Розроблено послідовний рекурсивний алгоритм фільтра Калмана для фільтрації даних в області шумів відмінних від гаусовського розподілу для використання у вимірювальній техніці. Відмінною рисою розробленого алгоритму фільтра Калмана для фільтрації даних з негаусовськими шумами є відсутність необхідності апріорного визначення статистичних характеристик шуму.

Була перевірена працездатність розробленої методики фільтрації Калмана шляхом обробки різних законів розподілу: шумів Коші, Парето, нормального і логістичного розподілів. Ефективність розробленої методики фільтрації підтверджується шляхом застосування фільтра при обробці експериментальних даних з різними законами розподілу шумів. Проведено апробацію розробленої методики фільтрації Калмана для даних, отриманих експериментально з урахуванням суперпозиції законів розподілу шумів. Апріорна оцінка помилки фільтрації при кількості ітерацій більше 30 прагне до нуля.

Розроблена методика фільтрації з використанням фільтра Калмана може бути використана при проведенні метрологічної атестації засобів вимірювальної техніки в умовах підприємства. В цій ситуації можливе зашумлення вимірювальної інформації різними шумами, в тому числі і тими, що не підкоряються закону розподілу Гауса. Фільтр може бути використаний при обробці даних систем контролю параметрів стану, що реалізуються за принципом порогового контролю величини.

Прикладним аспектом використання отриманого наукового результату є можливість розширення області застосування класичного фільтра Калмана в вимірювальній техніці. Це становить передумови для розробки універсального алгоритму фільтрації з використанням фільтра Калмана

Ключові слова: фільтр Калмана, рекурсивний алгоритм, Python, негауссовских шум, закон розподілу

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EXAMINING THE KALMAN FILTER IN THE FIELD OF NOISE AND INTERFERENCE WITH THE NON-GAUSSIAN DISTRIBUTION

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1. Introduction

The Kalman filter is a series recursive algorithm that uses a dynamic system model adopted to provide an estimate. The resulting estimation can be significantly adjusted as a result of the analysis of each new measurement sample of sensor over a time sequence [1].

The classic Kalman filter is an equation for the calculation of the first and second moment of *a posteriori* probability density (vector of mathematical expectation and variance matrix, including mutual) under given constraints. Since for the normal probability density mathematical expectation and variance matrix fully assign a probability density, we can say that the Kalman filter calculates the *a posteriori* probability density of the state vector at each point in time. And it means that it fully describes the state vector as a random vector magnitude [2]. The estimated values for mathematical expectations in this case are the optimal estimates for the mean square error criterion, which actually predetermines wide application of the Kalman filter.

The Kalman filter is widely used to solve numerous general engineering and econometric applied problems in the fields with a widespread distribution of Gaussian noise (economic forecasting, electronics, measuring equipment, radio engineering and communications) [3, 4]. Development of statistical models for indicators of processes in technology and economy [5], forecasting and determining the dynamics of economic indicators [6], cleaning of signals of measuring and radio engineering from noise and interference [7] is solved now employing the Kalman filter.

In a classic problem statement, the filter monitors a random signal generated by a linear recursion with additive white noise. The observed process is a linear combination of the signal and other white noise [9]. The impact of noise, interference on all elements of a device causes the emergence of random unique deviations of separate points of the static characteristic of the device [9]. In this case, an error of noise is a nonstationary random function of time. The most common normal (Gaussian) distribution in which the probability density of finding an object with the magnitude of attribute x depends on two parameters: the variance σ^2 and the offset μ, equal to the mathematical expectation x.

However, the opinion of the universal applicability of the normal distribution is a very stable delusion. Statistical models and methods based on Gaussianity (in particular the estimates of confidence intervals for selective medium) are often applied without a basic check, by default [10].

Therefore, the task to develop an effective procedure for filtering using the Kalman filter in the field of noises that differ from the Gaussian distribution in order to apply it in measuring instruments is a relevant scientific and applied task.

