Запропоновано технічні прийоми, за допомогою яких вдалося вирішити нелінійне рівняння алгебри Ріккаті для динамічних систем з ступенями свободи. Введено обмеження на структуру проектованого залізничного екіпажу. Як аналог взято симетричний екіпаж, в системі підвішування якого встановлені пружно-дисипативні зв'язки з лінійними характеристиками. Це дозволило розробити методику проектування системи підвішування залізничного екіпажу. Критерієм при виборі вагових коефіцієнтів якості було прийнято вимогу забезпечення комфортних умов пасажирам і локомотивної бригаді. Тому в системі повинен спостерігатися коливальний процес з малими амплітудами і частота власних коливань кузова не повинна перевищувати 2 Гц. Виконано декомпозицію методу динамічного програмування для безперервних стохастичних систем, що дозволило розробити методологію поетапного проектування системи підвішування. Методика придатна для використання при проектуванні підвіски екіпажів звичайного і високошвидкісного руху. На першому етапі проектується пасивна система підвішування. Другий етап передбачає перевірку доцільності розробки пристроїв управління параметрами пружно-дисипативних зв'язків системи підвішування транспортних екіпажів за допомогою оптимальних фільтрів Калмана-Бьюсі. Шляхом моделювання доведено, що управління параметрами пружно-дисипативних зв'язків покращує динаміку транспортних екіпажів. Тільки управління демпфуванням більш ніж в два рази може зменшити прискорення центру мас кузова і, отже, зменшити динамічні навантаження в системі. Алгоритм Калмана-Бьюсі дозволяє отримувати оптимальні параметри пружно-дисипативних зв'язків системи підвішування складних динамічних систем. Методику можна використовувати самостійно і в складі методики поетапного проектування системи підвішування. Ілюстрація методики виконана на тестових прикладах. Методика реалізована в системі моделювання. Управління параметрами пружно-дисипативних зв'язків системи підвішування транспортних екіпажів дозволить, по-перше, створити комфортні умови роботи локомотивної бригади і пасажирам, по-друге, підвищити надійність роботи і безпеку руху рухомого складу за рахунок зменшення динамічних навантажень

Ключові слова: фільтр Калмана-Бьюсі, транспортний екіпаж, управління параметрами пружно-дисипативних зв'язків, складні динамічні системи

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DEVELOPMENT OF THE PROCEDURE FOR VERIFYING THE FEASIBILITY OF DESIGNING AN ACTIVE SUSPENSION SYSTEM FOR TRANSPORT CARRIAGES

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1. Introduction

To obtain an accurate estimation of railway carriage run at the entire range of operating speeds, it is required to control suspension system parameters in terms of a possible change in the process of movement. Control over suspension system is understood as the preparation of the entire system to pass above the existing track irregularities.

When discusing control over a suspension system of transport carriages, it must be taken into consideration that as recently as in the mid-1980s many transport scientists did not believe in the possibility of constructing controlled suspension systems. Therefore, the main task of analytical reviews of that period was to demonstrate the advantages of controlled vibration protection systems compared with traditional systems and to direct a scientific thought towards the development of various types of active suspension systems capable of providing the dynamic properties required for the carriage at a substantial increase in speed motion. It was believed that the optimal economic indicators are achieved at a maximum speed of 200–300 km/h. Stages at the development of the considered subject were outlined in papers [1–8]. Paper [1] addressed the task on determining a transfer function of the locomotive dynamical suspension system as a reaction to the vertical component of force vector in the area of contact between a wheel and a rail. The stochastic excitation at the input to a dynamic system was the spectral density of vertical displacements, velocities, or accelerations of a wheel rim wheel or a wheelset axle-box. The optimality criterion a standard deviation in the signal spectral density at the output from spectral density of the permissible output signal. The authors considered a closed dynamic system. They proposed to damp the undesired frequency oscillations by introducing into a suspension system anti-vibrators or specialized regulators in the form of an air cylinder with a piston, without changing, in this case, parameters of elastic-dissipative links of the locomotive itself. It is impossible to determine, based on the transfer function of a complex dynamic system, the parameters of elastic-dissipative links of a suspension system, which would make it easier to solve the task on providing the required dynamic properties to the carriage over an operating range of motion speed. Paper [2] proposes a new approach to the design of a car suspension. The author believes that the suspension system can be regarded as a control function that regulates the motion of sprung mass in line with the preset manner. Uni-mass systems with one or two degrees of freedom are considered as an example. Quality criterion is the quadratic functional whose physical essence is energy consumption in order to suppress harmful oscillations. The problem is solved using a matrix method of dynamic programming. The method makes it possible to determine the optimal parameters for a suspension system using models without external perturbation, which greatly facilitates the procedure of synthesis. The synthesizing function for a model with deterministic perturbation always contains the component, which dampens the external perturbation. Weights coefficients for a quality quadratic functional are assigned arbitrarily, that is, there is no guarantee for the physical feasibility of a designed passive suspension system. In paper [3], in order to determine the optimum parameters of elastic-dissipative links in a wheelset, the authors applied the Kalman-Bucy algorithm. Only a simple model shows the possibility of control over parameters of elastic-dissipative relations. Author of work [4] published an overview of active suspension systems in rolling stock at railroads in England, the United States, Japan, Germany. He notes that the research and engineering development department of the British railways undertook a theoretical research, tested engineering designs, performed laboratory and limited field testing of active suspension systems, which ensure vertical and transverse "running" stability. It is reported that they estimated under laboratory conditions performance of various controlling elements in active suspension systems:

– a transverse hydraulic drive;

– a pneumatic spring that can be adjusted in a vertical plane;

– vertical and transverse electromagnetic drives.

The Japanese national railroads and the company Hitachi developed a system to enhance the stability of railroad cars in vertical and transverse directions by using adjustable pneumatic cylinders. Two pneumatic cylinders are mounted vertically, and one cylinder – transverse to the car; they are controlled based on the body acceleration. In addition, the suspension system utilizes standard air springs. Theoretical study and tests using a physical car model demonstrated that amplitudes of all types of oscillations reduce by 50 %.

In the United States, a study into feasibility of using active suspension systems was conducted by the firm Westinghouse, which developed a theoretical framework and created a prototype of the electro-hydraulic suspension system to be used by a high-speed railroad car-laboratory at the Ministry of Transport of the United States. General Electric (GE) examined the application of active suspension for high-speed rolling stock of urban lines analytically and under laboratory conditions using a physical model. The company GE considered hydraulic and pneumatic controlling elements managed by electronic devices or by means of jet automation.

