UDC 621.311:621.314
DOI: 10.15587/1729-4061.2018.136398

IMPROVEMENT OF THE METHOD FOR ANALYSIS OF NONLINEAR ELECTROTECHNICAL SYSTEMS BASED ON THE SMALL PARAMETER METHOD

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Для аналітичного аналізу нелінійних електричних кіл електротехнічних систем був використаний метод малого параметра, реалізований в частотної області. Для можливості реалізації розрахунків в частотної області використовувався автоматизований метод формування ортогональних гармонійних складових електричних величин на базі алгоритму дискретної згортки. Характеристика нелінійного елемента була представлена поліноміальною функцією третього ступеня. Показано, що застосування методу малого параметра з його реалізацією в частотної області дозволяє спростити процес аналізу електричних кіл з нелінійними елементами в аналітичному за рахунок автоматизації розрахунків у математичному пакеті. Аналітичні і чисельні розрахунки кола з активно-індуктивним навантаженням продемонстрували достатню точність запропонованого методу, відносна похибка за основною гармонікою струму не перевищила 6 %. Проведений порівняльний аналіз запропонованого методу малого параметра з класичним методом малого параметра на прикладі розрахунку електричного кола з RL навантаженням показав, що розроблений метод забезпечує більшу адекватність результатів та вищу точність розрахунків у порівнянні із існуючим. Відносна похибка за амплітудою першої та третьої гармонік струму не перевищує 2,5 % та за фазою – $1,042 \cdot 10^{-3}$ %. Метод чисельного структурного моделювання застосовувався для визначення еталонних значень струму досліджуваного кола. Результати роботи можуть бути використані при розрахунках електротехнічних пристроїв, що містять напівпровідникові елементи і електричні апарати з нелінійними характеристиками. Також отримані результати дозволять удосконалити процеси активної компенсації вищих гармонік струму в електричних мережах з нелінійним навантаженням, що містить, а також розробляти засоби пасивної компенсації

Ключові слова: нелінійна система, електричне коло, аналіз, метод малого параметра, частотна область, автоматизований алгоритм

1. Introduction

At present, analysis of nonlinear electrical circuits (NEC) is one of the most important tasks of modern theoretical electrical engineering [1, 2]. NEC are an integral part of most electrotechnical and electronic devices. The need for complicating the power semiconductor converters and accounting of new nonlinear effects is becoming good stimuli for the development of the theory of calculation and analysis of NEC. The latter is used increasingly often, both in traditional areas of analysis and synthesis [3, 4], and in new directions of modeling, identification, etc. [2, 5]. Analysis of NEC often comes down to determining the currents by known parameters of a circuit [3]. However, the process of analysis of the researched nonlinear systems is accompanied by specific difficulties, among which we can separate the following:

– impossibility of using well-developed methods for analysis of linear circuits to solve nonlinear ones, due to emerging of phenomena not inherent in linear circuits [4, 5];

– necessity of an accurate approximation of a nonlinear characteristic to maximize the accuracy of calculations [6].

That is why researchers have to resort in practice to a combination of several methods, such as the wavelet transformations and iterative methods [7] of modeling the NEC operation, using a large number of methods of numerical calculation with a complicated set of initial conditions [8]. This in turn leads to a complication of calculation dependences and provokes an increase in calculation errors [3, 5, 6–8].

Today, the task of studying the processes occurring in circuits with semiconductor converters, which are the elements of a clearly pronounced nonlinearity, remains relevant. These are nonlinear circuits of power systems [9, 10], nonlinear circuits of power active filters [11], control systems, which are based on the p-q theory [12] and cross-vector [13] theories of instantaneous power. The main issues that are tackled by researchers in the analysis of such systems include: determining the patterns and an increase in effectiveness of energy conversion [11–14, 16–18], determining the nature of occurrence of reactive power [12, 15, 20], and development of effective methods for calculation of currents of researched circuits [21].

It should also be noted that the relevant problem today is selection of tools [19] of active and passive filtration [22, 23]. But the correct choice of the tools for passive filtration requires the analysis of indicators of operation of a connected nonlinear load, which is possible using the analytical methods of calculation of electrical circuits [21].

Given the above, development of the analytical method for calculation that is effective and simple in practical terms is reasonable. The developed method should provide an opportunity to conduct a qualitative analysis of a nonlinear electrotechnical system and will also make it possible to conduct an analysis of the mechanisms of the current components formation.

2. Literature review and problem statement

As it is known, the methods for analysis of nonlinear electrical circuits can be divided into analytical and numerical.

The use of the numerical methods greatly facilitates the task for researchers because they make it possible with a relative ease of implementation to obtain information about almost any processes in the NEC. Due to the rapid development of computer technology and existence of a large number of specialized software products, such as SAPWIN [24, 25], SPICE [26], SEQUEL [27], MATLAB [28–30], which allow implementation of calculations and numerical modeling, numerical methods the most frequently used. However, it should be noted that these methods have the following shortcomings: numerical solution of equations with significantly pronounced nonlinearity requires a large number of iterations, accompanied by considerable time consumption. Another drawback is the fact the accuracy of the calculation significantly depends on correctness of the chosen initial approximations and configuration of a scheme, as well as on a method and a pitch of integration.

