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PREDICTION OF FATIGUE STRENGTH UNDER CYCLIC LOADING

В даній статті продемонстрований подібний підхід оцінки тривалих граничних характеристик для одновимірного елемента конструкції-стержня. Показано, що процес втомного руйнування визначається процесом пошкоджуваності. Отримані оцінки справедливі для однорідного одноосного напруженого стану. Дана наближена формула застосовна також і для оцінки міцності і стійкості малих осередків тіл, що містять мікродефекти. Отримані результати можуть бути використані при розробці нанотехнологій

Ключові слова: втомна міцність, циклічне навантаження, пошкодження, напруга, повзучість, в'язкопружні, заліковування дефектів, руйнування

В данной статье продемонстрирован подобный подход оценки длительных предельных характеристик для одномерного элемента конструкции-стержня. Показано, что процесс усталостного разрушения определяется процессом повреждаемости. Полученные оценки справедливы для однородного одноосного напряженного состояния. Данная приближенная формула применима также и для оценки прочности и устойчивости малых ячеек тел, содержащих микродефекты. Полученные результаты могут быть использованы при разработке нанотехнологий

Ключевые слова: усталостная прочность, циклическое нагружение, повреждение, напряжение, ползучесть, вязкоупругость, залечивание дефектов, разрушение

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1. Introduction

Numerous problems of oil mechanics require for adequate methods of calculating and obtaining estimates of the endurance of structural elements in periodic loading conditions. Examples include drill pipes and oil-and-gas pipelines. The working conditions of their service are close to periodic. With time and increasing of loading cycles, the irreversible structural changes, formation and accumulation of various defects occur in the materials of constructions. As a consequence, the real endurance can be less than the calculated, and this can lead to an emergency stop of installation. To improve and reduce the overall characteristic, the theory of fault probability appears to be most promising.

2. Problem

In this paper, it is demonstrated the approach of evaluation of long-limiting characteristics for one-dimensional element of structure, i.e., for the rod.

The basic theory of damageability, having phenomenological character and found its confirmation, both theoretically and experimentally, suggests the introduction of a function that characterizes the level of damage in the material. Usually this function is associated with the density of existing and developing defects, with their relative volume. The existence of such functions makes it necessary to define an equation, called the kinetic equation describing the change of this function depending on the parameters characterizing the stress-strain state [1 – 7].

Another, alternative variant of damages account that excludes the need for the formulation of kinetic equations is a variant of the hereditary theory of damageability [8 – 10]. According to this theory, along with the reversible creep in the material, it takes place an irreversible process of damage accumulation, which is at sufficiently high load leads to destruction. For a uniaxial stress state, the strain equation is [8, 11]:

$$\phi(\varepsilon)=\sigma+L^*\sigma+M^*\sigma, \tag{1}$$

where $\phi(\varepsilon)$ is a nonlinear function of instantaneous deformation, σ is stress, L^* and M^* are integral operators of hereditary type, and L^* is the operator responsible for the reversible creep, and the second operator M^* for the accumulation of damage.

Creep operator is the operator of a continuous action and an integral operator of viscoelasticity of the following form:

$$L^*\sigma=\int_0^t L(t-\tau)\sigma(\tau)d\tau. \tag{2}$$

The operator of damageability M^* is an integral operator of discrete operation acting only at time area of active loading. In general, if the effect of healing of defects takes place, then this operator has the form:

$$M^*\sigma=\sum_{k=0}^{n} f(t_k^+)\int_{t_k^-}^{t_k^+} M(t_k^+-\tau)\cdot\sigma(\tau)d\tau+\int_{t_{n+1}^-}^{t} M(t-\tau)\cdot\sigma(\tau)d\tau, \tag{3}$$

here $M(t-\tau)$ is the kernel of damageability, $(t_k^-;t_k^+)$ are the time intervals of active breach, $f(t_k^+)$ is the function of healing the defects.

Strength condition according to the hereditary theory of damageability can be represented as:

$$\sigma + M^*\sigma = \sigma_0 , \quad (4)$$

where σ_0 is an instant ultimate strength, which can also be interpreted as the ultimate strength of defect-free material.