The German company MAN investigated the effectiveness of using active suspension system in order to increase motion speed at the expense of the kinematic stabilization of rolling stock motion.

The theoretical studies carried out in these countries have shown that active suspension systems, which employ as a feedback signal acceleration or an absolute speed, make it possible to significantly improve running performance and motion stability of railroad carriages.

Theoretical studies were performed using dynamical systems with one degree of freedom. A result of the study is the constructed amplitude-frequency characteristics, which are used to fine-tune a control system over a change in amplification coefficients. The coefficients are chosen by a simple brute-force method. Authors consider the optimal value for the degree of damping a system to be its critical value of 0.707. The papers analyzed fail to justify the choice of amplification coefficients and the optimal value for a degree of damping. At a damping degree of 0.707 the system will experience an aperiodic law of motion of the center of mass of the body, which is undesirable.

Paper [5] reviews works by authors from England, the United States, Japan, Germany. The theoretical part of the review indicates that most research is conducted using simple models; the problem on choosing the weight coefficients of quality criteria remains unsolved.

Article [6] explores a dual-mass mechanical system with two degrees of freedom and active damping. The resistance force of the damper is proportional to the speed of the vehicle body displacements. Under active control, a degree of damping equal to 0.707 is implemented.

Author of work [7] built a system for suspending a railroad car with active elements that operate based on the electromagnetic principle. The level of vertical accelerations of a car body with a single-stage suspension system decreased by 2 times. Paper [8] reported a law of active control over a dual-mass mechanical system with two degrees of freedom using a method of dynamic programming for continuous stochastic systems. The authors could not resolve the issue on the choice of weights for the functional of quality though the reported results of mathematical modeling of the operation of an active suspension system shows its effectiveness; the amplitudes of displacements and body accelerations were reduced by two times. Theoretical investigations and simulation results cited in papers [1–8] show that active suspension systems can significantly improve running performance and motion stability of railway carriages. The remaining unresolved issues include:

– substantiated choice of weight coefficients for quality criteria;

– dissemination of the applied methods to complex dynamic systems.

Active suspension systems are used in order to provide for the acceptable level of running stability at a substantial increase in motion speed or under normal speeds and poor condition of a railway track. Creation of devices that would control parameters of the elastic-dissipative links in a suspension system is expensive and is justified when designing the rolling stock for high-speed motion. The rolling stock with a design speed of up to 160 km/h will be fully operational with a passive suspension system with optimal parameters of elastic-dissipative relations.

Thus, it is necessary to devise a procedure for verifying the feasibility of design of a controlled suspension system, in order to avoid unnecessary expenditures for the development of active control devices.

2. Literature review and problem statement

The task on designing a suspension system can be reduced to the optimization one and solved for two statements: in the form of a mathematical programming problem and a management theory problem. In the first statement a mathematical model is represented by a system of nonlinear differential equations that describe the process of oscillations for the system "carriage track" under the action of a random perturbation. The problem is to identify system parameters that minimize an objective function, constructed in a special way, and the problem is then solved by numerical methods of optimization. In this case, the parameters are considered established if the objective function approaches its extreme value with a predefined degree of accuracy. Even for the case of a convex objective function, the search for its extremum takes longer than an hour of mainframe computers. In order to speed up the process, calculation is first performed using a simple model, and the obtained parameters are then introduced to a more complex model to continue the optimization. Search duration depends on the type of an objective function and the applied method of optimization. The most effective of the numerical optimization methods, a method of the Nelder-Mead deformed polyhedron made it possible, over a relatively short time, to solve a problem using a model of any complexity. Authors of papers [9–16] apply various techniques to fully describe the physical process at modeling and use numerical methods. Using numerical methods does not allow obtaining a law that controls parameters of elastic-dissipative relations; that can be noted as shortcomings of the papers considered below.

The application of active control over suspension has become a universally accepted technology for railroads. Article [9] describes this issue's previous practice, including the identification of basic configurations, as well as variants of technology and respective control strategies. The author also addresses longer-term trends in the development of suspension. Paper [10] describes a simulation study that provides a comprehensive comparison between the fully active and semi-active suspensions in order to improve the quality of vertical dynamics of railroad vehicles. The study includes the assessment of advantages of dynamics quality that could be theoretically achieved using idealized devices; it also explores the impact of real devices based on the technology of a hydraulic drive.

The task of active suppression of vibration using the concept of a slow absorber resonator (DR) with acceleration feedback was addressed in paper [11]. The authors performed a complete dynamic analysis of DR and its relationship using a mechanical system with one degree of freedom. Based on a given analysis, a procedure for choosing resonator parameters was proposed in order to ensure the desired characteristics of suppression and provide for the safe limits of stability.

Paper [12] suggests using a combined research method for deriving a function of amplitude pulsations of traction forces in order to reduce mechanical vibrations. The proposed method employs a finite element method (FEM) and a statistical regression method. A combination of methods makes it possible to solve optimization problems on determining the functions of maximization or minimization of the considered parameters.

A new approach to tackling the issue of modernizing an undercarriage suspension using a method of "negative" rigidity was proposed in paper [13]. The approach includes:

– analysis of the kinematic structure of the mechanism that generates a function of the suspension;

– detection of sources of the undesired structural redundancy in the design of a suspension;

– prediction of structural causes that generate a function of suspension in new designs.

The application of genetic optimization algorithm made it possible to develop a new design of suspension using active vibration damping. The study was described in paper [14]. Recent trends in the use of numerical methods when studying the issue under consideration are outlined in work [15]. The study includes a description of the suspension design based on FEM. The authors addressed the improvement of a traction drive and reduction in its compression force under thermal and geometrical constraints for a rubberized tubular drive with permanent magnets (TDPM), used in the side secondary suspension of a railroad car.

An adaptive control method over vibration was also considered in paper [16]. Estimation of vibration was based on a comparison to the information from the front module of levitation. This information was accepted as reference. Suppression of additional vibrations is executed base on it.

