K_1, K_2, K_3, ... of the frequency equation, according to the formula (22), we will have a range of oscillation frequencies of the rod with allowance for its own weight

$$p_j=\frac{K_j}{l^2}\sqrt{\frac{EI}{m}} \quad (j=1,2,3,...). \quad (23)$$

Thus, the problem of determining the frequencies is reduced to finding dimensionless oscillation coefficients.

5. Results of studies of oscillations of cantilever structures

The dynamic boundary conditions corresponding to the rod, whose lower end is restrained, and the upper end is free, have the form:

$$M(0,t)=0;\ \ Q(0,t)=0;\ \ y(l,t)=0;\ \ \varphi(l,t)=0.$$

Using the formulas (2), (3), we obtain equivalent boundary conditions in the amplitude form:

$$M(0)=0;\ \ Q(0)=0;\ \ v(l)=0;\ \ \varphi(l)=0.$$

For the implementation of the conditions at the end $x=l$, we use the formulas (4), (5), where we first consider the conditions at the end $x=0$. As a result, we will have the system of equations:

$$\begin{cases} X_1(l)v(0)+lX_2(l)\varphi(0)=0; \\ \frac{1}{l}\tilde{X}_1(l)v(0)+\tilde{X}_2(l)\varphi(0)=0. \end{cases} \quad (24)$$

The solvability condition of this system is given by the frequency equation

$$X_1(l)\tilde{X}_2(l)-\tilde{X}_1(l)X_2(l)=0. \quad (25)$$

Taking into account that the numerical series $X_1(l)$, $\tilde{X}_1(l)$, $X_2(l)$, $\tilde{X}_2(l)$ converge, and also based on the known theorems of mathematical analysis, we conclude that the left-hand side of the frequency equation (25) is a convergent numerical series. Using (8), (9) and applying the rules for the product and the sum of the series, we transform (25) to the form

$$\eta_0+\eta_1K^2+\eta_2K^4+\eta_3K^6+...=0, \quad (26)$$

where

$$\eta_0=1,$$

$$\eta_k=\sum_{j=0}^{k}(\beta_{1,j}(l)\tilde{\beta}_{2,k-j}(l)-\tilde{\beta}_{1,j}(l)\beta_{2,k-j}(l)) \quad (k=1,2,3,...). \quad (27)$$

When calculating η_k (k=1, 2, 3,...), we use the following relations, which obviously follow from the formulas (12), (13), (15):

$$\beta_{1,0}(l)=1;\ \ \beta_{2,0}(l)=c_{2,0,0}+\sum_{j=1}^{\infty}(-1)^j\alpha^j c_{2,0,j};$$

$$\tilde{\beta}_{1,0}(l)=0;\ \ \tilde{\beta}_{2,0}(l)=f_{2,0,0}c_{2,0,0}+\sum_{j=1}^{\infty}(-1)^j\alpha^j f_{2,0,j}c_{2,0,j};$$

$$\beta_{n,k}(l)=c_{n,k,0}+\sum_{j=1}^{\infty}(-1)^j\alpha^j c_{n,k,j};$$

$$\tilde{\beta}_{n,k}(l)=f_{n,k,0}c_{n,k,0}+\sum_{j=1}^{\infty}(-1)^j\alpha^j f_{n,k,j}c_{n,k,j};$$

$$(n=1,2)(k=1,2,3,...).$$

The equations of the form (26) are often found in mechanics. To find solutions of such equations, the method of comparing the roots calculated for different numbers of retained terms of the series is used [14]. Such an approach makes it possible to calculate the roots of an equation with any given accuracy.

For the main (natural) mode shapes, we take

$$v_j(x)=C_jV_j\left(\frac{x}{l}\right) \quad (j=1,2,3,...),$$

where C_j is the dimensional constant factor; $V_j\left(\frac{x}{l}\right)$ is the dimensionless function determining the law of the corresponding mode shape.

The formula (4) in the case under consideration can be presented in the form

$$v(x) = v(0)(X_1(x) - \zeta X_2(x)),$$

where $\zeta = -l\varphi(0)/v(0)$ is the dimensionless parameter, for which from the first equation of the system (24) we find

$$\zeta = X_1(l,K)/X_2(l,K).$$

From this, we conclude that

$$V_j\left(\frac{x}{l}\right) = (X_1(x,K_j) - \zeta_j X_2(x,K_j)),$$

$$\zeta_j = X_1(l,K_j)/X_2(l,K_j) \quad (j = 1,2,3,\ldots),$$

or

$$V_j\left(\frac{x}{l}\right) = \sum_{k=0}^{\infty} K_j^{2k}(\beta_{1,k}(x) - \zeta_j \beta_{2,k}(x)),$$

$$\zeta_j = \sum_{k=0}^{\infty} K_j^{2k}\beta_{1,k}(l) \Bigg/ \sum_{k=0}^{\infty} K_j^{2k}\beta_{2,k}(l). \tag{28}$$

