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CONSTRUCTION OF THE METHOD FOR BUILDING ANALYTICAL MEMBERSHIP FUNCTIONS IN ORDER TO APPLY OPERATIONS OF MATHEMATICAL ANALYSIS IN THE THEORY OF FUZZY SETS

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Розглянуто чотири методи знаходження параметрів аналітичних виразів сигмоїд, дані про які подані чисельно. Проведено порівняльний аналіз ефективності апроксимації сигмоїдами, за допомогою методу найменших квадратів, прямим розрахунком констант по значенням у точках порогу рівноваги та насичення, розвиненням Тейлора і сплайнами на прикладі із різними порогами рівноваги, чутливості, насичення. Продемонстровано, що простим за алгоритмом знаходження всього двох коефіцієнтів є пряме обчислення двох констант за точками порогів рівноваги чутливості або насичення. Показано, що при апроксимації сигмоїдами за методом найменших квадратів похибка апроксимуючої функції залежить від симетричності добору точок сітки по відношенню до порогу рівноваги. Досліджено алгоритми конструювання функцій належності, на основі двох базисних функцій – сигмоїд двох типів спалаху та спаду. Побудовано набір стандартних функцій належності трикутника, трапеції, прямокутника у вигляді операції добутку. Сформульовано умови, за яких утворюються криволінійні форми функцій належності та вплив на величину відхилень коефіцієнтів апроксимації, досліджено властивості повноти та достатності.

Продемонстровано, що такий шлях формування функцій належності за сукупністю чисельних значень, як сплайн апроксимація не дозволяє забезпечити вимогу до обмеженості інтервалу області значень.

Отримано загальний розв'язок задачі оптимізації за допомогою аналітичних функцій належності та виконано порівняння із результатами її розв'язку у постановці Беллмана-Заде.

Проаналізовано властивості трансформованих операцій над нечіткими множинами на прикладі задачі про оптимізацію. Продемонстровано, що розв'язок у такій новій постановці має дві переваги. По-перше, він отримується з допомогою операцій пошуку оптимуму методами класичного математичного аналізу із використанням умов стаціонарної точки та умов незмінності знаків других похідних. По-друге, його пошук здійснюється з використанням операцій диференціювання та знаходження кореня, навіть за умов нелінійності, загально відомими методами Ньютона-Канторовича або рекурентної апроксимації

Ключові слова: аналітичні функції належності, нечіткі операції, стандартний набір, алгоритм конструювання, властивості повноти, задача оптимізації

1. Introduction

Universal processes of design and operation of hierarchical economic and organizational-technical structures, as well as their development, leads to the search for generalized description methods, the creation of generalized algorithms of functioning and control over the hierarchically structured objects [1–4]. The characteristic feature of control approaches, which have been successfully implemented recently in various areas of economic and financial analysis, is the application of the operations of fuzzy sets [1]. No less important are the latest results obtained in the practical implementation of fuzzy sets tools for the formation of conclusions in the theory of decision-making [4]. A standard set – one-sided flash and recession, a triangle, a trapezoid, a rectangle – enables the decomposition of arbitrary membership functions [4]. However, the discrete character of assigning the membership functions, as well as other characteristics that are defined by using the expert assessments, complicates the simultaneous application of known analytical methods of optimal analysis [2, 3]. That manifests itself vividly in the design and operation of technical systems.

The development of airborne [5–7] and underwater [8–10] crewless and unmanned vehicles (CUV) has formed the preconditions for solving the dual-purpose problems, in air, above water, and under water [11]. One of the generalized methods for describing the functioning under rapidly changing operation conditions is the recurrent restructuring of analytical forms of model representation [12]. The latter contributes to the transition from the paradigm of classical mechanics that implies «description», mandating the determining of a trajectory in a state space, to the cybernetic «prescription» [13]. According to the paradigm of «prescription», one defines the initial and final states only, while a transition through intermediate states is regulated by a set of constraints [13]. As shown in papers [14, 15], a transition through intermediate states turns out to be effective at the implementation of theoretical bases of coordination as a comprehensive procedure [16]. However, the implementation of the uncertainty conditions when describing the processes of planning, preparation for the execution of governing activities or technological operations, as well as the search for and ensuring the activation of technological processes and control over parameters, inevitably results in the application of orders [14]. Such an application of a fuzzy-set representation and manipulating high-quality – linguistic variables – is the inevitable necessity for modern processes of control over organizational-technological objects [14]. At present, such variables are described in terms of fuzzy sets; operating them implies the apparatus of the fuzzy set theory only [17].

