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DETERMINING THE EFFECTIVE CHARACTERISTICS OF A COMPOSITE WITH HOLLOW FIBER AT LONGITUDINAL ELONGATION

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При розв'язанні задач механіки композитів зручно використовувати модель композита у вигляді суцільного однорідного середовища з ефективними сталими, що адекватно відображають його найбільш суттєві характеристики. У сучасному машинобудуванні та будівництві широке застосування знаходять композити, армовані порожнистими волокнами. На сьогодні невідомими є аналітичні залежності для ефективних пружних сталих таких композитних матеріалів з транстропними складовими. Задача отримання таких залежностей розв'язується у даній публікації.

Отримано аналітичні залежності для ефективного поздовжнього модуля пружності та коефіцієнта Пуассона односпрямованого волокнистого композита, що містить транстропні матрицю та порожнисте волокно. Композит моделюється суцільним однорідним транстропним матеріалом. На міжфазних поверхнях виконуються умови ідеального з'єднання. Для отримання аналітичних залежностей розв'язано дві крайові задачі: про поздовжнє розтягування складеного циліндра, компонентами якого є транстропні матриця та порожнисте волокно, та суцільного однорідного циліндра, що моделює транстропний композит. Використання умов узгодження переміщень та напружень, отриманих при розв'язанні цих задач, забезпечило можливість отримання формул ефективного поздовжнього модуля пружності та коефіцієнта Пуассона. Ці формули відображають залежності ефективних сталих від пружних характеристик матриці, волокна й об'ємних часток волокна та порожнини в ньому.

Проведено порівняння результатів розрахунків за отриманими формулами з результатами обчислень за раніше відомими співвідношеннями для ізотропних складових. Це порівняння показало, що їх відносне відхилення не перевищує одного відсотка. Застосування отриманих залежностей дозволяє проектувати конструкції з елементами, виготовленими з композиційних матеріалів

Ключові слова: односпрямований волокнистий композит, поздовжнє розтягнення, порожнисте волокно, ефективні пружні сталі

1. Introduction

Fibrous composites are widely used as structural materials in engineering, construction, and other sectors of industrial activities.

The application of such materials when designing various structures makes it possible to change the properties of structural elements by altering the composition of a composite, the concentration and arrangement of fibers. The properties of composites depend on the mechanical characteristics of constituent elements. Their combination creates a synergy that is manifested in the formation of new physical-mechanical properties in a composite, not inherent to its individual phase elements. When designing structures from composite materials, it is necessary to calculate the stressed-strained state, which requires data on the effective elastic constants in a composite. That predetermines the relevance of the task on defining these indicators.

It is possible to significantly extend the range of technical characteristics of composite materials by using hollow fibers for the reinforcement. In this regard, an important problem is the homogenization of a composite. Solving this problem implies obtaining the analytical dependences of effective elastic constants of a composite on the values of corresponding constants for its constituents, as well as the volumetric share of the content of fiber and the cavity inside it in a composite. Adequate evaluation of the mechanical characteristics of a composite material is not possible without taking into consideration the transtropic properties of components.

2. Literature review and problem statement

Determining the physical-mechanical properties of composites based on the properties of their structural elements is one of the most important tasks for modern mechanics of composite materials, solving which employs the analytical, numerical and experimental methods. There is a growing body of scientific research into the construction of effective mechanical characteristics for fibrous composites consisting of the matrix and fiber of different types. Specifically addressed are the anisotropic, plastic, viscous-elastic components of a composite, as well as the existence of thermal expansion, etc.

Paper [1] proposed, based on the precise theory of reinforcement, ratios for determining the effective elastic characteristics of unidirected composites on the basis of isotropic plastics, reinforced, in line with the hexagonal scheme, by long transtropic fibers. Study [2] proposed ratios for determining the longitudinal elastic modulus and a Poisson's coefficient for a bi-phase fibrous composite with transtropic components.

