Основними критеріями працездатності переважної більшості зубчастих механізмів є контактна витривалість сполучених поверхонь зубів і згинальна витривалість ніжок зубів. При цьому розрахунок по контактним напруженням є основним з точки зору визначення геометричних розмірів зубчастих механізмів, а розрахунок на вигин зубів є перевірочним.

Як відомо, для продовження довговічності і збільшення працездатності та несучої здатності зубчастих механізмів використовуються різні мастильні матеріали. Однак через недостатнє вивчення питання впливу мастила на контактну витривалість активних поверхонь зубів в традиційних методах розрахунку зубчастих передач (наприклад, згідно з ГОСТ 21354-87) коефіцієнт впливу мастила прирівнюється до одиниці, тобто розглядається ідеальний випадок, коли тертя відсутнє. Такий підхід призводить до неточної оцінки несучої здатності зубчастих передач, що може виявитися як причиною їх передчасного виходу з ладу, так і привести до завищення їх геометричних розмірів.

В роботі вирішена контактна задача стикання двох тіл довільної форми, близьких до напівплощин при кінцевому коефіцієнті тертя і було встановлено, що значення отриманої контактної напруги перевершує напругу, яка обчислена згідно з відомим рішенням Герца на 6 %.

Запропонована методика розрахунку зубчастих коліс на контактну міцність при кінцевому коефіцієнті тертя, без припущень про малість ділянки дотику і форми кордонів, дозволяє оцінити здатність навантаження зубчастих передач з урахуванням впливу мастила і наявності тертя між сполученими поверхнями зубів.

Отримано аналітичний вираз для коефіцієнта впливу мастила на підставі рішення контактної задачі тиску жорсткого штампа на пружну напівплощину за критерієм коефіцієнта тертя між сполученими поверхнями зубчастих передач. Це дозволяє оцінити справжню навантажувальну здатність зубчастих передач при впливі різних мастильних матеріалів, що має важливе теоретичне і практичне значення при проектуванні машин і агрегатів

Ключові слова: коефіцієнт впливу мастила, коефіцієнт тертя, контактна напруга, дотичні напруження, комплексні змінні, бігармонічна функція

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CALCULATION OF TOOTHED GEAR MECHANISMS IN MACHINES AND ASSEMBLIES CONSIDERING THE EFFECT OF LUBRICANTS

B. Ahmedov
PhD, Associate Professor*
E-mail: Ahmedov.Beyali@mail.ru

A. Najafov
Doctor of Technical Sciences, Professor*
E-mail: ali.najafov@rohe.az

A. Abdullayev
Doctor of Technical Sciences, Professor*
E-mail: ayazab@aztu.edu.az

I. Chalabi
PhD, Associate Professor*
E-mail: i_tschalabi@yahoo.de

F. Veliev
Doctor of Technical Sciences, Professor
Department of technological machines and equipment of the branch
Azerbaijan State University of Economics (UNEC)
Istiglyaliyat str., 6, Baku, Azerbaijan, AZ 1001
E-mail: fazil-uzbek@mail.ru

*Department of machine design
Azerbaijan Technical University
Huseyn Javid ave., 25, Baku, Azerbaijan, AZ 1073

1. Introduction

The most common type of transmission mechanisms in machines and assemblies are the toothed gears. When transferring the torque, the engagement is exposed to the action of a normal force (F_n)) and a friction force (F_s), which is associated with sliding (Fig. 1). Under the influence of these forces, the tooth enters the complex stressed state. The decisive effect on its efficiency is exerted by contact stresses (σ_H) and bending stresses (σ_F) that change over time in line with a certain intermittent cycle initiated from zero. Alternating stresses are the cause for a fatigue failure of teeth: the fracture of teeth that are exposed to the bending stresses and the spalling of working surfaces of teeth due to the contact stresses. The contact stress and friction in the gearing are also associated with wear, seizing, and other kinds of damage to the teeth surfaces. Because the contact stresses are the cause of a fatigue failure, the basic criterion of efficiency and calculation of closed transmissions is the contact strength of teeth working surfaces (σ_{max}).

However, when calculating the active surfaces of teeth for contact strength, the influence of a lubricant is almost ignored. Instead, a factor of the lubricant effect is introduced to the calculation, whose value as a random variable is accepted in most cases equal to unity. Therefore, it is a relevant and practically interesting to calculate toothed gears for contact strength of the active surfaces of teeth considering the effect of lubrication and a friction factor between the conjugated surfaces of teeth.

2. Literature review and problem statement

Paper [1] has devised a procedure for calculating the contact strength of toothed gears with respect to friction between teeth, and derived analytic expression to estimate the influence of a lubricant on their load capacity. However, the authors failed to account for the effect of the maximum shear stress on geometrical parameters of the toothed gears.

Study [2] developed a procedure for designing toothed gears, which uses maximum surface shear stresses as the estimation to assess the fatigue spalling processes. These stresses, in addition to the geometrical and mechanical parameters of toothed wheels, take into consideration the tribological properties of contact at the active surfaces of teeth. However, the geometrical parameters of a toothed gear are determined depending on the molecular effect of a friction coefficient, while the influence of the deformation action has not been studied.

Article [3] constructed an algorithm and developed software for detailed study into the impact of friction on the strength characteristics of regular toothed gears. This creates the prerequisites for taking the friction forces and load factors of the teeth into consideration in the form that most closely corresponds to actual conditions. The analysis performed shows that when taking the friction into account the dependences of forces and moments in toothed gears are the nonlinear functions that depend on the angular coordinate of the driving wheel and the current values for friction coefficients between teeth and in supports. In this case, the nature of non-linearity is determined not only by the kind of friction forces, but mainly by a change in the parameters of engagement when moving from a two-pair contact between teeth to the one-pair contact. However, the authors did not consider the influence of friction forces on load capacity of the toothed gears.

Paper [4] addressed the conditions for contact between cylindrical toothed gears, as well as the technological methods of influence when forming the involute surface of teeth, at which the teeth of one gear slip relative to the profile of the conjugated wheel and the friction-slip forces. The loss of energy for friction in the toothed gears reaches 10 % of the total energy loss to overcome friction. It is shown that in the force calculation of cylindrical toothed wheels it is necessary to take into consideration, in addition to dynamic loads, the efforts of rolling friction-slip in the toothed gears. However, the paper lacks the mechanism of influence of lubricants on load capacity of toothed gears; also, insufficiently studied is the interaction between contact-hydrodynamic parameters in terms of the friction factor.

Article [5] describes engineering methods, developed by the authors, for the calculation of geometrical parameters for a contact between smooth bodies of arbitrary shape and curvature in the presence of elastoplastic deformation in the contact area. The paper describes a procedure for applying the estimation dependences in order to solve engineering problems related to analysis, interpretation, and prediction of contact deformation at static and impact force interaction between machine components. However, the authors failed to account for the maximum shear stress associated with sliding.

Study [6] reports the necessary information about the materials and geometry of toothed gears, technological and operational requirements, disregarding which makes it impossible to perform calculations in line with modern techniques. The peculiarities in the calculation of efficiency of toothed gears are given, with respect to the malleability of their links. However, the author did not examine the influence of a friction force on load capacity of the toothed gears.

Paper [7] describes results of the finite element calculation of the contact interaction between two elastic circular cylinders of finite length with intersecting axes as a model of contact interaction between teeth of the involute straight-tooth toothed gear when the axes of the toothed wheels are skewed. It was established that the determining factor that breaks, when skewed, the Hertz relationships between the contact parameters, and which significantly affects its load capacity, is the ratio of the summary vector of the assigned load to the length of the contact area in the direction of forming cylinders. However, the simultaneous action of normal and tangential forces, associated with the relative sliding of teeth, was not considered in the paper.

