Розроблені математична модель та алгоритм просторового розрахунку комбінованих сталезалізобетонних шпренгельних систем дозволяють визначати параметри напружено-деформованого стану в елементах конструкції. Доведено, що отримана математична модель задовільняє трьом групам умов: рівноваги; сумісності деформацій, які пов'язують деформації і переміщення; фізичним умовам, які пов'язують зусилля і деформації. Завдяки розробленій методиці проведені теоретичні розрахунки зі встановлення реальних діаграм «згинальний момент–прогин», «поздовжня сила–прогин» в залежності від величини і місця прикладання зовнішнього навантаження. В отримані рівності, які визначають коефіцієнти при невідомих системи лінійних алгебраїчних рівнянь, входять параметри топології та жорсткістні характеристики елементів конструкції. Це дало можливість в рамках розробленого алгоритму вести пошук мінімуму цільової функції рівнонапруженого стану в елементах просторової конструкції.

Теоретично досліджено міцність та деформативність шпренгельних сталезалізобетонних конструкцій на симетричні та несиметричні навантаження з врахуванням постадійної роботи системи. Шляхом ітераційного пошуку мінімуму цільової функції рівнонапруженого стану в перерізах елементів просторової конструкції встановлено зниження нормальних напружень у порівнянні з конструкціями іншого типу. Отримані результати дозволили запроектувати реальні будівельні конструкції. Таким чином, є підстави стверджувати про практичне значення розробленої математичної моделі та алгоритму, які використані при проектуванні нових і перерахунку існуючих комбінованих сталезалізобетонних шпренгельних систем

Ключові слова: комбіновані системи, рівняння нерозривності деформацій, міцність, деформативність, сталезалізобетон, шпренгель

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THEORETICAL RESEARCH INTO SPATIAL WORK OF A STEEL-REINFORCED-CONCRETE STATICALLY INDETERMINATE COMBINED STRUCTURE

I. Ivanyk
PhD, Associate Professor*
E-mail: k-spb@ukr.net

S. Vikhot
PhD*
E-mail: vixot@ukr.net

Y. Vybranets
PhD, Assistant*
E-mail:yura.vybranets@qmail.com

Y. Ivanyk
PhD, Assistant*
E-mail: ivanuk0404@ukr.net

*Department of Building Production***
Department of Building Constructions and Bridges*
***Lviv Polytechnic National University
S. Bandery str., 12, Lviv, Ukraine, 79013

1. Introduction

Contemporary construction industry no longer has the necessity to manufacture modular structures of the same type. More widely applied are the light structures for overlapping or covering different spans, whose wide range includes the non-uniform ones. In the design of large-span load-bearing overlapping structures the use of beams is ineffective. The application of frames or other structures has some major disadvantages: labor-intensive production, total height, and so on. Along with the application of already tested iron-concrete structures, engineers create and implement into construction the new structural shapes and structures that are characterized by low material consumption. These include the steel-reinforced concrete structures that combine the best properties of the steel and reinforced-concrete structures. The structures of this type, given their spatial work, possess a significantly lower weight and differ from the steel ones by a lower consumption of metal.

In recent years, the calculation of rod systems has increasingly employed a deformation model (fibresectionmodel). It is universal, it accounts for the physical nonlinearity and can be used in order to estimate the arbitrary cross sections of rod elements made from different materials (steel, concrete, etc.) for any combination of loads and could be implemented at a personal computer.

The relevance of our research is emphasized by the need to devise a procedure and to develop an algorithm for calculating the spatial work of steel-reinforced concrete structures.

2. Literature review and problem statement

An analysis of the development of structural shape of load-bearing building structures [1] reveals that one of the promising directions for further evolution is the application of spatial combined steel-reinforced-concrete truss systems [2]. Underlying these structures are the rigid elements (rigidity beams), strengthened by truss metallic bongs.

Combined steel-reinforced-concrete truss structures represent complex spatial elements. Calculations are typically performed by separating spatial structures into a series of flat systems.

The work of the structures' elements under conditions of the non-uniaxial stressed state defines the deformative properties of the load bearing capacity of the elements as well. Requirements for the consideration of physical nonlinearity of metallic and steel-reinforced-concrete elements are stated in the standards and normative documents [3 5], instructions, and recommendations. However, taking these requirements into consideration in the practice of design is difficult. Insufficient development of relevant methods and calculation programs may result in the use of conventional methods of estimation to produce the incorrect assessment of the work of a structure in general, as well as to making irrational design decisions. Therefore, the spatial model is one of the promising directions for improving the theory of calculation of the combined steel-reinforced-concrete structures.

In order to assess the stressed-strained state of structural systems of the truss type, additional theoretical [4] and experimental research is required into the effect of both static and dynamic loads [5]. At the same time, there is a need to improve the estimation models that assess the reliability [6] and optimization of parameters of the truss systems. Determining the areas of rational application necessitates the combination of design, fabrication, installation, and operation of such structural elements [7]. The emergence of new calculation methods has led to the development of structural solutions for combined structures due to the creation of new, more advanced, geometrical shapes, the application of improved profiles and materials, the design of new techniques for the artificial regulation of efforts.

Advances in computer engineering at the end of last century contributed to the emergence of scientific schools that implemented the automated methods for calculating building structures [8, 9], etc.

However, a significant drawback of the above methods for the calculation of steel-reinforced-concrete truss systems is the impossibility to select the optimum cross-sections of structures in the process of estimation both at the elastic and elastic-plastic stages (during formation of cracks in a concrete slab).

The devised procedure for the calculation of spatial cross-ribbed reinforced concrete systems taking into consideration their physical nonlinearity [10] makes it possible to construct a mathematical model at the design stage and to perform further numerical experiments for the reinforced concrete structures whose axes are in the *XOY* plane.

Employment of the basic theoretical provisions (a combination in the mathematical model of the equations of deformation continuity and the equations of statics) enabled their application when constructing a mathematical model of the spatial steel-reinforced-concrete truss structures whose elements operate in the spatial *XOYZ* coordinate system [11].

3. The aim and objectives of the study

The aim of this work is to build a mathematical model and to develop an algorithm for the spatial calculation of combined steel-reinforced-concrete truss systems. This would make it possible to study the character of the elements' operation and to find the reserve of their carrying capacity.

To achieve the set aim, the following tasks have been solved:

– to conduct, based on the devised procedures for calculating the cross-ribbed reinforced-concrete [10] and combined metallic truss systems [11], a numerical study into determining the efforts and vertical displacements in spatial combined steel-reinforced-concrete truss systems;

– to examine theoretically the strength and deformability of the combined steel-reinforced-concrete truss structures in terms of symmetrical and asymmetrical loads;

– to propose and design the steel-reinforced-concrete truss systems applying the iterative search for a minimum of the objective function of the equally-stressed state in the elements of a spatial structure;

– to establish, based on an analysis of theoretical research, the reserve of the carrying capacity of structures and to devise recommendations for the calculation, design, and application of combined steel-reinforced-concrete truss structures.

