В результаті застосування кількісного підходу до розрахунку статичної стійкості системи тягового електропостачання було встановлено, що в процесі руху поїзда по реальній ділянці мають місце зони відсутності стійкості по напрузі. Точне вирішення завдання оцінки стійкості надзвичайно ускладнене необхідністю розрахунку нелінійних залежностей, котрі визначають режими роботи системи тягового електропостачання і електрорухомого складу.

В пропонованій роботі виконане конструювання системи чотирьох автономних нелінійних диференціальних рівнянь на основі експериментальних даних, що моделюють поведінку струму і напруги в контактній мережі. Також були розраховані області стійкості регуляторів напруги в тяговій мережі, що стабілізують напругу на струмоприймачах електрорухомого складу.

Одержані області стійкості регуляторів напруги дозволили оцінити запаси стійкості і встановити найбільш робастні з побудованих регуляторів. В результаті проведених досліджень виявлено, що нелінійний регулятор має кращі робастні властивості, чим лінійний. При цьому стійкість лінійного регулятора дуже вузька – Δk=0,000004, що на порядок менше, чим для нелінійного регулятора. При застосуванні нелінійного регулятора напруга в контактній мережі стабілізується в 3 рази швидше незалежно від місця його розташування.

Використання розробленого підходу дозволить здійснювати розрахунки областей стійкості варіантних схемних рішень тягової мережі при впровадженні швидкісного руху та звузити діапазон коливань напруги. Розроблені динамічна модель процесів електроспоживання в тяговій мережі та регулятор напруги можуть бути використані при побудові інтелектуальної, адаптивної системи тягового електропостачання для швидкісного руху

Ключові слова: система тягового електропостачання, регулятор напруги, область стійкості, нелінійний рекурентний аналіз

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DEVELOPMENT OF AN APPROACH TO ENSURE STABILITY OF THE TRACTION DIRECT CURRENT SYSTEM

V. Sychenko
Doctor of Technical Sciences, Professor*
E-mail: elpostz@i.ua

V. Kuznetsov
Doctor of Technical Sciences, Professor *
E-mail: vkuz@i.ua

Ye. Kosariev
Assistant *
E-mail: kosarev@e.diit.edu.ua

P. Hubskyi
Postgraduate student*
E-mail: peter.gybskiy@gmail.com

V. Belozyorov
Doctor of Physical and Mathematical Sciences, Professor**
E-mail: belozvye2017@gmail.com

V. Zaytsev
PhD, Associate Professor**
E-mail: vadymzaytsev65@gmail.com

M. Pulin
Deputy Chief of Power Supply
Regional branch of "Lviv railway" association "Ukrzaliznytsya"
Hoholia str., 1, Lviv, Ukraine, 79000

*Department of intelligent power supply systems
Dnipropetrovsk National University of Railway Transport named after academician V. Lazaryan
Lazaryan str., 2, Dnipro, Ukraine, 49010

**Department of Computer Technologies
Oles Honchar Dnipro National University
Gagarin ave., 72, Dnipro, Ukraine, 49010

1. Introduction

Introduction of high-speed motion necessitates the modernization of the traction power supply system in order to ensure the required voltage mode and the level of specific power. The process of consumption of electric energy in the traction network occurs in the presence of heterogeneous perturbations that are nonstationary in character. These include a change in the operation modes of electric rolling stock depending on patterns in the timetable of trains, the profile of a railroad and existing restrictions, as well as the existence of different kinds of transient processes. Given this, the pantograph voltage and the consumed power vary over a wide range, which could lead to a loss of stability in the work of the traction electric supply system (TES). An approach to the quantitative assessment of TES stability in terms of voltage was described in paper [1]. Results of the authors' analytical calculations proved the existence of zones in the traction network, which lack voltage stability. Hence, one could draw an obvious conclusion about the necessity to control voltage in the traction network. Under real conditions, controlling influences on the voltage level are executed by the low-cost trivial means for strengthening the traction network: via an increase in the wire section, the application of parallel connection points. However, these means are not capable of ensuring the required stability of TES in terms of voltage and do not make it possible to adjust voltage in the traction network.

The level of voltage in the traction network depends on many factors and, by changing in time, plane, and space, demonstrates a probabilistic character with complex inter-relationships. When implementing the high-speed motion, there is the task on the development of new approaches to control voltage in the intelligent, self-tuning power supply system of the distributed type, built on the modern element base [2]. This approach meets the modern trends in the development of energy engineering, which underlies the Smart Grid concept, which is at present the main technological and methodological basis to improve the efficiency of energy consumption [3]. One could formulate the following understanding of an intelligent TES: implementation of advanced technologies to transport electricity, diagnosing the state of equipment, large-scale monitoring and management of regimes applying the new tools and technologies to ensure reliability of electrical power transmission and the traction network controllability.

In order to improve the reliability of TES functioning, it is necessary to use the strengthening points for the traction network with voltage controllers. Regulation of power supplied by the strengthening points would improve the stability of TES. Thus, there is a need to develop new approaches to ensure the stability of TES in terms of voltage.

2. Literature review and problem statement

The effective work of the traction power systems largely depends on its ability to ensure reliable and uninterrupted power supply to traction, stationary and third-party customers. The failures associated with the disruption of stability result in significant damages. It is therefore necessary to pay attention to improving the stability of TES both at designing and during their operation.

There has been a significant increase in the interest in DC systems in relation to the evolution of electricity supply systems towards the use of the Smart Grid-DC systems. Researchers have focused primarily on the possibility of creating distributed systems of direct current, designing the architecture of systems, the maintenance of required voltage, network protection, and stability provision. In [4], these studies were categorized and analyzed; the authors showed approaches to modeling flexible hybrid AC/DC systems.