In contrast to numerical methods, analytical methods have a significant advantage. They allow us to conduct a qualitative analysis and give an adequate evaluation to the processes in the NEC and estimate the influence of all factors in the course of operation of a system, their contribution and importance [1, 3, 5, 9, 10, 21].

As it is known, the work of circuits is described by nonlinear differential equations, the solution of which allows determining the currents that flow in a circle.

Based on the analysis of the literary sources, to solve such problems, researchers often choose the small parameter method (SPM) [3, 5, 9, 10, 21], which belongs to the perturbation methods, as the most efficient, universal, and simple to implement. SPM is one of the most effective tools of modern applied mathematics, which is widely used in mechanics, physics, and other sciences that operate with differential equations. SPM allows obtaining analytical solutions of complex linear and nonlinear boundary value problems both for ordinary differential equations and for equations in partial ones. It can be used separately or in a combination with other methods [3, 5].

The main advantage of SPM is its effectiveness in solving complicated differential equations that describe the processes in NEC. That is why in this paper it is proposed to use SPM to determine a harmonic composition of current of nonlinear load, connected to the electrical supply system. This will make it possible to select correctly the parameters of a linear reactor to reduce the impact of higher harmonics in the current signal.

According to SPM [3, 9, 10], a non-linear differential equation is solved in the time domain by successive approximations, representing the sought parameter x as a series by powers of some coefficient μ, which is called the small parameter:

$$x = x_0 + \mu x_1 + \mu^2 x_2 + \ldots + \mu^n x_n, \tag{1}$$

where x_0 is the solution to an equation of zero approximation (the latter is obtained from the original, assuming all nonlinear terms are absent in them); x_1 is the equation of first correction, which takes into consideration the influence of non-linear terms in first approximation; x_2 is the solution of the equation of second correction, etc.

The author of paper [3] proved effectiveness of using SPM for analyzing NEC with minimal errors. According to SPM, an approximated solution of nonlinear equations, which describe processes in nonlinear circuits, is found in the form of the functional series by increasing powers of small parameter. Substituting such a series into the original equation and representing it with the system of equations by powers of small parameter, all the functions, included in the solution, are found.

Taking into consideration the above shortcomings of the described numerical methods of analysis of NEC, as well as the fact that SPM is usually implemented in the time domain, which is accompanied by complex trigonometric transformations and decreases its effectiveness, it is necessary to search for a new method or to improve the existing one. That is why in order to enhance efficiency of analytical calculations, it is proposed to implement SPM in the frequency domain using the automated method for forming orthogonal harmonic components of electrical magnitudes [20, 31] based on the algorithm of discrete convolution [32].

3. The aim and objectives of the study

The aim of the research is to develop an effective and simple in practical terms analytical method for analysis of nonlinear of electrotechnical system due to the implementation of calculations in frequency domain with a possibility of their automation, which will give the opportunity to perform a qualitative analysis of the researched system, specifically, will make it possible to assess the impact of the parameters of a circuit on the harmonic composition of current.

To accomplish the set goal, the following tasks were stated:

– to develop SPM through its implementation in frequency domain due to the use of the automated method for forming electrical magnitudes based on the algorithm of discrete convolution;

– to carry out a comparative analysis of SPM with implementation of calculations in frequency domain that is developed with the classic SPM in time domain and with the results of calculation of a mathematical model of the researched electrical circuit.

4. Analysis of calculation of nonlinear electrical circuits of electrotechinical systems with the use of SPM in the frequency domain

4. 1. Algorithm of the analytical method for NEC analysis in the frequency domain based on SPM

SPM in time domain is the prototype of SPM, implemented in frequency domain. It should be noted that the implementation in frequency or time domain does not affect accuracy. In this regard, only accuracy of approximation of VAC of nonlinear function, as well as the number of harmonics, which are analyzed are important.

But it will cause complications of analytical expressions. As a result, calculation in time domain will be time-consuming and accompanied by difficulty associated with multiple multiplication of the current and voltage signals that are assigned in the form of the Fourier series, trigonometric transformations of cosine sine functions, double angle functions, etc.

The ultimate result of trigonometric transformations should be expressions that contain functions $\cos(k\omega t)$ and $\sin(k\omega t)$, where k is the number of harmonics. In addition, trigonometric transformations are difficult to automate. All this can lead to obtaining an incorrect and unpredictable result. That is why it is proposed to develop the SPM through implementation in frequency domain. The main advantage of SPM in frequency domain is that trigonometric transformations, associated with multiplication of current and voltage signals, are implemented with the use of the operation of discrete convolution, which will ensure adaptation to software automation of analytical calculations. This allows us to significantly decrease time consumption and to simplify the process of analytical calculation. It also makes it possible to obtain the predicted result regardless of degree of an approximating polynomial, as well as the number of analyzed harmonics [9, 10, 13, 23].