Thus, the operations of the theory of fuzzy sets is in contradiction with existing methods for optimization and the mathematical analysis, and, along with the non-analytical character of membership functions and operations over them, becomes the main obstacle to optimization, thereby forming a problem that defies solution. Solving the task on ensuring the analyticity of a standard set of membership functions is the prerequisite for the application of the developed apparatus of functional and mathematical analysis and optimization methods in problems on the design of and control over organizational-technical systems.

2. Literature review and problem statement

An inseparable interconnection between quantitative and qualitative quantities, which is implemented through the conditional system of reference etalons defined by the chamber of Weights and Measures, establishes the possibilities and advantages of measuring spaces and the analytical quantitative description of quantities [17]. However, the benefits of quantitative representation of quantities [1] in the era of the fourth scientific-technical revolution are somewhat blurred due to the necessity to measure qualitative notions [2] and the scripts of activities [3]. Especially attractive for studying and describing is the effectiveness of phenomenon of well-coordinated activities manifested by the self-organization of herd animals and groups of people. Recently, the results of these investigations have been studied in more detail and are transferred onto the area of objective achievements, even when they are implemented at inanimate anthropogenic systems and structures [18]. The demonstration of the successful transfer of the principle of self-organization in order to construct the dual-purpose heterogeneous networks and to determine their technical and technological parameters is paper [18]. Another example is its application to study CUV, which are operated in air, on the ground, above- and underwater [19]. That study also recognized the need for and benefits of theoretical bases of coordination [14]. The introduction of coordination as a comprehensive procedure that is carried out along with the use of modern tools, such as adequacy, effectiveness, vector-indicator, and intelligent expansion into a Taylor's series, will open up new opportunities [16]. The basis and preconditions for the expectations and opinions about control possibilities at qualitative description of processes is work [20]. It simplified the processes of comparison and decision making by comparing the numerical representation of values for membership functions, which is also confirmed using the example of a green building [21]. In addition, despite the simplicity and attractiveness for numerical analysis, the implementation of this approach, by comparing the sections of domains of the values sets of the equivalent level, does not make it possible to employ the operations of differentiation [22]. The issue is exacerbated when stating a problem on optimal control, as is demonstrated in [23]. The process is particularly complicated when it is necessary [24], in order to synthesize controlling influence, to apply at the same time the quantitative variables [25] and the fuzzy ones. Handling the specialized controllers [26], when factors of a different physical nature are represented by linguistic estimates [27] or numerical values for membership functions, while the objective function is nonlinear, is complicated [28]. There are examples of using a general approach to control the non-linear objects, demonstrated in paper [29]. They are implemented through quasi-linearization or by applying recurrent approximation [30]. However, they cannot be implemented without analytical expressions for membership functions because the implementation of algorithms for optimal control requires differentiation to form expressions for the derivates of first and second order from the objective functions and the Lagrange functions [30]. No less important are examples of application of control procedures, which are based on the results of processing information flows of 3-dimensional images [31], during stabilization of the miniature airborne CUV [32]. Another example of the implementation of fuzzy control systems is the navigation system of quadrocopters, which is demonstrated, for example, in paper [33]. An important step in the development of navigation algorithms is the simultaneous formation and representation of a trajectory at the image of the terrain map that is being formed [34]. The latter also makes it possible to use it additionally as a platform for the robotic research and training [35]. Additional control over features of images in the system of localization and control improves quality of navigation [36]. Control over navigation systems of ground-based mobile robotic vehicles also meets the need to modify the formal fuzzy model of motion based on the assigned trajectory taking obstacles into consideration [37]. One of the conclusions of this work is the need to differentiate membership functions, which could be overcome, as proposed, by using the method of «ultra-fast annealing». This method, according to estimates by the authors, is labor-intensive, and its convergence, in addition, has not been proven [37]. The introduction of uncertainties and the application of methods from the theory of fuzzy sets and decision-making, which was successfully proven in paper [38], would simplify construction of the trajectory and to execute motion control. A more effective technique is to combine a method of direct relative orientation [39] and the method to rapidly determine the angle [40]. Also promising is the navigation method based on an analysis of the stream of video images [41]. Especially so if one relies on the active determining of coordinates according to data from a laser rangefinder at points, which are recorded on a video camera and the idea of multi-agent approximation [42]. To a larger extent, the characteristics of algorithms depend on success in improving the quality and security of processes to code-decode video frames and the quality of compression [43] or representing through the operations of matrix algebra [44]. In addition, it should be noted that the quality of navigation systems operation becomes dependent on the stability of wireless communication between movable agents of cyber-physical systems [45]. Stability and security of communication and navigation will be largely provided under conditions of achieving considerable success in the development of hardware-oriented software and high-speed architectures of coding – decoding. In addition, the approximation efficiency is a decisive factor, which would employ simple and analytical expressions, and whose accuracy would not be inferior to the spline approximation.