To define the thermoelastic characteristics and characteristics of thermal conductivity of a composite with spherical and cylindrical inclusions, paper [3] applies a variational asymptotic method (VAMUCH). This same method is used to determine the elastic-plastic, electro-magnetoelastic and other properties of composites in [4].

Paper [5] considers asymptotic methods for the calculation of non-homogenous composite materials with respect to the micromechanical effects caused by the peculiarities of internal structure.

A procedure for calculating the effective viscoelastic characteristics of composites under steady cyclic oscillations, based on the method for solving local problems on viscoelasticity using a periodicity cell of composites, is described in [6]. The authors gave examples of the numerical simulation of viscoelastic characteristics of unidirected-reinforced composites. Paper [7] addressed issues related to numerical determination of the effective thermo-viscoelastic characteristics of unidirected composites with a polymeric matrix based on the properties of the components. Based on a method that exploits the Volterra principle, the authors solve the problem on determining the viscoelastic mechanical characteristics of a composite applying known characteristics of their viscoelastic components [8].

Using a method of homogenization, paper [9] determined effective elastic properties of composites with different shapes and distributions of inclusions (with spherical and elliptical particles, fibers of cylindrical shape, including the semi-spherical ends). Authors of [10] developed two, relatively new, approaches to the homogenization of multiphase composites, namely the effective self-correcting scheme (ESCS) and the direct derivative from interaction (IDD) [10]. The effectiveness of these procedures is compared with classical approaches and with the relevant results acquired from modeling using a method of finite elements.

The influence of a heterogeneous transition layer that forms between the matrix and the fiber, on the stressed-strained state of unidirected fibrous composites was investigated in [11]. In this case, the Poisson ratio and a coefficient of thermal expansion of the inter-phase were considered to be constant, while the longitudinal modulus of elasticity such that changes according to the linear and power laws.

In [12], authors constructed an improved model of the sliding lag in order to study the influence of surface roughness on mechanical properties of the unidirected fibrous polymeric composites with a stage structure.

In [1–12], the object of research is the composite materials, reinforced by solid fibers; however, the authors failed to take into consideration the existence of a cavity in the fibers, which, at homogenization, somewhat complicates mathematical models and the derivation of analytical relations for effective mechanical characteristics. In many structures, a special role belongs not only to the resistance to deformations, but the weight and cost of applied materials as well, which is why it is a relevant task to study the hollow fibers.

Paper [13] reports a comparative analysis of the application of hollow and solid glass fibers in the design of composite materials.

At present, there are formulae for determining the elastic characteristics in the case of isotropy of components of a bi-phase composite, reinforced by hollow fibers. These formulae define the longitudinal modulus of elasticity and a Poisson's coefficient [14]:

$$E_x = E_f V_f \left(1-q^2\right) + E_m V_m, \quad (1)$$

$$\mu_{xy} = \mu_m - \frac{V_f\left(\mu_m-\mu_f\right)\left(\chi_m+1\right)\left(1-q^2\right)}{\left(1-q^2\right)\left(1+V_m+V_f\chi_m\right)+V_m\left(\chi_f-1+2q^2\right)\dfrac{G_m}{G_f}}, \quad (2)$$

where E_f, E_m, E_x is the longitudinal elasticity modulus of the fiber, matrix, and composite, respectively; μ_f, μ_m, μ_{xy} is the Poisson's coefficient of the fiber, matrix, and composite, respectively; V_f, V_m is the volumetric share of the fiber and matrix, respectively; G_f, G_m is the shear modulus of the fiber and matrix, respectively; q is the ratio of the diameter of a cavity to the outer diameter of the fiber, $\chi_f = 3-4\mu_f$, $\chi_m = 3-4\mu_m$.

