Узагальнена математична модель взаємодії диска з ґрунтом побудована при загальних припущеннях щодо режиму руху дискового ножа в ґрунті, а саме, диск може рухатися в ґрунті в режимі ковзання, буксування або кочення без ковзання і буксування. Раніше побудовані моделі випливають з неї, як окремі випадки, при певних значеннях параметрів. Однак у зв'язку з обчислювальною складністю цієї моделі для випадку вільного обертового дискового ножа, що складається з необхідності попереднього чисельного рішення трансцендентного рівняння для визначення режиму руху диска, узагальнена математична модель не знайшла широкого застосування. Тому на основі комп'ютерного експерименту за допомогою методу найменших квадратів була побудована аналітична двовимірна апроксимація узагальненої моделі взаємодії диска з ґрунтом, що є новою моделлю апроксимаційного типу.

Отримано явний вираз для кінематичного параметра вільнообертового дискового ножа, що визначає режим його руху. Встановлено, що цей параметр є раціональною функцією відносного заглиблення диска і безрозмірного динамічного коефіцієнта, що характеризує властивості ґрунту. Також отримані явні вирази проекцій рівнодіючої реакцій ґрунту на лезо дискового ножа і його бічні поверхні в залежності від даних безрозмірних параметрів. Встановлено, що горизонтальна складова реакції, яка визначає тяговий опір диска, також є раціональною функцією відносного заглиблення і безрозмірного динамічного коефіцієнта. Встановлено, що величина кінематичного параметра істотно впливає на величину і напрямок рівнодіючої реакцій ґрунту на диск. Отримані вирази дозволяють значно спростити експерименти по визначенню рівнодіючої реакцій ґрунту на вільнообертовий дисковий ніж і скоротити їх необхідну кількість. Ці вирази дозволяють здійснювати розрахунки на міцність ґрунтообробних робочих органів з дисками і визначати їх оптимальні параметри за критеріями міцності і мінімальних питомих енергетичних витрат з точністю, достатньою для інженерної практики. Адекватність отриманих виразів підтверджено порівнянням з експериментальними даними по динамометруванню дискового ножа

Ключові слова: вільнообертовий диск, взаємодія з ґрунтом, силові характеристики, аналітична апроксимація, явні вирази

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AN APPROXIMATING MATHEMATICAL MODEL OF INTERACTION BETWEEN A FREELY ROTATING DISK AND SOIL

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1. Introduction

Disk working bodies are widely used in tillage. This is because they enable tillage even in those conditions under which tillage by means of non-rotational working bodies is insufficient or impossible. Flat disks with both passive and active drive are used as primary and secondary tools in disk shallow ploughs, common ploughs, tuber harvesters and other machines. Therefore, theoretical study of interaction of disk working organs with soil is an important task.

Knowledge of overall power characteristics of their working bodies is necessary for strength calculations of tillage equipment. Such parameters can be obtained either by spatial dynamometry or building mathematical models of interaction of the working bodies of this equipment with soil. Such models make it possible to substitute computational experiments for expensive full-scale experiments. Up-to-date computers make it possible to perform necessary calculations even for complicated mathematical models.

It is important that these models provide an opportunity for a rational choice of structural and operational parameters of the disk working bodies by statement and solution of optimization problems using one or several evaluation criteria. These solutions make it possible to find a combination of parameters that provides high-quality tillage with minimal energy consumption.

2. Literature review and problem statement

The disk movement mode is determined by the value of its kinematic λ parameter equal to the ratio of its peripheral velocity to the velocity of translational movement. Experiments show that the driven disk power characteristics substantially depend on this parameter [1–4]. In the event of a disk with an active drive, the value of λ parameter is considered to be of specified value.

Forces of normal soil pressure and forces of soil friction act on the side faces of the flat disk knife (DK) moving in its own plane in soil and forces of soil crushing resistance act on the edge of the disk blade. According to Coulomb's law, forces of soil friction acting on an elementary area of the side disk face are directed toward the relative velocity of movement of a soil particle interacting with the area in question, that is, opposite to the vector of absolute velocity of a certain point on this area since soil particles move mainly in the direction perpendicular to the DK plane. Consequently, direction of elementary friction forces depends on the magnitude of the kinematic parameter, λ. As is known, an arbitrary flat system of forces can be replaced by a resultant vector of forces applied in the center of reduction and a pair with the resultant moment of forces applied in the center of reduction. Since these power characteristics depend on distribution of elementary friction forces, they also depend on the λ parameter. For a sharp blade, reaction of soil applied to an elementary segment of the DK blade is also directed against absolute velocity of some point in this blade segment. Therefore, power characteristics of the knife blade are also a function of kinematic parameters of the disk.

In the case of a disk with an active drive, the value of λ parameter is specified. Experiments in a soil channel [1] showed that with an increase in λ from 1 to 3, magnitude of the disk tractive resistance decreases by 74 to 100 % at different angles of attack and small disk inclination angles. Based on field experiments, it was shown in [2] that the specific energy consumption of a driven single-sided disk plough with $\lambda=1.42$ is 50.76 % less than with $\lambda=2.62$. Experiments in a soil channel at large λ (44 to 74) have made it possible to establish a significant reduction in the disk tractive force (by 2.6 to 3.4) compared to the freely rotating disk [3]. According to laboratory studies, when λ increases from 2.7 to 5.4, power consumed by the disk in the soil channel increases by 25–35 % depending on the disk penetration depth and the angle of attack [4].