Thus, the papers considered above allow us to ague on that uncertainty, as the main feature, will dominate in the nearest future over the tasks on control over organizational-technical systems. Certainly, the examples of successful implementation of fuzzy systems are the systems of support and decision-making in maritime technology [47]. Further development will be accompanied by an unequivocal transition to the implementation of the concept of a modular cyber-physical system for early diagnosis of industrial equipment that is based on the standards of Industry 4.0 and in particular the methodology of the Internet of Things [48]. A special role in this case will belong to a neural-fuzzy observer, whose operation in turn would require the enlargement of computer libraries of fuzzy models to operate fuzzy asymmetrical numbers [49]. In the same time, the simplicity of successful construction of software that is based on the implementation of fuzzy-set controllers and practical control over acceleration and lifting force for a drone has been demonstrated in paper [50]. Despite the overall efficiency of fuzzy-set control over a single CUV, the main problem to control the maneuver of a group of CUV is the impossibility of applying optimization methods for the nonlinear dynamics models, which predetermines the non-analytical character of membership functions. Development of analytical tools and an analytical method of recurrent approximation, whose effectiveness has been proven for various technical and biological systems, opens up new opportunities [11, 16, 19, 30, 51]. In this regard, combining the advantages of the two paradigms of control, the first of which is based on the fuzzy-set, and the second is on the analytical representation of informational flows [52], is a reasonable, logically justified task. The development of the specified field of designing control systems for mobile collective cyber-systems would be ensured only when using the specialized methods of network technologies for calibration [53].

Thus, combined implementation of the methods of classical and fuzzy control faces the non-analytic character and the non-differential nature of membership functions. In addition, the form to assign a standard set of membership functions (one-sided flash and recession, triangular, rectangular, and trapezoidal) that contain gap points prevents, when using analytical methods, the formation of solutions to systems and equations in the processes of synthesis of controllers and the synthesis of controlling influence.

3. The aim and objectives of the study

The aim of this study is to construct a method for building the membership functions that would meet the conditions for analyticity and would be able to ensure the analytical formation of the magnitude of control influence based on the optimal selection of the nonlinear model functioning.

To accomplish the aim, the following tasks have been set:

– to explore the effectiveness of methods for finding the constants of analytical membership functions using the numerical data based on the approximation;

– to construct a system of base analytical membership functions and an algorithm of the structure of the standard set of membership functions that are commonly used;

– to examine the properties of membership functions – continuity, differentiation, completeness and sufficiency and applicability to solving optimization problems.

