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INVARIANTS AND FIRST INTEGRALS FOR A SPECIAL CASE OF A CONTROLLED PROCESS IN AN ACTIVE AVIATION SYSTEM

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Знайдено Гамільтоніани теоретично розглянуто-го частинного випадку варіаційної задачі керування в активній авіаційній системі, діючій в умовах багатальтернативності та конфліктів. Виконано математичне моделювання: для канонічних розподілів індивідуальних переваг, отриманих за допомогою рівнянь Ейлера-Лагранжа; для диференційних рівнянь другого порядку, з метою відшукати екстремалі керуючих функцій. Побудовано відповідні діаграми

Ключові слова: Гамільтоніан, постійна, багатальтернативність, конфлікт, безпека, функціонал, перевага, ентропія, варіація

Найдены Гамильтонианы теоретически рассмотренного частного случая вариационной задачи управления в активной авиационной системе, действующей в условиях многоальтернативности и конфликтов. Выполнено математическое моделирование: для канонических распределений индивидуальных предпочтений, полученных с помощью уравнений Эйлера-Лагранжа; для дифференциальных уравнений второго порядка, с целью отыскания экстремалей управляющих функций. Построены соответствующие диаграммы

Ключевые слова: Гамильтониан, постоянная, многоальтернативность, конфликт, безопасность, функционал, предпочтение, энтропия, вариация

1. Introduction

Solution to the related variational problems is a key for researches in the direction of control of active systems. Problem-resource approach invoke considering multi-alternativeness and conflictability of controlled by the system's active element situations.

Urgency of the researches. It is always important to find some value or values that remain invariant or constant in the given problem formulation. Conflictability and multi-alternativeness need those values to be discovered and analyzed.

Analysis of the latest researches and publications. We feel the necessity to continue the researches initiated in [1] and are going to add some new theoretical results concerning invariants and conservative values.

Subjective analysis [1-4] and calculus of variations [5] is the basis for the researches. The possible areas of the researches results applications are in the fields of aircraft powerplant [6] and ship propulsion [7, 8] operation, economics [9, 10], and environmental management science [10].

The task setting. The purpose of this paper is to uncover invariants, important dependences, regularities, and identities for a few special cases of the variational problems of control applicable to an active aviation system.

2. The main content (material)

Theoretical considerations of the particulars allow us revealing the important invariants that helps understand

the laws of thinking for making the needed controlling decisions.

2.1. The problem formulation. The postulated functional, in the context of the prototype one [1-3]:

$$\Phi_{\pi} = \int_{t_0}^{t_1} \left(-\sum_{i=1}^N \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^N \pi_i(t) F_i + \gamma \left[\sum_{i=1}^N \pi_i(t) - 1 \right] \right) dt, \quad (1)$$

where t – time; $-\sum_{i=1}^N \pi_i(t) \ln \pi_i(t)$ – entropy of subjective preferences of $\pi_i(t)$; N – number of the achievable alternatives; β, γ – structural parameters (can be considered in different situations as Lagrange coefficients or weight coefficients, reflecting some peculiarities of the *responsible for making decisions individual's psych through the endogenous parameters* of it); F_i – efficiency function related to the i th reachable alternative; $\sum_{i=1}^N \pi_i(t) - 1$ – normalizing condition.

In the simplest problem setting we consider $x(t)$ and $\dot{x}(t)$ as the efficiency functions of the two achievable alternatives with the corresponding preferences of $\pi_1(t), \pi_2(t)$. With respect to particular combinations of $x(t), \dot{x}(t), x(t)\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$, we will get eleven variants of the functional (1) with the subjective efficiency functions that include the elementary efficiency functions in the view of the linear combinations in the cases of:

- 1) $x(t)$ and $\dot{x}(t)$;
- 2) $x(t)$ and $x(t)\dot{x}(t)$;
- 3) $x(t)$ and $\frac{\dot{x}(t)}{x(t)}$; 4) $\dot{x}(t)$ and $x(t)\dot{x}(t)$;
- 5) $\dot{x}(t)$ and $\frac{\dot{x}(t)}{x(t)}$;
- 6) $x(t)\dot{x}(t)$ and $\frac{\dot{x}(t)}{x(t)}$;
- 7) $x(t)$, $\dot{x}(t)$, and $x(t)\dot{x}(t)$;
- 8) $x(t)$, $\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$;
- 9) $x(t)$, $x(t)\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$;
- 10) $\dot{x}(t)$, $x(t)\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$;
- 11) $x(t)$, $\dot{x}(t)$, $x(t)\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$.