A mathematical model of interaction of a freely rotating disk knife (FRDK) with soil when the disk is half-deepened was constructed in [5, 6]. It was assumed that the disk moves in soil with slippage ($\lambda<1$) and moreover, interaction of the disk blade with soil was not taken into account. Both assumptions do not represent the facts. The mode of DK motion is determined by soil properties and relative penetration of the DK, so it can also move in the soil with slippage. Consideration of the soil reaction forces acting on the disk blade is necessary since the resultant reaction to the DK blade may exceed the resulting soil reaction acting on the side faces of the disk [7]. In addition, it is really impossible to submerge the DK to its center since the disk should be mounted on a shaft.

In another mathematical model of disk and soil interaction, it was assumed with the aim of simplification, that the FRDK moves in soil without slippage and skidding ($\lambda=1$) [8, 9]. This greatly simplifies the model but leads to significant errors when establishing power characteristics of the disk.

A mathematical model of interaction of a driven disk with soil was constructed in [10] under assumption that the DK moves in a slip mode ($\lambda>1$). Drive disks and planter DKs move in this mode.

Generalized all-mode model of interaction of a flat disk with soil [11, 12] includes all abovementioned models as particular cases. The generalized model makes it possible to determine the resultant soil reaction acting both on the DK blade and the side faces of the disk at any penetration of the DK and any λ value. With the help of this model, the slip-and-skid phenomenon of the disk behavior in soil was theoretically explained. The model makes it possible to determine components of the resultant forces of soil reactions acting on the FRDK taking into account interaction of both disk blade and its side faces with soil [12, 13]. In this case, the mode of FRDK motion can be arbitrary.

The fact that the mode of the FRDK motion significantly affects its power characteristics was observed earlier in [14]. However, this effect has not been quantified.

Many works of contemporary researchers study DK interaction with soil. Dependence of power required for driving the disk on operating parameters is studied in [15]. However, only the power required to overcome friction forces acting on the side face of the DK is taken into account and the power required to overcome forces of soil crushing is not taken into account. Studies [16, 17] are devoted to the study of interaction of a hexagonal freely rotating disk with soil but interaction of the DK side faces with soil is not taken into account. In studies [18, 19] devoted to asphalt cutting with a disk, it is assumed without sufficient substantiation that «the disk conditionally works with no slippage» and moreover, interaction of the DK side faces with asphalt is not taken into account. Studies [20, 21] are devoted to oblique soil cutting with FRDK, however, the disk kinematic parameter is assumed to be specified and no indication is given as to proceeding from what considerations it can be determined.

Since angular velocity of the FRDK is established as a result of its interaction with soil, it is incorrect to assume that the kinematic parameter of the disk, λ, which determines mode of the disk movement is a specified value. This was assumed in the above-mentioned studies, except for the studies [5, 6] and the generalized mathematical model. With this approach, the λ parameter must either be determined experimentally or assumed to be ≈1. Angular acceleration of the

DK is zero at a steady motion and a constant angular velocity. Consequently, the sum of the resultant moment of the soil friction forces acting on the side face of the disk and the resultant moment of the forces of soil resistance to crushing with the disk blade is zero. Namely proceeding from this condition, magnitude of the DK kinematic parameter is found which determines mode of the DK motion (as is in [5, 6] and in the generalized model of the disk interaction with soil).

Experimental methods are widely used in the studies of FRDK interaction with soil. Various designs of single-disk furrow-openers were studied in field conditions: with a smooth cutting edge, with cut-outs, toothed and a furrow-opener with two disks [26]. An increase in traction resistance and deepening force was found for all types of furrow-openers with an increase in depth of cutting and translational velocity of the disks. Based on the field experiments, it was shown [27] that traction resistance and deepening force of a two-disk furrow-opener increased with an increase in disk diameter, the depth of cutting and the translational velocity. The experiments carried out in a soil channel [28] for two cutting depths (0.08 and 0.1 m) showed that traction resistance and deepening force of single-disk furrow-openers with smooth, notched and toothed cutting edges did not change significantly. Disadvantages of a purely experimental approach to studying disk and soil interaction should be attributed to its complexity and cost.

A mathematical model of interaction of a toothed disk with soil constructed in [29] did not take into account rotational motion of the DK. To test the model, laboratory experiments were conducted in a soil channel. The model overestimated 2.5–3 times the resultant moment of resistance to cutting soil having plant residues with the DK teeth. On the basis of the universal equation of soil displacement [30], magnitude of traction resistance of a single-disk furrower was estimated depending on the cutting depth and the disk thickness. Rotational motion of the DK was also not taken into account. Tractive resistance of a two-disk furrower and required power are estimated in [31] through specific resistance.