4. Statement of the task to examine the methods of formation of analytical membership functions using numerical data based on approximation

Consider the quantitative one-dimensional numerical set that has, for the stated problem, a known or a conditionally selected maximum value. We accept its maximum value to be a scale and then introduce the relative dimensionless quantitative magnitude, which will now be normalized and located in a strictly defined range $x\in[0,1]$. We shall assign in a two-dimensional space of actual numbers for an arbitrary qualitative magnitude a set A, whose membership function $\mu_A(x)$ has a value domain belonging to the range $x\in[0,1]$.

Assume that $\mu_A(x)$ acquires the shape of a sigmoid and is a continuous and differentiated function whose values are given in Table 1. It is also characterized by three points: the threshold of sensitivity, equilibrium, and a saturation boundary value or the threshold of early saturation – with respective values for abscissas and ordinates: $(x_a, \mu_A(x_a))$; $(x_c, \mu_A(x_c))$; $(x_b, \mu_A(x_b))$.

We shall state the problem on the approximation of numerically given functions using expression:

$$\mu(x)=\frac{1}{1+A\exp\left(-k\left(x-x_c\right)\right)}. \tag{1}$$

This problem can be solved by several methods.

The first method. The introduction of denotations and new coordinates X and Y:

$$X=x-x_c,\ \ Y=\ln\left[\frac{1}{\mu(x)}-1\right];\ \ a=\ln A; A=\exp(a), \tag{2}$$

would reduce (1), by using the method of coordinates smoothing, to a straight line:

$$Y=a-kX,$$

and we shall search for constants a and k by employing a method of least squares:

$$S_1=\sum_{1=1}^{9}Y_i;\ \ S_2=\sum_{1=1}^{9}X_i;\ \ S_3=\sum_{1=1}^{9}Y_iX_i;\ \ S_4=\sum_{1=1}^{9}X_i^2;$$

$$k=\frac{9S_3-S_1S_2}{S_2S_2-9S_4};\ \ a=\frac{S_1+kS_2}{9}. \tag{3}$$

Table 1

Values for the sigmoid membership function of the relative angular velocity of propeller for different inflection coordinates x_c

x	$\mu(x)$						
	x_c=0.2	x_c=0.3	x_c=0.4	x_c=0.5	x_c=0.6	x_c=0.7	x_c=0.8
0	0	0	0	0	0	0	0
0.1	0.001	0.0005	0.00033	0.00025	0.0002	0.00016	0.0001428
0.2	0.5	0.001	0.00066	0.00050	0.0004	0.00033	0.0002856
0.3	0.999	0.5	0.001	0.00075	0.0006	0.00050	0.0004284
0.4	0.9991428	0.999	0.5	0.001	0.0008	0.00066	0.0005712
0.5	0.9992856	0.99916	0.999	0.5	0.001	0.00083	0.0007140
0.6	0.9994284	0.99933	0.9992	0.999	0.5	0.001	0.008568
0.7	0.9995712	0.99950	0.9994	0.99925	0.999	0.5	0.001
0.8	0.9997140	0.99966	0.9996	0.99950	0.99933	0.999	0.5
0.9	0.9998568	0.99983	0.9998	0.99975	0.99966	0.9995	0.999
1	1	1	1	1	1	1	1

The second method. Direct calculation of the new variable k as the solution to equation (1) after calculating its values for each of the coordinates x_c та x_b:

$$k=\frac{1}{(x_b-x_c)}\ln\left[\frac{\mu(x_b)A}{1-\mu(x_b)}\right];\ \ A=\frac{1}{\mu(x_c)}-1. \tag{4}$$

The third method. A direct method for computing the first and second derivatives, search for the type of approximating functions and a direct approximation.

The fourth method. Spline approximation using piecewise functions or the set of them under conditions for derivatives coalescence at grid points.