2. Literature review and problem statement

For the non-Gaussian noise, the most commonly used approach implies the approximation of noise applying the noise models, and the filters are then employed that exploit algorithms developed for the Gaussian noise models [11].

There are several varieties of the Kalman filter that differ in the approximations used to linearize dynamic models [12]:

– the extended Kalman filter (EKF) that applies to non-linear models, it performs the linearization through decomposition into a Taylor series;

– the unscented Kalman filter (UKF) that is used in problems for which a simple linearization cannot be applied, it performs «linearization» using a sigma conversion.

Paper [13] proposes for a situation when the signals are often non-linear in dynamics and have an abnormal noise to use the extended Kalman filter. The effectiveness of development is confirmed for the case when the noise variance is not too large (that is, a linear approximation is adequate). However, authors of the study did not consider the region of noises, which are not characterized by parameters of the Gaussian distribution. That is why the developed extended Kalman filter can be applied for filtering the non-Gaussian noise.

The author of [14], given the lack of filtering procedure using the Kalman filter and the impossibility of its application to the non-Gaussian noise domain, synthesized a recurrent non-linear filter whose order is determined from the conditions for obtaining estimates at a rate of measurements acquisition. The paper describes the Gaussian and linearized approximations to an arbitrary order filter, however, the problem on filtering the non-Gaussian noises was never solved.

In paper [15], authors note that modifications of the Kalman filter cannot solve the problems on the non-linear filtering, as the filter is built on only two statistical characteristics of a process: mathematical expectation and a covariance function. This is due to the fact that the study addressed only the two specified statistical characteristics of the filtering process. In this case, the phenomena of superposition of distribution laws under which the given statistical characteristics do not characterize the noise parameters before and after filtering at all, were not taken into account in study [15]. However, there are data that suggest the possibility of obtaining the Kalman filter with nonlinear additional filters, which would make it possible to extend the scope of filter application in measurement technology [16]. We did not find any data in the scientific literature about implementing the Kalman filter that performs the filtering of the non-Gaussian noise.

Therefore, the development of a procedure of filtering using the Kalman filter in the field of the non-Gaussian noise would substantially extend the scope of filter application: signal processing when conducting metrological certification, control over parameters.

3. The aim and objectives of the study

The aim of this study is to develop an effective procedure of filtering using the Kalman filter in the field of noises that are different from the Gaussian distribution in order to apply it in measurement instruments.

To accomplish the aim, the following tasks have been set:

– to perform a comparative analysis of the laws of measurement errors distribution employing the software that makes it possible to simulate the noise effect that is governed by the considered distributions;

– to test the effectiveness of the Kalman filtering procedure by employing different laws of noise distribution;

– to verify the developed procedure of filtering for data obtained experimentally, with respect to the superposition of laws of noise distribution.

4. Investigating the Kalman filter in the field of noises that differ from the Gaussian distribution

Paper [17] reported a model of simple scalar implementation of the Kalman filter by the general-purpose programming tools Python. Simulation confirmed the possibility for applying the developed software implementation of the adaptive Kalman filter to compensate for the effects of amplitude and phase distortions in a data transmission channel. The data abode by the Gaussian distribution. No study was conducted outside the Gaussian domain.

To fulfill the tasks set in this work, the main challenge is the substantiated identification of laws for data measurement error distribution for the non-Gaussian noise.

From the standpoint of probability theory, the form of a numerical distribution law is characterized by its counterexcess with a coefficient, which is determined by the standard deviation σ and the fourth central moment μ_4. According to the information theory, a distribution law is characterized by the value of entropy coefficient $\kappa=\Delta/\sigma$. For all possible existing laws of distribution, the value of a psi coefficient ranges from 0 to 1, and k – from 0 to 2.076, which is why the identification of distribution laws for the non-Gaussian noise is conveniently considered in the (psi,k)-plane, in which each law is identified by a certain point [18].

A comparative analysis and identification of the measurement error distribution laws were carried out by means of Python. At present, Python is an ideal language in order to rapidly write different applications running on the most common platforms [19]. Python is a freely available software package, which enables wide use of development results.