The issue of kinematic mode of FRDK motion in soil was studied earlier by many researchers. It was rather of theoretical than practical interest [22–24]. But, as further studies have shown, the exact value of kinematic parameter for FRDK has a significant impact on its power characteristics [12, 13]. Influence of kinematic mode of motion on power characteristics of the disk was taken into account in a generalized mathematical model of interaction of the DK with soil. This model has not found wide application because of its relative complexity: power characteristics of the disk are determined in it by means of elliptic integrals. A simplified model of FRDK interaction with soil was built relatively recently [25]. It has made it possible to obtain approximate expressions for elliptic integrals under condition that $\lambda \approx 1$. However, this model does not allow one to set up explicit expressions for power characteristics of a freely rotating disk through its parameters. Since in order to find power characteristics of the FRDK, it is necessary to first solve transcendental equation numerically to determine kinematic parameter of the disk and then, as well numerically, determine its power characteristics. To overcome these difficulties, a two-dimensional analytic approximation of a generalized mathematical model of DK interaction with soil can be used for a freely rotating disk. This approximation can be constructed by conducting a computational experiment based on a generalized model of DK interaction with soil.

3. The aim and objectives of the study

This work objective was to build a mathematical model of FRDK interaction with soil which would make it possible to determine kinematic parameter and power characteristics of the disk in an explicit form with accuracy sufficient for engineering practice by using a two-dimensional approximation of the generalized mathematical model.

To achieve this objective, it was necessary to solve the following tasks:

– build analytical approximations of the consumed power necessary to overcome soil friction forces acting on the DK side face and the consumed power necessary to overcome forces of resistance to crushing soil with the DK blades;

– build an analytical approximation of the FRDK kinematic parameter;

– build an analytical approximation of horizontal and vertical components of the resultant soil reaction forces acting on the FRDK.

4. Analytical approximation of power characteristics of interaction between a freely rotating disk and soil

4. 1. Basic assumptions for the mathematical model

Translational and rotational motion of a flat tillage disk determines its functioning. If it is rotated forcibly, through the tractor's power take-off shaft, then such a disk is called an active rotary tool. If a disk knife rotates due to its interaction with soil, then its rotation is usually called free and the disk itself is freely rotating. If the freely rotating DK undergoes braking, then it is called braked DK.

The instantaneous center of DK rotation, point C, can be located either above its lower point D, below this point or coincide with it (Fig. 1). In the first case, the DK is «skidding», its lower point D moves backward at some velocity opposite to the velocity of its center. In the second case, the DK «slides», that is, the point D moves in the direction of the disk center. In the third case, the disk performs a pure rolling motion with a zero velocity of point D.

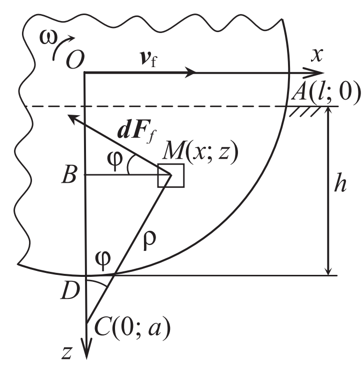

Fig. 1. Diagram for determining the resultant vector of the soil friction forces acting on the side face of the disk and their resultant moment relative to the disk rotation axis

Freely rotating disks are widely used in tillage, therefore, starting from classical studies [5, 6], a significant number of studies by well-known researchers, both experimental and theoretical, have been devoted to the study of the slip-skid phenomenon of the FRDK motion in soil. Moreover, conclusions in theoretical studies were most conflicting. One part of researchers believes that such a disk should move in soil with a slight slip. To prove this, they present not entirely accurate and convincing theoretical arguments [7]. Another part of researchers, on the contrary, tries to prove that only slippage takes place during the FRDK motion in soil [22–23]. At the same time, experimental studies of various researchers [8, 24] show that FRDK can move in soil in all indicated modes.

The importance of correct answer to the question about the mode of the FRDK motion is determined by the fact (as shown by our theoretical and experimental studies [13]) that DK power characteristics significantly depend on the mode of its functioning. And, therefore, the often-accepted assumption that the FRDK rolls in soil without slippage and skidding leads to significant errors in determining its power characteristics.

Let us build a simplified mathematical model of FRDK interaction with soil of an approximation type based on a generalized mathematical model of disk interaction with soil as it was developed in [11–13]. This model was built with the following assumptions:

1) a flat disk of radius r penetrates in soil to a constant depth h;

2) the DK advances at a velocity v_f equal to the velocity of a tillage tool or machine and is forcibly rotated at a given angular velocity ω;

3) soil is sufficiently homogeneous: soil pressure on the side faces of the knife segment deepened in soil can be replaced by the mean value of p and the soil resistance to cutting with the blade per unit of its length can be replaced by the mean value of Q.

All these assumptions, naturally, are also preserved within the framework of the proposed approximation model of FRDK interaction with soil which makes it possible to obtain explicit expressions for the power characteristics of the disk through its geometrical and operation parameters.

An important feature of the generalized mathematical model which makes it possible to build such an approximation model of FRDK interaction with soil is that it is an all-mode model. This model allows one to determine power characteristics of the disk knife when it moves in soil in any mode: skidding ($\lambda>1$), slippage ($\lambda<1$) and rolling with no slippage and skidding ($\lambda=1$) which is typical of a freely rotating disk.

4. 2. Determination of the power characteristics of friction forces acting on the side faces of the disk knife

The approximation model of the FRDK interaction with soil is built on the basis of a two-dimensional approximation with polynomials of the integral expressions of the DK power characteristics in the generalized mathematical model of the disk interaction with soil built earlier in [11–13].

When moving in soil, the disk knife is affected by soil resistance to cutting within the arc AD and the friction forces arising on the side faces of the ADG segment (Fig. 1).