The algorithm of analysis of NEC in an analytical form based on SPM in the frequency domain, shown in Fig. 1, includes the following stages:

1. We write down the expression that determines the form of nonlinearity of the researched electrical circuit:

$$\mu = f(i), \tag{2}$$

where μ is the nonlinear parameter. According to the classic SPM, nonlinearity is represented in the form of instantaneous resistance, which nonlinearly depends on current.

2. Using the automated method of forming components of instantaneous variables of electrical circuit in frequency domain based on the operation of discrete convolution, we form the equation with the use of the second law of Kirchhoff. The expression for voltage on a nonlinear element is written down in the left part, while the differences of voltages on the power source and all consumers, except for the nonlinear one, are written down in the right part.

3. To solve the resulting nonlinear equation, current is represented in the form of a power series by nonlinear parameter μ:

$$I = a_0 + a_1\mu + a_2\mu^2 \ldots + a_n\mu^n, \tag{3}$$

where a_0, a_1, a_2, a_n are the corresponding coefficients of the power series; n is the order of the power series, and of the corresponding number of the coefficient.

It should be noted that for a reliable analysis of the processes occurring in electrical circuits, as well as for taking into account the features, introduced by nonlinear elements, it is necessary to select a higher power of the series that describes the current than the order of the power series that describes the nonlinear element.

4. To represent (3) in the frequency domain, additional designations are introduced. According to SPM, it was accepted that orthogonal cosine a_{a0} and sine a_{b0} elements of coefficient a_0, which is a generating solution and describes the work of the linear part of the circuit, is determined by the first harmonic:

$$a_{a0} = \begin{pmatrix} 0 \\ A_0 \\ 0 \end{pmatrix}; \quad a_{b0} = \begin{pmatrix} 0 \\ B_0 \\ 0 \end{pmatrix}, \tag{4}$$

where A_0, B_0 are the coefficients that depend on parameters of a circuit, cosine and sine, respectively.

According to SPM, which is implemented in frequency domain, harmonic composition of coefficients a_0, a_1, and a_n of power series (2) is formed by the following principle. Coefficient a_1 must include harmonic composition, which corresponds to harmonic composition of coefficient a_0^{h+1} raised to power m in frequency domain, where h is the maximum power of current in the expression that describes current on the nonlinear element.

In other words, the cosine $\mathrm{a}_{\mathrm{a}1}$ and sine $\mathrm{a}_{\mathrm{b}1}$ components of a_1 in the general case in frequency domain will be made up of the sum of coefficients $\left(\sum_{k=0}^{K} A_{1m}\right)_m$ and $\left(\sum_{k=0}^{K} B_{1m}\right)_m$, where A_{1k}, B_{1k} are the components of coefficient a_1, which depends on the parameters of a circuit; k is the number of harmonic component that forms the coefficient; K is the minimal number of the harmonic component of current, which form the coefficient, m is the number of harmonic of current.

In its turn, harmonic composition of coefficient a_2 will be determined as convolution $a_0^h \cdot a_1$ in frequency domain.

Thus, a_n will be formed according to the maximum degree of current in the expression that describes voltage on the nonlinear element and its harmonic structure:

$$a_{an} = \left(\sum_{k=0}^{K} A_{nm}\right)_m;$$

$$a_{bn} = \left(\sum_{k=0}^{K} B_{nm}\right)_m. \tag{5}$$

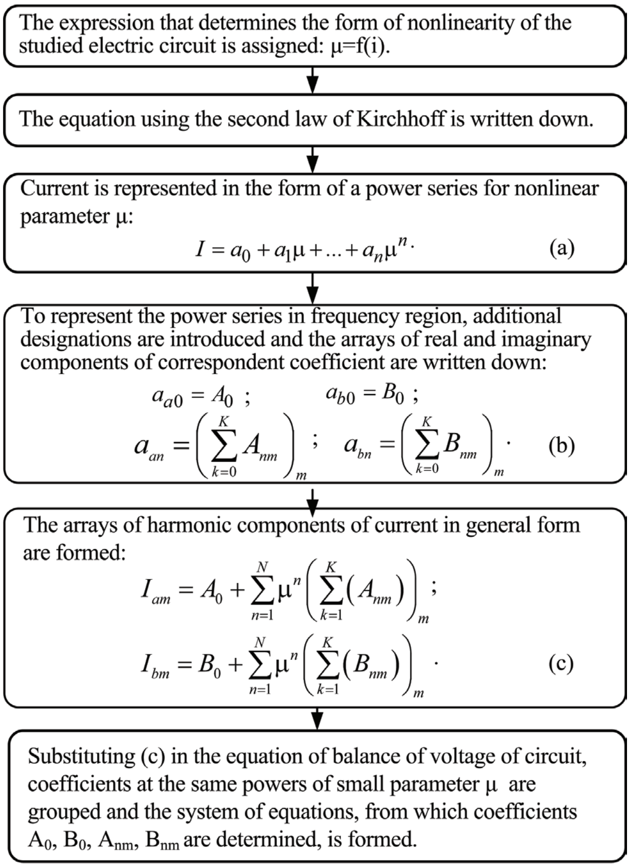
5. Arrays of harmonic components of current in the general form are formed:

$$I_{am} = A_0 + \sum_{n=1}^{N} \mu^n \left(\sum_{k=1}^{K} (A_{nm})\right)_m;$$

$$I_{bm} = B_0 + \sum_{n=1}^{N} \mu^n \left(\sum_{k=1}^{K} (B_{nm})\right)_m, \tag{6}$$

where N is the maximum order of the power series.