4. Materials and methods to study the spatial steel-reinforced-concrete combined systems

In order to calculate the complex spatial statically-indeterminate combined truss systems (Fig. 1), we shall use the mixed method of construction mechanics.

a

b

Fig. 1. Combined steel-reinforced-concrete truss structure (intermediate slab at the industrial complex "Business Center Pidzamche", B. Khmelnitsky Street, 176, Lviv, Ukraine): *a* — at the stage of fabrication; *b* — physical appearance of the combined steel-reinforced-concrete truss structure

Consider the spatial statically indeterminate combined steel-reinforced-concrete truss structure (SRTS), shown in Fig. 2, loaded with evenly distributed load q.

Such a structure is considered to be optimal on condition that its constituent elements possess elastic and rigid characteristics that are sufficient to ensure the carrying capacity both of the individual elements and the structure in general. We accept that the system is a linearly-deformed one, which is why, in order to determine efforts and displacements within it, it is advisable to apply the principle of force independence.

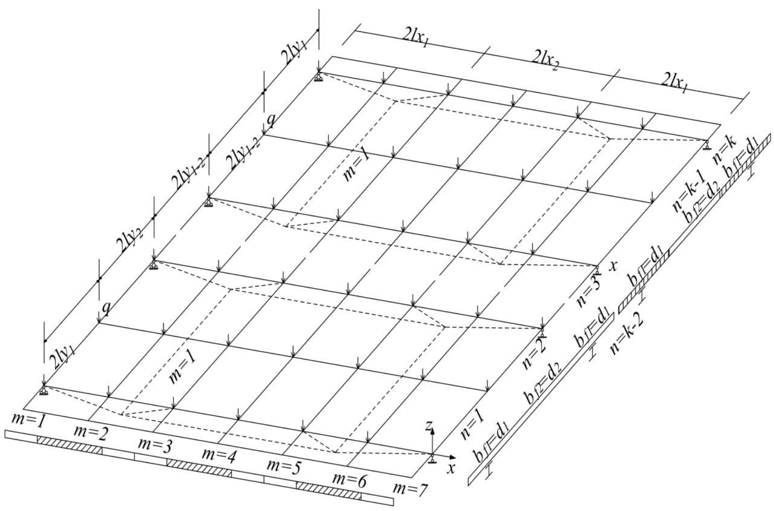

Fig. 2. Discrete physical model of the combined steel-reinforced-concrete truss structure

The steel-reinforced-concrete structure is composed of longitudinal metallic statically-indeterminate combined structures that are combined into a joint spatial work by using a monolithic reinforced concrete slab the size of $L\text{x}b_f$. The combined slab-beam system is replaced with a simplified discrete physical model in the form of a statically-indeterminate system [10]. Elements of a given structure represent the geometrical axes of steel-reinforced concrete rigidity beams and metallic trusses, whose rigidity in a static scheme meets their actual rigidity (Fig. 2).

We shall conditionally split SRTS into $n=k$ main longitudinal beams and $m=$ transverse beams. It should be noted that the main longitudinal beams $n=1, 3, ..., k-2, k$ are the combined steel-reinforced-concrete structures. Elements of the system are composed of a steel rolled rigidity beam, steel elements of the truss suspension, as well as a reinforced concrete slab with a width of b_{f1}. Parameters for the monolithic reinforced concrete slab are derived in accordance with requirements from [3–5].

Main longitudinal beams $n=2, 4,..., k-3, k-1$ are the elements of the reinforced concrete slab with a width of $b_{fi}=l_{xki}-b_{fi}$ (Fig. 2).

To solve the system, we shall use a procedure for the introduction of imaginary hinges for characteristic cross sections: at the intersections of rods from the cross-ribbed system [11]. The estimation scheme of such a structure will take the form shown in Fig. 3–5.

The estimation scheme of the structure, when introducing the imaginary hinges to the upper belt, is modeled as a frame, in which in the upper belt the main efforts are the unknown bending moments, transverse and longitudinal forces; in the bottom part of the frame, there are longitudinal efforts only (Fig. 3).

The result of SRTS deformations under load is a change in the position of elastic axes of rod elements. The proposed procedure is based on replacing the actual elastic bent axis of the beam with a fake one by introducing the imaginary hinges and by simultaneously applying bending moments at the characteristic points. In the actual beam, these points are matched by the places of application of external forces or the cross sections of a change in the rigidity characteristics. It is assumed that the simultaneous introduction of imaginary hinges and the corresponding bending moments does not alter the position of the actual elastic axis. Thus, the elastic bent axis is described as the axis with $n=0,..., i$ number of intermediate hinges, which, due to the deformation, were displaced along the vertical direction, respectively, by the magnitude of y_{np} ($n=0,...,$).

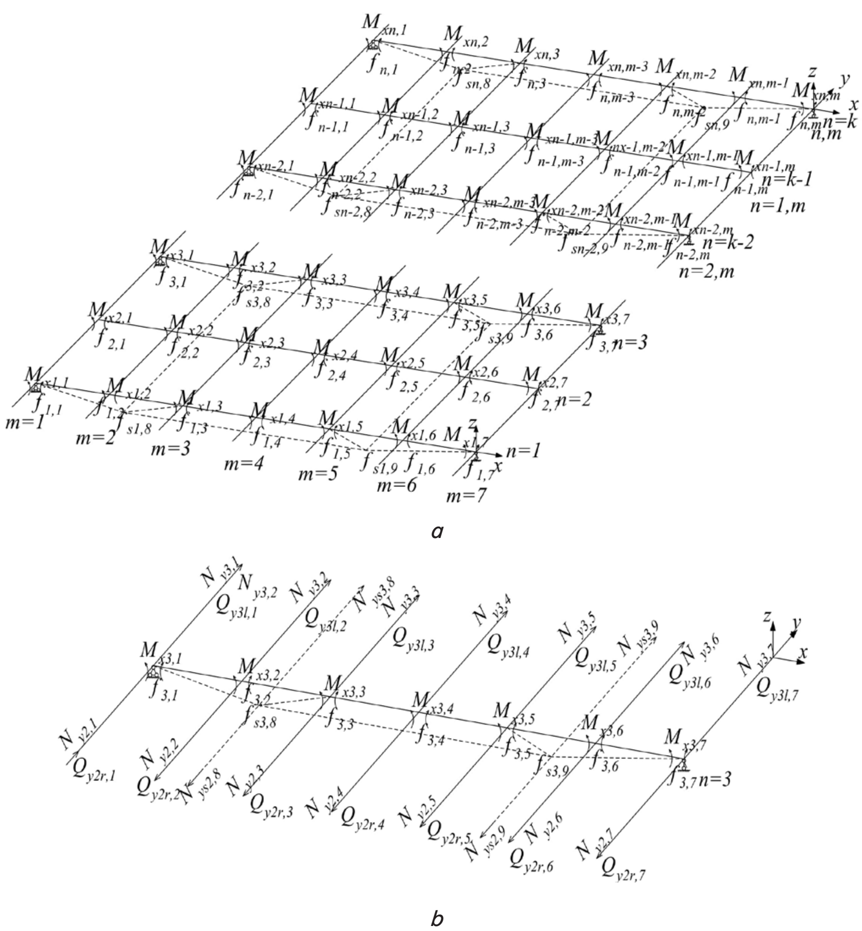

Fig. 3. Estimation scheme of SRTS along the Ox direction: *a* — general estimation scheme; *b* — selected fragment of the scheme

In order to derive a system of linear algebraic equations of static equilibrium, we shall employ methods for separating or cutting the nodes for the frame, as well as the equations of deformation continuity, which are recorded for each characteristic node.