5. Forming a standard set of membership functions that are the most sought after in the problems of control over organizational-technical systems

Suppose that a sigmoid flash function is described by expression:

$$\mu_{up}(x)=\frac{1}{1+A_1\exp(-k_1(x-x_{c1}))}, \tag{5}$$

and the sigmoid recession function takes the form:

$$\mu_{down}(x)=\frac{1}{1+A_2\exp(k_2(x-x_{c2}))}. \tag{6}$$

Under conditions when $x_{b1}=x_{b2}$, the product of two sigmoid functions (5) and (6) yields a triangular membership function – one of the five in a standard set:

$$\mu_{triangle}(x)=\prod_{i=1}^{2}\frac{1}{1+A_i\exp\left((-1)^i k_i(x-x_{ci})\right)}. \tag{7}$$

Under conditions when $x_{b2}=x_{b1}+d$, the product of two sigmoid functions (5) and (6) produces the second one from a standard set – a trapezoidal membership function with a length of the upper base of magnitude d:

$$\mu_{trapezium}(x)=\prod_{i=1}^{2}\frac{1}{1+A_i\exp\left((-1)^i k_i(x-x_{ci})\right)}. \tag{8}$$

It should be noted that if x_{a1} tends to x_{b1} and at the same time x_{a2} tends to x_{b2}, the trapezoid tends to the rectangle. In addition, if a segment between the thresholds of sensitivity and saturation has a finite and variable curvature, then the shapes formed of them will be a triangle, a trapezoid, a quadrilateral with curved sides. Thus, the algorithm of the structure of a standard set of membership functions is divided into the following stages. Based on analysis, the data on a membership function are split into the links of flash and recession, for which one finds such parameters that make it possible to describe the sigmoid of flash and recession, and, based on them, one constructs a triangle's membership function using expressions (7) and trapezoids applying expression (8), which under the conditions specified above turns into a rectangle.

Thus, the standard set of membership functions: one-sided flash and recession; a triangle; a trapezoid; a rectangle – have been completely restored, which ensures a one hundred percent degree of completeness of the standard set of membership functions. The application of operations of decomposition and composition makes it possible to construct analytical forms of the more complex types of membership functions.

6. Solving the optimization problem in the Bellman-Zadeh statement

Suppose that a fuzzy goal is assigned on the universal set X by a membership function $\mu_G(x)$. In addition, there are the restrictions that define the set's membership of the undesired parameters $\mu_{Ci}(x)$, where i runs from unity to n. The effect of each constraint is taken into consideration by the weight coefficients λ_i, then a solution to such a problem shall be stated as operations to find a maximum from the transformed operation of intersection of sets [19] taking into consideration the weighting coefficients:

$$\max_x\left\{\prod_{i=1}^{n}\mu_G(x)\lambda_i\left[1-\mu_{Ci}(x)\right]\right\}. \tag{9}$$

It is easy to see from the analysis of product's boundary of expression (9) that at the point of the lowest value for one of the multipliers the intersection of sets is the upper limit [19], and hence the solution to the problem in the Bellman-Zadeh statement is a boundary one for it. However, in contrast to the problem in the Bellman-Zadeh statement, a point of maximum could be found in line with the classic method of finding an extremum from the expression that is recorded in braces in expression (9).

7. Modelling, comparison, and discussion of the suitability of methods to build the membership functions

To compare the effectiveness of methods for constructing a membership function using numerical data, we shall perform the approximation in line with the first and the second method. By employing the method of straightening of coordinates and the least squares method, based on data from Table 1, we shall calculate constants A and k. Comparison between source data (Table 1) and the values derived by approximating the membership functions $\mu(x)$ and the relative error $\varepsilon(x)$ are given in Table 2 for four x_c values.

An analysis of the comparison between the absolute magnitude of deviations and the relative error has revealed that a given approach produces good results under conditions for the equal representation of points. However, the values for the absolute and the relative error itself are significant. In most cases, its high magnitude is predetermined by the comparison to very low magnitudes of values for the membership function. The absolute error, which is small compared to scale, reaches hundreds of percent in the local measurement. And the algorithm itself is more complex in comparison with the algorithm for constructing a membership function in line with (4). The results of calculations, based on it, are given in Table 3.