Results of the above studies indicate that, first, the main modeling approaches at present stage are numerical calculation methods. It is obvious that such approaches make it possible to find acceptable solutions only if the resulting state of a process is known from experimental observations. Hence it follows that violation of these conditions will not allow obtaining optimal solutions for control systems over suspension system parameters.

Designing effective devices for active control over dynamic processes is impossible without a knowledge of the theoretical foundations – control laws. Solution to a general problem on active control is well known from the theory of optimal control; however, practical results were obtained only for the simplest analytical models. Authors of paper [8] applied, in order to model control over suspension system of a railroad carriage, an estimation scheme – a dual-mass mechanical system with two degrees of freedom. Optimality criterion was the quadratic functional

$$J = M[q_1(z_k - z_T)^2 + q_3\ddot{z}_k^2 + u^2],$$

in which the values of weights are equal to 10. q_1=q_2=10. The authors, however, failed to solve a task on choosing weight coefficients for the functional of quality; nevertheless, the reported results of mathematical modeling of the operation of an active suspension system show its effectiveness; the amplitudes of displacements and body accelerations reduced by two times.

The authors of work [8] conclude on the importance of solving the tasks on filtration and optimal control in systems of higher orders. They propose to continue research in the field of development of techniques that would lead to a decrease in the orders of systems and would simplify the structure of a suspension system.

When designing suspension systems for transport carriages, it is very effective to use a matrix method of dynamic programming for continuous dynamical systems. One can apply it to obtain analytical dependences for parameters of elastic-dissipative relations. The difficulty is that the algorithm of the method includes a matrix algebraic non-linear equation of the Riccati type whose solution can be obtained only for the simplest models of dynamic systems. Algorithms for problems on filtration and optimal control also contain matrix non-linear differential equations of the Riccati type. To solve these problems for complex dynamic systems, it is required to impose a constraint for the structure of a designed carriage. The analogue to consider is a carriage whose suspension system has established elastic-dissipative links with linear characteristics. In addition, the carriage is considered symmetrical. These techniques make it possible to reduce the matrices of rigid and dissipative parameters of a dynamical system to the block-diagonal form. The same form will be acquired by the matrices of estimation errors of a non-linear algebraic equation of the Riccati type. As a result, we obtain analytical dependences for determining the parameters of elastic-dissipative links for a passive suspension system.

To design active suspension systems, a stochastic dynamic programming method is applied, which includes a matrix method of dynamic programming and a method of optimal Kalman-Bucy filters. The initial dynamic system is a passive suspension system. Decomposition of a stochastic dynamic programming method makes it possible to carry out phased design of a suspension system:

– the first stage implies the development of a passive suspension system;

– the second stage implies establishing the principle of operation (active or passive control);

– the third stage implies compiling a technical assignment to design active control devices.

3. The aim and objectives of the study

The aim of present work is to devise a procedure for verifying the feasibility of designing an active suspension system for transport carriages based on the optimal Kalman-Bucy filters.

To accomplish the aim, the following tasks have been set:

– to develop techniques that would make it possible to solve the Riccati matrix non-linear equations for complex dynamic systems;

– to adapt the optimal Kalman-Bucy filters to designing the suspension systems for transport carriages;

– to consider a possibility to control parameters of elastic-dissipative links in a suspension system of transport carriages;

– to implement the procedure in the modeling system MVTU 3.7 using a simple model of the transport carriage.

4. Development of techniques and a procedure for designing complex dynamic systems

4. 1. Reducing the matrices of rigidity and dissipative parameters of a dynamical system to the block-diagonal form

When designing a passive suspension system, there is a problem related to solving a matrix nonlinear algebraic Riccati equation

$$P + A^{'}S + SA - SBG^{-1}B^{'}S = 0, \tag{1}$$

where S is the matrix of estimation errors; P, G are the matrices of weight coefficients for the quadratic functional of quality;

$$J = \int_0^\infty (X^{'}PX + U^{'}GU)\mathrm{d}t,$$

where U is the unknown synthesizing function. A physical meaning of the functional is energy consumption to suppress harmful vibrations.

For the considered class of systems, equation (1) can be solved analytically if one imposes a constraint to the structure of the designed object.

Imagine a matrix of estimation error S in the block form

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2^{'} & S_4 \end{bmatrix}$$

and write a Riccati equation in the expanded form

$$\begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix} + \begin{bmatrix} S_1 & S_2 \\ S_2^{'} & S_4 \end{bmatrix}\begin{bmatrix} 0 & E \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ E & 0 \end{bmatrix}\begin{bmatrix} S_1 & S_2 \\ S_2^{'} & S_4 \end{bmatrix} -$$
$$- \begin{bmatrix} S_1 & S_2 \\ S_2^{'} & S_4 \end{bmatrix}\begin{bmatrix} 0 \\ E \end{bmatrix}\begin{bmatrix} G^{-1} \end{bmatrix}\begin{bmatrix} 0 & E \end{bmatrix}\begin{bmatrix} S_1 & S_2 \\ S_2^{'} & S_4 \end{bmatrix} = 0.$$

Upon performing matrix operations we obtain

$$\begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix} + \begin{bmatrix} 0 & S_1 \\ 0 & S_2^{'} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ S_1 & S_2^{'} \end{bmatrix} - \begin{bmatrix} S_2G^{-1}S_2^{'} & S_2G^{-1}S_4 \\ S_2G^{-1}S_4 & S_4G^{-1}S_4 \end{bmatrix} = 0.$$

We then have

$$\alpha - S_2G^{-1}S_2^{'} = 0;$$

$$\gamma + 2S_2 - S_4G^{-1}S_4 = 0;$$

$$S_1 - S_2G^{-1}S_4 = 0. \tag{2}$$

At the known matrices of weight coefficients for functional α, γ, G, we determine from system (2) elements of the estimation error matrix S_1, S_2, S_4. If the matrix of estimation errors S has no null elements, the problem cannot be solved.

We shall impose a constraint on the structure of the designed train carriage. An analogue is the carriage whose suspension system has established elastic-dissipative links with linear characteristics. In addition, we assume that the carriage is symmetric. This means:

– the center of masses of the body is in the geometrical center of its horizontal plane;

– bogies are symmetrical with the central location of the pivot mount;

– supports of the body on a bogie are symmetrical to the pivot;

– supports of the same type have the same values for the parameters of elastic-dissipative links.