This results in the general view functional:

$$\Phi_{11} = \int_{t_0}^{t_1} \left(-\sum_{i=1}^{N-4} \pi_i(t) \ln \pi_i(t) + \beta \left[\alpha_1 \pi_1(t) x(t) + \alpha_2 \pi_2(t) \dot{x}(t) + \alpha_3 \pi_3(t) x(t) \dot{x}(t) + \alpha_4 \pi_4(t) \frac{\dot{x}(t)}{x(t)} \right] + \gamma \left[\sum_{i=1}^{N-4} \pi_i(t) - 1 \right] \right) dt, \quad (2)$$

where α_i – coefficients that consider the differences in the measurement units.

It is convenient to consider $\alpha_1 = 1$.

Functionals derived from (2) differ in:

1. Entropies, though, even when formally their expressions are the same, the preferences functions contain the different efficiency functions, thus, the entropies are not identical;

2. Cognitive [11] functions [1] with their general view of:

$$\beta \left[\pi_1(t) x(t) + \alpha_2 \pi_2(t) \dot{x}(t) + \alpha_3 \pi_3(t) x(t) \dot{x}(t) + \alpha_4 \pi_4(t) \frac{\dot{x}(t)}{x(t)} \right]; \quad (3)$$

and all that because of the diversity in the considered set of the reachable alternatives (the alternatives number and/or their qualities are different).

The last functional (2) and cognitive function (3) are the general ones and each of the particular forms of them derives from those with the corresponding preferences $\pi_i(t)$ and coefficients α_i .

Solving the functionals obtained from (2) with the application of the necessary conditions for them to have the extremums in the view of the system of the Euler-Lagrange equations, we find:

1. The corresponding expressions for the canonical distributions of the preferences [1] in their general compact view:

$$\pi_j = \frac{e^{\alpha_j \beta F_j}}{\sum_{i=1}^N e^{\alpha_i \beta F_i}}; \quad (4)$$

2. The Euler-Lagrange equation for the sought function in the general case of (2) in the transformed view [1, P. 58, (4)]:

$$\pi_1 = \alpha_2 \dot{\pi}_2 + \alpha_3 x \dot{\pi}_3 + \frac{\alpha_4}{x} \dot{\pi}_4; \quad (5)$$

3. The generalized differential equation of the second order from (5) with respect to (4) [1, P. 58, (6)]:

$$\ddot{x} = \frac{\pi_1 + A + B + C}{D + E + F}, \quad (6)$$

where

$$A = \alpha_2 \left\{ \beta \pi_2 \left(\pi_1 + \alpha_3 \dot{x} \pi_3 - \alpha_4 \frac{\dot{x}}{x^2} \pi_4 \right) \dot{x} \right\},$$

$$B = -\alpha_3 x \left\{ \beta \pi_3 \left[\alpha_3 \dot{x} (\pi_1 + \pi_2 + \pi_4) - \pi_1 + \alpha_4 \frac{\dot{x}}{x^2} \pi_4 \right] \dot{x} \right\},$$

$$C = -\frac{\alpha_4}{x} \left\{ \beta \pi_4 \left[-\alpha_4 \frac{\dot{x}}{x^2} (\pi_1 + \pi_2 + \pi_3) - \pi_1 - \alpha_3 \dot{x} \pi_3 \right] \dot{x} \right\},$$

$$D = \alpha_2 \left\{ \beta \pi_2 \left[\alpha_2 (\pi_1 + \pi_3 + \pi_4) - \alpha_3 x \pi_3 - \frac{\alpha_4}{x} \pi_4 \right] \right\},$$

$$E = \alpha_3 x \left\{ \beta \pi_3 \left[\alpha_3 x (\pi_1 + \pi_2 + \pi_4) - \alpha_2 \pi_2 - \frac{\alpha_4}{x} \pi_4 \right] \right\},$$

$$F = \frac{\alpha_4}{x} \left\{ \beta \pi_4 \left[\frac{\alpha_4}{x} (\pi_1 + \pi_2 + \pi_3) - \alpha_2 \pi_2 - \alpha_3 x \pi_3 \right] \right\}. \quad (7)$$

It is noticeable that in all cases of the functionals derived from (2) their under-integral functions do not depend explicitly upon the independent variable t , that is upon the time. In accordance with [5], for functionals which do not depend upon the time explicitly, the function of Hamilton (Hamiltonian) represents by itself the first integral of the corresponding canonical system of the equations by Euler-Lagrange (hence, its equivalent system of the differential equations of the first order, and thus, the system of the initial differential equations of the second order, formed from the considered system of the Euler-Lagrange equations) [5].