Thus, the algorithm for calculating the bending oscillations of vertical cantilever structures with allowance for their own weight is determined by the following sequence of operations:

1. For the given physicomechanical characteristics of the structure *EI, q, l*, using the formula (14), we calculate the value of the dimensionless parameter α.
2. Using the formula (27), we calculate the coefficients η_k (k=1, 2, 3, ...) and form the frequency equation (26).
3. Using the comparison method, we find the roots of the frequency equation.
4. Using the formula (23), we find oscillation frequencies, and by the formula (28), the corresponding laws of the fundamental mode shapes of the structure.

Example. Let us investigate bending oscillations of a tower made of steel elements (Fig. 3). Similar structures are used in various industries and construction: as transmission towers, components of drill strings, TV and radio towers, bearing elements of industrial buildings.

The tower is a spatial truss frame structure of height-uniform square section. The load-bearing columns of the tower are made of Ø 245×12 mm round tubes and are interconnected by the lattice, also made of steel elements. The distance between the axes of the columns on one side is 6 m. The height of the tower is l=35 m.

To perform calculations, we consider the structure in the rod model (Fig. 4).

In order to verify the results, calculations in the SCAD computer system in parallel with calculations by the author's method were also performed [15]. SCAD is a software system for analyzing structures by the finite element method. It is used for calculation of the stress-strain state, stability study and solution of problems of statics and dynamics.

First, we determine the necessary parameters of the given mechanical system. The tower shown in the rod scheme will have the following characteristics: EI=2.0601×10^8 kN/m^2; I=0.31646 m^4; q=2.9 kN/m.

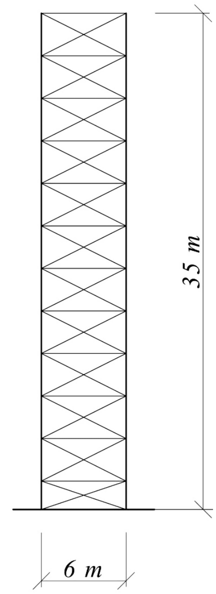

Fig. 3. Scheme of the through lattice structure of the tower

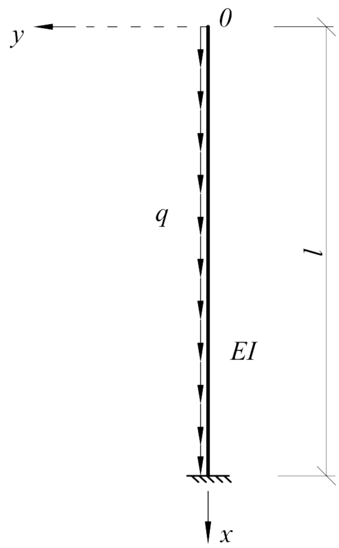

Fig. 4. Design scheme of the rod

We find the dimensionless parameter α by the formula (14):

$$\alpha = \frac{2.9 \cdot 35^3}{2.0601 \cdot 10^8 \cdot 0.31646} = 0.0019.$$

For the value α, we calculate the coefficients (27) and find the first three roots of the equation (26). As a result, we have the oscillation coefficients: K_1=3.5162; K_2=22.0340; K_3=61.6968.

After that, according to the formula (23), we calculate natural frequencies of the tower, which are presented in Table 1. The frequencies obtained as a result of the calculation in SCAD are also given there.

Similar calculations were also made for the tower height of l=50 m. The corresponding results are given in Table 2.

The laws of the fundamental mode shapes of the tower are determined by the formula (28).

Table 1

Comparison of natural frequencies of a 35 m tower

Frequency, s^{-1}	Author's method	SCAD	Error, %
p_1	42.6258	42.6220	0.009
p_2	267.1139	267.0457	0.026
p_3	747.9385	747.5797	0.048

Table 2

Comparison of natural frequencies of a 50 m tower

Frequency, s^{-1}	Author's method	SCAD	Error, %
p_1	20.8883	20.8865	0.009
p_2	130.8803	130.8887	0.008
p_3	366.4856	366.4856	0.0

Fig. 5–7 for the case of l=35 m show the graphs of the first three laws of oscillation, constructed in two ways, using both the formula (28) and SCAD.