Study [8] dealt with issues related to ensuring the resistance to the contact fatigue in the highly stressed toothed wheels in the transmissions of auto tractor vehicles. The influence of the microstructure of strengthened surface layers on resistance to the contact fatigue of cemented toothed wheels is shown. There is a procedure for the calculation and prediction of the toothed wheels' resource with respect to the quality of the structure of strengthened layers. The authors considered methods for determining the hardenability of cemented construction steels. They gave recommendations for choosing rational parameters for the technological regimes in the chemical-thermal treatment of toothed wheels in energy-intensive machines with high performance characteristics. However, the authors failed to study the stressed-deformed state of toothed gears when there is friction between the teeth of toothed gears.

Paper [9] provides the basic concepts of the mathematical theory of elasticity, and derives complete systems of equations, as well as proves the assumptions on these equations. In [10, 11, 14–17], there are basic data on the calculation and design of toothed gears and their components based on the main criteria for their operation efficiency. Studies [12, 13] report results obtained in the course of experimental study on the evaluation of influence of various parameters on a friction coefficient.

Paper [18] gives a comparative estimation of the service life of cylindrical toothed gears based on the two basic criteria of efficiency – the contact and flexural strength of teeth. However, the author did not study the influence of lubrication on load capacity of the toothed gears.

In works [19–21], in order to determine a factor of influence of lubrication K_L on contact endurance of the active surfaces of teeth, a probabilistic calculation method was employed. However, only the mathematical expectation and the variance coefficient K_L were taken into consideration rather than the estimation of the factor itself.

Thus, the issues on the influence of friction force on load capacity of the toothed gears under the influence of various lubricants remain to be solved.

3. The aim and objectives of the study

The aim of this work is to devise a procedure for calculating the toothed gears for contact strength with respect to friction between the teeth and the influence of lubrication.

To accomplish the aim, the following tasks have been set:

– to reveal the mechanics of change in shapes and dimensions in a contact between two bodies of arbitrary shape, close to half-planes;

– to solve a boundary problem on the pressure of a rigid stamp on the elastic half-plane in the presence of friction;

– to derive a holomorphic relationship between a contact stress and a friction coefficient;

– to derive an analytical expression to estimate the influence of a lubricant on the load capacity of toothed gears for contact stresses.

4. A problem on the contact between two bodies in the shape of half-planes given the finite friction coefficient

The existing calculation procedure for toothed gears in terms of contact stresses is based on the Hertz formula, derived from the solution to the contact problem from elasticity theory under certain restrictive assumptions:

– the area of contact between surfaces is very small;

– the friction coefficient between the conjugated surfaces is zero;

– given the appropriate choice of coordinate axes, the equations of non-deformed surfaces in the vicinity of a contact place can be at sufficient approximation represented in the form

$$z = Ax^2 + 2Bxy + Cy^2.$$

This paper examines the issue on solving the problem on the contact of two bodies of arbitrary shape, close to half-planes. When transferring the torque, the engagement is exposed to the action of a normal force (F_n)) and a friction force (F_s), which is associated with sliding. Friction force occurs when the toothed wheels' wheel profiles slide relative to each other. At the engagement pole, the friction forces are zero, but at the stem and top of the teeth they are maximal. Therefore, a slip velocity is proportional to the distance between the contact point and the pole. The sliding is accompanied by friction. Friction causes losses in the engagement and leads to tooth wear.

In the drive teeth, friction forces are directed from the initial circle, and in the driven ones – vice versa. At the constant diameters of wheels, the distance between the points of start and end of the engagement and the pole, and hence the slip velocity, increases with an increase in the height of the tooth or the engagement module. The low-module wheels with a larger number of teeth the sliding is smaller while the performance efficiency is higher than that of the large-module wheels with a small number of teeth.

Therefore, taking into consideration the presence of lubrication and friction at the conjugated teeth surfaces, the refined solution to the problem on contact between two bodies in the shape of half-planes at the finite friction coefficient is of practical importance.

In this case, the stated problem is a two-dimensional analog of the Hertz problem without assumptions about the smallness of the contact area and the shape of borders [9].

It is known that the load permissible in terms of the contact strength of teeth of the toothed wheels is defined basically by the hardness of a material. For a better joint work of teeth, it is typically recommended to assign the hardness of a gear that would exceed the wheel's hardness by not less than 10–15 Brinell units [10, 11]

$$HB_1 \geq HB_2 + (10 \ldots 15). \tag{1}$$

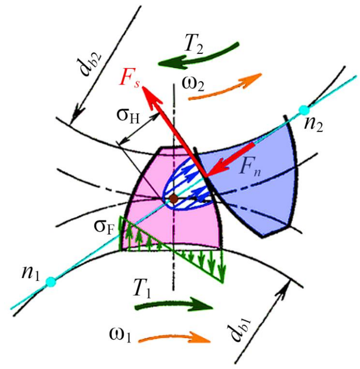

Fig. 1. Forces that act in the engagement of toothed wheels' teeth

This circumstance makes it possible to consider the conjugation of the surfaces of teeth of toothed wheels as the two elastic bodies (S_1 and S_2), similar in their shape to half-planes, which are in contact along sections "*ab*" (Fig. 2). The upper and bottom neighborhoods are to be distinguished by signs $(+)$ and $(-)$, respectively.

It is believed that the predefined or known conditions are the following:

– the shape of borders prior to deformation;

– the principal vector of external forces that pressed body S_1 to body S_2;

– strains and rotations of S_1 and S_2 at infinity are zero;

– coefficient of friction between the conjugated surfaces is non-zero, and has the finite value f;

– the section of contact *ab* is to be determined;

– body S_1 takes the lower half-plane, and body S_2 the upper one S^+.

Assume that at the border of the elastic half-plane, beneath the surface of the gear's tooth (stamp)

$$\tau = f \cdot q, \tag{2}$$

where q is the specific pressure; τ is the shear stress; f is the friction coefficient, considered constant.

Direct the Ox axis along the border of the elastic half-plane, and the Oy axis – perpendicular to it, so that the elastic body takes the lower half-plane $y<0$. It is obvious that with this choice of axes

$$q = -y_y^-; \quad \tau = X_y^-. \tag{3}$$

It is expected that the gear's tooth (stamp) comes into contact with a tooth of the wheel (elastic half-plane) along one continuous section $L = ab$ and can move only translationally.

It is considered that the assigned magnitude of the total pressure of a gear's tooth (stamp) at the surface of the tooth of the wheel (half-plane)

$$q_{\Sigma} = \int_L q(t)\,\mathrm{d}t, \tag{4}$$

In this case, the total tangential stress is, obviously, $\tau_\Sigma = f \cdot q_\Sigma$; the principal vector of external forces acting on the gear's tooth (stamp) and balanced by the reaction of the tooth (elastic half-plane) is

$$\left(F_x, F_y\right)=\left(\tau_\delta - q_\delta\right).$$

We believe that bodies S_1 and S_2 are matched with complex variable functions $\Phi_1(z)$ and $\Phi_2(z)$. The boundary conditions of our problem then take the form [1], [9]:

$$\left.\begin{array}{ll}\tau(t)=fq(t) & \text{on } L,\\ v=f(t)+\text{const} & \text{on } L,\\ \tau(t)=q(t)=0 & \text{on } OX,\end{array}\right\} \qquad (5)$$

where t is the abscissa of the point along the Ox; v is the projection of displacement onto the Oy; $f(t)$ is the assigned function that defines the gear's tooth profile (stamp); $y=f(x)$ is the equation of the tooth's profile.