In the given problem formulation, the Hamiltonians

$$H_{am} = -R^* + p \dot{x}. \quad (8)$$

where R^* – the under-integral function of the corresponding integral obtained from (2); p – canonical variable:

$$p = \frac{\partial R^*}{\partial \dot{x}}. \tag{9}$$

Here, on conditions of (9)

$$p_1 = \beta[\alpha\pi_2(t)], \quad p_2 = \beta[\alpha\pi_2(t)x(t)], \quad p_3 = \beta\left[\frac{\alpha\pi_2(t)}{x(t)}\right],$$

$$p_4 = \beta[\pi_1(t) + \alpha\pi_2(t)x(t)],$$

$$p_5 = \beta\left[\pi_1(t) + \frac{\alpha\pi_2(t)}{x(t)}\right], \quad p_6 = \beta\left[\pi_1(t)x(t) + \frac{\alpha\pi_2(t)}{x(t)}\right],$$

$$p_7 = \beta[\alpha_2\pi_2(t) + \alpha_3\pi_3(t)x(t)],$$

$$p_8 = \beta\left[\alpha_2\pi_2(t) + \frac{\alpha_3\pi_3(t)}{x(t)}\right], \quad p_9 = \beta\left[\alpha_2\pi_2(t)x(t) + \frac{\alpha_3\pi_3(t)}{x(t)}\right],$$

$$p_{10} = \beta\left[\pi_1(t) + \alpha_2\pi_2(t)x(t) + \frac{\alpha_3\pi_3(t)}{x(t)}\right],$$

$$p_{11} = \beta\left[\alpha_2\pi_2(t) + \alpha_3\pi_3(t)x(t) + \frac{\alpha_4\pi_4(t)}{x(t)}\right]. \tag{10}$$

Then, substituting (10) for (8)

$$H_{am_1} = \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=2} \pi_i(t) - 1\right],$$

$$H_{am_2} = \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=2} \pi_i(t) - 1\right],$$

$$H_{am_3} = \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=2} \pi_i(t) - 1\right],$$

$$H_{am_4} = \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) - \gamma\left[\sum_{i=1}^{N=2} \pi_i(t) - 1\right],$$

$$H_{am_5} = \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) - \gamma\left[\sum_{i=1}^{N=2} \pi_i(t) - 1\right],$$

$$H_{am_6} = \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) - \gamma\left[\sum_{i=1}^{N=2} \pi_i(t) - 1\right],$$

$$H_{am_7} = \sum_{i=1}^{N=3} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=3} \pi_i(t) - 1\right],$$

$$H_{am_8} = \sum_{i=1}^{N=3} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=3} \pi_i(t) - 1\right],$$

$$H_{am_9} = \sum_{i=1}^{N=3} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=3} \pi_i(t) - 1\right],$$

$$H_{am_{10}} = \sum_{i=1}^{N=3} \pi_i(t) \ln \pi_i(t) - \gamma\left[\sum_{i=1}^{N=3} \pi_i(t) - 1\right],$$

$$H_{am_{11}} = \sum_{i=1}^{N=4} \pi_i(t) \ln \pi_i(t) - \beta[\pi_1(t)x(t)] - \gamma\left[\sum_{i=1}^{N=4} \pi_i(t) - 1\right]. \tag{11}$$

2.2. The problem solution. The obtained expressions for the desired constant values (11) in some respect look like the same, although, those regularities are just the formal identical symbolizations (and nothing more) of absolutely different, not identical, however, conserved values.

As it is, the expressions for $H_{am_1} - H_{am_3}$ are indicated in their right-hand parts the same, but the corresponding preferences functions connected to the related alternatives are different, and they must be taken in the proper view from (4) for the considered functional of (2) which includes the special cognitive function derived from (3).

On that the same reasons $H_{am_4} - H_{am_6}$, $H_{am_7} - H_{am_9}$ are different as well.

2.3. Practical application of the problem solution. For the first variant of (2) the solution yields the results for the preferences functions:

$$\pi_1 = \frac{e^{\beta x}}{e^{\beta x} + e^{\alpha\beta x}}, \quad \pi_2 = \frac{e^{\alpha\beta x}}{e^{\beta x} + e^{\alpha\beta x}}. \tag{12}$$

The expressions of (12) are the special cases of (4), and they can be found from there as well.

On condition of the Euler-Lagrange equation for the sought function

$$\pi_1 = \alpha\pi_2. \tag{13}$$

The equation (13) is also the particular case of (5) and can be obtained from there as well.

Substituting (12) into (13) we get

$$\ddot{x} - \frac{\dot{x}}{\alpha} - \frac{1 + e^{\beta(x-\alpha x)}}{\alpha^2\beta} = 0. \tag{14}$$

And the ordinary differential equation of the second order (14), as well as in its turn, is the particular case of the generalized expressions of (6), (7); it derives from them for the related values.