An analysis of results from Table 3 indicates that the magnitude of the absolute and the relative error when applying the second method is less. We especially point to that the curves pass through the thresholds of sensitivity and saturation, as well as the equilibrium points, thereby ensuring the qualitative and quantitative representation of its behavior. At these points, the relative error is almost zero. The method itself, when compared with the method of least squares, is a simple since it is calculated through the points of threshold and the values for these membership functions are only two constants A and k.

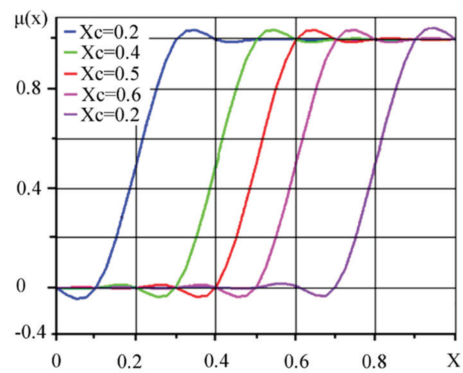

Fig. 1. The character of representation of a membership function based on the cubic spline approximation [46]

Table 2

Values for the relative error of sigmoid membership function for different inflection coordinates x_c that are calculated in line with (3)

x	$x_c=0.2$; $A=0.501517594$; $k=-14.9349013$		$x_c=0.4$; $A=2.071067933$; $k=-24.55844382$		$x_c=0.6$; $A=0.48659115$; $k=-24.53101123$		$x_c=0.8$; $A=1.374079281$; $k=-15.32287194$	
	$\mu(x)$	$\varepsilon(x)$	$\mu(x)$	$\varepsilon(x)$	$\mu(x)$	$\varepsilon(x)$	$\mu(x)$	$\varepsilon(x)$
0.1	0.3093	−308.30	0.0003	0.076408	9.6E−06	0.9515	1.6E−05	0.888055
0.2	0.6659	−0.3319	0.0035	−4.3654	0.00011	0.71866	7.3E−05	0.740931
0.3	0.8988	0.1003	0.0398	−38.776	0.0013	−1.1778	0.00034	0.200772
0.4	0.9753	0.0238	0.3256	0.348761	0.0149	−17.726	0.00159	−1.77113
0.5	0.9944	0.0049	0.8491	0.150022	0.1502	−149.23	0.00729	−9.20289
0.6	0.9987	0.0007	0.9849	0.014226	0.6726	−0.3454	0.0328	−2.8342
0.7	0.9997	−0.0001	0.9987	0.000706	0.9598	0.0392	0.13586	−134.865
0.8	0.9999	−0.0002	0.9999	−0.00029	0.9964	0.00292	0.42129	0.157568
0.9	0.9999	−0.0001	0.9999	−0.00019	0.9996	−3E−05	0.7711	0.228133

Table 3

Values for the relative error of sigmoid membership function for different inflection coordinates x_c, which are calculated in line with (4)

x	$\varepsilon(x)$						
	$x_c=0.2$	$x_c=0.3$	$x_c=0.4$	$x_c=0.5$	$x_c=0.6$	$x_c=0.7$	$x_c=0.8$
0.1	1.301E−15	0.997996	0.999997	0.999999	1	1	1
0.2	0	−4.77E−15	0.99848	0.99999	0.999999	1	1
0.3	1.111E−16	0	3.903E−15	0.9987	0.99999	1	1
0.4	−0.000857	1.111E−16	0	−4.3E−16	0.99874	0.9999	0.9999
0.5	−0.000715	−0.000839	1.111E−16	0	−4.34E−16	0.9997	0.99999
0.6	−0.000572	−0.000670	−0.000799	1.11E−16	0	0.5	0.99988
0.7	−0.000429	−0.000500	−0.000600	−0.00075	1.11E−16	0	7.58E−15
0.8	−0.000286	−0.000340	−0.000400	−0.00050	−0.00067	−0.0005	0
0.9	−0.000143	−0.000170	−0.000200	−0.00025	−0.00034	−0.0005	1.11E−16