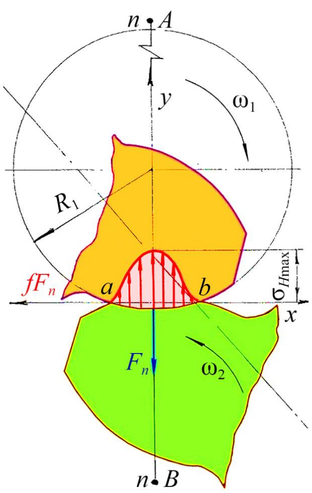

Fig. 2. Estimation scheme to build the refined method for calculating toothed gears in terms of contact strength

Next, the biharmonic function $U(x,y)$ of two variables x and y is represented by means of two functions of the complex variable

$$z=x+iy \text{ and } \bar{z}=x-iy.$$

In this case, the equilibrium equation takes the form

$$\frac{\partial U}{\partial z}=\frac{\partial U}{\partial x}\frac{\partial x}{\partial z}+\frac{\partial U}{\partial y}\frac{\partial y}{\partial z}=\frac{1}{2}\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)U,$$

$$\frac{\partial U}{\partial \bar{z}}=\frac{1}{2}\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)U. \qquad (6)$$

For x and y

$$\frac{\partial U}{\partial x}=\left(\frac{\partial}{\partial z}+i\frac{\partial}{\partial \bar{z}}\right)U,$$

$$\frac{\partial U}{\partial y}=\left(\frac{\partial}{\partial z}-i\frac{\partial}{\partial \bar{z}}\right)U. \qquad (7)$$

The consequence of these formulae is the expression of second derivatives, the Laplacian and the biharmonic operator

$$\left.\begin{array}{l}\dfrac{\partial^2 U}{\partial x^2}=\left(\dfrac{\partial^2}{\partial z^2}+2\dfrac{\partial^2}{\partial z\partial \bar{z}}+\dfrac{\partial^2}{\partial \bar{z}^2}\right)U,\\ \dfrac{\partial^2 U}{\partial y^2}=-\left(\dfrac{\partial^2}{\partial z^2}-2\dfrac{\partial^2}{\partial z\partial \bar{z}}+\dfrac{\partial^2}{\partial \bar{z}^2}\right)U,\\ \dfrac{\partial^2 U}{\partial x\partial y}=i\left(\dfrac{\partial^2}{\partial z^2}-\dfrac{\partial^2}{\partial \bar{z}^2}\right)U,\\ \nabla^2 U=4\dfrac{\partial^2 U}{\partial z^2\partial \bar{z}},\\ \nabla^2 U=16\dfrac{\partial^4 U}{\partial z^2\partial \bar{z}^2}.\end{array}\right\} \qquad (8)$$

With respect to displacement vectors

$$\nabla^2 U=4\frac{\partial^2 U}{\partial z\partial \bar{z}},$$

$$\nabla^2 U+it=4\psi(z), \quad \nabla^2 U-it=4\overline{\psi(z)}$$

from these equations

$$\frac{1}{2}\nabla^2 U=2\frac{\partial^2 U}{\partial z\partial \bar{z}}=\psi(z)+\overline{\psi(z)}.$$

The integration for z, that introduces the additively inbound function from $\bar{z}$, denoted as $\chi'(z)$, produces

$$2\frac{\partial U}{dz}=\psi(z)+\overline{\psi(z)}+\overline{\chi'(z)}.$$

Another integration leads to the desired representation

$$2U=\bar{z}\psi(z)+z\overline{\psi(z)}+\overline{\chi'(z)}+\chi'(z).$$

The function, introduced to the right side while integrating for $\bar{z}$, is equal to $\chi^*(z)$, because U is the real function. Then

$$U=\frac{1}{2}\left[\bar{z}\psi(z)+z\overline{\psi(z)}+\chi'(z)+\overline{\chi'(z)}\right]. \qquad (9)$$

This formula was first given by Goursat. Hereafter, we shall use for function U another expression, the expressions for its partial derivatives.

It is easily calculated

$$\left.\begin{array}{l}2\dfrac{\partial U}{\partial x}=\psi(z)+\bar{z}\psi'(z)+\overline{\psi(z)}+z\overline{\psi'(z)}+\chi'(z)+\overline{\chi'(z)},\\ 2\dfrac{\partial U}{\partial y}=i\left[-\psi(z)+\bar{z}\psi'(z)+\overline{\psi(z)}-z\overline{\psi'(z)}+\chi'(z)-\overline{\chi'(z)}\right].\end{array}\right\} \qquad (10)$$

Instead of considering expressions for $\dfrac{\partial U}{\partial x}$ and $\dfrac{\partial U}{\partial y}$, it is more convenient to consider the expression for complex combination

$$\frac{\partial U}{\partial x}+i\frac{\partial U}{\partial y}=\psi(z)+z\overline{\psi'(z)}+\overline{\varphi(z)}, \qquad (11)$$

here for brevity we put it as

$$\varphi(z)=\frac{d\chi}{dz}.$$

It should be noted that any expression of the form (9) represents a biharmonic function if $\psi(z)$ and $\chi(z)$ are the holomorphic functions of the complex variable z.

Indeed, by differentiating the first equation (10) for x and the second one for y and summing, we obtain

$$\Delta U = 2\left[\psi'(z)+\overline{\psi'(z)}\right] = 4Re\left[\psi'(z)\right], \tag{12}$$

hence it follows that ΔU is the harmonic function. Therefore,

$$\Delta^2 U = \Delta(\Delta U) = 0.$$

Under effort $(X_n ds, Y_n ds)$, acting on element ds of the contour's arc ab from the side of a positive normal $(X_n ds, Y_n ds)$:

$$\left.\begin{aligned} &X_n = X_x\cos(n,x)+X_y\cos(n,y)= \\ &-\frac{\partial^2 U}{\partial y^2}\cos(n,x)-\frac{\partial^2 U}{\partial x\partial y}\cos(n,y), \\ &Y_n = Y_x\cos(n,x)+Y_y\cos(n,y)= \\ &=\frac{\partial^2 U}{\partial y^2}\cos(n,y)-\frac{\partial^2 U}{\partial x\partial y}\cos(n,x). \end{aligned}\right\} \tag{13}$$

One can easily see

$$\cos(n,x)=\cos(t_0,y)=\frac{dy}{ds},$$

$$\cos(n,y)=-\cos(t_0,x)=-\frac{dx}{ds},$$

where t_0 is the positive direction of the tangent. By introducing these values to (13), we obtain the abscissa of the point along axis Ox;

$$X_n=\frac{d}{ds}\left(\frac{\partial U}{\partial y}\right),\quad Y_n=-\frac{d}{ds}\left(\frac{\partial U}{\partial x}\right), \tag{14}$$

or, in the complex form

$$X_n+iY_n=\frac{d}{ds}\left(\frac{\partial U}{\partial y}-i\frac{\partial U}{\partial x}\right)=-i\frac{d}{ds}\left(\frac{\partial U}{\partial x}+i\frac{\partial U}{\partial y}\right) \tag{15}$$

or

$$(X_n+iY_n)ds=-id\left(\frac{\partial U}{\partial x}+i\frac{\partial U}{\partial y}\right). \tag{16}$$

With respect to this expression in formula (15), we obtain

$$(X_n+iY_n)ds=-id\left[\psi(z)+z\overline{\psi'(z)}+\overline{\varphi(z)}\right]. \tag{17}$$

If we give the element ds, first, the direction of axis Oy, and then the direction of axis Ox, we obtain

$$ds=dy,\quad dz=idy,\quad dz=-idy,\quad X_n=X,\quad Y_n=X_y,$$

$$ds=dx,\quad dz=d\bar{z}=dx,\quad X_n=-X_y,\quad Y_n=-Y_y.$$

$$X_x+iX_y=\psi'(z)+\overline{\psi'(z)}-\overline{z\psi''(z)}-\overline{\varphi'(z)}, \tag{18}$$

$$Y_y-iY_y=\psi'(z)+\overline{\psi'(z)}+\overline{z\psi''(z)}+\overline{\varphi'(z)}. \tag{19}$$