For uniaxial stress state conditions determining the intervals of active loading can be written in the following form:

$$\sigma \rangle 0; \ \dot{\sigma} \rangle 0 , \quad (5)$$

where the dot over the stress function indicates the derivative by time.

The function of defects healing $f(t_k^+)$ characterizes the part of stored volume of accumulated damage during unloading.

It depends on level of damage accumulated in the previous range of active loading. Currently, there is no reliable experimental data on determination of this function. According to the physical nature of this function, it varies from zero, with the full healing of defects, up to the one at the absence of effect of healing the defects. Therefore, during the initial approach, it is natural to specify this function as the discrete-continuous:

$$f(t_k^+) = f(t_k) = \begin{cases} 1 & \text{at absence of defects healing effect} \\ 0 & \text{at complete healing of defects} \end{cases} . \quad (6)$$

In paper [2], it is made an attempt to evaluate the fatigue damage under cyclic loading:

$$\sigma(t) = \sigma_m + \sigma_a \cdot \sin \omega t , \quad (7)$$

where σ_m is an average stress of loading cycle, σ_a is its amplitude, ω is the frequency of cyclic loading.

In this work, it was assumed that the damage accumulation process is continuous, as both for the active and the passive loadings. Below the process of fatigue failure is analyzed with taking into account the specifics of damageability operator (3), i.e., with taking into account the discrete nature of the process of damage accumulation, and at the presence of defects healing phenomenon as well.

In the strength condition (4), the operator of damage is the same as in the deformation ratio (1), i.e., has the form (3). Suppose that in one-dimensional body under cyclic loading, it is realized one-dimensional stress characterized by the stress of the form (7).

First, let us define the time intervals of active loading. Subjecting function (4) to the condition (5), we obtain the following values:

$$t_k^- = \frac{2\pi(k-1)}{\omega}; \quad t_k^+ = \frac{0,5\pi + 2\pi(k-1)}{\omega} , \quad (8)$$

where k is an arbitrary natural number.

To determine the damage time, we substitute representation for stress (7) in the strength condition. Then, taking into account the principle of linear superposition (4), we get:

$$\sigma_m + \sigma_a \cdot \sin \omega t + \sigma_m \int_0^t M(\tau) d\tau + \\ + \sigma_a \left\{ \sum_{k=1}^{n} f_k \int_{t_k^-}^{t_k^+} M(t_k^+ - \tau) \sin \omega\tau d\tau + \int_{t_{n+1}^-}^{t} M(t-\tau) \sin \omega\tau d\tau \right\} = \sigma_0 . \quad (9)$$

For simplicity and to identify the qualitative picture of the considered process, we assume that the kernel of damageability is constant, i.e., $M(t-\tau) = \beta = \text{const}$. Then (9) has the form:

$$\sigma_m + \beta\sigma_m t + \sigma_a \frac{\beta}{\omega}\left(1 + \sum_{k=1}^{n} f_k\right) + \sigma_a \sqrt{1 + \left(\frac{\beta}{\omega}\right)} \sin(\omega t - \phi) = \sigma_0 , \quad (10)$$

where $tg\phi = \beta\omega^{-1}$.

To obtain an approximate estimation of the number of loading cycles up to failure, we assume $t \approx 2\pi n\omega^{-1}$, and $f_k = 1$, which corresponds to the absence of the defects healing phenomenon. Then:

$$n_{kp} = \frac{\sigma_0 - \sigma_m - \sigma_a\left(\beta\omega^{-1} + \sqrt{1+\beta^2\omega^{-2}}\right)}{\beta\omega^{-1}(2\pi\sigma_m + \sigma_a)} . \quad (11)$$

The obtained formula determines the dependence of cycles number up to failure n_{kp} from the average stress σ_m , amplitude σ_a , frequency ω , and the parameter of damageability β .

Simple analysis shows that these dependences are inversely proportional, besides dependence from frequency, which has the character of direct proportionality. They qualitatively conform to the experimental ones, in particular, with well-known curves of Wohler.

It should be noted that the rejection of the principle of forces linear superposition in the denominator of the formula (11), 2π must be replaced by 0.5π .