Thus, we obtain a spatial cross-ribbed structure, which is composed:

a) along the longitudinal direction *OX*, of the steel-reinforced-concrete and reinforced-concrete elements;

b) along the transverse direction *OУ,* of *m*= reinforced-concrete elements.

The load on the slab is considered to be a nodal one, where $P_i=q^*l_{xki}^*l_{ymj}$. The estimation scheme of SRTS is formed by introducing imaginary hinges along the length of the longitudinal and transverse beams. The places of installation of the imaginary hinges give rise to bending moments M_x and M_y, as well as the elastic reactions R_{nx}, R_{ny}. The load P_i changes the position of neutral axes $f_i=f(x)$, $f_j=(y)$. The magnitude of deflection of nodes along the length of the beams is f_x aбo f_y, respectively. Note that at the intersection of beams at nodes the deflections are the same $f_{xi}=f_{yi}$.

In contrast to the cross-ribbed structures that are in the horizontal *XOY* plane, SRTS whose elements are in different planes is exposed in the cross sections to longitudinal efforts N_i.

The sum of all longitudinal efforts in the steel-reinforced-concrete part ΣN_i equals the sum of longitudinal efforts that occur in the elements of the suspension.

To solve the system, it is necessary to construct a mathematical model. We shall conditionally divide the cross-ribbed system into separate nodes and panels (sections) between them (Fig. 5).

Mathematical model of the structure takes the form:

– of the equations of static that reflect the balance of the individually selected node;

– of the equations of deformation continuity, which reflect the equality of the bending nodal moments along the corresponding direction and vertical displacements.

The canonical equation used is the equation of *i*-th efforts, which, for the elastic-sagging support $s_{e,f}$ in the direction of the *OX* axis, taking into consideration the joint action of bending moments, as well as longitudinal and transverse forces, takes the following form:

$$\delta_{xe-2,f} X_{xe-2,f}+\delta_{xe-1,f} X_{xe-1,f}+\delta_{xe,f} X_{xe,f}+\delta_{xe+1,f} X_{xe+1,f}+ +\delta_{xe+2,f} X_{xe+2,f}+...+=\Delta_{e,f}\,P=0. \quad (1)$$

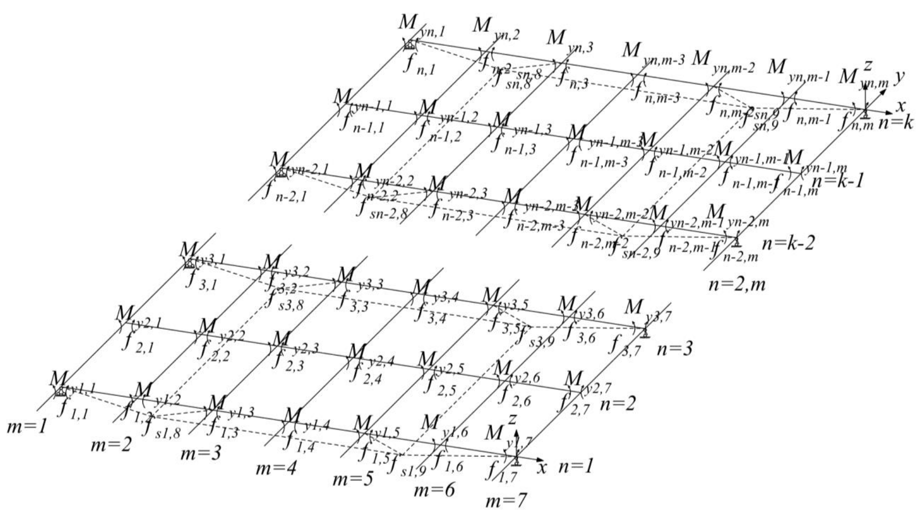

Fig. 4. Estimation scheme of SRTS along the Oy direction

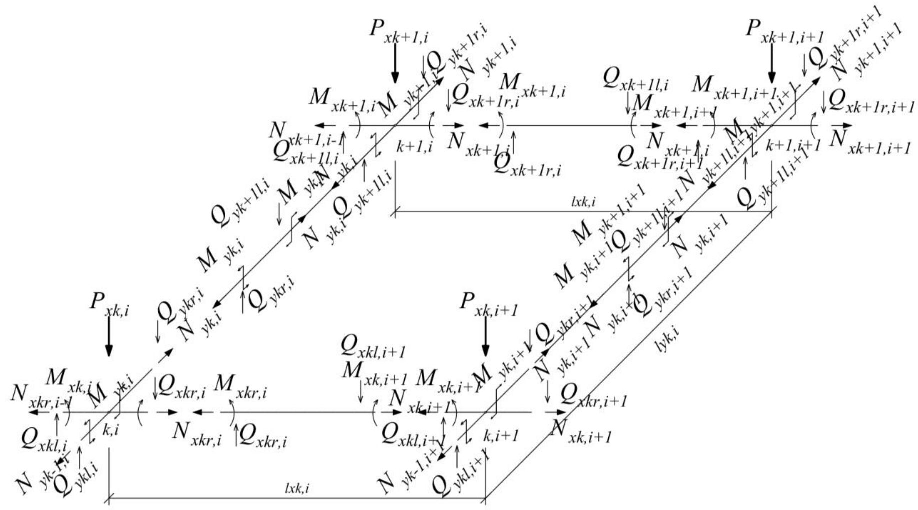

Fig. 5. Fragment of the estimation scheme of SRTS

Along direction of the *OY* axis, taking into consideration the joint action of bending moments and transverse forces, it takes the following form:

$$\delta_{ye-2,f} X_{ye-2,f}+\delta_{ye-1,f} X_{ye-1,f}+ +\delta_{ye,f} X_{ye,f}+\delta_{ye+1,f} X_{ye+1,f}+ \delta_{ye+2,f} X_{ye+2,f}+...+\Delta_{e,f} P=0. \quad (2)$$