Result of the analysis is shown in Fig. 1. The chart displays the most common measurement error distribution laws divided into two groups.

The plane in the lower left corner shows the Pareto, Poisson, Cauchy law, and the upper right corner exhibits a group of laws, similar to the Gaussian laws by their information indicators.

The data represented on the plane can be complemented through the introduction of the unused distribution laws. To investigate the effect of the Kalman filtering, we selected four distribution laws, which are in extreme positions on the plane. For the further analysis, we selected the Pareto, Cauchy laws (extreme left) and the logistic and normal distribution (extreme right).

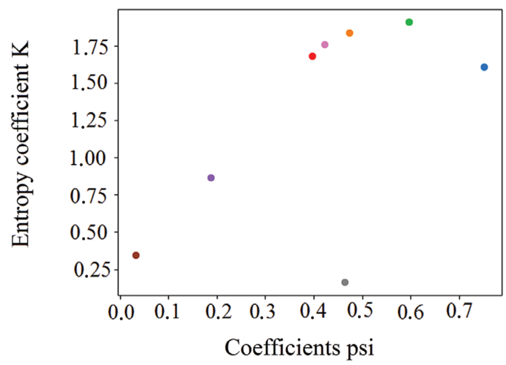


Fig. 1. A comparative analysis of the measurement error distribution laws: ● – uniform distribution; ● – logistic distribution; ● – normal distribution; ● – Erlang distribution; ● – Pareto distribution; ● – Cauchy distribution; ● – logistic distribution-1; ● – Poisson distribution

To study behavior of the Kalman filter model at different distributions of noise, we developed software that makes it possible to simulate noise exposure that is governed by the examined distributions.

We shall consider work of the scalar Kalman filter algorithm when changing a constant.

Because there is always a single constant, a system's model can be represented in the form:

$$x_k = x_{k-1} + w_k, \qquad (1)$$

where x_k is the prediction of system state at the current point in time; x_{k-1} is the prediction of system state at a preceding point in time; w_k is the control influence.

For a given model, the transfer matrix degenerates to unity, and the control matrix to zero. The measurement model takes the form:

$$y_k = y_{k-1} + v_k. \qquad (2)$$

In model (2), v_k is an error of measurement, which is characterized by a covariance matrix R, *a posteriori* matrix P for the accuracy of the estimate obtained and a variance of random process Q.

For model (2), a measurement matrix converts into unity, while covariance matrices P, Q, R transform into variances [20]. At the next k-th step, prior to receiving measurement results, the scalar Kalman filter attempts, in line with formula (1), to estimate the new state of the system:

$$x^{\wedge}_{k/(k-1)} = x^{\wedge}_{(k-1)/(k-1)}. \qquad (3)$$

Equation (3) shows that the *a priori* estimate at the next step is equal to the *a posteriori* estimate performed at the preceding step.

In this case, the *a priori* estimate of error variance is expressed by:

$$P_{k/(k-1)} = P_{(k-1)/(k-1)} \cdot + Q_k. \qquad (4)$$

Base on the *a priori* estimate of state $x^{\wedge}_{k/(k-1)}$, it is possible to calculate the forecast of measurement:

$$y^{\wedge}_k = x^{\wedge}_{(k-1)/(k-1)}. \qquad (5)$$

Once we have the next measurement of magnitude y_k, the filter calculates the error of its own prediction for the k-th measurement from expression:

$$e_k = y_k - y^{\wedge}_k = y_k - x^{\wedge}_{(k)/(k-1)}. \qquad (6)$$

The filter adjusts its estimation of the state of the system by choosing a point located somewhere between the initial estimate $x^{\wedge}_{(k)/(k-1)}$ and the point that corresponds to the new measurement y_k:

$$x^{\wedge}_{k/(k-1)} = x^{\wedge}_{(k-1)/(k-1)} + G_k e_k, \qquad (7)$$

where G_k is the filter gain coefficient. The estimate of the error variance is also adjusted:

$$P_{k/(k)} = (1 - G_k) P_{(k)/(k-1)}. \qquad (8)$$

Thus, variance e_k is equal to:

$$P_{k/(k)} = P_{(k)/(k-1)} + R_k. \qquad (9)$$

The filter gain coefficient at which the minimum error in the estimation of the system's state is reached, is derived from ratio:

$$G_k = P_{(k)/(k-1)} / S_k. \qquad (10)$$

We shall apply the resulting algorithm to evaluate the effectiveness of the Kalman filtering. Let us consider the work of the Kalman filter to suppress noise with the Pareto distribution. The Pareto distribution is a two-parameter family of absolutely continuous distributions.

The graphical part of evaluating the effectiveness of the Kalman filtering with the Pareto distribution is shown in Fig. 2. The data obtained indicate that the Kalman filter suppresses Pareto noises; a burst at the onset of filter's work is explained by the limited distribution density of the random component [21].

Let us study effectiveness of the Kalman filter to suppress noise with the Cauchy distribution. The graphical part of evaluating the effectiveness of the Kalman filtering with the Cauchy distribution is shown in Fig. 3.

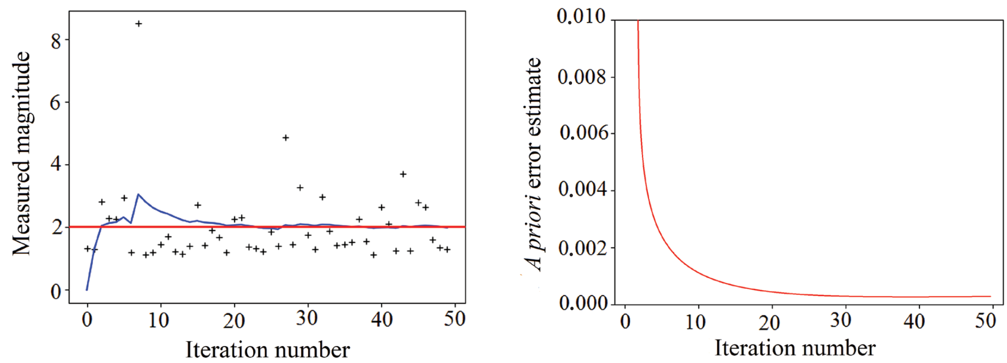
The Kalman filter suppresses Cauchy noises, a burst at the start is due to a random component distribution density.

Filtering efficiency was examined using the developed scalar software implementation of the Kalman filter for two laws of distribution from the left bottom corner in the plane of law distribution based on indicators *psi*, *k* (Fig. 1). To draw a final conclusion about the possibility of applying the Kalman filtering to the non- Gaussian noise, we shall investigate the effectiveness of filter application for laws from the upper right corner in the plane of law distribution based on indicators *psi*, *k* (Fig. 1).

Let us consider work of the Kalman filter to suppress noise with a normal distribution. The density of a normal distribution is determined from ratio:

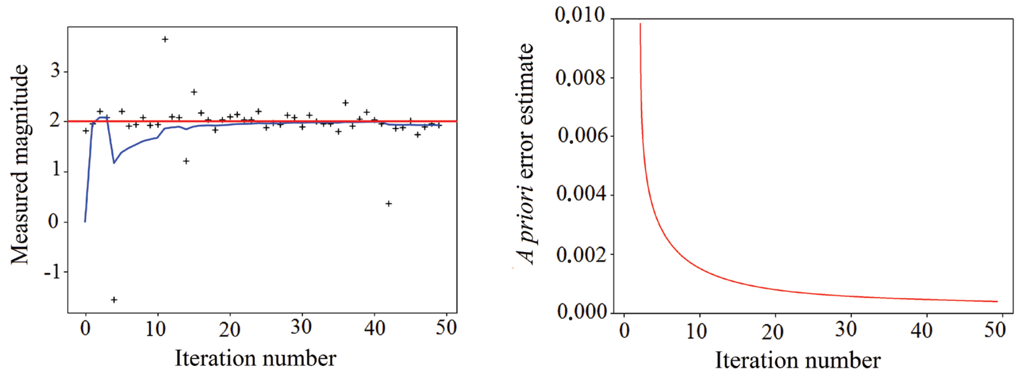
$$f(x) = \frac{\exp\left(-\frac{x^2}{2}\right)}{\sqrt{2\pi}}.$$

The graphical part of evaluating the effectiveness of the Kalman filtering with a normal distribution is shown in Fig. 4.