6. (6) are substituted in the equation of balance of voltages of the circuit, coefficients at equal powers of small parameter μ are grouped, and the system of equations, from which we determine coefficients A_0, B_0, A_{nm}, B_{nm}, is formed.



The expression that determines the form of nonlinearity of the studied electric circuit is assigned: μ=f(i).

The equation using the second law of Kirchhoff is written down.

Current is represented in the form of a power series for nonlinear parameter μ:

$$I = a_0 + a_1\mu + \ldots + a_n\mu^n . \qquad \text{(a)}$$

To represent the power series in frequency region, additional designations are introduced and the arrays of real and imaginary components of correspondent coefficient are written down:

$$a_{a0} = A_0 \ ; \qquad a_{b0} = B_0 \ ;$$

$$a_{an} = \left(\sum_{k=0}^{K} A_{nm}\right)_m ; \quad a_{bn} = \left(\sum_{k=0}^{K} B_{nm}\right)_m . \qquad \text{(b)}$$

The arrays of harmonic components of current in general form are formed:

$$I_{am} = A_0 + \sum_{n=1}^{N} \mu^n \left(\sum_{k=1}^{K} (A_{nm})\right)_m ;$$

$$I_{bm} = B_0 + \sum_{n=1}^{N} \mu^n \left(\sum_{k=1}^{K} (B_{nm})\right)_m . \qquad \text{(c)}$$

Substituting (c) in the equation of balance of voltage of circuit, coefficients at the same powers of small parameter μ are grouped and the system of equations, from which coefficients A_0, B_0, A_{nm}, B_{nm} are determined, is formed.

Fig. 1. The algorithm implementation of SPM in the frequency domain

4. 2. Calculation of the electrical circuit with nonlinear inductance using SPM in the frequency domain

Consider the example of calculation of the electrical circuit, composed of sequentially connected nonlinear inductance and linear active resistance (Fig. 2).

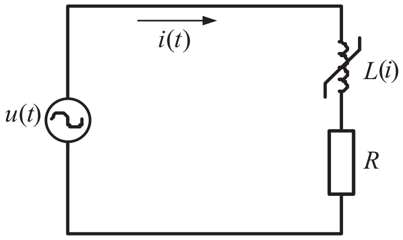


Fig. 2. Electrical circuit with nonlinear inductance

Power voltage $u_S(t)$ is assigned by sine component of the first harmonic: $u_S(t)=U_{b1}\sin(\omega t)$. For analysis, current is represented in the form of the first and the third harmonic components:

$$i(t)=I_{a1}\cos(\omega t)+I_{b1}\sin(\omega t)+ \\ +I_{a3}\cos(3\omega t)+I_{b3}\sin(3\omega t).$$

For an analysis of equivalent scheme of the researched circuit (Fig. 2), the following numerical values of parameters of the circuit were accepted: $U_{b1}=-12.5$ W, $R=50$ Ohm.

As we know, NEC are described by differential equations, the solution of which allows analysis of the system. For the researched system, shown in Fig. 2, the differential equation of the balance of voltages with dependence of parameters of a circuit on time that describes its operation of the system:

$$u_S(t)=i(t)R+\frac{d\psi(t)}{dt}, \qquad (7)$$

where ψ is the current linkage.

Nonlinear dependence of current on current linkage is approximated by the polynomial of third degree:

$$i(\psi)=a\psi+b\psi^3, \qquad (8)$$

where a, b are the corresponding coefficients of a polynomial. During the studies that were conducted with the aim of selecting the optimal function to describe a nonlinear characteristic in the software package CurveExpert, it was discovered that function (8), that is, a polynomial of third degree in the absence of quadratic term has the highest coefficient of coincidence with the curve of dependence of current on current linkage.

As a result of numerical construction (8), the magnetization curve of nonlinear inductance was obtained (Fig. 3), in this case, numerical values of coefficients were accepted as $a=0.1$, and $b=40$.