As is known [3, 4], the resultant of elementary forces of soil friction on the side faces of both the passive and active flat disk is directed horizontally. To determine the resultant of friction forces, use a moving coordinate system associated with the disk (Fig. 1), the origin of which is located in the center of the DK with its Ox axis laying horizontally in the direction of its movement, and the Oz axis is vertically directed downwards. Select an elementary area $dxdz$ on the side face of the disk containing point $M(x; z)$. The elementary friction force acting on this area is opposite to the vector of absolute velocity of the point M in this area, that is, perpendicular to the segment connecting this area point with the instantaneous center of rotation C. Therefore, its projection on the Ox axis has the following value:

$$dF_{sx}=fp\cos\varphi dxdz, \qquad (1)$$

where f is the coefficient of soil friction on the side faces of the disk, p is the mean soil pressure on the side faces of the DK, φ is the angle between the vertical diameter of the disk and the polar radius ρ connecting the instantaneous center of the disk velocities, the point C, with the point M in the elementary area (Fig. 1).

Integrate the last relation (1) over the circular segment S of a knife penetrated in soil to obtain, due to symmetry, the horizontal component of the soil friction forces acting on the side faces of the disk:

$$F_{sx}=-2fp\iint_S \cos\varphi \mathrm{d}x\mathrm{d}z.$$

Derive the following from the triangle MCB (Fig. 1).

$$\cos\varphi=\frac{(a-z)}{\sqrt{(z-a)^2+x^2}},\quad \sin\varphi=\frac{x}{\sqrt{(z-a)^2+x^2}}, \qquad (2)$$

where $a=r/\lambda$ is the distance from the disk center to its instantaneous center of rotation, the point C.

Since the circular segment penetrated in soil is bounded by a straight line with equation $z=r-h$ where $(r-h)$ is the distance from the disk center to the daylight surface of soil and a semi-circle with equation $z=\sqrt{r^2-x^2}$, substitute (2) in (1) and move from the double integral to the iterated integral to obtain the following [12]:

$$F_{sx}=4fp\int_0^{l\cdot r}\mathrm{d}x\int_{r-h}^{\sqrt{r^2-x^2}}\frac{(z-a)}{\sqrt{(z-a)^2+x^2}}\mathrm{d}z,$$

where $l=\sqrt{2\xi-\xi^2}$, $\xi=h/r$ is the relative disk penetration depth.

In [12], the following expression was obtained for the projection of the resultant friction force in the form of a definite integral by calculating the internal integral with respect to the variable z in the iterated integral and replacing the variable $t=x/r$ in the external integral:

$$F_{sx}=4fpr^2\int_0^l(\sqrt{\mu^2-2\mu\sqrt{1-t^2}+1}-\sqrt{(1-\xi-\mu)^2+t^2})\mathrm{d}t, \qquad (3)$$

where $\mu=1/\lambda$.

Equality (3) defines the Ox component of the resultant soil reaction on the side faces of the DK as a function of two dimensionless parameters λ and ξ which determine the mode of DK operation, its radius r and the empirical coefficient p.

Consider direction of DK rotation a positive direction of moments of forces. In this case, the elementary friction force acting on the elementary area containing point $M(x; z)$ creates a moment about the axis of rotation of the disk (Fig. 1):

$$dm_O=dF_f\cdot z\cos\varphi-dF_f\cdot x\sin\varphi, \qquad (4)$$

where $dF_f=fpdxdz$ is the magnitude of the elementary friction force.

Integrate equality (4) over the circular segment S of the knife penetrated in soil to obtain the resultant moment of the soil friction forces acting on the side faces of the disk relative to the DK rotation axis (center O):

$$m_O = 2fp\iint_S (z\cos\varphi - x\sin\varphi)\mathrm{d}x\mathrm{d}z.$$

Substitution of relations (2) into this double integral and a series of transformations give the following equality:

$$m_O = -2fp\iint_S \frac{x^2+(z-a)z}{\sqrt{x^2+(z-a)^2}}\mathrm{d}x\mathrm{d}z, \quad (5)$$

which, in turn, takes the following form after a series of transformations:

$$m_O = -2fp\iint_S \sqrt{x^2+(z-a)^2}\mathrm{d}x\mathrm{d}z + {}$$
$$+ 2fpa\iint_S \frac{(a-z)}{\sqrt{x^2+(z-a)^2}}\mathrm{d}x\mathrm{d}z = m_C - F_{sx}a, \quad (6)$$

where

$$m_C = -2fp\iint_S \sqrt{x^2+(z-a)^2}\mathrm{d}x\mathrm{d}z$$

is the resultant moment of the soil friction forces acting on the side faces of the disk relative to its instantaneous center of rotation, point C.