Adding and subtracting equations (18) and (19), and replacing in the second result i with $-i$, we obtain

$$\begin{aligned} &X_x+Y_y=2\left[\psi'(z)-\overline{\psi'(z)}\right]=4Re\psi'(z)= \\ &=4Re\Phi(z)=2\left[\Phi(z)+\overline{\Phi(z)}\right], \end{aligned} \tag{20}$$

$$\begin{aligned} &Y_y-X_x+2iX_y=2\left[\bar{z}\psi''(z)+\varphi'(z)\right]= \\ &=2\left[(\bar{z}-z)\Phi'(z)-\Phi(z)-\bar{\Phi}(z)\right], \end{aligned} \tag{21}$$

$$Y_y-iX_y=\Phi(z)-\Phi(\bar{z})+(z-\bar{z})\overline{\Phi'(z)}. \tag{22}$$

The principal vector (resultant force) acting on the finite arc ab, and the main moment (M) of the examined efforts relative to the coordinate origin, take the form:

$$\begin{aligned} &X+iY=\int_{AB}(X_n+iY_n)\mathrm{d}s= \\ &=-i\left[\frac{\partial U}{\partial x}+i\frac{\partial U}{\partial y}\right]_a^b=-i\left[\psi(z)+\overline{z\psi'(z)}+\overline{\varphi(z)}\right]_a^b, \end{aligned} \tag{23}$$

$$M=\int_{ab}(xY_n-yX_n)\mathrm{d}s=-\int_{ab}\left\{xd\frac{\partial U}{\partial x}+yd\frac{\partial U}{\partial y}\right\},$$

hence, by integrating by parts, we find:

$$\begin{aligned} &M=-\left[x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}\right]_a^b+\int_{ab}\left\{\frac{\partial U}{\partial x}\mathrm{d}x+\frac{\partial U}{\partial y}\mathrm{d}y\right\}= \\ &=-\left[x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}\right]_a^b+|U|_a^b. \end{aligned} \tag{24}$$

Note

$$x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}=Re\left\{z\left(\frac{\partial U}{dx}-i\frac{\partial U}{\partial y}\right)\right\},$$

$$\frac{\partial U}{dx}-i\frac{\partial U}{\partial y}=\overline{\psi(z)}+\bar{z}\psi'(z)+\varphi(z)$$

and

$$U=Re\left[\bar{z}\psi(z)+\chi(z)\right]$$

then we finally obtain

$$M=Re\left[\chi(z)-z\varphi(z)-z\cdot\bar{z}\psi'(z)\right]_a^b. \tag{25}$$

In order to express the stress and displacement components in the integrated form, we apply the following formula

$$2\mu^*(u+iv)=-\left(\frac{\partial U}{dx}-i\frac{\partial U}{\partial y}\right)+\frac{2(\lambda^*+2\mu^*)}{\lambda^*+\mu^*}\psi(z), \tag{26}$$

according to formula (11)

$$2\mu^*(u+iv)=\chi^*\Phi(z)-\Phi(\bar{z})-(z-\bar{z})\overline{\Phi'(z)}, \tag{27}$$

where

$$\chi^* = \frac{\lambda^* + 3\mu^*}{\lambda^* + \mu^*} = 3 - 4\nu,$$

μ^*, λ^* are the first and second Lamé parameters:

$$\lambda^* = \frac{\nu E}{(1+\nu)(1-2\nu)};$$

$$\mu^* = \frac{E}{2(1+\nu)}$$

(here: E is the Young modulus, ν is the Poisson ratio).

At the above assumptions and denotations, the boundary conditions (5) can be recorded as:

$$\left.\begin{aligned} &(1-if)\Phi^+(t)+(1+if)\bar{\Phi}^+(t)= \\ &=(1-if)\Phi^-(t)+(1+if)\bar{\Phi}^-(t), \\ &\chi^*\Phi^-(t)+\Phi^+(t)-\chi^*\bar{\Phi}^+(t)-\bar{\Phi}^-(t)=4i\mu^* f'(t). \end{aligned}\right\} \quad (28)$$

The first equation in the system of equations (28) shows that the function

$$(1-if)\Phi(z)+(1+if)\bar{\Phi}(z)$$

is holomorphic over the entire plane and, since it must vanish at infinity, then over the entire plane

$$(1-if)\Phi(z)+(1+if)\bar{\Phi}(z)=0. \quad (29)$$

By expressing $\bar{\Phi}(z)$ via $\Phi(z)$ and introducing (28) to the second equation (28), we obtain the boundary condition for $\Phi(z)$

$$\Phi^+(t)=g\Phi^-(t)+f_0(t), \quad (30)$$

where

$$g=-\frac{\chi^*+1+if(\chi^*-1)}{\chi^*+1-if(\chi^*-1)};$$

$$f_0(t)=\frac{4i\mu^*(1+if)f'(t)}{\chi^*+1-if(\chi^*-1)},$$

noting that $\chi^*>1$ and $f>0$, the previous expressions can be simplified by introducing a constant α^*, defined from conditions

$$\alpha^*=\frac{1}{\pi}\operatorname{arctg}\left(f\frac{\chi^*-1}{\chi^*+1}\right), \quad 0\le\alpha^*\le 0{,}5. \quad (31)$$

Then

$$\chi^*+1\pm if(\chi^*-1)=$$
$$=\sqrt{(\chi^*+1)^2+f^2(\chi^*-1)^2}\,e^{\pm\pi i\alpha^*}=\frac{(\chi^*+1)e^{\pm\pi i\alpha^*}}{\cos\pi\alpha^*},$$

and, consequently,

$$g=-e^{2\pi i\alpha^*},$$

$$f_0(t)=\frac{4i\mu^*(1+if)f'(t)e^{\pi i\alpha^*}\cos\pi\alpha}{\chi^*+1}f'(t). \quad (32)$$

From the accepted condition ($0\le c\le 1,$ where c is a positive constant) one must take the value of the logarithm for which

$$0\le Re\frac{lng}{2\pi i}<1.$$

Hence, for condition $e^{2\pi i\gamma}=g$

$$\gamma=\frac{\ln g}{2\pi i}=\frac{1}{2}+\alpha^*. \quad (33)$$

To solve this problem, we shall apply the Cauchy-type integral

$$F_0(z)=\frac{1}{2\pi i}\int_a^b\frac{f(t)\mathrm{d}t}{t-z}, \quad (34)$$

where $F_0(z)$ is a piecewise holomorphic function. According to the Liouville's theorem, $F(z)=C=\text{const}$ in all planes. Then the general solution to the problem is determined in the form $F(z)=F_0(z)+C,$ meaning

$$F(z)=\frac{1}{2\pi i}\int_a^b\frac{f(t)\mathrm{d}t}{t-z}+C, \quad (35)$$

where $f(t)$ denotes a surge $F(z)$ along the line of surges L, that is

$$f(t)=F^+(t)-gF^-(t), \quad (36)$$

and C is a constant.

If the pole of an arbitrary order is at infinity, the particular solution to problem $X_0(z)$ can be found from the following equation

$$X_0(z)=\sum_{i=1}^{n}(z-a_j)^{-\gamma}(z-b_j)^{\gamma-1}. \quad (37)$$

If we trace the evolution of argument $z-a_n$, or $z-b_n$, when z originates from point t of the arc, a_nb_n describes a closed path consisting of the lengths of arcs of the upper and lower half-planes

$$\left.\begin{aligned} X_0^-(t)=e^{-2\pi i\gamma}X_0^+(t), \\ X_0^+(t)=e^{2\pi i\gamma}X_0^-(t). \end{aligned}\right\} \quad (38)$$

For the solution to the homogeneous problem, which may have a pole at infinity, the function $X_0(z)$ must satisfy condition [9]

$$X_0^+(t)=gX_0^-(t),$$

hence

$$g=\frac{X_0^+(t)}{X_0^-(t)}.$$

By introducing this expression to boundary condition (36), we obtain

$$\frac{F^+(t)}{X_0^+(t)}-\frac{F^-(t)}{X_0^-(t)}=\frac{f(t)}{X_0^+(t)}.$$

Then

$$F(z)=\frac{X_0(z)}{2\pi i}\int_a^b \frac{f(t)\mathrm{d}t}{X_0^+(t)(t-z)}+C_0X_0(z). \quad (39)$$