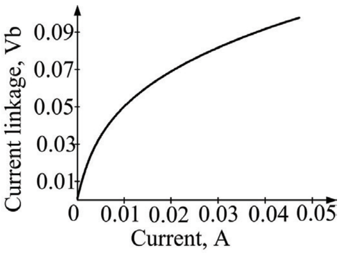


Fig. 3. Magnetization curve of nonlinear inductance

To have the possibility of realization of analysis of the researched electrical circuit using SPM in the frequency domain, differential equation (7) is written down in the frequency domain:

$$\begin{cases} U_{Sa}=U_{Ra}+U_{La}; \\ U_{Sb}=U_{Rb}+U_{Lb}, \end{cases} \qquad (9)$$

where U_{Sa}, U_{Ra}, U_{La} are the cosine component of power voltage, voltage on active resistance R, voltage on nonlinear inductance, respectively; U_{Sb}, U_{Rb}, U_{Lb} are the sine component of power voltage, voltage on active resistance R, voltage on nonlinear inductance, respectively.

Cosine and sine components of power voltage (10), voltage on active resistance (10) in frequency domain were represented according to SPM, implemented in the frequency domain.

$$U_{Sa}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}; \quad U_{Sb}=\begin{pmatrix}0\\U_{Sb1}\\0\\0\end{pmatrix}. \qquad (10)$$

$$U_{Ra}=\begin{pmatrix} 0 \\ -R\left[\begin{matrix} -\psi_{a1}a-3\psi_{a1}^{3}b-3\psi_{a1}\psi_{b1}^{2}b-6\psi_{a1}\psi_{a3}^{2}b- \\ -6\psi_{a1}\psi_{b3}^{2}b-3\psi_{a1}^{2}\psi_{a3}b-6\psi_{a1}\psi_{b3}\psi_{b1}b+ \\ +3\psi_{b1}^{2}\psi_{a3}b \end{matrix}\right] \\ 0 \\ R\left[\begin{matrix} \psi_{a3}a+\psi_{a1}^{3}b-3\psi_{a1}\psi_{b1}^{2}b+6\psi_{a1}^{3}\psi_{a3}^{2}b+ \\ +6\psi_{a3}\psi_{b1}^{2}b+3\psi_{a3}^{3}b++3\psi_{b3}^{2}\psi_{a3}b \end{matrix}\right] \\ 0 \\ -3bR\left[\begin{matrix} -\psi_{a3}\psi_{a1}^{2}+2\psi_{a1}\psi_{b3}\psi_{b1}+\psi_{a3}\psi_{b1}^{2}- \\ -\psi_{a1}\psi_{a3}^{2}-2\psi_{a3}\psi_{b3}\psi_{b1}+\psi_{b3}^{3}\psi_{a1} \end{matrix}\right] \\ 0 \\ -3bR\left[-\psi_{a1}\psi_{a3}^{2}+\psi_{a1}\psi_{b3}^{2}+2\psi_{a3}\psi_{b3}\psi_{b1}\right] \\ 0 \\ bR\psi_{a3}\left[\psi_{a3}^{2}-3\psi_{b3}^{2}\right] \end{pmatrix};$$

$$U_{Rb}=\begin{pmatrix} 0 \\ R\left[\begin{matrix} \psi_{b1}a+3\psi_{a1}^{2}\psi_{b1}b+3\psi_{b1}^{3}b+6\psi_{b1}\psi_{a3}^{2}b+ \\ +6\psi_{b1}\psi_{b3}^{2}b-6\psi_{a1}\psi_{a3}\psi_{b1}b-3\psi_{b3}\psi_{b1}^{2}b+ \\ +3\psi_{a1}^{2}\psi_{b3}b \end{matrix}\right] \\ 0 \\ -R\left[\begin{matrix} -\psi_{b3}a-3\psi_{a1}^{3}\psi_{b1}b+3\psi_{b1}^{3}b-6\psi_{b1}^{2}\psi_{b3}b- \\ -6\psi_{b3}\psi_{a1}^{2}b-3\psi_{a3}^{3}\psi_{b3}b-3\psi_{b3}^{2}b \end{matrix}\right] \\ 0 \\ -3Rb\left[\begin{matrix} -2\psi_{b1}\psi_{a1}\psi_{a3}+\psi_{b3}\psi_{b1}^{2}-\psi_{a1}^{3}\psi_{b3}- \\ -2\psi_{a1}\psi_{b3}\psi_{a3}-\psi_{b3}^{2}\psi_{b1}+\psi_{a3}^{3}\psi_{b1} \end{matrix}\right] \\ 0 \\ 3bR\left[\psi_{b1}\psi_{a3}^{2}-2\psi_{b3}^{2}\psi_{b1}+2\psi_{a1}\psi_{b3}\psi_{a3}\right] \\ 0 \\ bR\psi_{b3}\left[3\psi_{a3}^{2}-\psi_{b3}^{2}\right] \end{pmatrix}. \qquad (11)$$

According to (7), voltage on nonlinear inductance is determined as d$\psi(t)$/dt, that is why for further representation in frequency domain, cosine and sine components of current linkage, assigned by the first and third harmonics are written down first:

$$\psi_{a}=\begin{pmatrix} 0 \\ \psi_{a1} \\ 0 \\ \psi_{a3} \end{pmatrix}; \quad \psi_{a}=\begin{pmatrix} 0 \\ \psi_{b1} \\ 0 \\ \psi_{b3} \end{pmatrix}, \qquad (12)$$

where ψ_{a1}, ψ_{a3} are the cosine components of the first and third harmonics of current linkage, respectively; ψ_{b1}, ψ_{b3} are the sine component of the first and third harmonics of current linkage, respectively.