The effectiveness of the use of composite materials, reinforced by oriented hollow fibers, was also examined in paper [15]. In [16], effective elastic components of the composite materials with hollow fibers are determined by a method of sequential regularization. Specifically, the authors derived the following formula to determine the longitudinal elasticity module:

$$E_1^0 = \xi\left(1-q^2\right)E_a + (1-\xi)E + \frac{8G\xi(1-\xi)\left(1-q^2\right)\left(\nu_a-\nu\right)^2}{\left(1-q^2\right)(2-\xi+\chi\xi)+(1-\xi)\left(\chi_a-1+2q^2\right)\dfrac{G}{G_a}}, \quad (3)$$

where E_a, E, E_1^0 is the longitudinal elasticity modulus of the fiber, matrix, and composite, respectively, ν_a, ν is the Poisson's ratio of the fiber and matrix, respectively, G_a, G is the shear modulus of the fiber and matrix, respectively, $q^2 = \varepsilon^2/a^2$, a is the outer radius of the fiber, ε is the radius of the cavity inside the fiber, ξ is the volumetric share of the fiber, $\chi_a = 3-4\nu_a$, $\chi = 3-4\nu$.

Note that in [14–16] components of a composite are taken to be isotropic, but for many materials this assumption leads to inaccuracies in modeling.

Paper [17] derived analytical expressions for the effective modules of volumetric compression in unidirected-reinforced materials, whose components are the transtropic matrix and the hollow or solid cylindrical fibers of various diameters. The authors failed to obtain formulae for the effective elastic constants of such materials.

The application of hollow polyester fibers for the reinforcement of a composite compared with solid substances was investigated in [18]. The finite element method is used in the paper in order to model the behavior of composites and to study the mode of failure when tested for impact. The result is that the impact resistance of the composite with hollow fibers is higher than that with the solid ones.

Paper [19] reviewed experimental studies into behavior of composites with the epoxy matrix, reinforced by the unidirectional hollow, solid and mixed, polyester fibers, when tested for impact.

The results of experimental research into the influence of the reinforcement of composites by solid glass and hollow fibers for the resistance to deformation at elongation, compression, bending, as well as the impact resistance of composites, are highlighted in [20].

Papers [18–20] provide numerical data acquired from numerical or physical experiments, which makes it impossible to directly apply these data for new composite materials.

One of the ways to obtain effective characteristics is to investigate the representation of the volumetric element of a composite material with the transtropic matrix and the hollow fiber.

3. The aim and objectives of the study

The aim of this study is to derive analytical ratios for the effective longitudinal elasticity modulus and a Poisson's coefficient of the composite with the transtropic matrix and hollow fiber, by aligning the displacements components of a homogeneous composite and its components.

To accomplish the aim, the following tasks have been set:

– to determine the components of the stressed-strained state of the matrix, fibers, and a homogenous composite at longitudinal elongation of the elementary cell;

– to compare the results obtained based on the proposed analytical ratios with analogous results obtained based on known formulae for isotropic components.

4. Determining the effective elastic constants of a composite material with hollow fiber at longitudinal elongation

Principal assumptions:

– materials of the matrix and hollow fiber are transtropic, the planes of isotropy for the matrix and fibers coincide and are perpendicular to the axis of the fiber;

– a composite material will be considered as homogeneous and transtropic with the plane of isotropy that is perpendicular to the axis of the fiber;

– the relationship between stresses and deformations are described by the Hooke's law;

– there is a perfect adhesion between the materials of the matrix and the fiber;

– we consider the hexagonal arrangement of fibers in a unidirectional composite.

We approximate the volume of an elementary hexagonal cell by the volume of a cylinder. The radius in this case is taken such that the volumetric content of fiber in the hexagonal cell and the volumetric content of fiber in the cylindrical cell are the same (Fig. 1).

The representative element of the fibrous unidirectional composite material that has the transtropic properties is then represented in the form of a combination of two transtropic cylinders of infinite length, which model the matrix and hollow fiber, respectively.