Write differential equations for the oscillatory process of a carriage in the absence of external disturbances

$$M\ddot{q}+F\dot{q}+Cq=0, \tag{3}$$

where M is the matrix of inertial coefficients the size of $n \times n$; C, F are the matrices, respectively, of rigid and dissipation parameters the size of $n \times n$; q, $\dot{q}$, $\ddot{q}$ are the vectors, respectively, of the generalized coordinates, velocities, and accelerations. The assumptions accepted lead to that the matrices of rigid and dissipative parameters take the block-diagonal form (Fig. 1), where block 1, for example, describes vertical oscillations of the carriage; block 2 – transverse vibrations, side pitching and wagging; block 3 – oscillations of twitching.

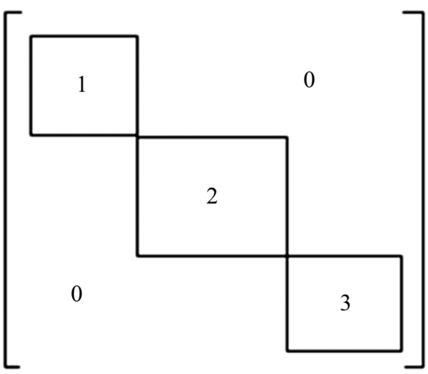

Fig. 1. Matrix calculation scheme

Upon determining a synthesizing function we obtain a system of differential equations

$$M\ddot{q}+MG^{-1}S_4\dot{q}+MG^{-1}S_2^{'}q=0. \tag{4}$$

Comparing matrix coefficients at the generalized coordinates and velocities in systems (3) and (4) we obtain dependences for determining the rigid and dissipative matrices of the designed system

$$C=MG^{-1}S_2^{'};\;\; F=MG^{-1}S_4. \tag{5}$$

Matrices of estimation errors $S_2^{'}$, S_4 are also block-diagonal; there is a dependence between matrix elements, similar to the dependence between elements of matrices of rigid or dissipative parameters.

Each obtained totality of synthesized parameters must be physically feasible. That is achieved by a reasonable selection of weight coefficients for the quadratic functional of quality. When solving this problem, we considered the main purpose of the designed system. A suspension system for the carriage is to be created, one of the purposes of which is to provide comfortable conditions for passengers and a locomotive crew in operation. To this end, the system is to experience an oscillatory process with small amplitudes while the presence of an aperiodic law of body motion is undesired. To eliminate resonance phenomena in the human body when traveling, the oscillation frequency of a car body shall not exceed 2 Hz. Ensuring the required dynamic properties of the carriage is achieved by checking the obtained totalities of synthesized parameters under conditions of the system operation. To this end, additional functional constraints on the dynamic indicators of a railroad carriage are imposed.

4. 2. Adaptation of the Kalman-Bucy algorithm to designing a suspension system for transport carriages

A theory of the optimal Kalman-Bucy filters can be practically implemented using only simple models due to the impossibility to determine, in a general case, a correlation error matrix of optimal estimates for a nonlinear matrix differential Riccati equation.

We shall impose constraints considered earlier on the structure of a dynamical system. The initial system is a system with optimal parameters for a suspension system, obtained using a matrix method of dynamic programming.

The state of the investigated dynamic system is described by equations

$$\begin{cases}\dfrac{dX(t)}{dt}=F(t)X(t)+L(t)N(t), \\ Y(t)=H(t)X(t)+V(t),\end{cases} \qquad X(t_0)=\bar{X}_0, \tag{6}$$

where F is the matrix of system parameters the size of $n \times n$; $X(t)$ is the system state vector; $N(t)$ is r-dimensional vector of external disturbances; L is the identity matrix at vector $N(t)$ the size of $n \times r$; $Y(t)$ is the vector of the observed components of the system state vector; $V(t)$ is l-dimensional vector whose elements are the measurement errors of components of the system state vector; H is the identity matrix that describes the relationship between the observed vector and the system state vector. Elements of the input perturbation vector are associated linearly with random white noise type functions and have null mathematical expectations and correlation functions derived from equality:

$$M[N_J(t)N_l(t)]=q_{Jl}\cdot\delta(t-\tau),\;\; (j,l=1,2,...,r),$$

where $\delta(t-\tau)$ is the Dirac delta-function. We introduce matrix $Q=[q_{Jl}(t)]$, whose elements are the variance of correlation functions of the system input disturbance.

The system state vector is measured by devices, this is taken into consideration in the calculation by introducing the second equation of system (6). Measurement errors are assumed to be random white noise type processes with null mathematical expectations. The correlation functions of the measurement error vector components are determined from formula:

$$M[V_J(t)V_m(t)]=r_{Jm}\cdot\delta(t-\tau),\;\; (j,m=1,2,...,l).$$

Matrix $R(t)=[r_{jm}(t)]$, whose elements are the measurement error variances, is symmetric positively defined by the size of $l \times l$. Components of the external perturbation vector are not correlated to the components of a measurement error vector.

The initial state of the system is not dependent on external influence and measurement errors. The vector of initial state is a normal vector magnitude for which we assign mathematical expectation $M[X(t_0)]=X_0$ and a correlation matrix $K(t_0)=K_0$. Elements of the correlation matrix are the desired variances of the state vector components.

If elements of the matrices are constant, the system (6) is stationary. In the event a disturbance is equal to zero, the system is called free.

The task of optimal filtration is finding the best estimate for the vector of state of a dynamic system, characterized by the first equation (6), according to measurements of the observed signal over time interval $[t_0, T]$. Dynamic system that determines the estimate of the system state vector is called a filter. Estimate of the system state vector is an actual output signal of the filter. The required (desired) output signal is the vector of the system state. A difference between the required output signal and the actual output signal is called an error of estimation or a filter error.

Optimality criterion is the minimum of an error variance, provided that the estimate of the system state vector is an unbiased estimation of the system state vector. To this end, the following condition must be met:

$$M[\hat{X}(t)] = M[X(t)], \quad t \ge t_0,$$

hence

$$M[E(t)] = M[X(t) - \hat{X}(t)] = 0.$$

Features of the criterion:

– a system that meets a given criterion maintains the deviation within the specified limits over the maximum possible period;

– if it is required that the variable should be less than a certain specified value, the dynamic system that satisfies the criterion for a variance minimum has the lowest probability of violating this constraint.