Stability analysis of TES is run based on experimental data that simulate behavior of current and voltage in the contact system. Studies reveal that currents and voltages in the contact system are nonstationary in character. When modeling, these magnitudes are conveniently represented by time series. The most effective tool for a time series analysis is the recurrent analysis. The most fruitful approach to solving this problem has been shown by Weber and Marwan [5–7]. Paper [5] described methods and the most successful practice of theoretical and practical aspects of the recurrent analysis, which is applied in a time series analysis. The paper showed modern achievements of recurrent analysis in medicine, geophysics, astronomy, radio electronics, hydro- and aerodynamics, space engineering. However, the methods of this analysis have so far not been used to study processes in traction networks. In principle, the measured time series are generated by a basic dynamical system that defines some (or all) variables in the state of this system over time in accordance with a set of deterministic rules. These rules are typically represented by a set of differential equations, with (or without) the effect of noise. We shall assume that the right sides of autonomous differential equations represent the quadratic polynomials from several variables. The unknowns in the quadratic right sides in a system of differential equations could be derived by a special procedure, based on the least square method, described in papers [6, 7]. Articles [8, 9] addressed approaches to the description of dynamic systems based on a multivariate time series when exposed to noise.

When analyzing stability of the traction power systems, it is also required to determine the system's stability reserve (level of robustness). Usually, when examining this issue, one builds the regions of stability. A comprehensive study that describes the method of recurrent regions for different applications is [10]. Paper [11] reported a toolset to analyze the stability of dynamic systems based on the Lyapunov method. The theoretical provisions reported in a given paper require adaptation to traction networks, to account for the specificity of traction power supply.

Voltage stability assessment is a major issue in the analysis of stability of energy systems. Papers [12–14] proposed a variety of voltage stability indices (VSI). These indices could be used to optimize the decentralized power supply systems (DG), to identify weak areas and to take countermeasures to improve stability. Authors of [12] examined VSI on various aspects, such as the concept of critical values, assumptions, and so forth. Paper [13] describes methods for monitoring the stability of power system in terms of voltage under conditions of incomplete information based on artificial neural networks. It is worth noting that issues of stability within energy systems have been given much greater attention to than those in traction power systems. One of the methods to improve stability of power supply systems in terms of voltage is the application of voltage regulators (VR). In [14], it is proposed to employ a genetic algorithm to determine the number, location, and rated power of VR. Theoretical provisions from [12–14] do not take into consideration the need for the modernization of traction power supply systems in the nearest future.

At present, there are many publications that address promising traction systems. Thus, for example, paper [15] proposed a segment technology for the traction network power, according to which a section of the traction power is divided into several segments, and a synchronous measurement technology is used in order to rapidly and precisely determine malfunctions and their location.

Modernizing the structure and circuitry of traction power systems is needed to ensure the stability of promising modes of power consumption related to the expansion of a high-speed motion polygon and, as a consequence, an increase in load. The modernization should be carried out in strict accordance with the regulations and, in the case of European railroads, in accordance with the technical specifications for interoperability [16]. The intelligent systems of traction power supply of the new generation will render issues on ensuring the assigned level of reliability and stability under all modes of operation even more relevant.

Expansion of a high-speed motion polygon and, consequently, an increase in load, require modernization of the structure of electricity supply system in order to ensure stability for the promising modes of electricity consumption. Therefore, it is necessary to devise a generalized approach to assessing the stability of TES and to selecting regulators that would provide it.

3. The aim and objectives of the study

The aim of this work is to devise approaches to ensure stability of the system of traction DC supply using a regulator to stabilize voltage in a traction network.

To accomplish the aim, the following tasks have been set:

– to construct a system of autonomous nonlinear differential equations based on experimental data that simulate a change in the current and voltage in a contact network;

– to design regulators that stabilize voltage in the pantographs of electric rolling stock;

– to assess the regions of stability for voltage regulators in the system of traction direct current supply.

4. Modeling the dynamics of current and voltage in a contact electric network

Let

$$x_0 = x(t_0), x_1 = x(t_1), \ldots x_n = x(t_n), \qquad (1)$$

be a finite sequence of numerical values for a certain scalar dynamic variable, measured with a constant time step Δt at moments $t_i = t_0 + i\Delta t$; $x_i = x(t_i)$; $i = 0, 1, \ldots, n$. Sequence (1) is called a time series [5–7]. In principle, the measured time series are generated by the basic dynamic system that defines some (or all) variables of this system's state over time in accordance with a set of deterministic rules. These rules are typically represented by a set of differential equations, with (or without) the effect of noise. It is known that any such set of differential equations could be transformed by using a Cauchy method into a set of autonomous equations of first order. Dynamic variables in all obtained first-order equations form the phase space and the number of such variables determines the dimensionality of this phase space, which we shall denote through n. We shall assume that it is possible to measure voltage and direct current (two variables), as well as, if possible, other dynamic characteristics of TES. We assume that, among these characteristics, there could be time-dependent derivatives from voltage and current. If the derivatives cannot be measured, it is assumed that there are smooth enough approximations of these derivatives [6]. We shall assume that the right sides of autonomous differential equations represent the quadratic polynomials from several variables. In addition, we employ a special procedure for determining the unknowns in the quadratic right sides of the system of differential equations, which was proposed in [5, 6]. This procedure is based on the method of least squares and the fact that it is possible to calculate quite accurately components $x_1(t_i)$, $x_2(t_i)$,..., $x_n(t_i)$ of vector $x(t)$ from a time series [5]) and its derivative dx/dt.

The result of experimental study is the constructed temporal dependences for current and voltage in a contact network at the electrified section NDY-P. To this end, we performed 12,470 measurements at a 1-second interval. Charts for the derived dependences are shown in Fig. 1, 2.

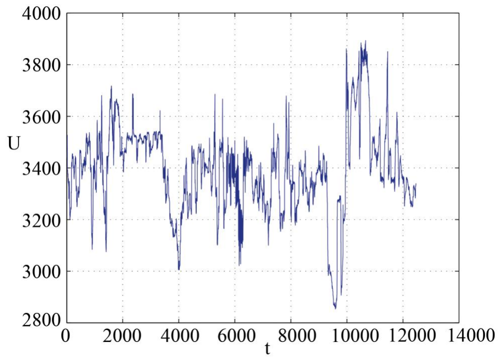

Fig. 1. Voltage at pantograph U(t)