Then, a derivative is determined from the obtained arrays.

$$d\psi_{a}=\begin{pmatrix} 0 \\ -\psi_{b1}\omega \\ 0 \\ -3\psi_{b3}\omega \end{pmatrix}; \quad d\psi_{b}=\begin{pmatrix} 0 \\ \psi_{a1}\omega \\ 0 \\ 3\psi_{a3}\omega \end{pmatrix}. \qquad (13)$$

According to (8), using an automated algorithm of formation of components of electrical magnitudes in frequency domain, the cosine and sine components of current linkage in third power are determined. Then, the arrays of cosine and sine components of current, flowing in the researched circuit, are formed:

$$I_{am}=\begin{pmatrix} 0 \\ \psi_{a1}a+3\psi_{a1}^{3}b+3\psi_{a1}\psi_{b1}^{2}b+6\psi_{a1}\psi_{a3}^{2}b+ \\ +6\psi_{a1}\psi_{b3}^{2}b+3\psi_{a1}^{2}\psi_{a3}b+6\psi_{a1}\psi_{b3}\psi_{b1}b-3\psi_{b1}^{2}\psi_{a3} \\ 0 \\ \psi_{a3}a+\psi_{a1}^{3}b-3\psi_{a1}\psi_{b1}^{2}b+6\psi_{a1}^{2}\psi_{a3}b+ \\ +6\psi_{a3}\psi_{b1}^{2}b+3\psi_{a3}^{3}b+3\psi_{b3}^{2}\psi_{a3}b \\ 0 \\ -3b\left[\begin{matrix} -\psi_{a3}\psi_{a1}^{2}+2\psi_{a1}\psi_{b3}\psi_{b1}+\psi_{a3}\psi_{b1}^{2}- \\ \psi_{a1}\psi_{a3}^{2}-2\psi_{a3}\psi_{b3}\psi_{b1}+\psi_{b3}^{3}\psi_{a1} \end{matrix}\right] \\ 0 \\ -3b\left[-\psi_{a1}\psi_{a3}^{2}+\psi_{a1}\psi_{b3}^{2}+2\psi_{a3}\psi_{b3}\psi_{b1}\right] \\ 0 \\ b\psi_{a3}\left[\psi_{a3}^{2}-3\psi_{b3}^{2}\right] \end{pmatrix};$$

$$I_{bm}=\begin{pmatrix} 0 \\ \psi_{b1}a+3\psi_{a1}^{2}\psi_{b1}b+3\psi_{b1}^{3}b+6\psi_{b1}\psi_{a3}^{2}b+ \\ +6\psi_{b1}\psi_{b3}^{2}b-6\psi_{a1}\psi_{a3}\psi_{b1}b- \\ -3\psi_{b3}\psi_{b1}^{2}b+3\psi_{a1}^{2}\psi_{b3}b \\ 0 \\ \psi_{b3}a+3\psi_{a1}^{3}\psi_{b1}b-3\psi_{b1}^{3}b+6\psi_{b1}^{2}\psi_{b3}b+ \\ +6\psi_{b3}\psi_{a1}^{2}b+3\psi_{a3}^{3}\psi_{b3}b+3\psi_{b3}^{2}b \\ 0 \\ 3b\left[\begin{matrix} 2\psi_{b1}\psi_{a1}\psi_{a3}-\psi_{b3}\psi_{b1}^{2}+\psi_{a1}^{3}\psi_{b3}+ \\ +2\psi_{a1}\psi_{b3}\psi_{a3}+\psi_{b3}^{2}\psi_{b1}-\psi_{a3}^{3}\psi_{b1} \end{matrix}\right] \\ 0 \\ 3b\left[\psi_{b1}\psi_{a3}^{2}-2\psi_{b3}^{2}\psi_{b1}+2\psi_{a1}\psi_{b3}\psi_{a3}\right] \\ 0 \\ b\psi_{b3}\left[-3\psi_{a3}^{2}+\psi_{b3}^{2}\right] \end{pmatrix}. \qquad (14)$$

According to the algorithm of SPM (Fig. 1) and taking into account expression (7), we will represent current linkage in the form of a polynomial of the first degree by nonlinear parameter b:

$$\psi=a_{0}+a_{1}b, \qquad (15)$$

where a_0, a_1 are the corresponding coefficients of the polynomial.

According to the harmonic composition of current linkage $\psi(t)$ and algorithm of implementation of the SPM in frequency domain, coefficient a_0 will be determined from the following expression:

$$a_{0}=A_{0}\cos(\omega t)+B_{0}\sin(\omega t), \qquad (16)$$

since they are the generating solutions.