Limited endurance limit from (11) is given by expression

$$\sigma = \frac{\sigma_0}{1 + \frac{\beta}{\omega}\left(2\pi + \frac{1-R}{1+R}\right)n + \frac{1-R}{1+R}\left(\frac{\beta}{\omega} + \sqrt{1 + \left(\frac{\beta}{\omega}\right)^2}\right)} , \quad (12)$$

where $R = (\sigma_m - \sigma_a)(\sigma_m + \sigma_a)^{-1}$ is the cycle asymmetry coefficient.

In the absence of damageability, from (11), it follows the classical failure criterion by maximum stress

$$\sigma_m + \sigma_a = \sigma_0 . \quad (13)$$

This shows that the process of fatigue failure is directly determined by the process of damageability, with the non-monotonic and complex dependence character mostly depending on stress variability by time.

The obtained estimates are valid for a homogeneous uniaxial stress state. If we refuse from this limitation, then we first solve the corresponding boundary problem for the one-dimensional body.

As a part of the rod theory, the one-dimensional equation of motion has the form:

$$\frac{\partial\sigma}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} , \quad (14)$$

where u is movement associated with the deformation ε by Cauchy relation

$$\varepsilon=\frac{\partial u}{\partial x}. \quad (15)$$

Let us take strain ratio (1) in simplified form, assuming that percentage of strain associated with creep is negligible small. In addition, assume that the function of instantaneous deformation is linear $\phi(\varepsilon)=E\omega$, where E is the Young's modulus of elasticity.

Substituting (1) with subject to accepted simplifications and formula (15) in (14), we obtain the following integrodifferential equation for displacement function u(x, t):

$$c_0^2u''(x,t)=\ddot{u}(x,t)+ +\sum_{k=1}^{n}f_k\int_{t_k^-}^{t_k^+}M(t_k^+-\tau)\ddot{u}(x,\tau)d\tau+\int_{t_{n+1}^-}^{t}M(t-\tau)\ddot{u}(x,\tau)d\tau, \quad (16)$$

where $c_0^2=\frac{E}{\rho}$ where ρ is the density. Prime denotes differentiation with respect to spatial coordinates x, and the dot above the function is differentiation with respect to time.

Suppose that one face of the rod of length l subjected to periodic displacement of the given amplitude a and frequency ω, and the other face is free from effort. These boundary conditions are written as:

$$\begin{cases}u(0;t)=a\cdot\cos\omega t,\\ u'(l;t)=0\end{cases}. \quad (17)$$

We are interested in steady-state oscillations, and therefore there is no need for the initial conditions.

As before, for simplicity, and visibility of the results, we accept the kernel of damageability operator as constant $M(t-\tau)=\beta=\text{const}$.

Conditions determining the intervals of active loading times are taken in the form:

$$u(x;t)\rangle 0;\quad \dot{u}(x;t)\rangle 0. \quad (18)$$

Solution of equation (16) will be found by Fourier variable separation method:

$$u(x;t)=\nu(x)\cos\omega t. \quad (19)$$

Determining regarding to this expression by conditions (18) the times t_k^- and t_k^+, and then substituting them into equation (16) and integrating, we obtain:

$$c_0^2\nu''(x)\cos\omega t+\left[\omega^2\cos\omega t+\beta\left(1+\sin\omega t+\sum_{k=1}^{n}f_k\right)\right]\nu(x)=0. \quad (20)$$

Due to the mathematical complexity of this equation and to obtain visual engineering estimates, we preliminary multiply equation (20) by $\cos\omega t$ and average the obtained by last interval of active loading from $\frac{(2\pi+1.5)n}{\omega}$ up to $\frac{2\pi(n+1)}{\omega}$ then we find

$$\nu''(x)+\left(\frac{\omega\chi}{c}\right)^2\nu(x)=0, \quad (21)$$

where

$$\chi=1+\frac{4\beta}{\pi\omega}\left(\frac{1}{2}+\sum_{k=1}^{n}f_k\right). \quad (22)$$