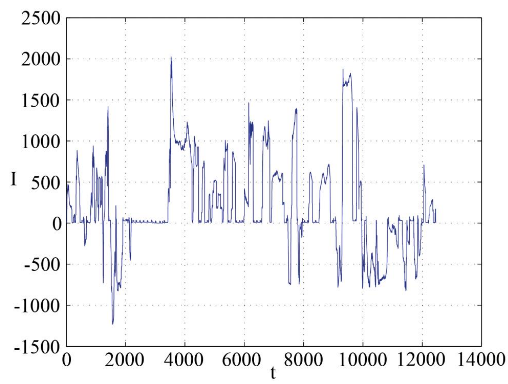

Fig. 2. Current consumption by electric rolling stock I(t)

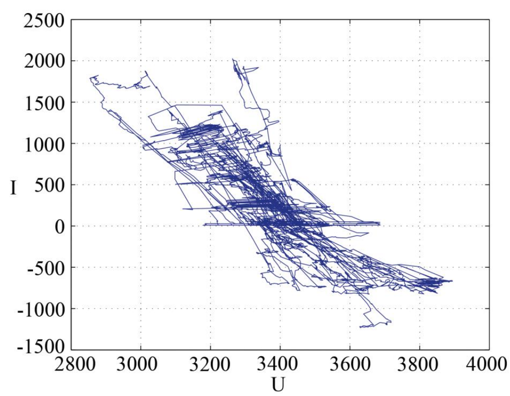

Fig. 3. Volt-ampere (*U–I*) characteristic of a contact network (experimental data)

Preliminary analysis of the acquired data reveals that they are nonstationary in character. Note that non-stationarity may manifest itself through the emergence of a deterministic or stochastic trend, changing over time with variance and covariance. There are two main objectives in a time series analysis: identification of the character of a time series and forecasting (prediction of the future values for a time series based on the current and past values). Both objectives require that a time series model should be identified and described more or less formally [5–8].

To successfully complete the modelling of processes shown in Fig. 1–3, it is required to apply the methods of a nonlinear recurrence analysis [5, 10]. The first step of modelling implies determining the dimensionality of the nesting phase space that hosts the process. Determining a given dimensionality is shown in Fig. 4.

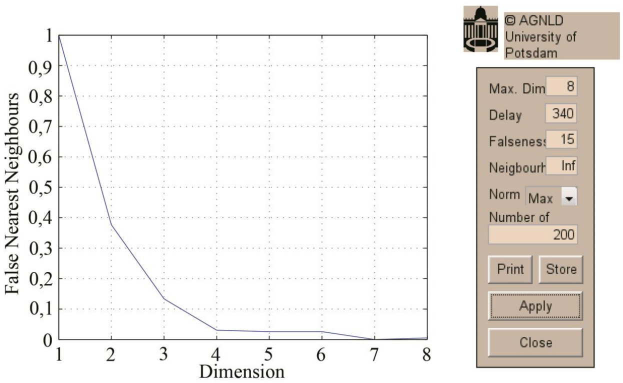

Fig. 4. Search for the dimensionality of nesting space using a method of false nearest neighbors [1–3]

Next, we shall use the method of least squares [8, 9]. First, based on data from Fig. 1, 2, we calculate dimensionality of the phase space that hosts the process [10]. This dimensionality is equal to n=4. Next, the application of the method of least squares leads to the following system of differential equations:

$$\begin{cases} \dot{x}(t)=y(t), \\ \dot{y}(t)=0.0193-0.0072x(t)+0.0218y(t)-\epsilon_1 z(t)+ \\ +0.0057u(t)-\epsilon_2 x(t)y(t)+0.000422x^2(t), \\ \dot{z}(t)=u(t), \\ \dot{u}(t)=0.0294-0.0145x(t)-0.8506y(t)-0.0019z(t)- \\ -0.0095u(t)+0.2380x(t)y(t)+0.0017x^2(t). \end{cases} \qquad (2)$$

Where $x(t)=U(t)$, $z(t)=I(t)$, and ϵ_1, ϵ_2 are control parameters; voltage $U(t)$ and current $I(t)$ is measured in kilovolts (kV) and kiloamperes (kA). The behavior of the solutions of system (2) is shown in Fig. 5–8.

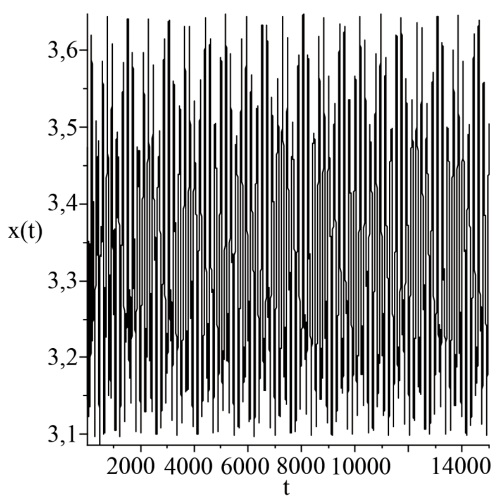

Fig. 5. Behavior of voltage $U(t)$ when ϵ_2=0.0039 and ϵ_1=0.0008 (quasi-periodic behavior)

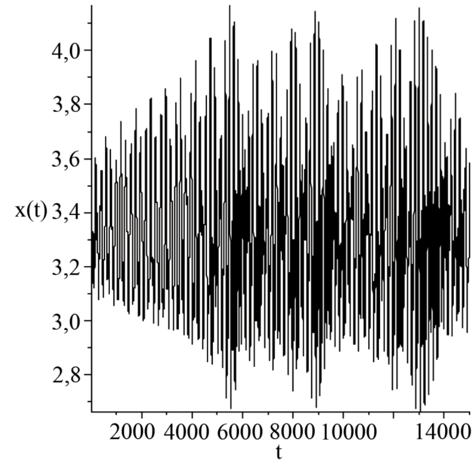

Fig. 6. Behavior of voltage $U(t)$ when ϵ_2=0.0039 and ϵ_1=0.00088 (chaotic behavior)

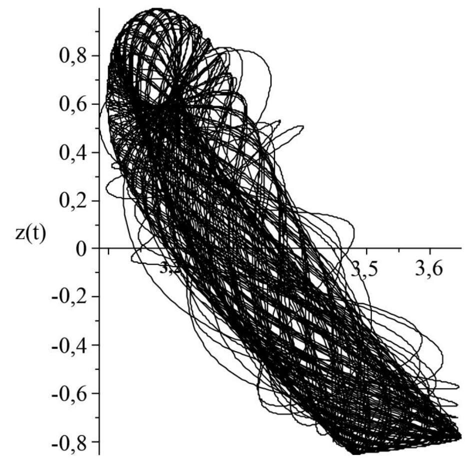

Fig. 7. Behavior of U–I characteristic when ϵ2=0.0039 and ϵ1=0.0008 (quasi-periodic behavior)