In accordance with the rule of formation of harmonic composition a_1 for a given electrical circuit, after analyzing a_0^3, we obtain that a_1 is formed by third harmonic in addition to first harmonic:

$$a_{1}=A_{11}\cos(\omega t)+A_{13}\cos(3\omega t)+$$
$$+B_{11}\sin(\omega t)+B_{13}\sin(3\omega t), \qquad (17)$$

where A_0, B_0, A_{11}, B_{11}, A_{13}, B_{13} are the coefficients that depend on parameters of a circuit.

Then the polynomial that describes the current linkage in the researched circuit will take the form:

$$\psi = A_0\cos(\omega t) + B_0\sin(\omega t) + \left[\begin{matrix} A_{11}\cos(\omega t) + A_{13}\cos(3\omega t) + \\ +B_{11}\sin(\omega t) + B_{13}\sin(3\omega t)\end{matrix}\right]b. \tag{18}$$

Coefficients in (18) are grouped by the appropriate trigonometric functions and frequencies, and the arrays of cosine and sine harmonic components of current linkage in frequency domain are written:

$$\psi_a = \begin{pmatrix} 0 \\ A_0 + A_{11}b \\ 0 \\ A_{13}b \end{pmatrix}; \quad \psi_b = \begin{pmatrix} 0 \\ B_0 + B_{11}b \\ 0 \\ B_{13}b \end{pmatrix}. \tag{19}$$

I. e., $\psi_{a1}=A_0+A_{11}b$, $\psi_{b1}=B_0+B_{11}b$, $\psi_{a3}=A_{13}b$, $\psi_{b3}=B_{13}b$.

The above expressions are substituted in the equation of balances of voltages in frequency domain, and coefficients at the same powers of the nonlinear parameter are equaled. It should be noted that we will take into consideration only coefficients of zero and first degrees b. And then, the system of equations is obtained:

$$\begin{cases} U_{b1} = \omega A_0 + aRB_0; \\ 0 = aRA_0 - \omega B_0; \\ 0 = 3RB_0^3 + \omega A_{11} + 3RB_0A_0^2 + aRB_{11}; \\ 0 = aRA_{13} + RA_0^3 - 3\omega B_{13} - 3RA_0B_0^2; \\ 0 = aRB_{13} + 3RB_0A_0^2 + 3\omega A_{13} - RB_0^3; \\ U_{b1} = A_0\omega + aB_0R + \left(3RB_0^3 + 3RB_0A_0^2 + \omega A_{11} + aRB_{11}\right)b; \\ 0 = -B_0\omega + aA_0R + \left(3RA_0^3 - \omega B_{11} + aRA_{11} + 3RA_0B_0^2\right)b; \\ 0 = \left(aRA_{13} + RA_0^3 - 3\omega B_{13} - 3RA_0B_0^2\right)b; \\ 0 = \left(aRB_{13} - RB_0^3 + 3\omega A_{13} + 3RB_0A_0^2\right)b, \end{cases}$$

from which coefficients A_0, B_0, A_{11}, B_{11}, A_{13}, B_{13} are determined:

$$\begin{cases} A_0 = \dfrac{U_{b1}\omega}{\omega^2 + R^2a^2}; \\ B_0 = \dfrac{U_{b1}Ra\left(Ra - \omega^2\right)}{-\omega^4 + R^4a^4}; \\ A_{11} = \dfrac{-6U_{b1}{}^3R^2a}{\left(\omega^3 + \omega R^2a^2\right)^2}; \\ B_{11} = \dfrac{3U_{b1}{}^3R}{\left(\omega^2 + R^2a^2\right)^2}; \\ A_{13} = \dfrac{\left(3\omega^4 - 12R^2a^2\omega^2 + R^4a^4\right)U_{b1}{}^3aR^2}{\left(9\omega^2 + R^2a^2\right)\left(\omega^2 + R^2a^2\right)^3}; \\ B_{13} = \dfrac{\left(3\omega^4 - 12R^2a^2\omega^2 + R^4a^4\right)U_{b1}{}^3R}{\left(9\omega^2 + R^2a^2\right)\left(\omega^2 + R^2a^2\right)^3}. \end{cases}$$

The obtained coefficients have the following numerical values: $A_0=-0.04$, $B_0=-6.337\cdot10^{-4}$, $A_{11}=9.593\cdot10^{-7}$, $B_{11}=-3.012\cdot10^{-5}$, $A_{13}=-1.776\cdot10^{-7}$, $B_{13}=-3.343\cdot10^{-6}$.

Substituting the obtained analytical values of the coefficients in arrays (18), we determine analytical dependences and numerical values of harmonic components of current linkage of the researched ψ_{a1}, ψ_{b1}, ψ_{a3}, ψ_{b3} in the function of coefficients A_0, B_0, A_{11}, B_{11}, A_{13}, B_{13}. Substituting ψ_{a1}, ψ_{b1}, ψ_{a3}, ψ_{b3} in expressions (14), analytical dependences and numerical values of harmonic components of current of the researched circuit are determined.