It was proven by Kalman that the solution to the problem of optimal filtration is an estimate of the system state vector, which is described by differential equation

$$\frac{d\hat{X}}{dt} = F(t)\hat{X}(t) + C(t)[Y(t) - H(t)\hat{X}(t)],$$

$$\hat{X}(t_0) = \bar{X}_0. \quad (7)$$

Expression (7) is called a differential equation of the filter, which is "disturbed" by the observed signal and yields at the output the best linear estimation of the vector of state of a dynamical system. Matrix coefficient for the gain in optimal filter is derived from expression

$$C(t) = K(t)H'(t)R^{-1}(t),$$

where R^{-1} is the inverse matrix relative to the matrix of measurement error variances; $K(t)$ is the correlation matrix of an optimal estimate error. To determine a correlation matrix of the optimal estimate error, R. Kalman derived a nonlinear differential equation

$$\begin{gathered}\frac{dK}{dt} = F(t)K(t) + K(t)F'(t) - \\ -K(t)H'(t)R^{-1}(t)H(t)K(t) + L(t)Q(t)L'(t),\end{gathered} \quad K(t_0), \quad (8)$$

where Q is the matrix of external perturbation variances.

Thus, a solution to the problem on optimal filtration is derived from the system of equations (6)–(8), which form the Kalman algorithm. An optimal filter parameters matrix is formed from equation (7)

$$F_{opt} = F(t) - C(t)H(t) = F - KH'R^{-1}H. \quad (9)$$

When implementing a Kalman filter, it is important to ensure its stability. R. Kalman and R. Bucy demonstrated that the optimal filter is a uniformly asymptotically stable dynamic system. In this case, a solution to the correlation equation represents a steady computational process, insensitive to minor errors.

As an example, let us determine a matrix of parameters for the optimal filter of a carriage whose estimation scheme is shown in Fig. 2.

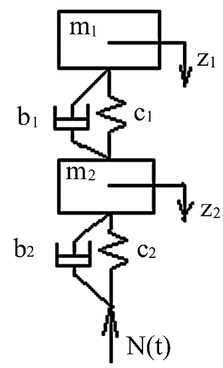

Fig. 2. A carriage estimation scheme

Denotations: m_1, m_2 is the sprung mass of the body and the bogie, respectively; c_1, c_2 is the rigidity of the upper and bottom degrees of suspension; b_1, b_2 are the resistance coefficients of the oscillation dampers, mounted in a suspension system; $N(t)$ is an accidental disturbance from the track.

Mathematical model of an oscillatory process takes the form:

$$m_1\ddot{z}_1 + b_1(\dot{z}_1 - \dot{z}_2) + c_1(z_1 - z_2) = 0;$$

$$m_2\ddot{z}_2 + b_2\dot{z}_2 + c_2 z_2 - b_1(\dot{z}_1 - \dot{z}_2) - c_1(z_1 - z_2) = N(t).$$

To represent the system of differential equations in the form of Cauchy, we shall introduce the following denotations (Table 1):

Table 1

Denotation of variables for recording in the form of Cauchy

z_1	z_2	$\dot{z}_1$	$\dot{z}_2$	$\ddot{z}_1$	$\ddot{z}_2$
x_1	x_2	$\dot{x}_1$	$\dot{x}_2$		
		x_3	x_4	$\dot{x}_3$	$\dot{x}_4$

$$\dot{x}_1 = x_3;$$

$$\dot{x}_2 = x_4;$$

$$\dot{x}_3 = -\frac{1}{m_1}\left(b_1\left(x_3 - x_4\right) + c_1\left(x_1 - x_2\right)\right);$$

$$\dot{x}_4 = -\frac{1}{m_2}(b_1(x_4 - x_3) + b_2 x_4 + c_1(x_2 - x_1) + c_2 x_2 - N(t))$$

or

$$\dot{X} = A_1 X + LV_0,$$

where

$$A_1 = \begin{bmatrix} 0 & E \\ -M^{-1}C & -M^{-1}F \end{bmatrix}; \quad V_0 = M^{-1}N;$$

$$N = \begin{bmatrix} 0 \\ N(t) \end{bmatrix}; \; L = \begin{bmatrix} 0 \\ E \end{bmatrix}; \; X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix};$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}; \; F = \begin{bmatrix} b_1 & -b_1 \\ -b_1 & b_1 + b_2 \end{bmatrix}; \; C = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix}.$$

A matrix of parameters for the optimum filter will be derived from the Kalman-Bucy algorithm. For the model of carriage under consideration

$$\dot{X} = A_1 X + L V_0;$$

$$Y = HX + V_H,$$

where $H=E$;

$$V_H = \begin{bmatrix} V_q \\ V_{\dot{q}} \end{bmatrix},$$

V_0, V_H are the white noises (an object noise and a measurement noise), with a null mathematical expectation and variance of external disturbances and measurement errors. The initial state of the system is a Gaussian random variable with the system state vector mathematical expectation and a correlation matrix of the estimation error.

A matrix differential equation for the optimal filter, disturbed by the observed signal, takes the form

$$\dot{\hat{X}} = (A_1 - KH' R^{-1} H)\hat{X} + KH' R^{-1} Y, \quad \hat{X}(t_0) = \bar{X}_0,$$

where $\hat{X}$ is the system state vector estimation; K is the correlation matrix of estimation error, derived from a solution to the variance equation of Riccati type

$$\dot{K} = A_1 K + K A_1' - KH' R^{-1} HK + LQL', \quad K(t_0) = K_0.$$

We obtain for the considered case

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & D \end{bmatrix}; \; R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}; \; R_1 = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}; \; R_2 = \begin{bmatrix} r_3 & 0 \\ 0 & r_4 \end{bmatrix};$$

$$K_0 = \begin{bmatrix} D_{x_1} & 0 & 0 & 0 \\ 0 & D_{x_2} & 0 & 0 \\ 0 & 0 & D_{x_3} & 0 \\ 0 & 0 & 0 & D_{x_4} \end{bmatrix},$$

where D is the variance of accelerations of a wheel center; D_{Xi} are the required variances of the system state vector components; r_i are the variances of errors in the measurements of displacements and accelerations.