In this formula, expression $(z-a)^{-\frac{1}{2}-\alpha^*}(b-z)^{-\frac{1}{2}+\alpha^*}$ could be taken as the function $X_0(z)$

$$\Phi(z)=\frac{2\mu^*(1+if)e^{\pi i\alpha^*}\cos\pi\alpha}{\pi(\chi^*+1)(z-a)^{\frac{1}{2}+\alpha^*}(b-z)^{\frac{1}{2}-\alpha^*}}\times$$
$$\times\int_a^b \frac{(t-a)^{\frac{1}{2}+\alpha^*}(b-t)^{\frac{1}{2}-\alpha^*}}{t-z}f'(t)dt+$$
$$+\frac{C_0}{(z-a)^{\frac{1}{2}+\alpha^*}(b-z)^{\frac{1}{2}-\alpha^*}}, \quad (40)$$

where C_0 is a constant, and $(z-a)^{\frac{1}{2}+\alpha^*}(b-z)^{\frac{1}{2}-\alpha^*}$ implies a branch, holomorphic outside segment ab and acquiring on the upper side of this segment the valid positive values $(t-a)^{\frac{1}{2}+\alpha^*}(b-t)^{\frac{1}{2}-\alpha^*}$. The specified branch is characterized by

$$\lim_{z\to\infty}\frac{(z-a)^{\frac{1}{2}+\alpha^*}(b-z)^{\frac{1}{2}-\alpha^*}}{z}=-ie^{\pi i\alpha^*}.$$

The constant C_0 is determined from formula

$$\lim_{z\to\infty} z\Phi(z)=\frac{-T_0+iP_0}{2\pi}=\frac{iP_0(1+if)}{2\pi},$$

hence

$$C_0=\frac{P_0(1+if)e^{\pi i\alpha^*}}{2\pi}.$$

Formula (40) then takes the form

$$\Phi(z)=\frac{2\mu^*(1+if)e^{\pi i\alpha^*}\cos\pi\alpha}{\pi(\chi^*+1)(z-a)^{\frac{1}{2}+\alpha^*}(b-z)^{\frac{1}{2}-\alpha^*}}\times$$
$$\times\int_a^b \frac{(t-a)^{\frac{1}{2}+\alpha^*}(b-t)^{\frac{1}{2}-\alpha^*}}{t-z}f'(t)\mathrm{d}t+$$
$$+\frac{P_0(1+if)e^{\pi i\alpha^*}}{2\pi(z-a)^{\frac{1}{2}+\alpha^*}(b-z)^{\frac{1}{2}-\alpha^*}}. \quad (41)$$

Pressure $q(t)$ at points t_0 of the gear's tooth (stamp) is determined based on formula (22)

$$q(t_0)+i\tau(t_0)=q(t_0)(1+if)=\Phi^+(t_0)-\Phi^-(t_0). \quad (42)$$

By computing the last difference based on the Sachotsky-Plemel formulae, we obtain the refined expression to determine the contact stress:

$$q(t_0)=\sigma^*_{H\max}(t_0)==-\frac{4\mu^*\sin(\pi\alpha^*)\cos(\pi\alpha^*)}{\chi^*+1}f'(t_0)+$$
$$+\frac{P_0\cos(\pi\alpha^*)}{\pi(t_0-a)^{\frac{1}{2}+\alpha^*}(b-t_0)^{\frac{1}{2}-\alpha^*}}=\frac{4\mu^*\cos^2(\pi\alpha^*)}{\pi(\chi^*+1)(t_0-a)^{\frac{1}{2}+\alpha^*}(b-t_0)^{\frac{1}{2}-\alpha^*}}\times$$
$$\times\int_a^b \frac{(t-a)^{\frac{1}{2}+\alpha^*}(b-t)^{\frac{1}{2}-\alpha^*}}{t-t_0}f'(t)\mathrm{d}t. \quad (43)$$

It is obvious that when $f=0$ (then $\alpha^*=0$) we obtain a solution for the perfect case where there is no friction between the surfaces conjugated.

To determine distance ab, we apply the following formula [9]

$$\int_a^b \frac{tf'(t)\mathrm{d}t}{\sqrt{(t-a)(b-t)}}=KP_0, \quad (44)$$

where P_0 is the magnitude of the principal vector of external forces that press a gear to the wheel.

If function $f(t)$ is even, that is $f(-t)=f(t)$, then, for reasons of symmetry, we can take $a=-l$ and $b=+l$, in advance, where l is determined from the following ratio

$$\int_0^l \frac{tf'(t)\mathrm{d}t}{\sqrt{l^2-t^2}}=\frac{1}{2}K^*P_0, \quad (45)$$

where

$$K^*=\frac{\chi_1+1}{4\mu_1}+\frac{\chi_2+1}{4\mu_2}$$

is the all-side compression module.

For

$$\chi^*_1=\chi^*_2=\chi^* \text{ and } \mu^*_1=\mu^*_2=\mu^*$$

$$K^*=\frac{\chi+1}{2\mu}=\frac{2(1-\nu)}{\mu}=\frac{2(1-\nu)}{\frac{E}{2(1+\nu)}}=$$
$$=\frac{4(1-\nu^2)}{\frac{2E_1E_2}{E_1+E_2}}=\frac{2(1-\nu^2)(E_1+E_2)}{E_1E_2}.$$

Then

$$\int_0^l \frac{tf'(t)dt}{\sqrt{l^2-t^2}}=(1-\nu^2)\frac{E_1+E_2}{E_1E_2}P_0. \quad (46)$$

If the body of the teeth is limited by circles of radii R_1 and R_2, larger in comparison with the contact areas, then, believing

$$f(t)=\frac{t^2}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{t^2}{2\rho_r}, \quad (47)$$

we obtain solution (43) in the form

$$\sigma^*_{H\max}(t_0)=\frac{\cos^2(\pi\alpha^*)}{\pi k^{**}l}\times$$
$$\times\frac{1}{\rho_r}\int_a^b \sqrt{l^2-t^2}\left(\frac{l+t}{l-t}\right)^{\alpha^*}dt+P_0\frac{\cos(\pi\alpha^*)}{\pi l}, \quad (48)$$

where

$$k^{**}=\frac{2(1-\nu^2)}{E_r}. \quad (49)$$

The integral in this formula is not expressed by a combination of elementary functions and, therefore, we represent it in the form of a Taylor series

$$\left(\frac{l+t}{l-t}\right)^{\alpha^*}=1+\frac{2\alpha^*}{l}t+\frac{2\alpha^{*2}}{l^2}t^2+\frac{2}{3}\frac{\left(2\alpha^{*3}+\alpha^*\right)}{l^3}t^3\ldots \quad (50)$$

Then

$$\int_0^l\sqrt{l^2-t^2}\left(\frac{t+l}{l-t}\right)^{\alpha^*}\mathrm{d}t=\pi l^2\left(\frac{1}{4}+\frac{\alpha^{*2}}{8}+\frac{2\alpha^*}{3\pi}+\frac{4\left(\alpha^*+2\alpha^{*3}\right)}{45\pi}\right). \quad (51)$$

By introducing the latter to (48) to determine the greatest value of the contact stress, we obtain the following formula:

$$\sigma^*_{H\max}(t_0)=\frac{\cos^2\left(\pi\alpha^*\right)}{\pi k^* l}\frac{1}{\rho_r}\pi l^2\times$$
$$\times\left(\frac{1}{4}+\frac{\alpha^{*2}}{8}+\frac{2\alpha^{*2}}{3\pi}+\frac{4\left(\alpha^*+2\alpha^{*3}\right)}{45\pi}\right)+P_0\frac{\cos\left(\pi\alpha^*\right)}{\pi l}=$$
$$=\frac{\cos^2\left(\pi\alpha^*\right)}{k^*}\frac{l}{\rho_r}\left(\frac{1}{4}+\frac{\alpha^{*2}}{8}+\frac{2\alpha^*}{3\pi}+\frac{4\left(\alpha^*+2\alpha^{*3}\right)}{45\pi}\right)+$$
$$+P_0\frac{\cos\left(\pi\alpha^*\right)}{\pi l}. \quad (52)$$