When calculating the coefficients and free terms in equations (1), (2), we considered the interaction of the elements of bending moments, longitudinal and transverse forces, in the cross sections. The coefficients for equations (1), (2) were derived by multiplying the corresponding curves, for example:

$$\delta_{xe-2,f}=\frac{2\overline{N}_{xe-2,f-2}*\overline{N}_{xe,f-2}*l_{xe-2}}{6EA_{xe-2}}+\frac{2\overline{N}_{xe-2,f-1}*\overline{N}_{xe,f-2}*l_{xe-1}}{6EA_{xe-1}}+ +\frac{2\overline{N}_{xe-2,f}*\overline{N}_{xe,f}*l_{xe-2}}{6EA_{xe}}+\frac{\overline{N}_{xe-2,f+1}*\overline{N}_{xe,f+1}*h}{6\sin aEA_{xe+1}}+ +\frac{\overline{N}_{xe-2,f+2}*\overline{N}_{xe,f+2}*h}{6\sin\beta EA_{xe+2}}+\frac{\overline{N}_{xe-2,f+3}*\overline{N}_{xe,f+3}*h}{6\sin\beta EA_{xe+3}}+ +\frac{\overline{N}_{xe-2,f+4}*\overline{N}_{xe,f+4}*h}{6\sin aEA_{xe+4}}+\frac{\overline{N}_{xe-2,f+5}*\overline{N}_{xe,f+5}*(l-2h*\mathrm{ctg}a)}{6EA_{xe+5}}, \quad (3)$$

$$\delta_{xe-1,f}=\frac{l_{xe-1}}{6EI_{xe-1}}-\frac{\mu}{l_{xe-1}EA_{xe-1}}+\frac{2\overline{N}_{xe-1,f-2}*\overline{N}_{xe,f-2}*l_{xe-2}}{6EA_{xe-2}}+ +\frac{2\overline{N}_{xe-1,f-1}*\overline{N}_{xe,f-1}*l_{xe-1}}{6EA_{xe-1}}+\frac{2\overline{N}_{xe-1,f}*\overline{N}_{xe,f}*l_{xe}}{6EA_{xe}}+\frac{\overline{N}_{xe-1,f+1}*\overline{N}_{xe,f+1}*h}{6\sin aEA_{xe+1}}+ +\frac{\overline{N}_{xe-1,f+2}*\overline{N}_{xe,f+2}*h}{6\sin\beta EA_{xe+2}}+\frac{\overline{N}_{xe-1,f+3}*\overline{N}_{xe,f+3}*h}{6\sin\beta EA_{xe+3}}+ +\frac{\overline{N}_{xe-1,f+4}*\overline{N}_{xe,ef+4}*h}{6\sin aEA_{xe+4}}+\frac{\overline{N}_{xe-1,f+5}*\overline{N}_{xe,f+5}*h(l-2h*\mathrm{ctg}a)}{6EA_{xe+5}}, \quad (4)$$

$$\delta_{xe,f}=\frac{2l_{xe,f-1}}{3EI_{xe-1}}+\frac{2\mu}{l_{xe-1}6A_{xe-1}}+\frac{2\overline{N}^2_{xe,f-2}*l_{xe-2}}{6EA_{xe-2}}+ +\frac{2\overline{N}^2_{xe,f-1}*l_{xe-1}}{6EA_{xe-1}}+\frac{2\overline{N}^2_{xe,f}*l_{xe}}{6EA_{xe}}+\frac{\overline{N}^2_{xe,f+1}*h}{6\sin aEA_{xe+1}}+ +\frac{\overline{N}^2_{xe,f+2}*h}{6\sin\beta EA_{xe+2}}+\frac{\overline{N}^2_{xe,f+3}*h}{6\sin\beta EA_{xe+3}}+ \frac{\overline{N}^2_{xe,f+4}*h}{6\sin aEA_{xe+4}}+\frac{\overline{N}^2_{xe,f+5}*(l-2h*ctga)}{6EA_{xe+5}}, \quad (5)$$

$$\delta_{xe,f+1}=\frac{l_{xe-1}}{6EI_{xe-1}}-\frac{2\mu}{l_{xe-1}6A_{xe-1}}+\frac{2\overline{N}_{xe,f-2}*\overline{N}_{xe+1,f+1}*l_{xe-2}}{6EA_{xe-2}}+ +\frac{2\overline{N}_{xe,f-1}*\overline{N}_{xe+1,f-1}*l_{xe-1}}{6EA_{xe-1}}+\frac{2\overline{N}_{xe,f}*\overline{N}_{xe+1,f}*l_{xe-2}}{6EA_{xe}}+ +\frac{\overline{N}_{xe,f+1}*\overline{N}_{xe+1,f+1}*h}{6\sin aEA_{xe+1}}+\frac{\overline{N}_{xe,f+2}*\overline{N}_{xe+1,f+2}*h}{6\sin\beta EA_{xe+2}}+ +\frac{\overline{N}_{xe,f+3}*\overline{N}_{xe+1,f+3}*h}{6\sin\beta EA_{xe+3}}+\frac{\overline{N}_{xe,f+4}*\overline{N}_{xe+1,f+4}*h}{6\sin aEA_{xe+4}}+ +\frac{\overline{N}_{xe,f+5}*\overline{N}_{xe+1,f+5}*(l-2h*ctga)}{6EA_{xe+5}}, \quad (6)$$

$$\delta_{xe,f+2}=\frac{2\overline{N}_{xe,f-2}*\overline{N}_{xe+2,f-2}*l_{xe-2}}{6EA_{xe-2}}+\frac{2\overline{N}_{xe,f-1}*\overline{N}_{xe+2,f-1}*l_{xe-1}}{6EA_{xe-1}}+ +\frac{2\overline{N}_{xe,f}*\overline{N}_{xe+2,f}*l_{xe}}{6EA_{xe}}+\frac{\overline{N}_{xe,f+1}*\overline{N}_{xe+2,f+1}*h}{6\sin aEA_{xe+1}}+ +\frac{\overline{N}_{xe,f+2}*\overline{N}_{xe+2,f+2}*h}{6\sin\beta EA_{xe+2}}+\frac{\overline{N}_{xe,f+3}*\overline{N}_{xe+2,f+3}*h}{6\sin\beta EA_{xe+3}}+ +\frac{\overline{N}_{xe,f+4}*\overline{N}_{xe+2,f+4}*h}{6\sin\beta EA_{xe+4}}+\frac{\overline{N}_{xe,f+5}*\overline{N}_{xe+2,f+5}*(l-2h*ctga)}{6\sin aEA_{xe+5}}. \quad (7)$$

Coefficients $\delta_{xe+1,f}$, $\delta_{xe+2,f}$, $\delta^*_{xe+1,f}$, $\delta^*_{xe+2,f}$ are recorded in a similar fashion, similar to coefficients $\delta_{xe-1,f}$, $\delta_{xe-2,f}$, $\delta^*_{xe-1,f}$ and $\delta^*_{xe-2,f}$. A free term in the general form is expressed by formula:

$$\Delta_{e,fP}=\Sigma\int\frac{X_{xe,f}X_{xe,fP}}{\mathrm{EI}_{xe,f}}+\Sigma\frac{R_{xe,f}*R_{xe,fP}}{\mathrm{EA}_{xe,f}}=\frac{\mathrm{B}^{\Phi}_{xe,f}}{\mathrm{EI}_{xe,f}}+\frac{\mathrm{K}^{\Phi}_{xe+1,f}}{\mathrm{EI}_{xe+1,f}}+ +\frac{c_{xe-1,f}R_{xe-1,f}}{l_{xe,f}}-c_{xe,f}R_{xe,f}\left(\frac{1}{l_{xe,f}}+\frac{1}{l_{xe+1,f}}\right)+\frac{c_{xe+1,f}R_{xe+1,f}}{l_{xe+1,f}}, \quad (8)$$

where $R_{xe-1,f}$, $R_{xe,f}$, $R_{xe+1,f}$ denote the elastic reactions of supports *e*−1,*f*, *e*,*f*, *e*+1,*f*, derived as a result of distribution of the external load in a spatial rod model;

$$B_{xe,f}^{\Phi}=K_{xe,f}^{\Phi}=0$$

denote the fake reactions of supports *e*,*f* в *e*,*f* і *e*+1,*f* at spans, equal to zero given the nodal application of external effort *P*.