Move from the double integral in m_C to the iterated integral to get:

$$m_C = -4fp\int_{r-h}^{r}\mathrm{d}z\int_0^{\sqrt{r^2-z^2}}\sqrt{x^2+(z-a)^2}\mathrm{d}x. \quad (7)$$

It was shown in [12] that calculation of the internal integral with respect to the variable x in the iterated integral and replacement of the variable $t=x/r$ in the external integral leads to the following expression for the resultant moment of friction relative to the instantaneous center velocities, m_C, in the form of the following definite integral:

$$m_C = -2fpr^3\int_{1-\xi}^{1}\left(\begin{array}{l}\sqrt{1-t^2}\sqrt{\mu^2-2\mu t+1} + \\ +(\mu-t)^2\ln\dfrac{\sqrt{1-t^2}+\sqrt{\mu^2-2\mu t+1}}{|\mu-t|}\end{array}\right)\mathrm{d}t. \quad (8)$$

Thus, equalities (6), (8) and (3) determine the resultant moment of the friction forces relative to the axis of rotation of the disk as a function of two dimensionless parameters λ and ξ which determine the mode of the DK operation, its radius r and empirical coefficient p.

To overcome the friction force acting on the elementary area containing point $M(x;\, z)$, it is necessary to consume power (Fig. 1):

$$dW_s = \mathbf{v}\cdot d\mathbf{F}_f = fp\,v dxdz = fp\omega\sqrt{x^2+(z-a)^2}\mathrm{d}x\mathrm{d}z, \quad (9)$$

where $\mathbf{v}$ is the velocity vector and v is the value of the point M velocity.

Integrate this equality over the circular segment S of a knife penetrated in soil to obtain the power W_s to be consumed to overcome the friction forces acting on the DK side face:

$$W_s = 2fp\omega\iint_S \sqrt{x^2+(z-a)^2}\mathrm{d}x\mathrm{d}z = -m_C\omega. \quad (10)$$

Since $\omega\cdot r = v_f(r/a) = \lambda v_f$, then substitution of (8) in the last equality yields:

$$W_s = -2fpr^2 v_f\int_{1-\xi}^{1}\left(\begin{array}{l}\sqrt{1-t^2}\sqrt{1+\lambda^2-2\lambda t} + \dfrac{(1-\lambda t)^2}{\lambda}\times \\ \times\ln\dfrac{\lambda\sqrt{1-t^2}+\sqrt{1+\lambda^2-2\lambda t}}{|1-\lambda t|}\end{array}\right)\mathrm{d}t. \quad (11)$$

The obtained equality (11) determines the power W_s as a function of two dimensionless parameters λ and ξ determining mode of the DK operation, its radius r, velocity of translational motion, v_f, and empirical coefficient, p.

The integrals in the right-hand sides of equalities (3), (8) and (11) are not expressed by means of a finite number of operations through elementary functions. They are expressed through normal elliptic integrals in the Legendre form [12]. These integrals can also be easily found using one of the well-known numerical methods.

4. 3. Determining power characteristics of the soil reaction forces acting on the disk knife blade

The elementary soil reaction to crushing and wedging, dR_b, acting on some infinitely small arc of the blade, dL, is directed opposite to the absolute velocity vector, $\boldsymbol{v}$, of an arbitrary point M of this arc (Fig. 2). As follows from Fig. 2, projection on the Ox component of the soil reaction force acting on the elementary arc of the DK blade will be:

$$dR_{bx} = Qrd\vartheta\cdot\cos\varphi, \quad (12)$$

where ϑ is the angle between the lower vertical radius and the radius-vector CM of the point M under consideration, φ is the angle between this radius and the polar radius connecting this point with the instantaneous center of velocities, point C, (Fig. 2).

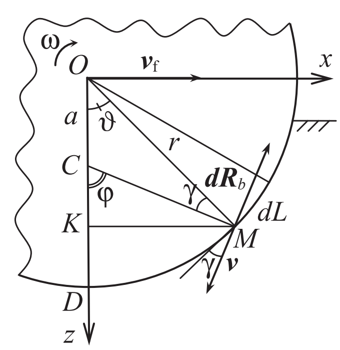

Fig. 2. Diagram for determining the resultant vector of soil reaction forces acting on the disk blade and their resultant moment relative to the axis of disk rotation

Since (Fig. 2):

$$\cos\varphi = \frac{r\cos\vartheta - a}{\sqrt{a^2+r^2-2ar\cos\vartheta}} = \frac{\lambda\cos\vartheta - 1}{\sqrt{1+\lambda^2-2\lambda\cos\vartheta}},$$

then, substitute this expression in (12) and integrate the resulting equality along the blade arc cutting soil to obtain the following for the sought *Ox* component of the resultant vector of forces of the soil reaction forces acting on the DK blade:

$$R_{bx} = Qr\int_0^{\vartheta_0} \frac{\lambda\cos\vartheta - 1}{\sqrt{1+\lambda^2 - 2\lambda\cos\vartheta}}\,d\vartheta, \qquad (13)$$

where $\vartheta_0=\arccos(1-\xi)$.