The matrix of parameters for an optimum filter

$$F_{opt} = A_1 - KH' R^{-1} H =$$

$$= \begin{bmatrix} 0 - R_1^{-1} K_1 & E - R_2^{-1} K_2 \\ -M^{-1}C - R_1^{-1} K_2' & -M^{-1}F - R_2^{-1} K_4 \end{bmatrix},$$

where

$$K_1 = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}; \; K_2 = \begin{bmatrix} K_{13} & K_{14} \\ K_{23} & K_{24} \end{bmatrix}; \; K_4 = \begin{bmatrix} K_{33} & K_{34} \\ K_{43} & K_{44} \end{bmatrix}.$$

For a qualitative analysis, it will suffice to check dynamic indicators of the source systems and their corresponding optimal filters for the effect of a harmonic disturbance. Table 2 gives results of simulation of locomotives with a two-stage suspension system.

Table 2

Simulation results

Type of locomotive	$z_{1\max}$, m	$\ddot{z}_{1\max}$, m/s^2	$z_{2\max}$, m	$\ddot{z}_{2\max}$, m/s^2	Degree of damping b_1/b_{1kp}	Degree of damping b_2/b_{2kp}
VL80^K	0.013	2.591	0.017	1.334	0.15	0.15
VL80^K filter	0.008	1.896	0.007	1.03	0.41	0.07
2TE121	0.031	3.076	0.016	1.55	0.145	0.04
2TE121 filter	0.004	0.635	0.006	1.049	0.546	0.149
TEM7	0.019	1.345	0.016	1.098	0.444	0.038
TEM7 filter	0.011	0.546	0.011	0.505	0.623	0.085
GE125	0.028	2.7	0.011	1.09	0.122	0.082
GE125 filter	0.008	0.935	0.008	1.02	0.401	0.13

Comparison of degrees of damping of locomotives with the original suspension system and optimum filters clearly shows that oscillation dampers are needed also at the bottom stage of suspension. In this case, a passive suspension system will be sufficient as the design speed of locomotives does not exceed 160 km/h.

When designing high-speed transport, it is possible to achieve excellent estimation of the carriage run if the passive system is complemented by devices for active control over parameters of elastic-dissipative links.

4. 3. A possibility to control parameters of elastic-dissipative links in a suspension system for transport carriages

A differential equation of the oscillatory process of a carriage model with a single-stage suspension system, represented in the form of a dynamic system with one degree of freedom, is recorded in the form

$$m\ddot{z} + b\dot{z} + cz = N(t)$$

or

$$\ddot{z} + a_1\dot{z} + a_2 z = n(t), \tag{10}$$

where $a_1=b/m$; $a_2=c/m$; $n(t)=N(t)/m$; $N(t)$ is the external random disturbance from the track. To proceed to the Kalman-Bucy algorithm, we reduce equation (10) to the normal Cauchy form, taking $x_1=z$, $x_2=\dot{z}$

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_2 x_1 - a_1 x_2 + n(t). \end{cases} \tag{11}$$

Let us measure displacements and velocities of the center of body masses when a carriage is in motion, that is, all components of the system state vector are "monitored". In this case, a dynamic system will be represented by differential equations

$$\frac{dX(t)}{dt} = FX(t) + LN(t); \tag{12}$$

$$Y(t) = HX(t) + V(t),$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

is a vector-column of the system state;

$$F = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}$$

is a matrix of system parameters;

$$N = \begin{bmatrix} 0 \\ n(t) \end{bmatrix}$$

is the vector-column of external disturbances;

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is the identity matrix at vector *N(t)*;

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is the identity matrix that describes the relationship between the observed vector and the state vector;

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

is a vector-column of measurement errors.

To derive a correlation matrix of the estimation error and a matrix gain coefficient of the optimal filter, we shall employ a Riccati equation

$$\frac{dK}{dt} = FK(t) + K(t)F' - K(t)H'R^{-1}HK(t) + LQL', \tag{13}$$

where

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}; \quad Q = \begin{bmatrix} 0 & 0 \\ 0 & q \end{bmatrix};$$

$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}; \quad R^{-1} = \begin{bmatrix} 1/r_1 & 0 \\ 0 & 1/r_2 \end{bmatrix}.$$

Substituting the matrices in equation (13) and performing matrix operations, we shall obtain a system of Riccati equations

$$\dot{K}_{11} = 2K_{12} - K_{11}^2 / r_1 - K_{12}^2 / r_2;$$

$$\dot{K}_{12} = K_{22} - a_2 K_{11} - a_1 K_{12} - K_{11} K_{12} / r_1 - K_{12} K_{22} / r_2;$$

$$\dot{K}_{22} = -2(a_2 K_{12} + a_1 K_{22}) - K_{12}^2 / r_1 - K_{22}^2 / r_2 + q.$$

Expression for the filter misalignment factor

$$C(t) = K(t)H'R^{-1} = \begin{bmatrix} K_{11}/r_1 & K_{12}/r_2 \\ K_{21}/r_1 & K_{22}/r_2 \end{bmatrix}. \tag{14}$$

Then the matrix of parameters for an optimum filter takes the form

$$F_{opt} = F - C(t)H = \begin{bmatrix} -K_{11}/r_1 & 1 - K_{12}/r_2 \\ -a_2 - K_{12}/r_1 & -a_1 - K_{22}/r_2 \end{bmatrix}. \tag{15}$$

By analyzing the matrix of parameters for an optimal filter, we conclude that the rigidity and coefficient of resistance of oscillation dampers should change in this case. If the values of elements in the correlation matrix of an estimation error approach zero or are equal to zero, the initial system will be close to optimal.

Write the matrix of parameters for an optimal filter in the following form

$$F_{opt} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}.$$

Characteristic equation of an optimal filter

$$D = \begin{vmatrix} A_1 - \lambda & A_2 \\ A_3 & A_4 - \lambda \end{vmatrix} = 0$$

or

$$\lambda^2 - (A_1 + A_4)\lambda + A_1 A_4 - A_2 A_3 = 0.$$

Therefore, the rigidity and resistance coefficient of oscillation absorbers can be determined from formulae

$$c = m(A_1 A_4 - A_2 A_3);$$

$$b = -m(A_1 + A_4). \tag{16}$$

To obtain numerical values for parameters, it is required to solve a system of Riccati equations and substitute in (15) values for the elements of the correlation matrix of an estimation error corresponding to the established process.