The practice (8)-(10) yields the Hamiltonian H_{am_i} of (11).

2.4. The researches results. Mathematical modeling for the accepted data: $t_0 = 0$; $t_1 = 120$; $\alpha = 95$; $\beta = 0.1$ show the calculation experiment results illustrated in fig. 1, 2.

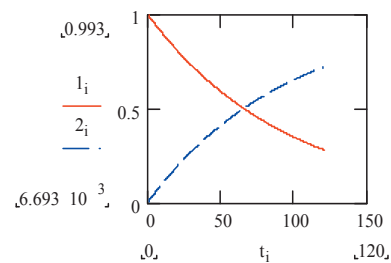


Fig. 1. Preferences functions

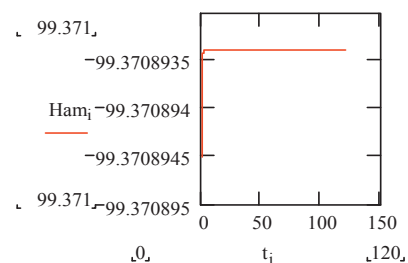


Fig. 2. Hamiltonian

3. Conclusions

Herein we have found the sought after value which remains constant during the controlled by the active element process.

Prospects of further researches.

One of the further researches directions is considering functionals with the under-integral functions which contain the independent variables in the explicit view.

References

1. Kasyanov V., Goncharenko A. (2012). Variational principle in the problem of ship propulsion and power plant operation with respect to subjective preferences. Scientific proceedings of Kherson state maritime academy: Scientific journal. – Kherson, Ukraine: Publishing house of Kherson state maritime academy, № 2(7), 56-61.
2. Kasyanov V., Goncharenko A. (2013). Light and shadow. Proportions of shadow economy. Entropy approach: monograph. Kyiv, Ukraine: Kafedra, 86.
3. Kasyanov V. (2007). Subjective analysis: monograph. Kyiv, Ukraine: National Aviation University, 512.
4. Kasyanov V. (2003). Elements of subjective analysis: monograph. Kyiv, Ukraine: National Aviation University, 224.
5. Gel'fand I., Fomin S. (1961). Calculus of variations. Moscow, USSR: State Publishing House of Physics-Mathematics Literature, 228.
6. Kroes M.J., Wild T.W. (1994). Aircraft powerplants. 7th ed. New York, New York, USA: GLENCOE Macmillan/McGraw-Hill, International Editions, 694.
7. Kuiken K. (2008). Diesel engines for ship propulsion and power plants from 0 to 100,000 kW: in 2 parts. Part I. Onnen, The Netherlands: Target Global Energy Training, 509.
8. Kuiken K. (2008). Diesel engines for ship propulsion and power plants from 0 to 100,000 kW: in 2 parts. Part II. Onnen, The Netherlands: Target Global Energy Training, 442.
9. Silberberg E., Suen W. (2001). The structure of economics. A mathematical analysis. 3rd ed. New York: McGraw-Hill Higher Education, 668.
10. Kolstad Ch.D. (2000). Environmental Economics. New York: Oxford Univ. Press, 400.
11. Random house Webster's unabridged dictionary. 2nd ed. (1999). New-York, USA: Random House, 2230.

Запропоновано підхід щодо раціоналізації процесу перевезення зернових культур. Особлива увага приділяється раціоналізації технології перевезень для зменшення часу простою комбайнів і транспортних засобів. Встановлено, що час збору врожаю залежить від площі полів та від довжини їздки з вантажем, оскільки зі збільшенням площі поля, збільшується відстань доставки зерна

Ключові слова: компенсатор, технологія, технологічний ланцюг

Предложен подход к рационализации процесса перевозки зерновых культур. Особое внимание уделяется рационализации технологии перевозок для уменьшения времени простоя комбайнов и транспортных средств. Установлено, что время сбора урожая зависит от площади полей и от длины ездки с грузом, так как с увеличением площади поля, увеличивается расстояние доставки зерна

Ключевые слова: компенсатор, технология, технологическая цепочка

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РАЦІОНАЛІЗАЦІЯ ТЕХНОЛОГІЇ ПЕРЕВЕЗЕНЬ ЗЕРНА

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1. Вступ

Важливою складовою виробничого процесу є транспортне обслуговування. У сфері сільськогосподарського виробництва України автомобільним транс-

портом перевозиться вантажів у декілька разів більше, ніж усіма видами транспорту разом взятими.

Вдосконалення форм та методів заготівель сільськогосподарської продукції, розвиток прямих зв'язків орендаторів з переробними і торговими підприємства-