Equation (1) with respect to expressions (3) to (8) will take the form:

– for the first truss structure:

$$\delta_{11}X_1+\delta_{12}X_2+\delta_{13}X_3+\delta_{14}X_4+ +\delta_{15}X_5+\delta_{16}X_6+\delta_{17}X_7-\frac{2y_1}{l_1}+\frac{y_2}{l_2}=0;$$

$$\delta_{21}X_1+\delta_{22}X_2+\delta_{23}X_3+\delta_{24}X_4+\delta_{25}X_5+ +\delta_{26}X_6+\delta_{27}X_7+\frac{y_1}{l_1}-\frac{y_2(l_1+l_2)}{l_1*l_2}+\frac{y_3}{l_2}=0;$$

$$\delta_{31}X_1+\delta_{32}X_2+\delta_{33}X_3+\delta_{34}X_4+\delta_{35}X_5+ +\delta_{36}X_6+\delta_{37}X_7+\frac{y_2}{l_2}-\frac{2y_3}{l_2}+\frac{y_4}{l_2}=0;$$

$$\delta_{41}X_1+\delta_{42}X_2+\delta_{43}X_3+\delta_{44}X_4+\delta_{45}X_5+ +\delta_{46}X_6+\delta_{47}X_7+\frac{y_3}{l_2}-\frac{y_4(l_1+l_2)}{l_1*l_2}+\frac{y_5}{l_1}=0;$$

$$\delta_{51}X_1+\delta_{52}X_2+\delta_{53}X_3+\delta_{54}X_4+ +\delta_{55}X_5+\delta_{56}X_6+\delta_{57}X_7+\frac{y_4}{l_1}-\frac{2y_5}{l_1}=0;$$

$$\delta_{61}X_1+\delta_{62}X_2+\delta_{63}X_3+\delta_{64}X_4+ +\delta_{65}X_5+\delta_{66}X_6+\delta_{67}X_7+y_6=0;$$

$$\delta_{71}X_1+\delta_{72}X_2+\delta_{73}X_3+\delta_{74}X_4+ +\delta_{75}X_5+\delta_{76}X_6+\delta_{77}X_7+y_7=0. \quad (9)$$

– for the second truss structure:

$$\delta_{11}X_1+\delta_{12}X_2+\delta_{13}X_3+\delta_{14}X_4+$$
$$+\delta_{15}X_5+\delta_{16}X_6+\delta_{17}X_7-\frac{2y_1}{l_1}+\frac{y_2}{l_2}=0;$$

$$\delta_{21}X_1+\delta_{22}X_2+\delta_{23}X_3+\delta_{24}X_4+\delta_{25}X_5+$$
$$+\delta_{26}X_6+\delta_{27}X_7+\frac{y_1}{l_1}-\frac{y_2(l_1+l_2)}{l_1*l_2}+\frac{y_3}{l_2}=0;$$

$$\delta_{31}X_1+\delta_{32}X_2+\delta_{33}X_3+\delta_{34}X_4+\delta_{35}X_5+$$
$$+\delta_{36}X_6+\delta_{37}X_7+\frac{y_2}{l_2}-\frac{2y_3}{l_2}+\frac{y_4}{l_2}=0;$$

$$\delta_{41}X_1+\delta_{42}X_2+\delta_{43}X_3+\delta_{44}X_4+\delta_{45}X_5+$$
$$+\delta_{46}X_6+\delta_{47}X_7+\frac{y_3}{l_2}-\frac{y_4(l_1+l_2)}{l_1*l_2}+\frac{y_5}{l_1}=0;$$

$$\delta_{51}X_1+\delta_{52}X_2+\delta_{53}X_3+\delta_{54}X_4+\delta_{55}X_5+$$
$$+\delta_{56}X_6+\delta_{57}X_7+\frac{y_4}{l_1}-\frac{2y_5}{l_1}=0;$$

$$\delta_{61}X_1+\delta_{62}X_2+\delta_{63}X_3+\delta_{64}X_4+$$
$$+\delta_{65}X_5+\delta_{66}X_6+\delta_{67}X_7+y_6=0;$$

$$\delta_{71}X_1+\delta_{72}X_2+\delta_{73}X_3+\delta_{74}X_4+$$
$$+\delta_{75}X_5+\delta_{76}X_6+\delta_{77}X_7+y_7=0. \qquad (10)$$

Similarly recorded are the equations of the *i*-th efforts for structures along the other direction.

Deflection in an elastic-sagging support at the uncut structure along the *OX* direction in a general case is equal to the total reaction of a given support, multiplied by the coefficient of pliability $c_{xe,f}$:

$$z_{xe,f}=c_{xe,f}*\begin{bmatrix}X_{xe-1,f}/l_{xe,f}-\\-X_{xe,f}*(1/l_{xe,f}+1/l_{xe+1,f})+\\+X_{xe+1,f}/l_{xe+1,f}\end{bmatrix}+c_{xe,f}*R_{xe,f}, \qquad (11)$$

along the *OY* direction:

$$z_{ye,f}=c_{ye,f}*\begin{bmatrix}X_{ye1,f-1}/l_{ye,f}-\\-X_{ye,f}*\left(1/l_{ye,f}+1/l_{ye,f+1}\right)+\\+X_{ye,f+1}/l_{y1,f+1}\end{bmatrix}+c_{ye,f}*R_{ye,f}. \qquad (12)$$

Expressions (11) and (12) reflect the same deflection at node *e, f*. By equating the right sides of these expressions, we obtain

$$c_{xe,f}*\begin{bmatrix}X_{xe-1,f}/l_{xe,f}-\\-X_{xe,f}*\left(1/l_{xe,f}+1/l_{xe+1,f}\right)+\\+X_{xe+1,f}/l_{xe+1,f}\end{bmatrix}-$$
$$-c_{ye,f}*\begin{bmatrix}X_{ye1,f-1}/l_{ye,f}-\\-X_{ye,f}*\left(1/l_{ye,f}+1/l_{ye,f+1}\right)+\\+X_{ye,f+1}/l_{y1,f+1}\end{bmatrix}+$$
$$+c_{xe,f}*R_{xe,f}-c_{ye,f}*R_{ye,f}=0; \qquad (13)$$

is the equation that represents the static equilibrium of efforts at node *e, f* with respect to the pliability of supports.