The procedure is universal and is suitable for use when designing or modernizing any transport vehicles. As an example, we obtain the matrix of parameters for an optimal filter for the diesel locomotive 2TE10L (Ukraine, PO "Luganskteplovoz") taking into consideration that in the process of motion we monitor the speed of displacement of the center of the body masses.

Data for calculation are the sprung mass of the locomotive body *m*=129 t; resistance coefficient of hydraulic oscillation dampers, mounted in the suspension system *b*=446.34 kNs/m; rigidity of the suspension system *c*=17,157 kN/m.

Differential equation of the oscillatory process of a carriage model

$$m\ddot{z} + b\dot{z} + cz = N(t) \ \text{ or } \ \ddot{z} + a_1\dot{z} + a_2 z = n(t),$$

where

$a_1=b/m=3.46$; $a_2=c/m=133$; $n(t)=N(t)/m$.

Taking $x_1=z$, $x_2=\dot{z}$, we proceed to recording in the normal Cauchy form

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -133x_1 - 3,46x_2 + n(t). \end{cases}$$

Represent a model according to the Kalman-Bucy

$$\frac{dX(t)}{dt} = FX(t) + LN(t); \;\; Y(t) = HX(t) + V(t),$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \;\; F = \begin{bmatrix} 0 & 1 \\ -133 & -3,46 \end{bmatrix}; \;\; N = \begin{bmatrix} 0 \\ n(t) \end{bmatrix};$$

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \;\; H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \;\; V = \begin{bmatrix} 0 \\ v_2 \end{bmatrix}.$$

The correlation matrix of an estimation error is determined from the matrix Riccati equation

$$\frac{dK}{dt} = FK(t) + K(t)F^{'} - K(t)H^{'}R^{-1}HK(t) + LQL^{'},$$

where

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}; \;\; Q = \begin{bmatrix} 0 & 0 \\ 0 & q \end{bmatrix};$$

$$R = \begin{bmatrix} 0 & 0 \\ 0 & r_2 \end{bmatrix}; \;\; R^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 1/r_2 \end{bmatrix}.$$

We accept for the calculation $r_2=0.002$ m^2/s^2; $q=0.5$ m^2/s^4;

$$K(t_0) = \begin{bmatrix} 0 & 0 \\ 0 & 0,005 \end{bmatrix}.$$

The process of solving the Riccati equations is a rapidly convergent one. Under the established state at $t=1$ s we obtained: $K_{11}=0.000191$; $K_{12}=0$; $K_{22}=0.02545$.

Matrix of parameters for an optimal filter

$$F_{opt} = \begin{bmatrix} 0 & 1 \\ -133 & -16,185 \end{bmatrix}.$$

Note. When modeling dynamics of the diesel locomotive TE127 (Ukraine, PO "Luganskteplovoz") under the action of a random perturbation (spectrum from the All-Union Scientific Research Institute of Railroad Transport), a variance of the wheel's center accelerations was obtained of $q=20.2$ m^2/s^4 at speed 35 m/s. [19].

In this case,

$r_2=0.02$ m^2/s^2; $K(t_0) = \begin{bmatrix} 0 & 0 \\ 0 & 0,05 \end{bmatrix}.$

Under a steady mode at $t=1$ s we obtain: $K_{11}=0.0013$; $K_{12}=0$; $K_{22}=0.1938$.

Matrix of parameters for an optimal filter

$$F_{opt} = \begin{bmatrix} 0 & 1 \\ -133 & -13,15 \end{bmatrix}.$$

Comparing the matrices of the original system parameters and the optimal filter we note that the value for a parameter related to the work of an oscillation damper significantly changed.

Let us consider a possibility to control the damping. A system of Riccati equations for this case is

$$\dot{K}_{11} = 2K_{12} - K_{12}^2 / r_2;$$

$$\dot{K}_{12} = K_{22} - a_2K_{11} - a_1K_{12} - K_{12}K_{22} / r_2;$$

$$\dot{K}_{22} = -2(a_2K_{12} + a_1K_{22}) - K_{22}^2 / r_2 + q. \quad (17)$$

Under a steady mode, elements of the correlation matrix of an estimation error are constant, which is why derivatives from them are equal to zero. Then

$$2K_{12} - K_{12}^2 / r_2 = 0;$$

$$K_{22} - a_2K_{11} - a_1K_{12} - K_{12}K_{22} / r_2 = 0;$$

$$-2(a_2K_{12} + a_1K_{22}) - K_{22}^2 / r_2 + q = 0. \quad (18)$$

$$K_{12} = 0; \quad K_{22} = r_2(-a_1 \pm \sqrt{a_1^2 + q / r_2}); \quad K_{11} = K_{22} / a_2. \quad (19)$$

$$F_{opt} = \begin{bmatrix} -K_{11}/r_1 & 1 - K_{12}/r_2 \\ -a_2 - K_{12}/r_1 & -a_1 - K_{22}/r_2 \end{bmatrix}. \quad (20)$$

Therefore, the resistance coefficient of oscillation dampers is

$$b = m\sqrt{a_1^2 + q / r_2}, \quad (21)$$

where r_2 is the variance of errors in the displacement speed measurements, which does not depend on the speed of locomotive motion and is determined by the quality of the device. Therefore, the resistance coefficient of oscillation dampers depends only on the variance of disturbance acceleration of the wheel's center. By establishing the actual dependence of variance of disturbance acceleration of the wheel's center on motion speed, it is possible to achieve an excellent estimation of the locomotive run throughout the entire range of operating speeds. In practice, it is possible to improve dynamics of the examined locomotives through a transition to controlled pneumatic suspension.

5. Results of studying carriage oscillation in the simulation system MVTU 3.7

We shall perform a qualitative analysis of dynamic indicators of the diesel locomotive 2TE10L with the existing parameters of the suspension system and parameters for the optimal filter under the action of a harmonic disturbance.

A differential equation for the vertical oscillations of a simple model is

$$m\ddot{z} + b\dot{z} + cz = ch\sin\omega t,$$

where m is the sprung mass of the locomotive body, b is a resistance coefficient of hydraulic oscillation dampers, mounted in the suspension system, c is the rigidity of a suspension system, h is the maximum amplitude of track irregularities, ω is the frequency of repeated track irregularities, z, $\dot{z}$, $\ddot{z}$ are the displacement, speed, and acceleration of the center of body masses.