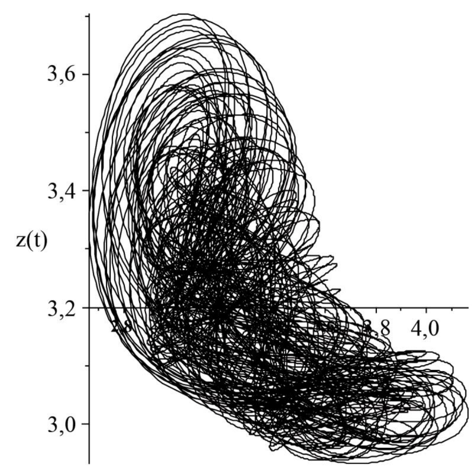

Fig. 8. Behavior of $U-I$ characteristic when ϵ_2=0.0039 and ϵ_1=0.00088 (chaotic behavior)

It is obvious that dynamics of system (2) could be changed using the controlling parameters ϵ1 and ϵ2. These changes could be demonstrated more effectively by using the Lyapunov exponents (Fig. 9, 10) [5–7, 10].

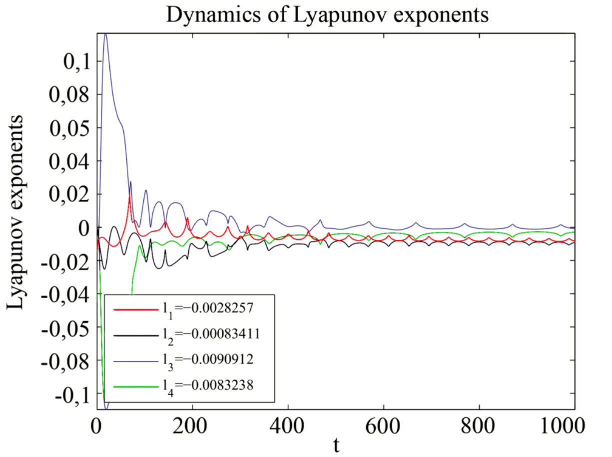

Fig. 9. Lyapunov exponents for system (4) when $\epsilon_1=0.00088$, $\epsilon_2=0.0039$

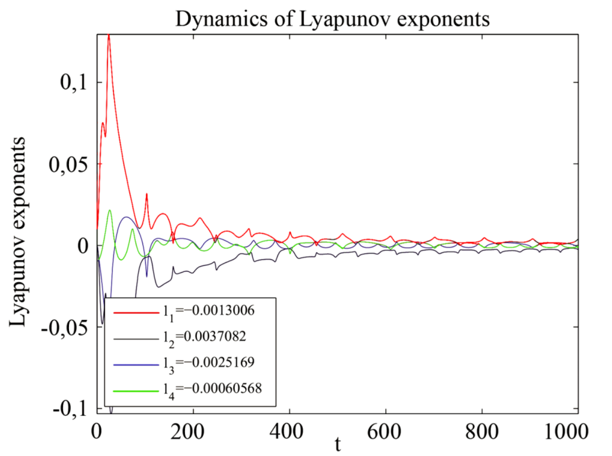

Fig. 10. Lyapunov exponents for system (4) when $\epsilon_1=0.00088$, $\epsilon_2=0.01$

In Fig. 5, *a*, one of the Lyapunov exponents is positive. This indicates the existence of chaotic processes in system (2). In Fig. 5, *b*, all four Lyapunov exponents are negative. It testifies to the stabilization of all processes in system (2).

4. 1. Voltage stabilization in system (2)

Consider the following generalization of system (2):

$$\begin{cases} \dot{x}(t)=y(t), \\ \dot{y}(t)=a_{10}+a_{11}x(t)+a_{12}y(t)+a_{13}z(t)+ \\ +a_{14}u(t)+b_{12}x(t)y(t)+b_{11}x^2(t), \\ \dot{z}(t)=u(t), \\ \dot{u}(t)=a_{20}+a_{21}x(t)+a_{22}y(t)+a_{23}z(t)+ \\ +a_{24}u(t)+b_{21}x(t)y(t)+b_{22}x^2(t), \end{cases} \tag{3}$$

where $a_{10},..., b_{11}, a_{20},..., b_{22}$ are valid parameters.

At certain values of parameters, this system describes the dynamics of changes in voltage and current in a contact network (refer to equation (2)). Note that voltage $U(t)=x(t)$ and current $I(t)=z(t)$ are measured using a mobile laboratory that moves at constant velocity v along a contact network. Note that in order to study processes in the contact network, it is more convenient to employ a dynamic model in which voltage $U(t)=x(t)$ and current $I(t)=z(t)$ are represented as functions $U(s)=x(s)$, $I(s)=z(s)$ of distance s from some starting point. Such a representation is very convenient when the stabilization of voltage in a contact network is implemented from some fixed points along the route of the train, for example, at traction substations or gain points. Furthermore, we shall assume that control over voltage is executed using the regulator $Uinput(s)=f(U(s), U'(s), I(s), I'(s))$, where $f(...)$ is the real function of its arguments. Thus, the transition from model (3), where $x(t)$ and $z(t)$ are represented as a function of time t, has been achieved through the replacement of independent variable t with independent variable s, according to formula $s=vt$. In this case, $y(t)\to vy(s)$, $u(t)\to vu(s)$, and system (3) moves to the following system:

$$\begin{cases} \dot{x}(s)=y(s), \\ \dot{y}(s)=(a_{10}+a_{11}x(s)+a_{12}vy(s)+a_{13}z(s)+ \\ +a_{14}vu(s)+b_{12}vx(s)y(s)+b_{11}x^2(s)+U_{input}(s))/v^2, \\ \dot{z}(s)=u(s), \\ \dot{u}(s)=(a_{20}+a_{21}x(s)+a_{22}vy(s)+a_{23}z(s)+ \\ +a_{24}vu(s)+b_{21}vx(s)y(s)+b_{22}x^2(s)+I_{input}(s))/v^2, \end{cases} \tag{4}$$

where velocity v=const is measured in m/s and $U(s)=x(s)$, $I(s)=z(s)$ are some functions of distance. (For simplicity, we kept the former designations for dependent variables x and z in the newly derived system. Variables $x(t)$ and $z(t)$ are replaced with variables $x(s)$ *and* $z(s)$).