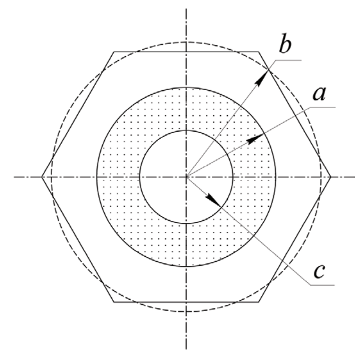

Fig. 1. Hexagonal cell: *a* – radius of the fiber; *b* – radius of the matrix; *c* – radius of the cavity

Let f and g be, accordingly, the volumetric content of fiber and the cavity inside it a composite's material. Considering that the area occupied by the matrix in the elementary cell and the area the fiber occupies inside it are of the same height, we the obtain the following ratios:

$$g=\frac{\pi c^2}{\pi b^2}=\frac{c^2}{b^2},\quad f=\frac{\pi(a^2-c^2)}{\pi b^2}=\frac{a^2-c^2}{b^2}. \tag{4}$$

Consider the joint longitudinal elongation (Fig. 2, *a*) of the hollow cylinder ($c\leq r\leq a$) that simulates the fiber, and the hollow cylinder ($a\leq r\leq b$) that simulates the matrix. Proceed to the cylindrical coordinate system $Ozr\theta$. The index 1 then will correspond directly to the z axis direction, direction 2–r, direction 3–θ.

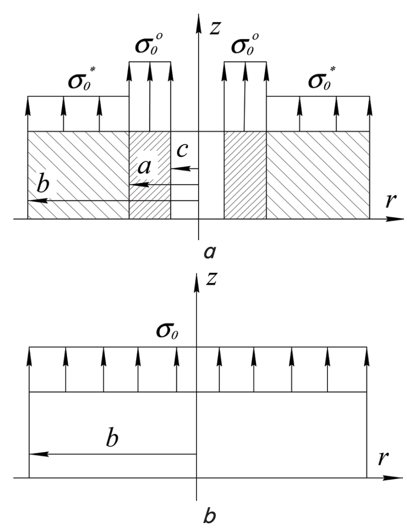

Fig. 2. Longitudinal elongation: *a* – joint deformation of the matrix and hollow fiber; *b* – deformation of the composite

We shall confine ourselves to the case when conditions for the perfect connection are satisfied at the interphase surface of the composite. We believe that the following conditions are met for stresses and displacements:

$$\sigma_{rr}^{\circ}(a)=\sigma_{rr}^{*}(a),$$

$$u_r^{\circ}(a)=u_r^{*}(a),$$

$$u_z^{\circ}(h)=u_z^{*}(h), \qquad (5)$$

and the outer surface of the matrix and the inner surface of the hollow fiber are free from stresses:

$$\sigma_{rr}^{*}(b)=0,\ \ \sigma_{rr}^{\circ}(c)=0. \qquad (6)$$

Hereinafter symbol denotes the magnitudes that are related to fiber, and symbol * – magnitudes related to the matrix.

Radial displacements of the transtropic fiber are described by the following ratio:

$$u_r(r)=C_1 r+\frac{C_2}{r}, \qquad (7)$$

where C_1 and C_2 are the constants that are determined from the boundary conditions, $u_r(r)$ is the solution to equation:

$$\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}=0, \qquad (8)$$

which we obtain from equation of equilibrium for the axial-symmetrical stressed-strained state.

Axial displacements are described by ratio [21]:

$$u_z(z)=\left(\frac{\sigma_0\left(1-\nu_{23}-2\nu_{12}\nu_{21}\right)}{E_1\left(1-\nu_{23}\right)}-\frac{2C_1\nu_{21}}{\left(1-\nu_{23}\right)}\right)z. \qquad (9)$$

For the displacements and stresses of points in the transtropic matrix, by changing C_1 with A, and C_2 with B, we find the ratio:

$$u_r^{*}(r)=Ar+\frac{B}{r}, \qquad (10)$$

$$u_z^{*}(z)=\frac{1}{\left(1-\nu_{23}^{*}\right)}\left(\frac{\sigma_0^{*}\left(1-\nu_{23}^{*}-2\nu_{12}^{*}\nu_{21}^{*}\right)}{E_1^{*}}-2A\nu_{21}^{*}\right)z, \qquad (11)$$