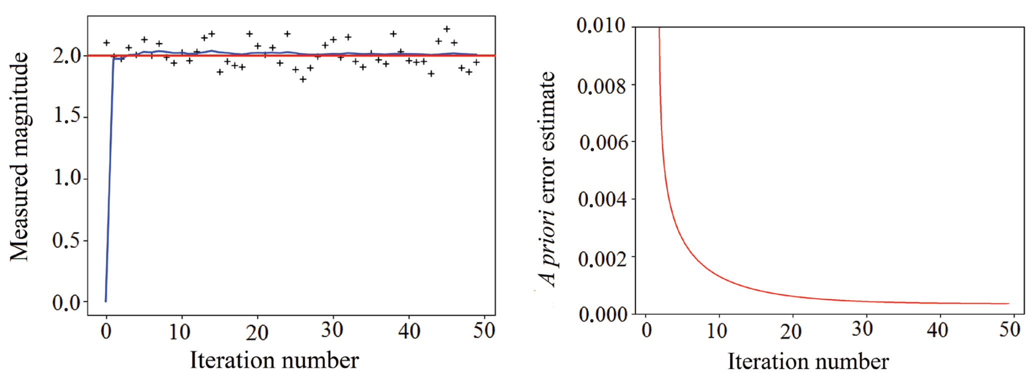
a *b*

Fig. 2. Graphical part of evaluating the effectiveness of the Kalman filtering with Pareto distribution: *a* – noise suppression errors with Pareto distribution (+ noisy measurements; —— – *a posteriori* estimate; —— – true value); *b* – errors in suppressing noise with Pareto distribution



a *b*

Fig. 3. Graphical part of evaluating the effectiveness of the Kalman filtering with Cauchy distribution: *a* – noise suppression errors with Cauchy distribution distribution (+ noisy measurements; —— – *a posteriori* estimate; —— – true value); *b* – errors in suppressing noise with Cauchy distribution



a *b*

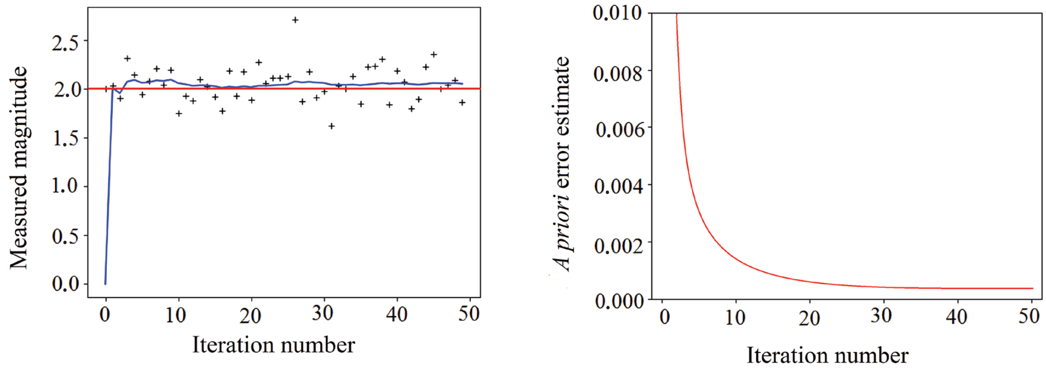
Fig. 4. Graphical part of evaluating the effectiveness of the Kalman filtering with a normal distribution: *a* – noise suppression with a normal distribution distribution (+ noisy measurements; —— – *a posteriori* estimate; —— – true value); *b* – errors when suppressing noise with a normal distribution

Let us consider work of the Kalman filter to suppress noise with a logistic distribution. In this case, the density of logistic distribution is derived from ratio:

$$f(x)=\frac{\exp(-x)}{(1-\exp(-x))^2}.$$

The graphical part of evaluating the effectiveness of the Kalman filtering with a logistic distribution is shown in Fig. 5. The study on the applicability of the Kalman filter in the field of noise with the non-Gaussian distribution suggests that the Gaussian noise distributions are suppressed by the Kalman filter with the same mistake as is the case for noises with Pareto or Cauchy distributions that are far from the Gaussian distribution.