The solution of equation (21) can be represented as:

$$\nu(x)=b\cos\left(\frac{\omega\chi}{c}x-\alpha\right). \quad (23)$$

Constants b and α are determined by the boundary conditions (17), which with taking into account the representation (19) have the form:

$$\nu(0)=a;\quad \nu'(0)=0. \quad (24)$$

Substituting (23) in (24), we get:

$$\alpha=\frac{\chi\omega l}{c_0};\ b=\frac{a}{\cos\alpha}. \quad (25)$$

Considering (23) into (19), we obtain the following representation for displacement:

$$u(x;t)=a\frac{\cos\frac{\omega\chi}{c_0}(l-x)}{\cos\frac{\omega\chi l}{c_0}}\cos\omega t. \quad (26)$$

Strain ratio, recorded here in the form of

$$Eu'(x;t)=\sigma(x;t)+M\sigma(x;t), \quad (27)$$

together with the criterion of strength, (4) gives the following representation for the strength criterion by means of the displacement function:

$$Eu'(x;t)=\sigma_0. \quad (28)$$

We strengthen this requirement by demanding its implementation for the maximum values of the left hand side of (28):

$$E\cdot\max u'(x;t)=\sigma_0. \quad (29)$$

This allows with taking into account representation for displacement (26) to obtain relatively simple expressions for the strength criterion:

$$\frac{Ea\chi\omega}{c_0}\text{tg}\frac{\omega\chi l}{c_0}=\sigma_0. \quad (30)$$

For real values of parameters, the quantity $\frac{\omega\chi l}{c_0}$ is small. We use this to further simplification of (30). We replace the tangent of the angle by the value of the last one. Then we find:

$$Eal\left(\frac{\chi\omega}{c_0}\right)^2 = \sigma_0 . \qquad (31)$$

Enter the nominal stress σ_H as follows:

$$\sigma_H = \frac{Ea\omega^2 l}{c_0^2} , \qquad (32)$$

$$1+\frac{4\beta}{\pi\omega}\left(\frac{1}{2}+\sum_{k=1}^{n_{kp}} f_k\right) = \left(\frac{\sigma_0}{\sigma_H}\right)^{1/2} , \qquad (33)$$

where n_{kp} is the number of cycles faults before failure.

At the absence of the phenomenon of damage accumulation, when $\beta = 0$, from (33), it follows the classic criterion of strength:

$$\sigma_H = \sigma_0 . \qquad (34)$$

In the presence of damages from (33), we obtain the following strengthening of this classic criterion of strength.

For complete healing of defects, $\beta \neq 0$; $f_k = 0$, from (33), we obtain the following representation for the strength criterion:

$$\sigma_H\left(1+\frac{2\beta}{\pi\chi}\right)^2 = \sigma_0 . \qquad (35)$$

At the absence of the effect of defects healing, when $f_k = 1$, from (33), we find the following formula for the critical number of loading cycles up to failure:

$$n_{kp} = \frac{\pi\omega}{4\beta}\left[\left(\frac{\sigma_0}{\sigma_H}\right)^{1/2} - 1\right] - \frac{1}{2} . \qquad (36)$$

This approximate formula for the number of loading gives an estimate of long-term strength for the inhomogeneous one-dimensional stress state, determining the fatigue resistance.

Equation (36) is also applied for estimation of the strength and stability of small cell bodies containing microdefects. The obtained results can be used in the development of nanotechnology, to accommodate both existing and possibly created micropores.

3. Conclusions

The given approximate formula for the number of loading gives the estimation of prolonged strength for one-dimensional stress state and defines the fatigue strength.

On the base of hereditary theory of damageability a way for defining the working life of a cyclically loaded bar was worked out. Quality analysis of the situation with regard to deficiencies healing process was given.

It was shown that this process may make a valuable contribution to the estimation of fatigue strength. Within the frames of the approximate engineering approach, a formula of dependence of ultimate number of cycles before failure on nominal stress is was obtained.

The obtained formula is applicable also for strength and stability estimation of small cells of bodies containing micro-deficiencies. The obtained results may be used by developing nanotechnologies in order to take into account both the existing and possibly the created micropores.

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