$$\sigma_{rr}^{*}(r)=E_2^{*}\left(\frac{\sigma_0^{*}\nu_{12}^{*}}{E_1^{*}\left(1-\nu_{23}^{*}\right)}+\frac{A}{1-\nu_{23}^{*}}-\frac{B}{r^2\left(1+\nu_{23}^{*}\right)}\right), \qquad (12)$$

$$\sigma_{\theta\theta}^{*}(r)=E_2^{*}\left(\frac{\sigma_0^{*}\nu_{12}^{*}}{E_1^{*}\left(1-\nu_{23}^{*}\right)}+\frac{A}{1-\nu_{23}^{*}}+\frac{B}{r^2\left(1+\nu_{23}^{*}\right)}\right). \qquad (13)$$

Similarly, we write the ratios describing the stressed-strained state of the hollow transtropic fiber (change C_1 with C, C_2 with D):

$$u_r^{\circ}(r)=Cr+\frac{D}{r}, \qquad (14)$$

$$u_z^{\circ}(z)=\frac{1}{\left(1-\nu_{23}^{\circ}\right)}\left(\frac{\sigma_0^{\circ}\left(1-\nu_{23}^{\circ}-2\nu_{12}^{\circ}\nu_{21}^{\circ}\right)}{E_1^{\circ}}-2C\nu_{21}^{\circ}\right)z, \qquad (15)$$

$$\sigma_{rr}^{\circ}(r)=E_2^{\circ}\left(\frac{\sigma_0^{\circ}\nu_{12}^{\circ}}{E_1^{\circ}\left(1-\nu_{23}^{\circ}\right)}+\frac{C}{1-\nu_{23}^{\circ}}-\frac{D}{r^2\left(1+\nu_{23}^{\circ}\right)}\right), \qquad (16)$$

$$\sigma_{\theta\theta}^{\circ}(r)=E_2^{\circ}\left(\frac{\sigma_0^{\circ}\nu_{12}^{\circ}}{E_1^{\circ}\left(1-\nu_{23}^{\circ}\right)}+\frac{C}{1-\nu_{23}^{\circ}}+\frac{D}{r^2\left(1+\nu_{23}^{\circ}\right)}\right). \qquad (17)$$

Based on the first two conditions (5) and boundary conditions (6), we find the constants A, B, C, and D. We obtain:

$$A=E_2^{\circ}(1-\nu_{23}^{*})f(f+g)\frac{\sigma_0^{\circ}\nu_{12}^{\circ}E_1^{*}-\sigma_0^{*}\nu_{12}^{*}E_1^{\circ}}{\alpha E_1^{\circ}E_1^{*}}-\frac{\sigma_0^{*}\nu_{12}^{*}}{E_1^{*}}, \qquad (18)$$

$$B=E_2^{\circ}(1+\nu_{23}^{*})fa^2\frac{\sigma_0^{\circ}\nu_{12}^{\circ}E_1^{*}-\sigma_0^{*}\nu_{12}^{*}E_1^{\circ}}{\alpha E_1^{\circ}E_1^{*}}, \qquad (19)$$

$$C=\left(1-\frac{g(1+\nu_{23}^{\circ})}{2g+f(1-\nu_{23}^{\circ})}\right)\left(E_2^{\circ}f\left(1+f+g+\nu_{23}^{*}(1-f-g)\right)\times\right.$$

$$\left.\times\frac{\sigma_0^{\circ}\nu_{12}^{\circ}E_1^{*}-\sigma_0^{*}\nu_{12}^{*}E_1^{\circ}}{\alpha E_1^{\circ}E_1^{*}}-\frac{\sigma_0^{*}\nu_{12}^{*}}{E_1^{*}}\right)-\frac{\sigma_0^{\circ}\nu_{12}^{\circ}}{E_1^{\circ}}\cdot\frac{g(1+\nu_{23}^{\circ})}{2g+f(1-\nu_{23}^{\circ})}, \qquad (20)$$

$$D=-E_2^{*}(1+\nu_{23}^{*})(f+g)(1-f-g)c^2\frac{\sigma_0^{\circ}\nu_{12}^{\circ}E_1^{*}-\sigma_0^{*}\nu_{12}^{*}E_1^{\circ}}{\alpha E_1^{\circ}E_1^{*}}, \qquad (21)$$

where

$$\alpha=(1-f-g)\left(\left(E_2^{*}\nu_{23}^{\circ}-E_2^{\circ}\nu_{23}^{*}\right)f-E_2^{*}(f+2g)\right)-$$
$$-E_2^{\circ}(1+f+g)f.$$