Note that when $\lambda=1$, an expression in another notation obtained earlier in [8] follows from (13):

$$R_{bx} = -2Qr\left(1-\cos\frac{\vartheta_0}{2}\right).$$

Similarly, the *Oz* axis component of the resultant vector of soil resistance to cutting is determined. Since (Fig. 2),

$$\sin\varphi = \frac{r\sin\vartheta}{\sqrt{a^2+r^2-2ar\cos\vartheta}} = \frac{\lambda\sin\vartheta}{\sqrt{1+\lambda^2-2\lambda\cos\vartheta}},$$

then, the following is obtained for the desired *Oz* axis component of the resultant vector of the soil reaction forces acting on the DK blade:

$$R_{bz} = -Qr\int_0^{\vartheta_0} \frac{\lambda\sin\vartheta d\vartheta}{\sqrt{1+\lambda^2-2\lambda\cos\vartheta}}.$$

Calculate the latter integral to get the following expression for this axis component [12]:

$$R_{bz} = Qr\left(|1-\lambda| - \sqrt{1+\lambda^2-2\lambda\cos\vartheta_0}\right). \qquad (14)$$

From which one more expression follows (at $\lambda=1$) obtained earlier in [8]:

$$|R_{bz}| = 2Qr\sin\frac{\vartheta_0}{2}.$$

Turning, for ease of use, in (14) from the angle ϑ_0 to the relative depth ξ, a final expression for the sought axis component is obtained:

$$R_{bz} = Qr\left(|1-\lambda| - \sqrt{(1-\lambda)^2+2\lambda\xi}\right). \qquad (15)$$

Magnitude of the elementary soil reaction to crushing with an elementary arc of the DK blade containing point $M(r\sin\vartheta, r\cos\vartheta)$ (Fig. 2) will be equal to:

$$dR_b = Q\cdot r\cdot d\vartheta.$$

Since the direction of disk rotation is taken as the positive direction of the moments of forces, then the elementary moment of this reaction relative to the axis of disk rotation (center *O*) will be:

$$dM_O = -r\cos\gamma\cdot dR_d = -r\cos(\varphi-\vartheta)\cdot Q\cdot r\cdot d\vartheta. \qquad (16)$$

The following is easily determined from the right triangle CKM (Fig. 2):

$$\cos(\varphi-\vartheta) = \frac{r-a\cos\vartheta}{\sqrt{a^2+r^2-2ar\cos\vartheta}} = \frac{\cos\vartheta-\lambda}{\sqrt{1+\lambda^2-2\lambda\cos\vartheta}}.$$

Integration of equality (16) along the soil cutting arc of the blade with taking into account the last relation gives the following expression [12] for the desired:

$$M_O = Qr^2\int_0^{\vartheta_0} \frac{\cos\vartheta-\lambda}{\sqrt{1+\lambda^2-2\lambda\cos\vartheta}}\,d\vartheta. \qquad (17)$$

To overcome the elementary soil reaction to crushing with an elementary arc of the DK blade containing point $M(r\sin\vartheta, r\cos\vartheta)$ (Fig. 2), power to be consumed is:

$$dW_b = \boldsymbol{v}d\mathbf{R}_b = Qr\omega\rho d\vartheta = Qrv_f\sqrt{1+\lambda^2-2\lambda\cos\vartheta}\,d\vartheta.$$

Having integrated this equality along the soil cutting arc of the blade, power consumption for soil crushing with the DK blade is determined:

$$W_b = Qrv_f\int_0^{\vartheta_0}\sqrt{1+\lambda^2-2\lambda\cos\vartheta}\,d\vartheta. \qquad (18)$$

The integrals in the right-hand sides of equalities (13), (15), (17) and (18) cannot be expressed using a finite number of operations through elementary functions. However, as shown in [12], they can be expressed in terms of normal elliptic integrals in the Legendre form. In addition, these integrals can be found by known numerical methods.

4. Determination of the power characteristics of the soil reaction forces acting on a freely rotating disk knife and their analytical approximation

In order to reduce the number of essential parameters of the studied dependences, introduce, following [12], dimensionless projections of the resulting soil reaction forces acting on the disk:

$$R_x^* = \frac{R_x}{Qr},\ R_z^* = \frac{R_z}{Qr},\ F_{sx}^* = \frac{F_{sx}}{4fpr^2},$$

$$R_{bx}^* = \frac{R_{bx}}{Qr},\ R_{bz}^* = \frac{R_{bz}}{Qr}, \qquad (19)$$

dimensionless resultant moments of soil reaction forces acting on the disk, its side faces and its blade:

$$M_{Ot}^* = \frac{M_{Ot}}{Qr^2},\ m_O^* = \frac{m_O}{4fpr^3},\ M_O^* = \frac{M_O}{Qr^2} \qquad (20)$$

and dimensionless power consumptions necessary to overcome the soil reaction forces, the friction forces acting on the side faces of the DK and the soil reaction forces actin on its blade:

$$W^* = \frac{W}{Qrv_f},\ W_s^* = \frac{W_s}{4fpr^2v_f},\ W_b^* = \frac{W_b}{Qrv_f}, \qquad (21)$$

where $W=W_b+W_s$.

Also, introduce, according to [12], the dimensionless dynamic coefficient $n=4fpr/Q$. This coefficient is equal to the doubled ratio of the module of the resultant elementary friction forces acting on the side face of a fully braked disk of a given radius r penetrated in soil to its center to the module of the horizontal component of the soil reaction forces acting on the DK blade. Then the projections of the resultant soil reaction forces acting on the DK, their resultant moment and total consumption of power necessary to move the disk in soil will be determined by the following expressions:

$$R_x = Qr(R^*_{bx} + nF^*_{sx}), \quad R_z = QrR^*_{bz}, \tag{22}$$

$$M_{Ot} = Qr^3(M^*_O + nm^*_O), \tag{23}$$

$$W = Qrv_f(W^*_b + nW^*_s). \tag{24}$$

It is not hard to find experimentally the empirical coefficient n by determining horizontal components of the forces of resistance to movement in soil of the fully braked disk of a given radius for two different penetration depths.