In equation (45)

$$f'(t)=\left(f(t)\right)'=\frac{t}{\rho_r}. \quad (53)$$

With respect to equation (45) and (47) in equation (44), we obtain

$$\frac{1}{\rho_r}\int_0^l\frac{t^2dt}{\sqrt{l^2-t^2}}=\left(1-\nu^2\right)\frac{P_0}{E_r}. \quad (54)$$

By substituting in this equation

$t=l\sin x$ and $dt=l\cos x dx$,

we obtain

$$\sqrt{l^2-t^2}=l\cos x,\quad x=\arcsin\frac{t}{l},$$

$$t=0\Rightarrow x=0;\quad t=l\Rightarrow x=\frac{\pi}{2}.$$

Then

$$\frac{1}{\rho_r}\int_0^l\frac{t^2dt}{\sqrt{l^2-t^2}}=\left(1-\nu^2\right)\frac{P_0}{E_r},$$

hence, we determine

$$l=2\sqrt{\frac{\left(1-\nu^2\right)P_0\rho_r}{\pi E_r}}. \quad (55)$$

Employing formulae (49), (55) in (52), we come to the following value:

$$\sigma^*_{H\max}(t_0)=\frac{\cos^2\left(\pi\alpha^*\right)}{k^*}\frac{l}{\rho_{np}}\left(\frac{1}{4}+\frac{\alpha^{*2}}{8}+\frac{2\alpha^*}{3\pi}+\frac{4\left(\alpha^*+2\alpha^{*3}\right)}{45\pi}\right)+P_0\frac{\cos\left(\pi\alpha^*\right)}{\pi l}=$$
$$=\left[\sqrt{2}\cos^2\left(\pi\alpha^*\right)\left(\frac{1}{4}+\frac{\alpha^{*2}}{8}+\frac{2\alpha^*}{3\pi}+\frac{4\left(\alpha^*+2\alpha^{*3}\right)}{45\pi}\right)+\frac{1}{\sqrt{2}}\cos\left(\pi\alpha^*\right)\right]\sigma_H\le[\sigma]_H,$$

where

$$\sigma_H=\sqrt{\frac{P_0 E}{2\pi\left(1-\nu^2\right)\rho_r}}$$

is the known Hertz solution [11, 14–17].

It should be noted that at $\alpha^*=0$ $(f=0)$ or $t_0=0$, the refined solution to the problem exceeds the known Hertz solution by 6 %, that is

$$\sigma^*_{H\max}(t_0)=\frac{3\sqrt{2}}{4}\sigma_H\approx 1{,}06\sigma_H.$$

5. Quantitative evaluation of the influence of a lubricant on load capacity of toothed gears

To quantify the effect of a lubricant on load capacity of the toothed gears, we compare values for the largest contact stress with its permissible value

$$\begin{bmatrix}\sqrt{2}\left(\frac{1}{4}+\frac{\alpha^{*2}}{8}+\frac{2\alpha^*}{3\pi}+\frac{4\left(\alpha^*+2\alpha^{*3}\right)}{45\pi}\right)\cos^2\left(\pi\alpha^*\right)+\\+\frac{1}{\sqrt{2}}\cos\left(\pi\alpha^*\right)\end{bmatrix}\sigma_H\le$$
$$\le\frac{\sigma_{H\lim b}Z_N}{S_{H\min}}Z_R Z_v Z_L Z_X Z_w, \quad (57)$$

where σ_{Hlimb} is the limit of contact resistance of the teeth surfaces corresponding to the basis number of stress cycles; S_{Hmin} is the minimum factor of safety; Z_N is a durability factor; Z_R is a coefficient that takes into consideration the influence of the initial roughness of the conjugated teeth surfaces; Z_v is a coefficient that takes into consideration the influence of circumferential speed; Z_L is a coefficient that takes into consideration the influence of a lubricant; Z_X is a coefficient that takes account the size of the toothed wheel; Z_w is a coefficient that takes into consideration the influence of difference in the hardness of materials of the conjugated surfaces of teeth.

By comparing the left and right sides of this expression, rejecting the terms of the second order of smallness, we obtain an expression to determine the coefficient of influence of a lubricant

$$Z_L=\frac{1}{\sqrt{2}}\begin{bmatrix}\left(0{,}5+0{,}48\alpha^*+0{,}25\alpha^{*2}+0{,}11\alpha^{*3}\right)\times\\\times\cos^2\left(\pi\alpha^*\right)+\cos\left(\pi\alpha^*\right)\end{bmatrix}. \quad (58)$$

Based on an analysis of expression (31), it was established that $\alpha^*\approx f$. Taking this fact into consideration, after the expansion of $\cos\left(\pi\alpha^*\right)$ into a Taylor power series, with some simplifications, we obtain the following analytical expression for the coefficient of influence of a lubricant

$$Z_L=$$
$$=12{,}575(f-0{,}732645)(f-0{,}467789)\times$$
$$\times\left(0{,}24607+0{,}940553f+f^2\right). \quad (59)$$

Based on the experimental study, carried out at the authors' highly sensitive roller installation that makes it possible to evaluate impact of the contact parameters on a friction coefficient, we obtained an empirical expression for the coefficient of friction. In this case, we used lubricants of the oil brands MS-20, MC-22, Tp-46, TP-30, Tp-22, I-40A, I-30A, and I-20A [12]:

$$f = 0{,}5215 \cdot 10^4 \frac{\left(1 - 0{,}065 V_{ó}\right) R_a}{\sigma_H^{0{,}223} \nu^{0{,}3} V_s^{0{,}334} \rho_r}, \tag{60}$$

where σ_H is the contact stress, MPa; ν is the kinematic viscosity of oil, m^2/s ; V_Σ is the total rolling speed, m/s; V_s is the slip velocity, m/s; R_a is the arithmetic mean deviation of the profile within the basic length, m; ρ_r is the reduced radius of the surfaces' curvature, m.

It is obvious that the less *f*, the larger Z_L as well as the related load-bearing capacity. An argument in favor of this conclusion is that the coefficient Z_L is directly included in the formula for calculating the contact endurance of the active surfaces of teeth – dependences (57), (63), (64), (67) – that affects load capacity of the toothed gears for a contact endurance criterion. For clarity, Fig. 3 shows a graph of dependence of a lubricant influence coefficient on the friction coefficient, calculated from formula (60).

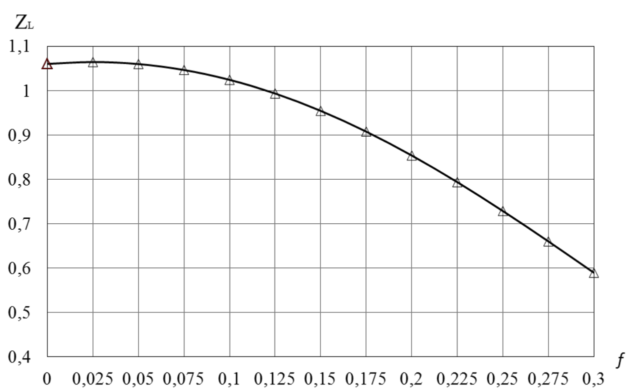

Fig. 3. Dependence graph of a lubricant influence coefficient on friction coefficient

After substituting (60) in (59) and some transforms, we obtain the analytical dependence of a lubricant influence coefficient on contact parameters

$$Z_L = 1{,}0605 + 0{,}177 \cdot 10^4 \frac{\left(1 - 0{,}065 V_{ó}\right) R_a}{\sigma_H^{0{,}223} \nu^{0{,}3} V_s^{0{,}334} \rho_r} \times$$
$$\times \left(1 - 10{,}45 \cdot 10^2 \frac{\left(1 - 0{,}065 V_{ó}\right) R_a}{\sigma_H^{0{,}223} \nu^{0{,}3} V_s^{0{,}334} \rho_r}\right). \tag{61}$$

Thus, the lubricant influence coefficient Z_L is linked in the general case to the contact stress, kinematic viscosity of oil, the roughness of surfaces, the total rolling speed, sliding speed and the reduced radius of curvature.