We find ratios σ_0^{*} and σ_0° from the third equality (5). By denoting:

$$d^{\circ}=\frac{\alpha+\beta\nu_{12}^{\circ}}{\alpha E_1^{\circ}}, \qquad (22)$$

$$d^{*}=\frac{\alpha+\beta\nu_{12}^{*}}{\alpha E_1^{*}}, \qquad (23)$$

where

$$\beta=2(f+g)\left(\nu_{21}^{\circ}E_2^{*}(1-f-g)+\nu_{21}^{*}E_2^{\circ}f\right),$$

we obtain:

$$d^{\circ}\sigma_0^{\circ}=d^{*}\sigma_0^{*}. \qquad (24)$$

Next, consider a similar problem for a homogeneous transtropic material that simulates the behavior of a composite (Fig. 2, *b*).

In this case, the field of stresses will be determined by the following ratios:

$$\sigma_{zz}=\sigma_0,\ \ \sigma_{rr}=0,$$

$$\sigma_{\theta\theta}=0,\ \ \sigma_{zr}=\sigma_{\theta z}=\sigma_{r\theta}=0. \qquad (25)$$

In order to match the conditions of equilibrium for both problems, it is required that the following condition should be satisfied:

$$\pi(a^2-c^2)\sigma_0^\circ+\pi\left(b^2-a^2\right)\sigma_0^*=\pi b^2\sigma_0. \qquad (26)$$

Proceeding to the volumetric shares of the components of a composite, we obtain:

$$\sigma_0^\circ f+\sigma_0^*\left(1-f-g\right)=\sigma_0. \qquad (27)$$

We obtain with respect to (24):

$$\sigma_0^*=\frac{\sigma_0 d^\circ}{fd^*+d^\circ\left(1-f-g\right)},$$

$$\sigma_0^\circ=\frac{\sigma_0 d^*}{fd^*+d^\circ\left(1-f-g\right)}. \qquad (28)$$

Displacements are determined from formulae:

$$u_r\left(r\right)=-\frac{\nu_{21}}{E_2}\sigma_0 r+C_1;\;\; u_z\left(z\right)=\frac{1}{E_1}\sigma_0 z+C_2. \qquad (29)$$

The constants C_1=C_2=0, given that the conditions $u_r(0)$=0 and $u_z(0)$=0 are satisfied for this problem, which is why the expressions for displacements take the form:

$$u_r\left(r\right)=-\frac{\nu_{21}}{E_2}\sigma_0 r;\;\; u_z\left(z\right)=\frac{1}{E_1}\sigma_0 z. \qquad (30)$$

The effective constants will be obtained using the conditions for displacements alignment, obtained as a result of solving the problems considered:

$$u_r\left(b\right)=u_r^*\left(b\right);\;\; u_z\left(h\right)=u_z^\circ\left(h\right)=u_z^*\left(h\right). \qquad (31)$$

Then the second ratio from ratios (31), with respect to (11) and (30), will be recorded in the form:

$$\frac{1}{\left(1-\nu_{23}^*\right)}\left(\frac{\sigma_0^*\left(1-\nu_{23}^*-2\nu_{12}^*\nu_{21}^*\right)}{E_1^*}-2A\nu_{21}^*\right)=\frac{1}{E_1}\sigma_0. \qquad (32)$$