We introduce a control law to system (4) according to formula:

$$\begin{aligned} &U_{input}(s)=(-0.00088+piecewise(s<10000,0,k_1))I(s)+ \\ &+(0.0057+piecewise(s<10000,0,k_2))v\dot{I}(s)+(-0.0039+ \\ &+piecewise(s<10000,0,k_3))vU(s)\dot{U}(s)\equiv \\ &\equiv(-0.00088+piecewise(s<10000,0,k_1))z(s)+ \\ &+(0.0057+piecewise(s<10000,0,k_2))vu(s)+ \\ &+piecewise(s<10000,0,k_2))vu(s)+(-0.0039+ \\ &+piecewise(s<10000,0,k_3))vx(s)y(s), \end{aligned} \tag{5}$$

where the value for valid function $f=piecewise(s<Dist; b; c)$ equals b if $s<a$ and c in the opposite case; $0<Dist<120{,}000$ meters.

The proposed control law indicates that control over voltage will be implemented using the feedback that is introduced to the system at a distance *Dist* meters from the starting point. Note that in Fig. 6 we measure current from some zero. This is a common practice of measurements when a contact network employs DC sources. Here, this zero level corresponds to a value of 1.5 kA.

It should also be specified that in accordance with acting regulations the minimum voltage level at the pantograph of TES should be 2.9 kV. However, paper [17] showed that for high-speed motion the voltage at a pantograph should be at a level of 3.5–3.6 kV over a range of change ±360 V. Similar results were obtained in paper [18]; it was proposed to raise the level of useful average voltage to 2.8 kV. In a given paper, the task on stabilizing was solved under the assigned constraints.

Consider the impact on the dynamics of system (4) exerted by control law (5) for velocity v=15 m/s, *Dist*=1000 (or 6000) meters, and different values for coefficients k_1, k_2, k_3 (Fig. 11–20).

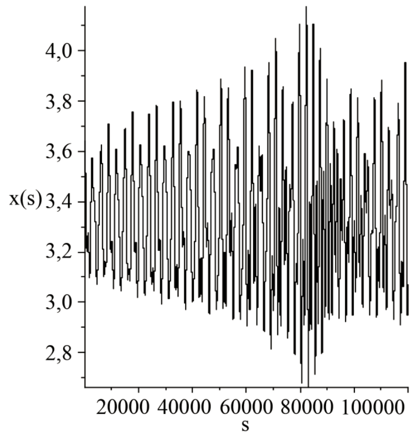

Fig. 11. Behavior of a distance function $U(s)$ for system (4) when $k_1=0.00018$, $k_2=-0.0003$, $k_3=0$, $\epsilon_1=0.00088$, $\epsilon_2=0.0039$, $Dist=10{,}000$ meters (quasi-periodic behavior). Linear control law

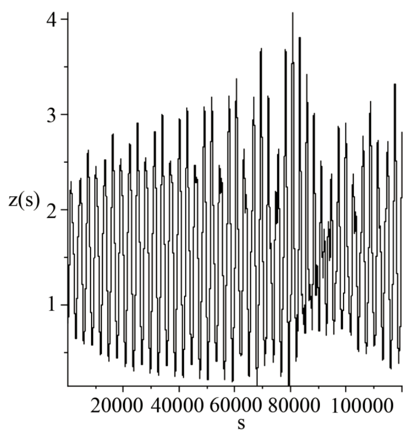

Fig. 12. Behavior of a distance function $I(s)$ for system (4) when $k_1=0.00018$, $k_2=-0.0003$, $k_3=0$, $\epsilon_1=0.00088$, $\epsilon_2=0.0039$, $Dist=10{,}000$ meters (quasi-periodic behavior). Linear control law

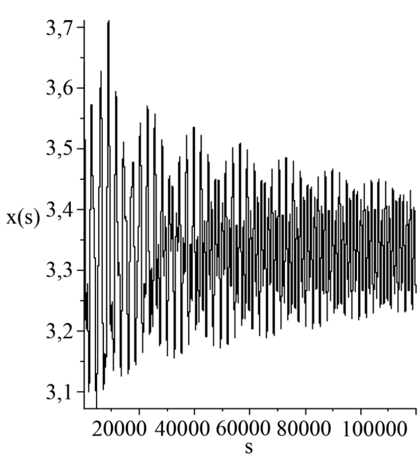

Fig. 13. Behavior of a distance function $U(s)$ for system (4) when $k_1=0.00018$, $k_2=k_3=0$, $\epsilon_1=0.00088$; $\epsilon_2=0.0039$, $Dist=10{,}000$ meters (quasi-periodic behavior). Linear control law

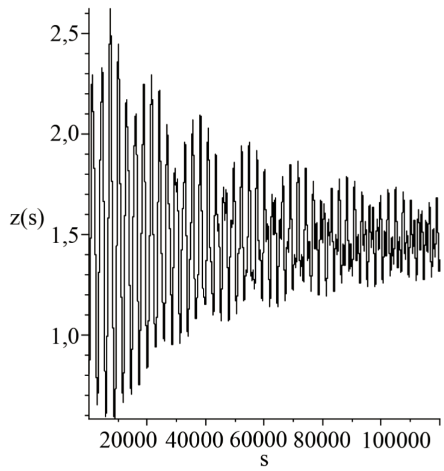

Fig. 14. Behavior of a distance function $I(s)$ for system (4) when $k_1=0.00018$, $k_2=k_3=0$, $\epsilon_1=0.00088$; $\epsilon_2=0.0039$, $Dist=10{,}000$ meters (quasi-periodic behavior). Linear control law