4. 3. A comparative analysis of the developed and classic SPM

The calculated values of harmonic components of current with the use of classic SPM and the proposed SPM using instantaneous resistance frequency domain were compared with the results of numerical modeling of the researched circuit in the environment of MATLAB package (Fig. 4).

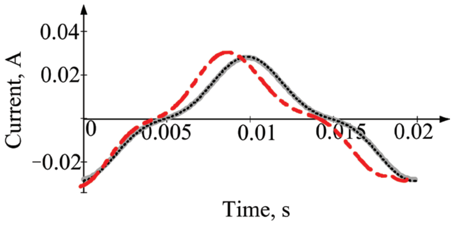


Fig. 4. Curves of current of the researched circuit: (⋯⋯) is the one, obtained using the proposed SPM, (– · ·) is the one, obtained using the classic SPM, (——) is the one, obtained as a result of calculation of the mathematical model

As evaluation criteria of accuracy assessment for the two compared methods, we selected the relative error by magnitude of harmonic components of the sought current and by amplitude and phase of first and third harmonics (Table 1). In addition, we assessed the degree of coincidence of calculated curves of the current signal with the curve, obtained as a result of the numerical experiment (Fig. 4), by coefficient of determination R^2.

Table 1

Results of comparative analysis of the developed and classic SPM

Magnitude	Relative error	Developed SPM, implemented in frequency domain	Classic SPM, implemented in time domain
Harmonic components of current	$\delta(I_{a1})$, %	6.313	8.5
	$\delta(I_{b1})$, %	0.39	4.7
	$\delta(I_{a3})$, %	2.316	8.1
	$\delta(I_{b3})$, %	0.46	24.2
Amplitude of first and third harmonics of current	$\delta(I_1)$, %	1.786	3.988
	$\delta(I_3)$, %	2.273	7.577
Phase of first and third harmonics of current	$\delta(\varphi_{I1})$, %	$1.041\cdot10^{-4}$	$2.41\cdot10^{-4}$
	$\delta(\varphi_{I3})$, %	$1.042\cdot10^{-3}$	$4.58\cdot10^{-3}$
Coefficient of determination	R^2	0.993	0.962

The conducted comparative analysis of the proposed and the classic method of the small parameter with the results of numerical calculation of the mathematical model of the electrical circuit with RL-load showed that the developed method provides greater adequacy of results and a higher accuracy of calculations.

5. Discussion of results of using the improved SPM with the implementation in the frequency domain for analysis of nonlinear electrical circuits

It should be noted that calculation accuracy depends on the accuracy of approximation of the characteristic of a nonlinear element. When implementing the analysis of the researched circuit, in order to avoid the awkwardness of analytical expressions, the polynomial dependence of the third degree was selected for calculation. This choice was determined by the conducted experimental research in the mathematical package with the aim of searching for a function that at the minimal number of terms will ensure acceptable accuracy of the description of a nonlinear characteristic.

In this case, the above numerical values of relative errors for values of the harmonic components of current do not exceed 6 % even at a relatively low quality of approximation.

Thus, the advantage of the SPM in frequency domain can be the fact that it is quite simple to implement, ensures high accuracy and effectiveness of obtaining numerical values of amplitude components of current and allows conducting an analysis in the analytical form of the processes of the researched electrical circuit with a nonlinear load.

The disadvantage of the method is the need to ignore higher harmonic components when solving the formed system of equations, which leads to a decrease in accuracy. In addition, to achieve the accurate approximation dependence of a nonlinear characteristic, it is necessary to use a polynomial function of the high degree, which will make the analytical expressions cumbersome.

Presented research shows the possibility of implementation of the SPM in frequency domain with its practical use, specifically: the presented method enables qualitative analysis of indicators of operation of nonlinear load and the current spectrum, which is a good basis for the development of the method for selection of the tools for active and passive filtering that was already partially presented in paper [21].

In subsequent studies, the obtained analytical expressions will make it possible to explore the processes of compensation of higher harmonics of current in electrical networks with nonlinear load, containing semiconductor converters for more precise selection of the tools for both passive and active filtering.

6. Conclusions

1. The method for analytical analysis using the method of the small parameter, implemented in frequency domain, was developed. It was shown that the method makes it possible to avoid complex trigonometric transformations and to simplify analytical calculations and maximally adapt them to automation through the use of the automated method of formation of components of instantaneous values in frequency domain, which will enable a decrease in time consumption.

2. The algorithm of calculation of nonlinear electrical circuits using the small parameter method in frequency domain with the help of orthogonal components of nonlinear resistance was developed, and effectiveness of its use was proved. A special feature of the presented algorithm is automation of analytical calculation in the environment of the mathematical package. Conducted comparative analysis of the proposed small parameter method with the classic small parameter method on the example of calculation of an electrical circuit with the RL loading showed that the developed method provides greater adequacy of results and higher precision calculations in comparison with the existing one. Relative error for harmonics of current does not exceed 6 %. Relative error by the amplitude of first and third harmonics of current does not exceed 2.5 % and relative error for phase does not exceed $1.042 \cdot 10^{-3}$ %.

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