Considering (18) and (28), following the transforms, we obtain a formula for determining the effective longitudinal elasticity modulus of the composite material with a transtropic matrix and hollow fiber:

$$E_1=E_1^*\frac{\alpha}{d^\circ\alpha-\gamma}(d^* f+d^\circ(1-f-g)), \qquad (33)$$

where

$$\gamma=2(f+g)f\left(\nu_{21}^\circ E_2^*\frac{\nu_{12}^*}{E_1^*}-\nu_{21}^* E_2^\circ\frac{\nu_{12}^*}{E_1^\circ}\right).$$

From the first condition from conditions (31), taking into consideration (18), (19) and (28), we obtain a ratio for the Poisson's coefficient ν_{12}:

$$\nu_{12}=\frac{\nu_{21}^*\nu_{12}^* d^\circ\alpha-\gamma}{\nu_{21}^*(d^\circ\alpha-\gamma)}. \qquad (34)$$

Thus, we have derived dependences for the effective elastic constants of a composite – longitudinal elasticity module E_1 and Poisson's coefficient ν_{12} – on the characteristics of the transtropic matrix and transtropic hollow fiber.

5. Numerical calculation of the effective elastic constants of composite materials with hollow fibers

It should be noted that at g=0 the formulae obtained completely coincide with formulae [2] for determining the elastic constants of the composite material with a transtropic matrix and fiber.

Let us compare values for the longitudinal elasticity module E_1 and the Poisson's coefficient ν_{12}, obtained from formulae (33) and (34), respectively, and from formulae (1) to (3), by varying the volumetric content of the fiber and cavity. First, we calculate the composite with isotropic components, the epoxy matrix EDT-10 and the fiber made from alumoborosilicate glass, whose elastic characteristics are: $E^\circ=7.31\cdot 10^4$ MPa, $\nu^\circ=0.25$, $E^*=2900$ MPa, $\nu^*=0.35$ [22].

Results of the calculations are summarized in Table 1, assuming that f+g=0.4.

Table 1

Values for the longitudinal elasticity module E_1 and the Poisson' coefficient ν_{12}, calculated from formulae (33), (34), and (1) to (3)

Volumetric share		E_1, MPa			ν_{12}	
g	f	(33)	(1)	(3)	(34)	(2)
0	0.4	30,989.12	30,980.00	30,989.12	0.3040	0.3040
0.05	0.35	27,334.08	27,325.00	27,334.08	0.3042	0.3042
0.1	0.3	23,679.94	23,670.00	23,679.03	0.3045	0.3045
0.15	0.25	20,025.00	20,015.00	20,023.96	0.3048	0.3048
0.2	0.2	16,368.86	16,360.00	16,368.86	0.3053	0.3053
0.25	0.15	12,715.18	12,705.00	12,713.69	0.3062	0.3062
0.3	0.1	9,060.26	9,050.00	9,058.38	0.3078	0.3077
0.35	0.05	5,405.01	5,395.00	5,402.57	0.3119	0.3118

It should be noted that the results obtained by different methods are almost the same. Specifically, the maximum relative error of calculating E_1 applying the proposed method, when compared with formula from [13], is 0.2 %; when compared with formula from [15], 0.05 %. The maximum relative error of calculating the Poisson's coefficient $\nu_{12,}$ when compared with method from [13], is equal to 0.03 %.

The dependence of the longitudinal elasticity module E_1 on volumetric share of the cavity demonstrates a clearly pronounced descending character, which is consistent with the physical meaning of these indicators. Values for the Poisson's coefficient ν_{12} grow with an increase in the volumetric share of the cavity.

A similar calculation of the effective elasticity constants E_1 and ν_{12} will be performed for the composite with an isotropic matrix and the transtropic fiber. We employ data for the composite UD PFRP with the fiber made of the high density polyethylene VHDPE Tenfor SN1A ($E_1^\circ=60.4$ GPa, $E_2^\circ=4.68$ GPa, $\nu_{12}^\circ=0.38$, $\nu_{23}^\circ=0.55$, $G_{12}^\circ=1.65$ GPa) and the matrix made from the epoxy resin Ciba-Geigy 913 ($E^*=5.55$ GPa, $\nu^*=0.37$) [1]. We shall construct dependence graphs of elastic constants on the volumetric share of cavity g at fixed values for the volumetric share of fiber f (Fig. 3).