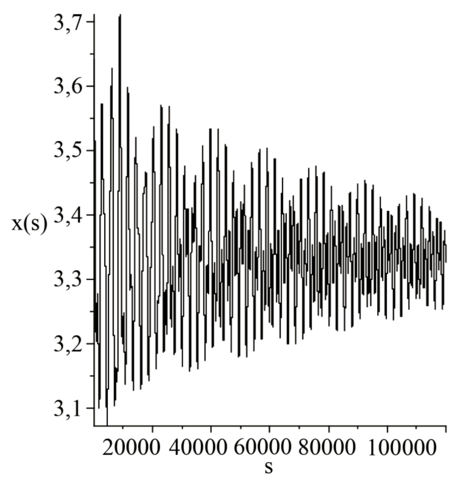

Fig. 15. Behavior of a distance function $U(s)$ for system (4) when $k_1=0.00018$, $k_2=-0.0003$, $k_3=0$, $\epsilon_1=0.00088$, $\epsilon_2=0.0039$, $Dist=10{,}000$ meters (quasi-periodic behavior). Linear control law

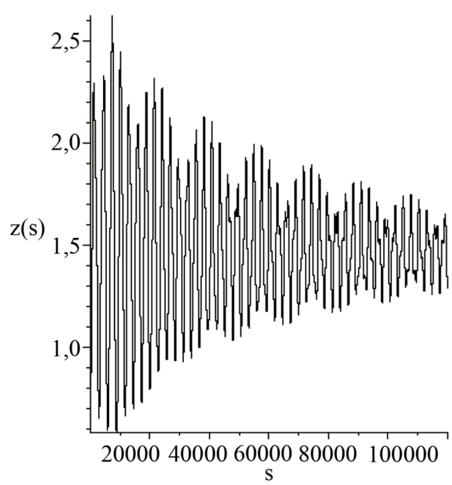

Fig. 16. Behavior of a distance function $I(s)$ for system (4) when $k_1=0.00018$, $k_2=-0.0003$, $k_3=0$, $\epsilon_1=0.00088$, $\epsilon_2=0.0039$, $Dist=10{,}000$ meters (quasi-periodic behavior). Linear control law

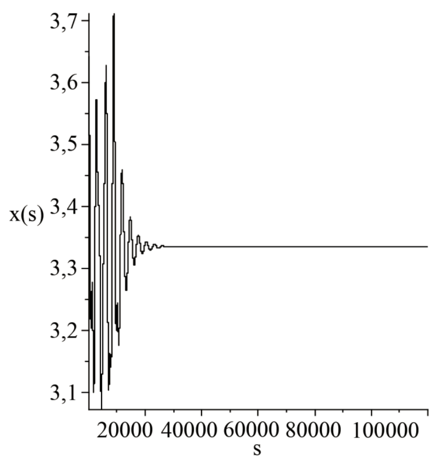

Fig. 17. Behavior of a distance function $U(s)$ for system (4) when k_1=0.00018, k_2=0, k_3=–0.0015, ϵ_1=0.00088, ϵ_2=0.0039, *Dist*=10,000 meters (voltage and current are stabilized). Non-linear control law

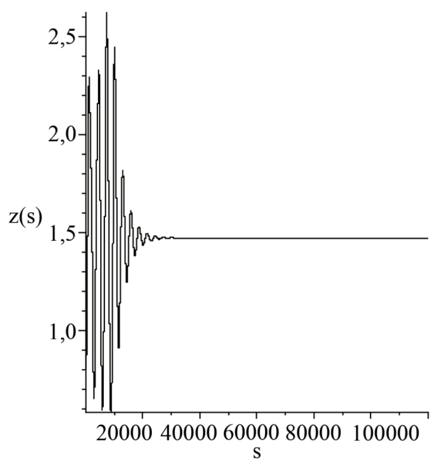

Fig. 18. Behavior of a distance function $I(s)$ for system (4) when k_1=0.00018, k_2=0, k_3=–0.0015, ϵ_1=0.00088, ϵ_2=0.0039, *Dist*=10,000 meters (voltage and current are stabilized). Non-linear control law

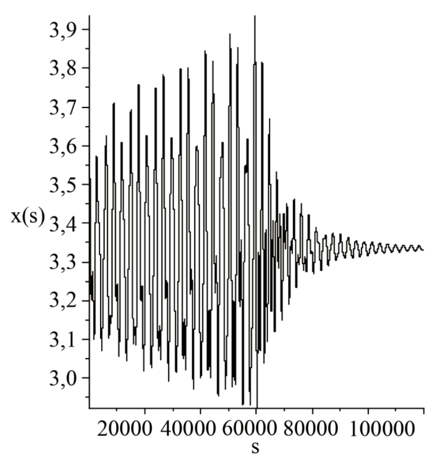

Fig. 19. Behavior of a distance function $U(s)$ for system (4) when k_1=0.00018, k_2=–0.0003, k_3=–0.0015, ϵ_1=0.00088, ϵ_2=0.0039, *Dist*=60,000 meters (voltage and current are stabilized). Non-linear control law

An analysis of Fig. 13–20 reveals that the linear regulator that uses only the proportional gain $U_{input}(s)=k_1 I(s)$ (Fig. 13, 14) or the aperiodic link $U_{input}(s)=k_1 I(s)+k_2 vI'(s)$ (Fig. 15, 16) is less effective than the non-linear regulators (Fig. 17–20).

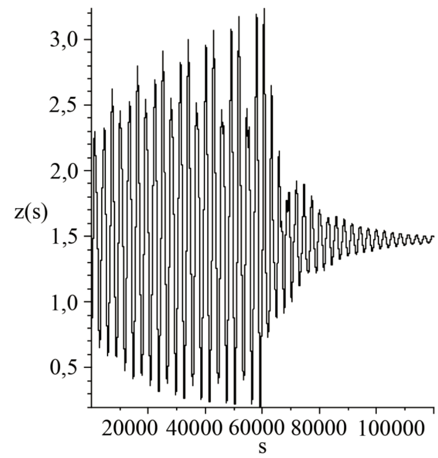

Fig. 20. Behavior of a distance function $I(s)$ for system (4) when k_1=0.00018, k_2=–0.0003, k_3=–0.0015, ϵ_1=0.00088, ϵ_2=0.0039, *Dist*=60,000 meters (voltage and current are stabilized). Non-linear control law

4. 2. Stability regions

An important issue that has not yet been considered in the present work is the problem of robustness (insensitivity to small perturbations) for the built regulator. A problem of robustness is rather difficult. Therefore, we shall consider only one aspect of this problem.