We shall study a model in the simulation system MVTU 3.7 for the following data: m=129 t; b=446.34 kNs/m; c=17,157 kN/m; h=0.02 m; ω=10 s^{-1}; z_0=0; $\dot{z}_0$=0. For the optimal filter, b=2,087.865 kNs/m.

In order to construct a structural scheme of the model we shall solve a differential equation relative to the senior derivative

$$\ddot{z} = (-1/m)(b\dot{z} + cz - ch\sin\omega t).$$

One can see that the acceleration is equal to the sum of three components; in order to obtain the displacements, it is required to perform a double integration. Fig. 3 shows a modeling scheme. The process of constructing modeling schemes, as well as the technology of work in a simulation system, are described in paper [19].

Fig. 4–7 show results of the simulation.

Comparison of simulation results shown in Fig. 4, 6, and Fig. 5, 7, shows that the application of analytical approaches using the Kalman-Bucy algorithm makes it possible to obtain optimal parameters for a suspension system of dynamic systems.

An analysis of dynamic indicators of the examined systems could be conveniently conducted in the modeling system MVTU 3.7. The library of the modeling system MVTU 3.7 includes a link of the general form that enables the construction of enlarged schemes and examination of transient processes of complex dynamic systems.

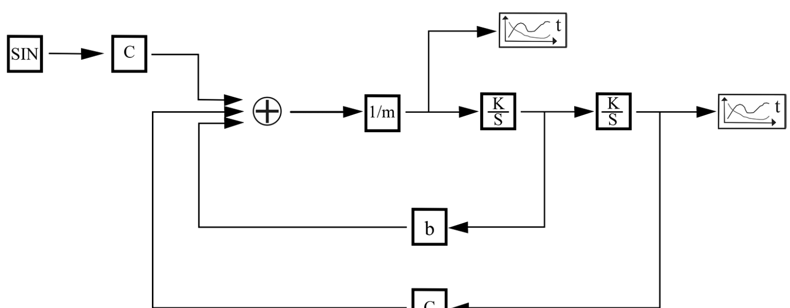

Fig. 3. Modeling scheme

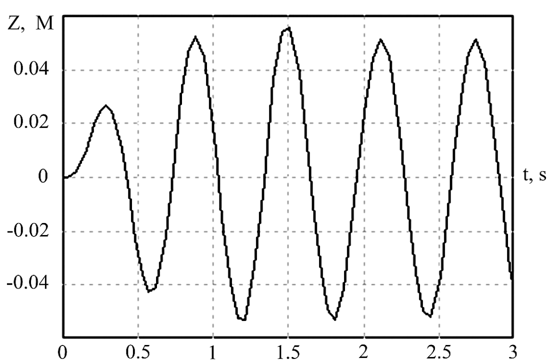

Fig. 4. Chart of displacements of the center of body masses

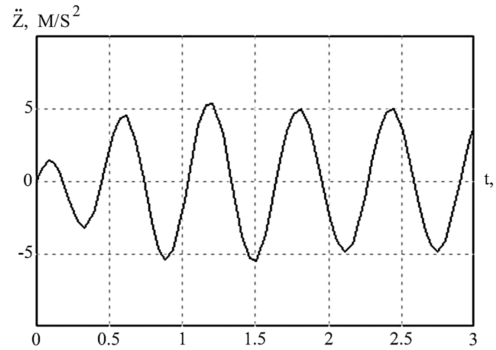

Fig. 5. Chart of accelerations of the center of body masses

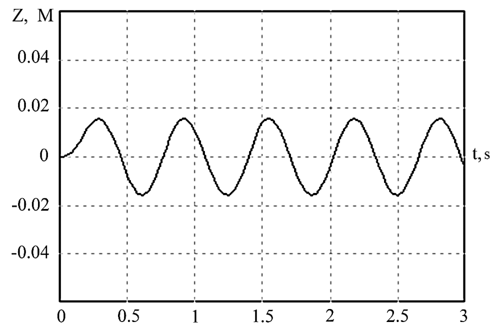

Fig. 6. Displacements of the center of body masses of the filter

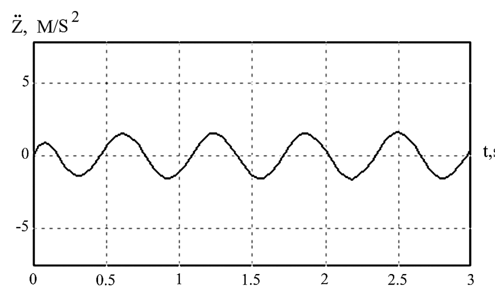

Fig. 7. Accelerations of the center of body masses of the filter

6. Discussion of results of theoretical study and simulation of a railroad carriage oscillations

The research results obtained demonstrate a procedure for testing the feasibility of designing an active suspension system for transport carriages.

The proposed procedure employs analytical method of calculation. To this end, we have developed techniques that make it possible to apply the Kalman-Bucy algorithm to complex dynamic systems. The techniques are as follows:

– we impose a constraint on the structure of the designed system, which implies the application of linear and symmetric estimation schemes;

– we have chosen weight coefficients for the square functional of quality in accordance with the requirements to sanitary-and-hygienic norms that ensure health and working capacity for a locomotive crew and passengers.

The viability of the procedure was checked using test models of a transport carriage. The obtained results on the simulation of control over parameters of elastic-dissipative links (Table 2, Fig. 4–7) testify to the improvement of dynamic indicators of rolling stock.

The procedure might prove useful when designing new carriages or modernizing the existing ones, thereby avoiding unnecessary expenditures for creating devices of active control over elastic-dissipative links. The procedure is aimed at building transport carriages for regular and high-speed motion.

This study is continuation of the earlier studies [20]. Still unresolved is the task on solving the systems of nonlinear differential Riccati equations in the programming environment MVTU 3.7 for complex dynamic systems.

7. Conclusions

1. At present, there exist the prerequisites for the development of various types of active suspension systems, capable of providing the required dynamic properties for a carriage given a substantial increase in motion speed.

2. We have devised techniques that allow the adaptation of the Kalman-Bucy method for designing complex dynamic systems.

3. When designing suspension systems for transport carriages, it is required to apply a procedure of phased design, which implies at the first stage designing a passive suspension system, at the second – verifying the principle of operation of a suspension system using the optimal Kalman-Bucy filters, that is, testing the feasibility of designing devices to control elastic-dissipative links.

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