Study results show that the influence of a friction coefficient on the lubricant influence coefficient and related load bearing capacity of the toothed gear is significant [13]. The proposed procedure makes it possible to solve the inverse problem: knowing the kinematical and energy parameters of toothed gears, one may choose, based on a lubricant influence coefficient (Z_L), a more efficient lubricating material.

6. Estimation of contact endurance of the toothed mechanisms under the influence of tangential stresses existing in the contact zone between active surfaces of teeth, with respect to a lubricant influence coefficient

Because a value of tangential stresses also depends on a friction force (F_s), associated with sliding, it is expedient to consider this field of research.

It is known that the design calculation of closed toothed gears is performed in terms of endurance based on the permissible contact stresses, in order to avoid fatigue spalling of the teeth working surfaces. According to acting standards, the magnitude of the maximum contact stress between the teeth of toothed gears is calculated according to the Hertz formula [11, 14–16]:

$$\sigma_H = Z_H Z_M Z_\varepsilon \sqrt{\frac{W_H}{d_{W1}} \frac{u \pm 1}{u}} \le \frac{\sigma_{Hlimb} Z_N}{S_H} Z_R Z_v Z_L Z_X Z_w, \tag{62}$$

where

$$Z_M = \sqrt{\frac{E_r}{\pi\left(1 - \nu^2\right)}}$$

is a coefficient that takes account of the mechanical properties of material of the conjugated toothed wheels (for steel toothed wheels, $Z_M = 275\,\text{MPa}^{1/2}$);

$$Z_H = \sqrt{\frac{2 cos\beta}{sin 2\alpha_w}}$$

is a coefficient that takes into consideration the shape of the conjugated surfaces of teeth in the engagement pole;

$$Z_\varepsilon = \sqrt{\frac{\left(4 - \varepsilon_\alpha\right)}{3}}$$

is a coefficient that takes account of the total length of contact lines; d_{W2} is the diameter of the initial wheel circumference, which is accepted when calculating as

$$d_{w2} \approx d_2 = \frac{2 a_w u}{u \pm 1};$$

d_2 is the diameter of the pitch circle of the wheel; u is the gear ratio;

$$W_H = \frac{F_t K_{H\alpha} \cdot K_{H\beta} K_{HV}}{b_w}$$

is the normal load per unit length of the contact line of the tooth;

$$F_t = \frac{2T_2}{d_2}$$

is the circumferential force; $b_w = \psi_{ba} \cdot a_w$ is the working width of the toothed gear's crown; $K_{H\alpha}$ is a coefficient that takes into consideration the distribution of load between the teeth; $K_{H\beta}$ is a coefficient that takes account of the uneven distribution of load along the length of contact lines; K_{HV} is a coefficient that takes into consideration the internal dynamic load;

$$E_{\text{пр}}=\frac{2E_1E_2}{E_1+E_2}$$

is the reduced modulus of elasticity of materials of toothed gears; E_1E_2 is the modulus of elasticity of a material of the gear and the wheel, respectively.

Considering these parameters in expression (62), we obtain the following formula

$$\sigma_H=Z_HZ_MZ_\varepsilon\sqrt{\frac{T_2K_{H\alpha}K_{H\beta}K_{HV}\left(u\pm1\right)^3}{2a_w^3u^2\psi_{ba}Z_L^2}}\leq$$
$$\leq\left[\sigma_H\right]=\frac{\sigma_{Hlimb}Z_N}{S_H}Z_RZ_vZ_XZ_w. \quad (63)$$

The study conducted demonstrate that normal contact stresses only indirectly affect the main types of toothed gears fracture. Therefore, the responsibility for the emergence and development of damage should lay with the tangential stresses, which act in the contact zone between active surfaces of the teeth. When a contact zone moves away from the pole line of a tooth, stresses $\tau_{\max}$ increase in magnitude and approach the surface layers, thereby intensifying the processes of destruction in them. Given this, the greatest surface tangential stresses $\tau_{\max,n}$, grow, the magnitude of which may exceed the magnitude of $\tau_{\max}$. The direction of interaction between these stresses also creates real conditions for the occurrence and development of fatigue cracks. As was mentioned, the value of these stresses depends on a friction force (F_s), associated with sliding, and the friction force depends on the coefficient of friction.

The study performed [10] shows that should a friction force act between two contacting surfaces, in addition to normal pressure, then the magnitude of the maximum tangential stress is determined from [10]:

$$\tau_{\max}=0{,}347\sigma_H=$$
$$=0{,}347\cdot Z_HZ_MZ_\varepsilon\sqrt{\frac{T_2K_{H\alpha}K_{H\beta}K_{HV}\left(u\pm1\right)^3}{2a_w^3u^2\psi_{ba}Z_L^2}}\leq\left[\tau\right], \quad (64)$$

where $\left[\tau_s\right]$ is the permissible tangential stress $\left[2\right]$

$$\left[\tau_s\right]=0{,}5c_2\left[\sigma_H\right]=\frac{\sigma_{Hlimb}Z_N}{S_H}Z_RZ_vZ_XZ_wc_2, \quad (65)$$

where c_2 is the coefficient of reliability, adopted based on the probability of non-failure operation of a toothed gear. If the probability of a non-failure operation of the toothed gear is

$$P(t)=0{,}9\ \Rightarrow\ c_2=1{,}0,$$

$$P(t)=0{,}95\ \Rightarrow\ c_2=0{,}897,$$

$$P(t)=0{,}96\ \Rightarrow\ c_2=0{,}865,$$

$$P(t)=0{,}97\ \Rightarrow\ c_2=0{,}827,$$

$$P(t)=0{,}99\ \Rightarrow\ c_2=0{,}702.$$

In the case of complex stressed state, the estimation stress then is a certain reduced (equivalent) stress obtained based on one of the theories of strength, the most acceptable for the examined stressed state of a material. For ductile materials

$$\sigma_{\text{эк}}=\sqrt{\sigma_{\text{H}}^2+4\tau_{\max}^2}=$$
$$=1{,}217\cdot Z_HZ_MZ_\varepsilon\sqrt{\frac{T_2K_{H\alpha}K_{H\beta}K_{HV}\left(u\pm1\right)^3}{2a_w^3u^2\psi_{ba}Z_L^2}}\leq\left[\sigma_H\right]. \quad (66)$$

In the design calculation, one typically determines the inter-axis spacing or a pitch diameter of the toothed wheel. The inter-axis spacing

$$a_w=K_a\left(u\pm1\right)\sqrt[3]{\frac{T_2K_{H\beta}}{\left[\sigma_H\right]^2Z_L^2u^2\psi_{ba}}}, \quad (67)$$

where K_a is an auxiliary coefficient, for spur gears

$$K_a=\sqrt[3]{\left(1{,}217\cdot Z_HZ_MZ_\varepsilon\right)^2 0{,}5K_{H\alpha}K_{HV}}=$$
$$=\sqrt[3]{\left(1{,}217\cdot275\cdot1{,}76\cdot0{,}9\right)^2\cdot0{,}5\cdot1{,}0\cdot1{,}25}\approx56\ \text{MPa}^{1/3}.$$

Here

$$Z_M=275\,\text{MPa}^{1/2};\quad Z_H=1{,}76;$$

$$Z_\varepsilon\approx0{,}9;\quad K_{H\alpha}=1{,}0;\quad K_{HV}=1{,}25.$$

For helical gears

$$K_a=\sqrt[3]{\left(1{,}217\cdot Z_HZ_MZ_\varepsilon\right)^2 0{,}5K_{H\alpha}K_{HV}}=$$
$$=\sqrt[3]{\left(1{,}217\cdot275\cdot1{,}71\cdot0{,}8\right)^2 0{,}5\cdot1{,}1}\approx49\ \text{MPa}^{1/3},$$

here

$$Z_M=275\,\text{MPa}^{1/2};\quad Z_H=1{,}76\cos\beta\approx1{,}71;$$

$$Z_\varepsilon=\sqrt{\frac{1}{\varepsilon_\alpha}}\approx0{,}8.$$

The helical gears operate more smoothly than the spur ones, which is why factor K_{HV} is less. Given this observation, we accept the product $K_{H\alpha}K_{HV}=1{,}1$.