Hereinafter the robustness of system (4) shall be understood as the stability resource of this system with the constructed regulator (the system is closed via feedback). This stability resource could be determined based on the construction of stability regions.

As shown in Fig. 11–20, the problem of voltage stabilization in a contact network does not depend on the place of introducing generation capacities. Therefore, we shall use system (2) to search for the regions of stability [11, 19].

Let ϵ_1=0.00088, ϵ_2=0.0039 and v=1. (In this case, system (4) becomes system (2)). We introduce to system (4) the control law $U_{input}=k_1U(t)$ _$U(t)=k_1x(t)y(t)$ and $I_{\text{input}}=$ $=k_2U(t)$_$U(t)=k_2x(t)y(t)$, where k_1, k_2 are valid parameters. We obtain then the following closed system:

$$\begin{cases} \dot{x}(t)=y(t), \\ \dot{y}(t)=0.0193-0.0072x(t)+0.0218y(t)-0.00088z(t)+ \\ +0.0057u(t)-(0.0039+k_1)x(t)y(t)+0.000422x^2(t), \\ \dot{z}(t)=u(t), \\ \dot{u}(t)=0.0294-0.0145x(t)-0.8506y(t)-0.0019z(t)- \\ -0.0095u(t)+(0.2380+k_2)x(t)y(t)+0.0017x^2(t). \end{cases} \quad (6)$$

The equilibrium point of system (6) at $k_1=k_2=0$ is the point with coordinates x^*=3.336576430, y^*=0, z^*= =– 0.02878756292, u^*=0. We shall search for the region of stability in system (4) in the neighborhood of the equilibrium position x^*, y^*, z^*, u^*). Calculate Jacobian $A=(a_{ij})$ [11] of system (4) at point (x^*, y^*, z^*, u^*). Here, the Jacobian matrix elements acquire the following values:

$$a_{11}=a_{13}=a_{14}=a_{31}=a_{32}=a_{33}=0,$$

$a_{12}=a_{34}=1$,

$a_{21}=-0.004383929493$,

$a_{23}=-0.00088$,

$a_{22}=0.00878735192+3.336576430k_1$,

$a_{41}=-0.00315564014$,

$a_{42}=-0.0564948097+3.336576430k_2$,

$a_{43}=-0.0019$,

$a_{44}=-0.0095$.

Let E be the identity matrix of order 4. The characteristic polynomial of matrix A will take the form

$$\det(\lambda E_4 - A) = \lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 =$$
$$= \lambda^4 + (0.00071264808 - 3.336576430k_1)\lambda^3 +$$
$$+(0.006522470065 - 0.03169747608k_1 -$$
$$-0.002936187258k_2)\lambda^2 + (-0.00000677692221 -$$
$$-0.006339495217k_1 + 0.002936187258k_2)\lambda +$$
$$+0.000005552502714.$$

Hurwitz polynomials [15] for matrix A are as follows:

$$\Delta_1(k_1,k_2) = c_1 = 0.00071264808 - 3.336576430k_1,$$

$$\Delta_2(k_1,k_2) = c_1c_2 - c_3 = (0.00071264808 - 3.336576430k_1)\times$$
$$\times(0.006522470065 - 0.03169747608k_1 -$$
$$-0.01901848565k_2) - (-0.00000677692221 -$$
$$-0.006339495217k_1 + 0.002936187258k_2),$$

$$\Delta_3(k_1,k_2) = c_1c_2c_3 - c_3^2 - c_1^2c_4 = (0.00071264808 -$$
$$-3.336576430k_1)(0.006522470065 - 0.03169747608k_1 -$$
$$-0.01901848565k_2)(-0.00000677692221 -$$
$$-0.006339495217k_1 + 0.002936187258k_2) -$$
$$-(-0.00000677692221 - 0.006339495217k_1 +$$
$$+0.002936187258k_2)^2 - (0.00071264808 -$$
$$-3.336576430k_1)^2 0.000005552502714,$$

$$\Delta_4(k_1,k_2) = c_4 = 0.000005552502714.$$

Stability region is defined by conditions

$\Delta_1(k_1, k_2)>0$,

$\Delta_2(k_1, k_2)>0$,

$\Delta_3(k_1, k_2)>0$,

$\Delta_4(k_1, k_2)>0$.

Now, we introduce to system (4) the control law

$U_{input}=k_1I(t)=k_1z(t)$

and

$I_{input}=k_2I(t)=k_2z(t)$,

where k_1, k_2 are the valid parameters. In this case, we also build the Hurwitz polynomials

$\Delta_1(k_1, k_2)>0$,

$\Delta_2(k_1, k_2)>0$,

$\Delta_3(k_1, k_2)>0$,

$\Delta_4(k_1, k_2)>0$.

In the plane of parameters k_1, k_2, stability region for a nonlinear law

$U_{input}=k_1U(t)\ U'(t)=k_1x(t)y(t)$

and

$I_{input}=k_2U(t)\ U'(t)=k_2x(t)y(t)$

is shown in Fig. 21.

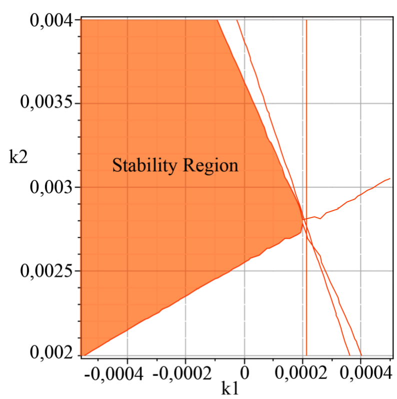

Fig. 21. Stability region for system (6) for a nonlinear control law Uinput=k1U(t) U'(t)=k1x(t)y(t) and Iinput=k2U(t) U'(t)=k2x(t)y(t))

In the plane of parameters k_1, k_2, stability region for control law $U_{input}=k_1I(t)=k_1z(t)$ and $I_{input}=k_2I(t)=k_2z(t)$ is shown in Fig. 22.