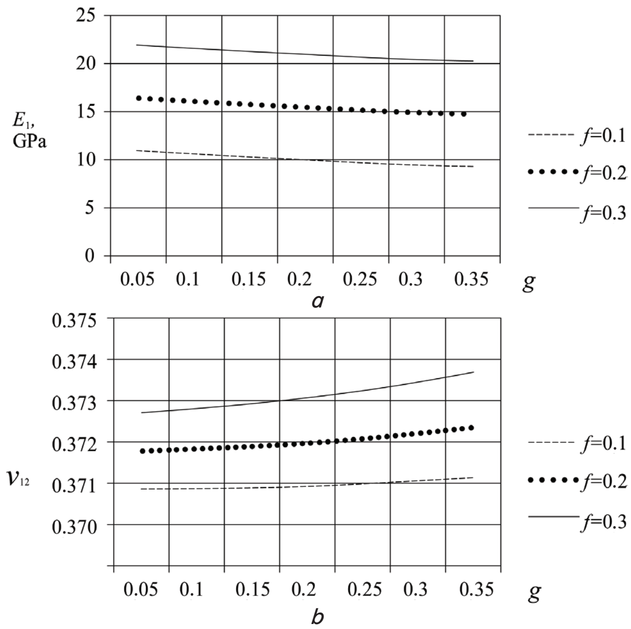

Fig. 3. Dependence of elastic characteristics, calculated from formulae (33) and (34), on volumetric share of the cavity in a fiber at fixed share of fiber: *a* — longitudinal elasticity module E_1; *b* — Poisson's coefficient ν_{12}

The dependences of longitudinal elasticity module on volumetric share of the cavity at the fixed values for f are descending in nature, close to the linear ones (Fig. 3, *a*). The Poisson's coefficient, in contrast, grows with an increase in g (Fig. 3, *b*).

6. Discussion of results of applying the proposed formulae

The presence in the derived analytical formulae of characteristics for the hollow fiber makes it possible to assess the impact of the existence of a cavity on values for the effective constants. That provides an opportunity to design the optimal composite materials in terms of strength, stability, thereby reducing the weight and cost of fabrication of such materials. Such a design process is aimed at increasing the elasticity modules, improving the resistance to deformation without increasing the absolute mass of the respective structural material. The application of the proposed analytical dependences for determining the effective characteristics of composites is appropriate in terms of time cost compared with the use of numerical and experimental methods.

Taking the transtropic properties of the components into consideration makes it possible to refine the effective elastic constants of the composite, which is rather important in some cases. Thus, for some types of fibers the longitudinal characteristics of the material differ from the transversal ones by one to two orders of magnitude.

In contrast to the experimental and numerical studies, the derived analytical ratios make it possible to perform their qualitative analysis and obtain the optimal values for parameters (volumetric content of fiber, the type of components, dimensions of the cavity).

The ratios derived are valid for the hexagonal arrangement of fibers, which limits their application for a wide range of composites.

The composite material that we considered is transtropic and its elastic properties are characterized by five constants. That is why the formulae for E_1 and ν_{12} are not enough to investigate the stressed-strained state of the structural elements made from it. In the future, it is planned to employ the proposed procedure for determining the transversal module of elasticity E_2, the Poisson's coefficient ν_{23}, and the shear module G_{12}.

7. Conclusions

1. The result of the study conducted is the derived formulae for the effective longitudinal elasticity module E_1 and the effective Poisson's coefficient ν_{12} for the composite with transtropic components. They reflect the dependence of these constants on elastic characteristics of the matrix, fiber, and volumetric shares of the fiber and the cavity inside a composite's material.

2. We have compared values for the elastic constants, calculated using the proposed procedure, with results obtained by previously known ratios for the isotropic components. The maximum relative error of calculating the module of elasticity and the Poisson's coefficient based on the proposed ratios, when compared with known formulae, does not exceed 0.2 %.

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