The equation of the static equilibrium of the node in a cross-ribbed truss system, expressed through the nodal bending moments, longitudinal and transverse forces, and external load, will take the form:

$$X_1*d_{1,1}+\ldots+X_{n-1}*d_{n,n-1}+X_n*d_{n,n}+$$
$$+X_{n+1}*d_{n,n+1}+\ldots+X_i*d_{i,i}+D_{nf}=0. \qquad (14)$$

The resulting equation is

$$X_1*\delta_{1,1}+\ldots+X_{n-1}*\delta_{n,n-1}+X_n*\delta_{n,n}+$$
$$+X_{n+1}*\delta_{n,n+1}+\ldots+X_i*\delta_{i,i}+\Delta_{nf}=0$$

with the number of longitudinal rigidity beams *f* from 1 to *n*. The boundary conditions for equations (13), (14) will be the following:

$$X_{x(e=0),f}=X_{x(e=e+1),f}=0,$$

$$X_{y(e=1),f}=X_{ye,(f=n)}=0. \qquad (15)$$

Thus, the assigned mathematical model of a spatial combined steel-reinforced-concrete truss structure satisfies three groups of conditions:

– conditions for equilibrium;

– conditions for the compatibility of deformations, linking deformations and displacements;

– physical conditions that bind efforts and deformations.

Solution to the finite systems of linear algebraic equations implies:

a) obtaining, through iterative search, a minimum of the objective function of the equally-stressed state in the elements of a spatial combined steel-reinforced-concrete truss system;

b) obtaining the distribution of efforts due to the action of external load, bending moments, longitudinal and transverse forces, vertical displacements, and the parameters for the stressed-strained state of the elements of a spatial combined steel-reinforced-concrete truss structure under the action of external nodal load;

c) obtaining the intensity of the load in the adopted scheme under specified parameters for the stressed-strained state of *all* elements in a spatial combined steel-reinforced-concrete truss system.

5. Results of theoretical research into a combined steel-reinforced-concrete truss structure

The resulting system of equations is sufficient for finding the unknowns of bending moments, transverse and longitudinal forces, and vertical displacements at each assigned node of the combined structure.

According to the above given procedure, we performed numerical simulation and calculation of the combined steel-reinforced-concrete truss d structures. The results obtained during calculation have been compared with experimental results, obtained during the experiment. In order to compare the economic feasibility of using a combined steel-reinforced-concrete truss structure, we calculated the 12-meter span structures. A step of main beams is 3 m, that of the secondary ones is 1.5 m, a load of 10 kN/m^2: a regular metallic beam (variant 1), a metallic truss beam (variant II), and a steel-reinforced-concrete SRTS (variant III) (Fig. 6). Unfastening from the plane of the action of effort in the upper belt are the secondary metallic beams and a concrete spanning slab. Metal belts are made along the lower belt.

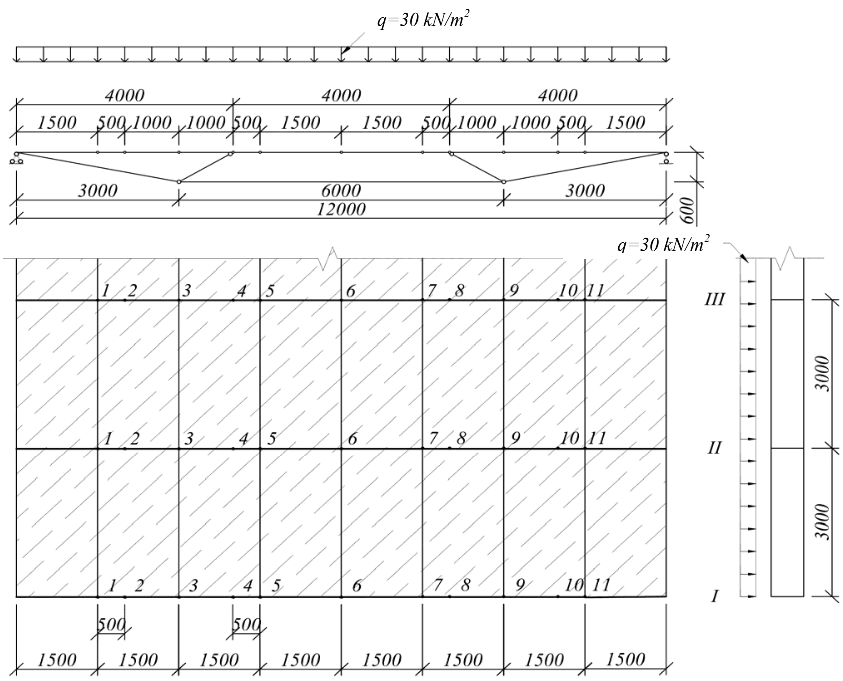

Fig. 6. Spatial calculation scheme of SRTS

Under the action of the uniformly distributed load, the upper part of the cross section, according to the stress diagram, is exposed to efforts received by the upper shelf of the H-beam and a reinforced concrete slab. The theoretical study that we conducted has shown that underloads at the intersections of the combined steel-reinforced-concrete truss structure, in comparison with metallic ones, makes it possible to reduce the magnitude of rolling a rigidity beam by 25 %.

Since the slab is reinforced, it can perceive the tensile efforts as well, such as in the presence of consoles or an uncut structure. However, that implies additional reinforcement. The total weight of metal in the structure (without taking into consideration the transverse beams: they are equal in all cases) is 630 kg.

Experimental study was conducted regarding the combined metallic systems, united in a joint work with the reinforced-concrete slab (Fig. 9, 10) [12].

Fig. 10. Graphs of dependence of stresses on the step-wise applied load in the elements of the metallic structure: CS-1...CS-3 – examined structures, SS-1 – a theoretical change in stress in the elements of the metallic structure, not united in a joint work with a slab

Based on the performed calculations for groups I and II of the boundary states, we have chosen a metallic rolled beam – H-beam No. 60. Weight per unit length is 108 kg/m, the weight of the entire structure is, respectively, 12·108=1,296 kg.

The estimation scheme of the steel truss structure is shown in Fig. 7. The rigid upper belt is made from the rolled H-beam, the elements of the suspension – from paired steel angles or other profiles. The connection of the beam to the suspension shall be considered as a hinge, similar for the frame's elements. The upper rigid belt works as a beam on elastic supports, which significantly reduces the spanning moments within a structure. By selecting, according to efforts (Fig. 8), the rolled profiles, we obtained: the upper belt is H-beam No. 36, the elements of suspension made from the paired steel angles – the lower part is of 100×10 mm, the extreme slopes are of 90×7 mm, the internal ones are of 65×50×5 mm (Fig. 8). The total weight of a given structure is 750 kg.