In order to compare and analyze the possibilities of the third method, we shall expand a membership function into a Taylor's series:

$$\mu(x)=\mu(x_0)+\mu'(x_0)\Delta x+ + \frac{1}{2}\mu''(x_0)\Delta x^2.$$

Suppose that in the interval of determining a membership function $\mu(x)$, its first $\mu'(x)$ and second $\mu''(x)$ derivatives have an error whose estimation is denoted $\|\varepsilon\|$, then the error of the third technique is estimated as:

$$\|\mu(x)(1+\varepsilon)\| \le \le \begin{bmatrix} |\mu(x_0)| + \\ +|\mu'(x_0)||\Delta x| + \\ +\frac{1}{2}|\mu''(x_2)|\Delta x^2 \end{bmatrix}_{\max} \times \times \left(1+\|\varepsilon\|\right). \qquad (10)$$

Thus, the third method is difficult to implement because it requires the approximation of the function and derivatives of the first and second order. In addition, the formula for error estimation (10) indicates that the maximum possible error is greater than that when using the first method to form sigmoid functions based on numerical data on the membership function.

In order to compare to the first two methods and to analyze capabilities of the fourth method, we shall confine ourselves to an example of the application of cubic spline approximation in line with algorithm [46]. In terms of the initial concept, the spline approximation of a function has gaps in derivatives at grid points, however, according to the results of solving the problem on finding the constants of approximation using variational methods this drawback is eliminated due to the condition of the merger of derivatives [46]. Comparison of numerical data for a membership function at different values for the sigmoid equilibrium coordinates x_c is shown in Fig. 1. As evidenced by the curves, cubic approximation is oscillatory in nature and its curves move beyond the values domain. In addition, the need to memorize for n grid points $4(n-1)$ values for the coefficients of approximation complicates the further application of such an approximation.

8. Discussion of results of investigating the techniques to construct the analytical membership functions

The research that we conducted into methods of construction of a system of base functions has demonstrated the advantage of the method for approximating sigmoids using the values at points of thresholds of sensitivity and saturation, as well as the point of equilibrium. The simplicity of expressions for finding the coefficients, a small error, and the qualitative and quantitative representation of sigmoid behavior, determine the advantage of the proposed technique. In addition, when constructed in such a way, the sigmoid function that is chosen as the base and is described by expression (1), has at every point in the interval of its determining $x\in[0, 1]$ a limit equal to the value for the function itself, and hence it is continuous. Additionally, based on the definition for any continuous function, it is differentiable. In addition, because the first and the following n of its derivatives in the determining interval are also not discontinuous, it is differentiated $n+1$ times. The functions that were created by it are the product, which is why, according to the rule of differentiation of the product of two functions, their derivative produces the analytic function, since it is the sum of the products of derivative from the multiplier and another multiplier, each of which is an analytical function.

Thus, the further direction of the implementation of results and continued study into formation of analytical membership functions is a practical combined application of the apparatus of fuzzy sets and mathematical analysis to solve the problems on synthesis of control influence on techno-economic systems or problems on the analytical construction of controllers.

9. Conclusions

1. The method for determining parameters of sigmoids based on information about the thresholds of sensitivity, saturation, and the point of equilibrium, is more effective in terms of the criteria of simplicity and accuracy than the method of least squares, expansion into a Taylor’s series, and spline approximation. It contains only two algebraic operations of direct calculation of their parameters and ensures a one hundred percent degree of completeness of the standard set of membership functions.

2. The introduced analytical sigmoids of flash and recession, combined with the simplified method for finding their parameters based on the information about the points of thresholds of sensitivity, saturation, and the equilibrium point, are simple in construction and are sufficient to build a standard set of five types of membership functions.

3. The proposed algebraic operation of multiplication over sigmoids of flash and recession is the base for the construction of a general set of membership functions for a triangle, a trapezoid, and a rectangle, with sides that are approaching straight lines according to the displacement of thresholds of sensitivity, saturation, and equilibrium.

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