The angular velocity of free rotation of the DK is established as a result of its interaction with soil. Since steady motion of the disk is considered, algebraic sum of the moments of all forces applied to the DK should be zero. As noted in [8], the moment of friction forces in the disk bearing is negligible. Therefore, the kinematic parameter, λ, is determined from the condition of equality to zero of the sum of the resultant moments of the soil reaction forces acting on the disk blade and its side faces which takes the following form [12, 25] when taking into account (23):

$$M^*_O + nm^*_O = 0. \tag{25}$$

The moment M^*_O in this equation is expressed through parameters λ and ξ with the help of (20) through the «non-taken» integral entering the equality (17). The moment m^*_O is expressed through these parameters with the help of (20) and (6) through «non-taken» integrals that are in equalities (8) and (3). Therefore, this equation (25) has to be solved numerically for different values of the dimensionless ξ and n parameters with respect to the λ parameter. As a result of numerical solution of this equation, the function $\lambda^* = \lambda^*(\xi, n)$ is found. This function determines the mode of the disk operation depending on its relative penetration depth, ξ, and the dynamic coefficient, n. Then, dimensionless projections on the coordinate axes of the resultant soil reaction forces are found by formulas (19), after by substituting this function into equations (3), (13) and (14).

Making replacement of variables $x = r\cdot u$, $y = r\cdot v$ in double integrals of equalities (5) and (10), the following two equalities are obtained:

$$m^*_O = -\frac{1}{2}\iint_{S'} \frac{\lambda u^2 + (\lambda v - 1)v}{\sqrt{\lambda^2 u^2 + (\lambda v - 1)^2}} \mathrm{d}u\mathrm{d}v,$$

$$W^*_s = \frac{1}{2}\iint_{S'} \sqrt{\lambda^2 u^2 + (\lambda v - 1)^2} \mathrm{d}u\mathrm{d}v,$$

where S' is the corresponding segment of the unit circle.

Partial derivation of the second of these equalities with respect to the λ parameter under the integral sign gives the following:

$$\frac{\partial W^*_b}{\partial \lambda} = -m^*_O. \tag{26}$$

The partial derivation under the integral sign of equality (18) taking into account equality (17) leads to the following equality:

$$\frac{\partial W^*_b}{\partial \lambda} = -M^*_O.$$

The following is obtained from the latter equality and equality (26):

$$\frac{\partial}{\partial \lambda}(W^*_b + nW^*_s) = -(M^*_O + nm^*_O). \tag{27}$$

It follows from equalities (27), (25) and (24) that the kinematic parameter of the FRDK can be found as the extremum point of the power consumptions necessary for overcoming the friction forces acting on its side face and the forces of soil crushing with the DK blade. Since it follows from equality (27) that:

$$\frac{\partial^2}{\partial \lambda^2}(W^*_b + nW^*_s) = -\frac{\partial}{\partial \lambda}(M^*_O + nm^*_O) > 0,$$

then this point of extremum is the point of minimum. This means that as a result of interaction with soil, the FRDK acquires such an angular rotational velocity at which the total power consumed to overcome the friction forces acting on its side face and the forces of soil crushing with the disk blade is minimal.

As is known, along with the basic requirement of adequacy, the requirement of relative simplicity of the model in relation to the chosen system of its characteristics, to some extent opposite to the requirement of adequacy, is also made to the mathematical models [32]. In order for the model to satisfy both requirements, it needs to be optimized: roughly speaking, optimization consists in that the model should be neither too complex nor too simplified. This can be achieved by constructing (on the basis of calculation of a completely formulated problem) some simplified mathematical model, e. g. model of an interpolation (approximation) type with its coefficients taken from the results of a computational experiment [33].

Use a relatively simple way of approximation of two-dimensional table-defined functions by generalized polynomials of a particular type [34] for approximation of W^*_b and W^*_s functions. Using the generalized mathematical model, calculate the table of values for each of these functions for given values of the kinematic parameter and relative penetration depth with the help of equalities (21), (18) and (11) of the following type (Table 1):

Table 1

Values for the approximated function

λ, ξ	ξ_1	ξ_2	...	ξ_j	...	ξ_m
λ_1	W_{11}	W_{12}	...	W_{1j}	...	W_{1m}
λ_2	W_{21}	W_{22}	...	W_{2j}	...	W_{2m}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
λ_i	W_{i1}	W_{i2}	...	W_{ij}	...	W_{im}
⋮	⋮	⋮	...	⋮	⋮	⋮
λ_n	W_{n1}	W_{n2}	...	W_{nj}	...	W_{nm}

Define an approximating polynomial in the form:

$$Q_q = a_1 f_1(\lambda)\varphi_1(\xi) + a_2 f_2(\lambda)\varphi_2(\xi) + \ldots + a_q f_q(\lambda)\varphi_q(\xi),$$

where $f_p(\lambda)$, $\varphi_p(\xi)$, $p = 1, 2, \ldots, q$ are the functions chosen proceeding from some considerations; a_p are unknown coefficients.