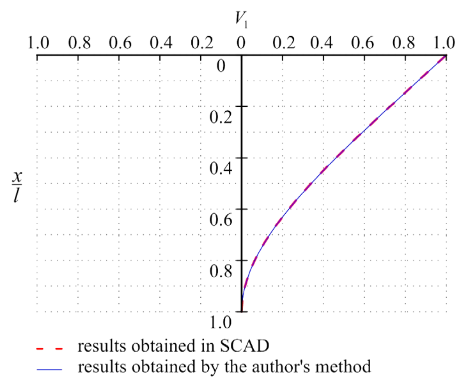

Fig. 5. The law of the mode shape I in the dimensionless coordinate system V_1 and $\frac{x}{l}$

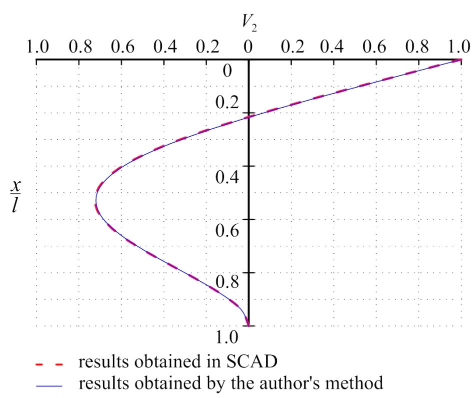

Fig. 6. The law of the mode shape II in the dimensionless coordinate system V_2 and $\frac{x}{l}$

As you can see, the graphs constructed in the software system almost completely coincide with the graphs constructed by the author's method. Represented in the same coordinate system, the graphs visually form a single line.

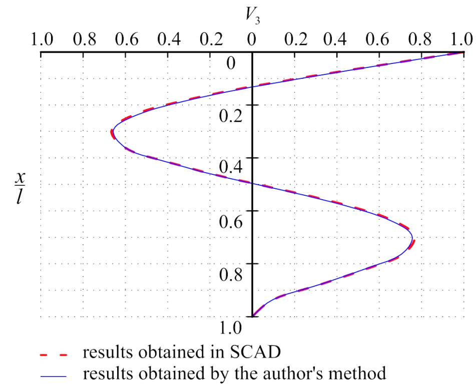

Fig. 7. The law of the mode shape III in the dimensionless coordinate system V_3 and $\frac{x}{l}$

6. Discussion of analytical and numerical results of studies of oscillations of cantilever structures

The above studies are a logical continuation of the research initiated in [13] and are entirely based on the results obtained there. In fact, the idea is the application of the general solutions obtained in [13] for the case of oscillations of vertical cantilever structures.

Due to the fact that the proposed research method is based on the exact solution of the differential equation, it allows obtaining information of a qualitative nature and forming the most reliable picture of oscillations. At the same time, the calculation procedure does not require the discretization of the structure, as is customary with approximate methods. Using explicit analytical formulas, the accuracy of calculations is increased. The method allows determining natural frequencies and mode shapes of cantilever structures with any predetermined accuracy.

As can be seen from the above data, the results of calculations by the author's method and the finite element method by means of the SCAD software system practically coincide. This confirms the reliability of calculations by the author's method.

We pay attention to the role of the dimensionless parameter α, which is an important characteristic of a mechanical system. In particular, as can be seen from the formula (14), the value of α=0 will correspond to the case of a design scheme in the form of a weightless rod. In this case, the fundamental functions (8) degenerate into the known Krylov functions [16]. Generally, the larger the parameter α, the more the values of the calculated oscillation parameters will differ, taking into account the own weight of the structure from the values of the similar parameters calculated without the own weight.

In general, the results obtained make it possible to simplify the procedure and improve the accuracy of calculations of bending oscillations of cantilever structures with allowance for the own weight. This allows designing these mechanical systems with the desired properties that are more economic at a given level of reliability.

In addition to cantilever structures, structures with other ways of end restraint are often found in the industry. Therefore, in the future this method can be extended to other cases of boundary conditions. The limitations of the method include the fact that it is applicable only to structures with uniform cross-section. To distribute it for the case of structures with variable cross-section, the exact solution of the corresponding differential equation is required.

The drawbacks of this work include the fact that it does not cover the actual issue of the effect of the magnitude of the longitudinal load q on oscillation frequencies of structures. The authors plan to devote a separate paper to this issue, which will become one of the directions for further development of this study.

7. Conclusions

1. The research is based on the partial differential oscillation equation of rod structures with variable coefficients. Based on the exact solution of this equation, the method for calculating bending oscillations of cantilever structures with allowance for their own weight is developed. For the method, it is fundamental that the exact solution is expressed in terms of dimensionless fundamental functions. The cross-section of the structure is assumed to be uniform.

2. The frequency equation of the problem is obtained in a dimensionless form. The roots of this equation, corresponding to the given mechanical and geometric parameters of the structure, can be found with any given accuracy. Through the dimensionless roots of this equation, oscillation frequencies of the structure are expressed.

3. Mode shapes of the structure are expressed through the dimensionless fundamental functions and roots of the frequency equation.

4. Comparison of frequencies and graphs of mode shapes of the lattice tower, obtained by the proposed method and the finite element method, indicates the reliability of calculations by the author's method.

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