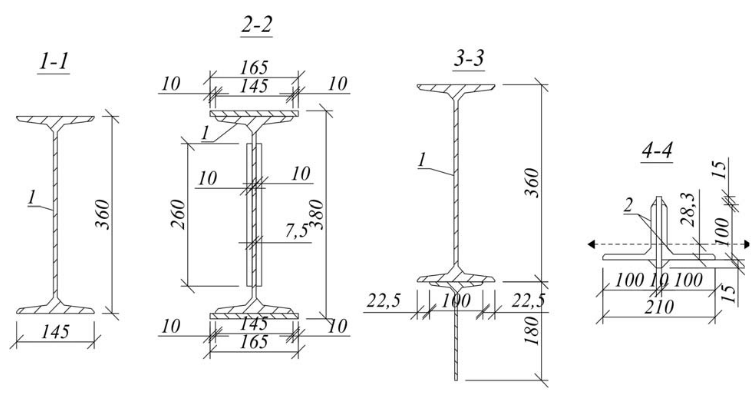

Fig. 8. Drawing of the metal truss structure with characteristic cross sections: 1 – H-beam No. 36, 2 – steel angles 100×10

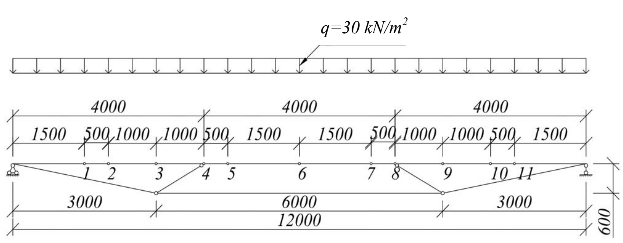

Fig. 7. Flat estimation scheme of the truss structure

Fig. 9. General view of the experimental structure

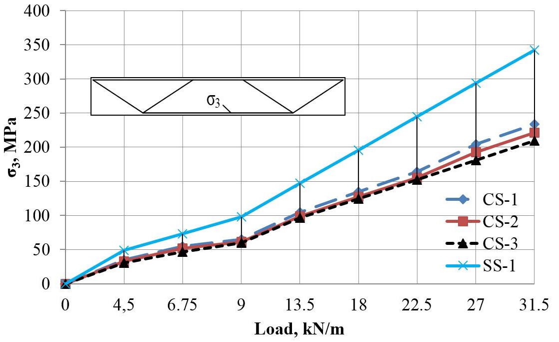

Fig. 10. Graphs of dependence of stresses on the step-wise applied load in the elements of the metallic structure: CS-1...CS-3 – examined structures, SS-1 – a theoretical change in stress in the elements of the metallic structure, not united in a joint work with a slab

6. Discussion of results of theoretical studies into a combined steel-reinforced-concrete truss structure

As shown by the numerical calculations that we performed, a spatial SRTS is more than twice lighter than the standard rolled steel beam, and is 16 % lighter than the metallic truss structure [13].

It should be noted that the normal concentrated effort in the compressed zone of concrete in the cross section of a rigidity beam is determined from dependence:

$$f_C=0,85*f_{Cd}*\beta_1*Y_B*_b, \qquad (16)$$

where f_{Cd} is the estimation value of concrete for compression strength; b is the width of a rectangular cross-section of rigidity beam; β_1 is the reduced coefficient of the compressed zone of concrete for its height Y_B [14–16].

When taking into consideration physical nonlinearity, the rigidity of the reduced cross section after cracking:

$$B(x)=h_{0,xef}z_{xef}\,/\left[\frac{\psi_s}{E_sA_s}+\frac{\psi_b}{(\phi_f+\xi)\lambda_bE_bb_{xef}h_{0,xef}}\right]; \qquad (17)$$

$$B(y)=h_{0,yef}z_{yef}\,/\left[\frac{\psi_s}{E_sA_s}+\frac{\psi_b}{(\phi_f+\xi)\lambda_bE_bb_{yef}h_{0,yef}}\right]. \qquad (18)$$

The resulting vector of the redistribution of efforts and displacement at nodes of the cross-ribbed combined steel-reinforced-concrete truss system in the case of the elastic-plastic work of individual elements forms the matrix of pliability for the subsequent approximation. The equations of deformation continuity (9), (10) with respect to properties (16) to (18) in the subsequent approximations will take the form:

– for the n-th beams along the OX direction, equation:

$$X_1*d_{1,1}+\ldots+X_{n-1}*d_{n,n-1}+X_n*d_{n,n}+$$
$$+X_{n+1}*d_{n,n+1}+\ldots+X_i*d_{i,i}+D_{nf}=0, \qquad (19)$$

– for the m-th beams along the OY axis:

$$X_{ye,f-1}\times d/(6B(y)_{ye,f-1})+2X_{ye,f}\times d/(3B(y)_{ye,f})+$$
$$+X_{ye,f+1}\times d/(6B(y)_{ye,f+1})+$$
$$+z_{ye,f-1}/d-2z_{ye,f}/d+z_{ye,f+1}/d=0. \qquad (20)$$

Iterative search is performed until the difference received as a result of intermediate solution for two successive calculations exceeds a certain magnitude Ω, which is the assigned accuracy of the calculation.

The constructed mathematical apparatus could be used to solve the inverse problem as well: finding efforts in the cross-ribbed system based on known experimental values for deflections or deformations.

The developed mathematical apparatus for calculating the truss steel-reinforced-concrete systems taking into consideration the factors of physical nonlinearity could be applied to truss structures of arbitrary shape, with arbitrary geometry and cross section reinforcements.

Combined application of methods of linear programming and methods of nonlinear-elastic systems, which in the process come down to computing the linear algebraic equations, makes it possible to speed up convergence of the iterative process and to reduce computation time at PC.

An analysis of the conducted theoretical study has shown that the proposed mathematical model allows the design of more complex spatial combined steel-reinforced-concrete truss structures. The calculation procedure provides an opportunity to account for the stage character of the system's work, including taking into consideration the preliminary stress.

The proposed principle for calculating a spatial statically-indeterminate combined steel-reinforced-concrete truss structure based on the accepted estimation scheme can be attributed to the universal one. Its application makes it possible, in a simpler fashion, at the initial design stage, by using the mathematical apparatus, to model the deformed-stressed state in the elements of the structure, which allows a more economical design of the structure in general.

The constructed mathematical model, as well as the performed numerical and field study of the steel-reinforced-concrete truss structures with a span of up to 6 m, demonstrated sufficient convergence of theoretical and experimental results. The proposed mathematical model is at the initial stage of testing. Therefore, a number of factors remain unresolved in terms of theory, including the influence of width of the reinforced concrete shelf in a steel-reinforced-concrete rigidity beam. According to the requirements of building regulations, in the middle of the span, or at an intermediate support, the total reduced width b_{eff} (Fig. 11) can be determined from

$$b_{eff}=b_0+\sum b_{ei}, \qquad (21)$$

where b_0 are the distances between the centers of protruding shear joints; b_{ei} is the magnitude of the actual width of the concrete shelf at each side of the wall, which is accepted as $L_e/8$, but not larger than the geometrical width b_i.

The value for b_i should be taken such that it is equal to the distance from the protruding sliding joint to a midpoint between the adjacent walls, positioned in the middle of the height of the concrete shelf, except for the open (extreme) facets, where b_i is the distance to the open facet. The length of L_e must be taken as the corresponding distance between the points of zero moments. For the standard uncut combined beams, whose calculation is defined by the diagram of bending moments due to different loads and for consoles, L_e can be accepted as shown in Fig. 11.