We shall verify the constructed filtering procedure employing data obtained experimentally.



a *b*

Fig. 5. Graphical part of evaluating the effectiveness of the Kalman filtering with a logistical distribution: *a* – noise suppression with a logistical distribution distribution (+ noisy measurements; —— – *a posteriori* estimate; —— – true value); *b* – errors when suppressing noise with a logistical distribution

5. Experimental data filtering using the Kalman filter

We used in our research an array of data acquired when controlling weight. The data array is composed of 55 weight measurement values (kg). The measurements were carried out using a specially prepared non-standard weight with a certified weight of 0.175 kg. The measurements were performed using a digital electronic scale with a vibro-frequency mechanical resonator under conditions of vibration and electromagnetic interference, distributed based on an unknown law or the superposition of laws.

y = [0.203, 0.154, 0.172, 0.192, 0.233, 0.181, 0.219, 0.153, 0.168, 0.132, 0.204, 0.165, 0.197, 0.205, 0.143, 0.201, 0.168, 0.147, 0.208, 0.195, 0.153, 0.193, 0.178, 0.162, 0.157, 0.228, 0.219, 0.125, 0.101, 0.211, 0.183, 0.147, 0.145, 0.181, 0.184, 0.139, 0.198, 0.185, 0.202, 0.238, 0.167, 0.204, 0.195, 0.172, 0.196, 0.178, 0.213, 0.175, 0.194, 0.178, 0.135, 0.178, 0.118, 0.186, 0.191].

Let us define the law of distribution of measurement errors in the specified sample; to this end, we map the results of its processing onto the plane of distribution laws in the *psi*, *k* coordinates (Fig. 6).

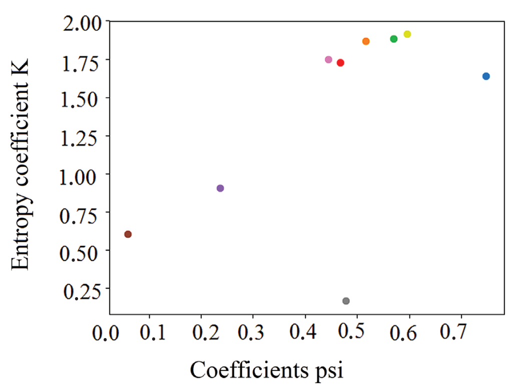
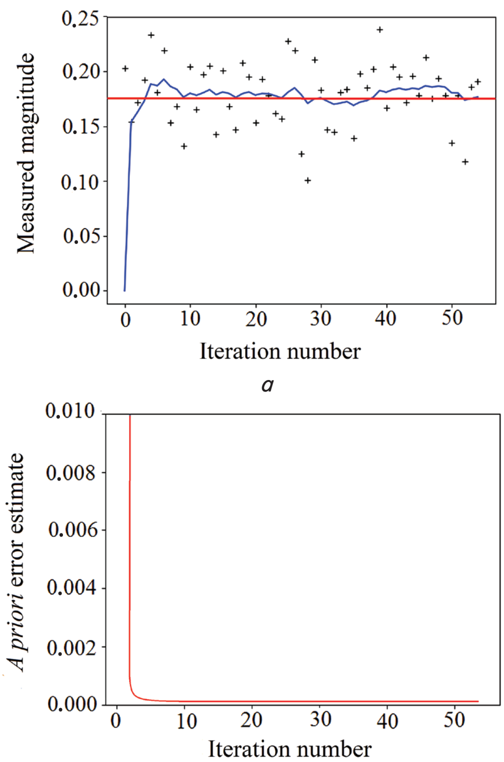