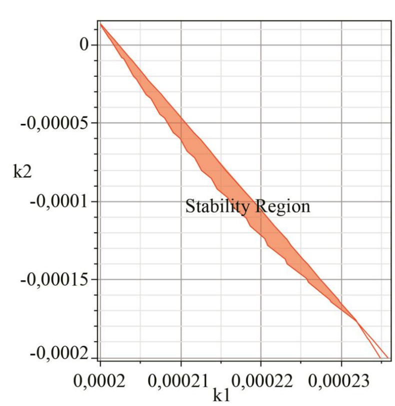

Fig. 22. Stability region for system (6) for linear control law $U_{input}=k_1I(t)=k_1z(t)$ and $I_{input}=k_2I(t)=k_2z(t)$

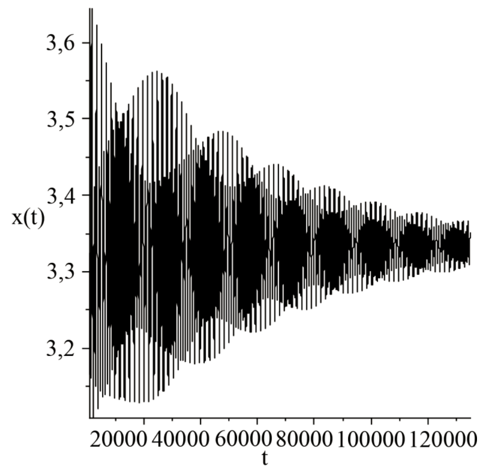

Fig. 23. Behavior of function $x(t)$ (voltage in a contact network) for system (6) at a linear control law $U_{input}=k_1z(t)$, $I_{input}=k_2z(t)$

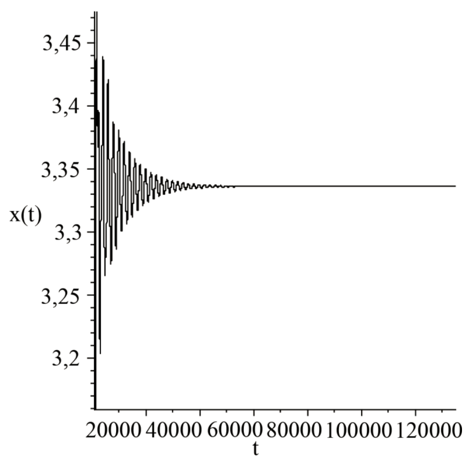

Fig. 24. Behavior of function $x(t)$ (voltage in a contact network) for system (6) at a non-linear control law $U_{input}=k_1x(t)y(t)$, $I_{input}=k_2x(t)y(t)$

Thus, as it follows from Fig. 21–23, the non-linear regulator possesses better robust properties than the linear regulator. In addition, the region of stability for the linear regulator is rather small. This fact does not make it possible to apply such regulators to solve the task of voltage stabilization in a contact network. One should also note that the stability regions were built for the linearized system (6). Hence, it follows that stability regions for the non-linear system (6) could happen to be wider than the regions shown in Fig. 21, 22.

5. Discussion of results of studying the approaches to ensuring the stability of the traction direct current system

The proposed dynamic model of the power consumption processes in a traction network and the structure of a non-linear voltage regulator could be used when building an intelligent, adaptive system of traction electric supply for high-speed motion. At the same time, one should indicate the necessity to conduct further research aimed to adapt the proposed procedure for the circuits of traction network of the distributed type, especially at the unsymmetrical location of generating points. That will necessitate the improvement of the algorithm for the functioning of a non-linear voltage regulator for high speed motion.

The received stability regions are quite narrow, because in a first approximation we investigated the linearized model. When building stability regions, we applied simpler control laws than the law (5). Formulae for control laws are indicated below the above figures. Stability regions were constructed only for the linearized control systems. This is explained by that the Hurwitz criterion applies only to linear systems. Using a given criterion for the linearized nonlinear systems either produces a much narrower stability region or yields nothing at all for determining the stability regions of the original nonlinear system. Application of this criterion could be seen as the first step in building the regions of stability for nonlinear systems.

More precise results could be achieved in the case of employing a non-linear model, when using Lyapunov functions and a synergistic regulator. We must specify that the model, devised for such an application, quite accurately represents energy-exchange processes in a traction network. The methodology of our study implied that the considered non-linear regulator is very sensitive to external perturbations caused by the hyperchaotic character of change in the examined parameters.

The control law was assigned by formula (5) using a discontinuous function. Control is not enabled until the value s= =10,000 m (s=60,000 m), it is enabled after covering this distance. A simpler notation is $U(in)=k1\cdot I+k2\cdot I'+k3\cdot U\cdot U'$ Here, $k1$ is the gain factor for current; $k2$ is the gain factor for the derivative from current; $k3$ is the gain factor for the function that is the product of voltage and the derivative from voltage. This nonlinearity is introduced in such a fashion that it has made it possible to stabilize voltage as quickly as possible, in contrast to other models examined in the synthesis of non-linear models. If k3=0, the control law is linear.

Conjugation of discontinuous models of TES between points of generation could lead to the need to change gain coefficients and to a decrease in the robustness of regulators. That is why, given such a complex system, it is not expedient to use regulators with the predefined parameters, but rather the synergistic regulators, adjusted to a change in parameters.

6. Conclusions

1. Based on experimental data, we have proposed a mathematical model of change in the current and voltage in a traction network, which makes it possible to estimate the quality of power consumption processes, as well as the regions of stable operation of a traction power system. The structure of the model was developed using a nonlinear recurrent analysis, which allowed us to reveal the hidden components of the process. For example, in addition to the measured current and voltage, we calculated their derivatives.

2. A comparative analysis of the structures of regulators with a linear and nonlinear control law was performed. The result of our study shows that the stability region for a nonlinear control law, due to the hyperchaotic properties of the system, is very narrow, of the order of Δk=0.00005, and for the linear Δk=0.000004, that is still an order of magnitude less. This phenomenon could be explained by the high dynamic sensitivity of the system to changes in external factors.

3. It is shown that the regulator with a nonlinear control law possesses a better robustness, since voltage in a contact network stabilizes even at t=4, and at linear control it enters the stabilization mode after t=12. In this case, the voltage stabilization zone in a traction network when using a non-linear regulator is 3–4 times wider in comparison with the linear regulator.

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