The actual width of the shelf at the extreme support can be determined from

$$b_{eff}=b_0+\sum\beta_ib_{ei}, \qquad (22)$$

at

$$\beta_i=(0{,}55+0{,}025L_e/b_{ei})\leq 1{,}0,$$

where b_{ei} is the actual width of the shelf in the middle of the extreme span; L_e is the equivalent extreme span.

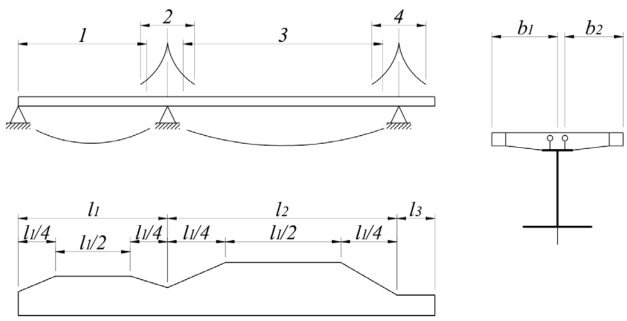

Fig. 11. Equivalent spans for the actual width of a concrete shelf (1 – L_e=0.85 L_1 for $b_{eff,1}$; 2 – L_e=0.25(L_1+L_2) for $b_{eff,2}$; 3 – L_e=0.7 L_2 for $b_{eff,1}$; 4 – L_e=2 L_1 for $b_{eff,2}$

In contrast to the reinforced concrete or metallic beam multispan structures, in which the action of the external load gives rise to the sign-alternating bending moments, the curves of the bending moments cross the neutral axes, which are considered the zero points.

In the steel-reinforced-concrete truss structures, the rigidity beam under the action of external forces is exposed both to bending moments and longitudinal forces. As shown by the performed theoretical study, depending on the topology of a structure, the physical-mechanical characteristics of elements, and the external load, the position of the zero point along the length of the rigidity beam varies. In some cases, the curve of bending moments along the length of the rigidity beam may be completely positive and, consequently, the requirements set out above are not applicable.

When fabricating a steel-reinforced-concrete truss structure, at the initial stage the neutral axis in the steel rigidity beam is located in the middle of the cross section of the beam. After the reinforced concrete slab acquires strength, the rigidity beam is regarded as a steel-reinforced-concrete element. In this case, the neutral axis is offset. This factor increases the estimated height of the reinforced concrete truss structure, the positions of the nodes at intersections of axes of elastic intermediate supports with the axis of the rigidity beam are displaced accordingly.

The above-specified factors necessitate the further research, both theoretical and experimental, including the use of large-sized models.

7. Conclusions

1. We have constructed a mathematical calculation model and developed an algorithm for determining efforts and vertical displacements in the spatial combined steel-reinforced-concrete truss systems. The theoretical results obtained make it possible to explore the deformed-stressed state in the structures' elements in comparison with existing methods of calculation taking into consideration the stages of operation at variable input parameters of topology and rigidity.

2. Based on an analysis of the conducted theoretical study, we have designed combined steel-reinforced-concrete truss structures under the action of symmetrical and asymmetrical loads for actual construction. The designed combined steel-reinforced-concrete truss structures use 16 % less materials compared with structures that are estimated by other known methods.

3. The developed algorithm makes it possible in the course of numerical calculation to search for a minimum of the objective function od the equally-stressed state in all elements of the spatial structure. Numerical calculation allowed us to carry out research through an iterative search for the domain of solution to the stated problem under acceptable boundary conditions.

4. The underloading of intersections in a combined steel-reinforced-concrete truss structures, in comparison with metallic ones, makes it possible to reduce the magnitude of rolling a rigidity beam by 25 %. The theoretical study that we conducted has shown that the reserve of carrying capacity of the spatial combined steel-reinforced-concrete truss structure, in comparison with the calculation for the two-axial stressed state, is 16 %.

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NUMERICAL METHODS FOR CONTACT ANALYSIS OF COMPLEX-SHAPED BODIES WITH ACCOUNT FOR NON-LINEAR INTERFACE LAYERS

M. Tkachuk
PhD, Senior Researcher*
E-mail: m.tkachuk@tmm-sapr.org
N. Skripchenko
PhD, Researcher*
E-mail: negrobova@gmail.com
M. Tkachuk
Doctor of Technical Sciences, Head of Department*
E-mail: tma@tmm-sapr.org
A. Grabovskiy
PhD, Senior Researcher*
E-mail: andrej8383@gmail.com
*Department of Theory and Systems of Automated Design of Mechanisms and Machines
National Technical University «Kharkiv Polytechnic Institute»
Kyrpychova str., 2, Kharkiv, Ukraine, 61002

З метою забезпечення високих технічних характеристик машин різноманітного призначення необхідно підвищувати міцність найбільш навантажених та відповідальних елементів конструкцій, якими є складнопрофільні деталі у процесі інтенсивних контактних навантажень. Для випадку близької форми поверхонь контактуючих тіл стають важливими чинники, які раніше не враховувалися. Це, зокрема, нелінійна контактна жорсткість поверхневих шарів деталей. Відповідно, в умовах непроникнення контактуючих тіл замість традиційних лінійних компонентів з'являються також нелінійні. Для дослідження контактної взаємодії тіл із урахуванням такого типу обмежень розроблено новий метод дослідження напружено-деформованого стану та забезпечення міцності деталей машин різноманітного призначення на основі модифікації варіаційного принципу Калькера. Створено і застосовано нелінійні моделі поведінки матеріалу поверхневих шарів контактуючих складнопрофільних тіл. Дискретизація розв'язувальних співвідношень здійснена за допомогою розробленого варіанту методу граничних елементів.

Побудовані моделі контактної взаємодії поєднують в собі фізичну та структурну нелінійність. Це забезпечує більш адекватне визначення напружено-деформованого стану контактуючих складнопрофільних тіл у порівнянні з традиційними підходами. На цій основі досліджені особливості розподілу контактного тиску при варіюванні форми зазору та властивостей проміжного шару між контактуючими тілами. З урахуванням результатів такого аналізу у подальшому можуть бути запропоновані більш достовірні рекомендації із обґрунтування проектно-технологічних рішень, які, у кінцевому підсумку, забезпечують підвищення технічних характеристик машин різноманітного призначення

Ключові слова: контактна взаємодія, варіаційний принцип Калькера, метод граничних елементів, шар Вінклера

1. Introduction

A large number of engineering constructions contain elements that are in the conditions of contact force and kinematic conjugation. For instance, it can be the modified working surfaces of gears, roller bearings, etc.

Traditional methods for modeling of contact interaction lead to significant errors in the obtained results or to overly cumbersome numerical models. Another important factor is the lack of adequate modeling of the contact conditions at the boundaries of the bodies. For instance, the impenetration condition for smooth bodies in linearized form is most commonly