7. Discussion of results of devising a calculation procedure for toothed gears with respect to the influence of a lubricant

The proposed procedure for the calculation of toothed gears in terms of contact strength at the finite friction coefficient, without the assumptions about the smallness of the contact area and shape of borders, makes it possible to estimate the load capacity of toothed gears considering the influence of lubrication and the existence of friction between the conjugated surfaces of teeth, which is almost disregarded in traditional calculation methods by equating the lubricant influence coefficient K_L to unity. In this case, the value of the resulting stress exceeds the stress, calculated according to the known Hertz solution, by 6 %, which would accordingly improve load capacity of the toothed mechanisms.

The study conducted demonstrates that normal contact stresses only indirectly affect the main types of fracture of toothed gears. Therefore, responsibility for the emergence and development of damage should lay with the tangential stresses, which act in the contact zone between active surfaces of the teeth. However, in the design calculation of toothed gears, the influence of the tangential stress and the effect of a lubricant are almost ignored. The proposed procedure makes it possible to more accurately assess the effect of a lubricating oil on load capacity of the toothed gears and to solve the inverse problem: knowing the kinematical and energy parameters of toothed gears, it is required to choose, based on a lubricant influence coefficient $Z_{L,}$ a more effective lubricating material. Practical literature on the estimation and design of gear mechanisms should pay more attention to the influence of lubrication on their load capacity, to recommendations on the choice of lubricants for gear mechanisms, to determining the precise value for a lubricant influence coefficient.

When estimating the coefficient of lubricant influence on load capacity of toothed gears, the range of input values for contact parameters is valid at the following ranges in the change in parameters and dimensionality of the input magnitudes:

$$\sigma_H \le 550\,\text{MPa};\quad 0 \le V_\Sigma \le 9{,}0\ \text{m/s};$$

$$2{,}0 \le V_S \le 12{,}0\ \ \text{m/s};$$

$$15\cdot 10^{-6} \le \nu \le 165\cdot 10^{-6}\ \ \text{m}^2/\text{s};$$

$$0{,}16\cdot 10^{-6} \le R_a \le 0{,}32\cdot 10^{-6}\,\text{m};\quad \rho \le 0{,}1\ \text{m}.$$

If one is to consider the problem outside these ranges, there is a need to conduct additional experiments using different lubricants.

The shortcoming of this study is in that each lubricant requires experiments to calculate a lubricant influence coefficient K_L. The statistical data that would be acquired must be added to existing reference books and tutorials on the calculation and design of toothed gear mechanisms.

7. Conclusions

1. We have solved the problem on the contact between two bodies of arbitrary shape, close to half-planes, at the finite friction coefficient, without assumptions about the smallness of the contact area and the shape of borders; the result revealed that the value of the stress, derived from the refined solution to the contact problem, exceeds the stress, calculated according to the known Hertz solution, by 6 %.

2. The proposed calculation procedure makes it possible to more accurately assess the load-carrying capacity of toothed gears, taking into consideration the influence of lubricants, which is of practical importance when designing machines and assemblies.

3. The result of the refined calculation of toothed wheels in terms of contact strength is the derived analytical expression for a lubricant influence coefficient based on the criterion of a friction coefficient between the conjugated surfaces of friction nodes.

4. Based on the study performed, we propose to take into consideration a lubricant influence coefficient directly to determine the geometrical parameters.

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GEOMETRICAL SYNTHESIS OF SPATIAL SIX-LINK GUIDING MECHANISMS

S. Kliuchkovskyi
PhD, Associate Professor*
E-mail: ksm65@meta.ua
M. Tykhan
Doctor of Technical Sciences, Associate Professor*
E-mail: tykhanm@ukr.net
T. Repetylo
PhD, Assistant*
E-mail: tarasrepetylo@gmail.com
Y. Hirnyak
Assistant*
E-mail: yurii.b.hirniak@lpnu.ua
*Department of precision mechanics devices
Lviv Polytechnic National University
S. Bandery str., 12, Lviv, Ukraine, 79013

Розглянуті питання геометричного синтезу просторових шарнірно-важільних шестиланкових механізмів з лінійним переміщенням кінцевої ланки, що виконують функцію напрямної. Виявлено варіанти компонування механізмів та конструктивні особливості, що забезпечують максимальний хід кінцевої ланки при мінімальних довжинах важелів. Встановлено геометричні параметри механізму в його узагальненому вигляді, які визначають кінематику та компонувальні схеми. Досліджено вплив геометричних параметрів і варіантів компонування на кінематичні параметри механізмів. Отримано залежності, які дозволяють визначити геометричні параметри базового механізму за заданим ходом кінцевої ланки та допустимими кутами передачі у шарнірах повідців. Подано параметричні залежності, які дозволяють провести точний розрахунок оптимальної геометрії механізму за критерієм мінімізації довжин повідців при допустимих кутах передачі і необхідному діапазоні переміщень. Запропонована схема розрахунку просторових розмірних ланцюгів для визначення форми деталей. 3D моделюванням виявлена варіативність геометричних параметрів, що дозволило сформулювати компонувальні варіанти механізму. Розроблено методику геометричного синтезу та змодельовано у відповідності до цієї методики варіанти просторових шарнірно–важільних шестиланкових механізмів в динаміці, що дозволило показати особливості руху ланок.

Проведені дослідження виявили можливі шляхи розробок нових варіантів просторових шарнірно-важільних шестиланкових механізмів та розкрили нові можливості при їхньому застосуванні у якості напрямного механізму. Результати досліджень можуть бути використані при розробленні платформ підйомників, маніпуляторах роботів, верстатобудуванні та мехатроніці

Ключові слова: механізм Саррюса, шестиланковий просторовий механізм, напрямний механізм, лінійне переміщення, геометричний синтез

1. Introduction

Lever mechanisms with a linear motion of moving parts are used in devices and equipment of various areas of mechanical engineering, including robotics, lifting machinery, machine tools, and mechatronics. Despite the complexity of the structure, multifaceted lever mechanisms of linear relocation in some devices have replaced guides with sliding carriages, telescopic mechanisms, and carriages on linear bearings. This is due to various factors: the requirements of layout and minimization of the dimensions, the need to protect open surfaces from friction, the tendency to jam carriages with translational pairs, etc.

The choice of the scheme and structure of these non-standard mechanisms can significantly affect the performance of the devices in general, their layout, dimensions, cost, etc.

Hinged lever mechanisms with translational or near-translational finite-element coupling provide for the implementation of compact structures in a folded state in the absence or minimum number of translational kinematic pairs [1, 2]. This avoids the drawbacks of translational pairs and uses standard camshafts with protected local friction surfaces in the design of rotary kinematics pairs. An increase in the course of the final link is achieved in these mechanisms by using levers.

Flat guide lever mechanisms with rotational pairs are found in devices for a steady movement of the working body of machine tools or platforms of lifts and manipulators of robots. In the designs of lifts and manipulators, flat lever pantograph mechanisms are widely used [3–5], which, however, are not devoid of translational pairs. In woodworking format-cutting machines, combined circuits are employed,