Fig. 6. A comparative analysis of the distribution laws of experimental data error: ● – uniform distribution; ● – logistic distribution; ● – normal distribution; ● – Erlang distribution; ● – Pareto distribution; ● – Cauchy distribution; ● – logistic distribution-1; ● – Poisson distribution; ● – unknown distribution

A comparative analysis reveals that the experimental sample have an error that is distributed based on the law close to the normal law. Thus, we can apply ratios for the normal distribution to the sample. We shall use the Kalman filter to suppress the normally distributed error of weight measurement (Fig. 7).



a

b

Fig. 7. Graphical part of evaluating the effectiveness of the Kalman filtering for experimental data: *a* – noise distribution (+ noisy measurements; —— – *a posteriori* estimate; —— – true value); *b* – errors in noise suppression

Evaluation results confirm the effectiveness of the use of the developed software implementation of the Kalman filter for experimental data whose distribution is outside the Gaussian field. The *a priori* estimate for a filtering error when the number of iterations exceeds 30 tends to zero.

6. Discussion of results of studying the Kalman filter

The constructed serial recursive algorithm of the Kalman Filter to filter the data in the field of noises that differ from the Gaussian distribution computes the forecast for measuring the magnitude with respect to the *a priori* estimate. A distinctive feature of a given filtering algorithm is the identification of a data measurement error distribution law for the non-Gaussian noises at the first stage.

The suggested technique for the identification of a distribution law makes it possible to use the Kalman filtering algorithm when processing noisy data in the cases when a distribution law is unknown. The identification of a distribution law is carried out by determining the point that corresponds to a given distribution in the (*psi*, *k*)-plane.

The classic Kalman filter employs the calculated values for mathematical expectations, which serve the optimal estimates for the mean square error criterion. In the case of data filtering in the field of the non-Gaussian noise distribution, the specified characteristics cannot be used, the consequence being the inapplicability of the Kalman filter.

The devised procedure of filtering using the Kalman filter could be used when executing the metrological attestation of measurement instruments under industrial conditions when there may be the noisy measuring information due to various noises, including those that are not governed by the Gaussian distribution law. The filter could be applied when processing the data from control systems over state parameters, implemented on the principle of a magnitude threshold control.

The effectiveness of the developed filtering procedure is confirmed by testing the filter when processing experimental data with different laws of noise distribution. To obtain a generic Kalman filter, it is required to undertake a study aimed at the applicability of the filtering technique for data from aggregate and combined measurements, and to construct a filtering algorithm for the multi-channel Kalman filter.

9. Conclusions

1. Here we report the development of a filtering procedure using the Kalman filter for the non-Gaussian noise distribution. A special feature of the proposed technique is the procedure for the identification of a distribution law for data measurement errors for the non-Gaussian noises at the first stage.

Through the proposed technique for the identification of a distribution law, it has become possible to use the Kalman filtering algorithm when processing noisy data in the cases when a distribution law is unknown. The filtering of such noises using the Kalman filter has not been achieved previously.

Its applicability was confirmed for the non-Gaussian distribution of noises, which has significantly expanded the scope of filter application.

2. We have proven the possibility of using the Kalman filter in measurement instruments when processing information that is distorted by interference of different origins and levels. It is established that the developed Kalman filter could work in the field of noise with the non-Gaussian distributions.

The effectiveness of the devised filtering procedure was tested by employing various laws of noise distribution. A special feature of the developed recursive serial algorithm of the Kalman Filter to filter data in the field of the non-Gaussian noise distribution is the absence of a need to determine *a priori* the statistical characteristics of noise.

3. We have verified the devised Kalman filtering procedure for the experimentally obtained data with respect to the superposition of noise distribution laws.

The developed filtering procedure has proved effective in terms of experimental data. The *a priori* estimate for a filtering error when the number of iterations exceeds 30 tends to zero.

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