To determine coefficients a_p, use the method of least squares, i. e. find them from the condition of the sum minimum:

$$\Delta = \sum_{i=1}^{n}\sum_{j=1}^{m}\left[\sum_{p=1}^{q} a_p f_p(\lambda_i)\phi_p(\xi_j) - W_{ij}\right]^2.$$

Equate the partial derivatives with respect to a_p to zero and obtain the system of equations for determining the unknown $a_1, a_2, \ldots, a_q$:

$$\begin{cases} c_{11}a_1 + c_{12}a_2 + \ldots + c_{1q}a_q = b_1, \\ c_{21}a_1 + c_{22}a_2 + \ldots + c_{2q}a_q = b_2, \\ \ldots \\ c_{q1}a_1 + c_{q2}a_2 + \ldots + c_{qq}a_q = b_q, \end{cases} \quad (28)$$

where $c_{\alpha\beta} = f_{\alpha\beta}\varphi_{\alpha\beta}$, $\alpha = 1, 2, \ldots, q$, $\beta = 1, 2, \ldots, q$,

$$f_{\alpha\beta} = \sum_{i=1}^{n} f_\alpha(\lambda_i) f_\beta(\lambda_i), \quad \varphi_{\alpha\beta} = \sum_{j=1}^{m} \varphi_\alpha(\xi_j)\varphi_\beta(\xi_j),$$

$$b_p = \sum_{i=1}^{n}\sum_{j=1}^{m} W_{ij} f_p(\lambda_i)\varphi_p(\xi_j), \; p = 1, 2, \ldots, q, \; (c_{\alpha\beta} = c_{\beta\alpha}).$$

Thus, the matrix of the system of equations (28) is symmetric.

5. Results of analytical approximation of a generalized mathematical model of interaction between a freely rotating disk and soil

To approximate dependence of dimensionless powers W_b^* and W_s^* on the λ and ξ parameters, the fourth-degree polynomials were chosen according to these two variables of a particular type, namely, the approximating polynomials were taken as follows:

$$a_1 + a_2\lambda + a_3\xi + a_4\lambda^2 + a_5\xi^2 + a_6\lambda\xi + a_7\lambda^2\xi + a_8\lambda\xi^2 + a_9\lambda^2\xi^2.$$

A program for calculating the table of values of approximated functions of two variables and finding coefficients of the polynomial approximating it using the system of equations (28) was compiled in the Maple math system.

As a result of calculations for powers W_b^* and W_s^* depending on λ and ξ, the following expressions were obtained for the polynomials approximating them on the basis of equalities (11) and (18):

$$W_b^* = 1.099 - 2.187l + 1.904x + 1.092l^2 -$$
$$- 0.920x^2 - 2.331lx + 1.406l^2x + 2.098lx^2 - 1.002l^2x^2,$$

$$W_s^* = 0.012 - 0.044l + 0.737x + 0.034l^2 +$$
$$+ 0.143x^2 - 1.291lx + 0.530l^2x + 0.787lx^2 - 0.309l^2x^2.$$

The computational experiment showed that the relative error of approximation of power consumption by these expressions did not exceed 1 % at $0.85 \le \lambda \le 1.15$ and $0.2 \le \xi \le 0.6$.

Thus, the approximating polynomial for the total power consumption required to overcome forces of soil reactions to the DK has the following form:

$$W^* = 1.099 - 2.187\lambda + 1.904\xi + 1.092\lambda^2 - 0.920\xi^2 -$$
$$- 2.331\lambda\xi + 1.406\lambda^2\xi + 2.098\lambda\xi^2 - 1.002\lambda^2\xi^2 +$$
$$+ n(0.012 - 0.044\lambda + 0.737\xi + 0.034\lambda^2 + 0.143\xi^2 -$$
$$- 1.291\lambda\xi + 0.530\lambda^2\xi + 0.787\lambda\xi^2 - 0.309\lambda^2\xi^2). \quad (29)$$

Total power consumption necessary for overcoming the soil reaction forces acting on DK are minimal for FRDK. Therefore, the kinematic parameter of the FRDK is easily determined from equation $\partial W^*/\partial\lambda = 0$ depending on the relative penetration depth and the dimensionless dynamic coefficient n which reflects soil properties.

$$\lambda^* = \frac{1.049\xi^2 - 1.166\xi - 1.094 + n(0.394\xi^2 - 0.646\xi - 0.022)}{1.002\xi^2 - 1.406\xi - 1.092 + n(0.309\xi^2 - 0.530\xi - 0.034)}. \quad (30)$$

Graphs of the kinematic coefficient λ^* as a function of relative penetration depth, ξ, for three different values of n are presented in Fig. 3.

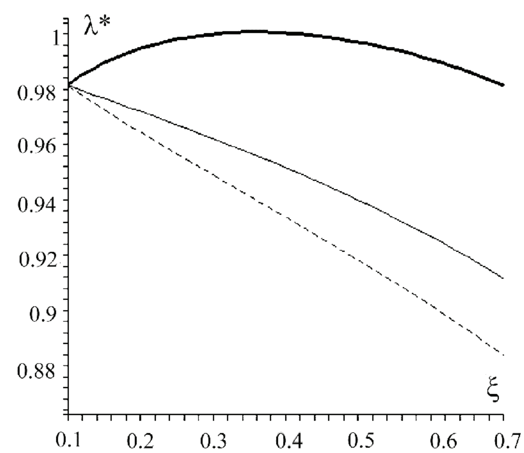

Fig. 3. Graphs of kinematic parameter of the disk knife as a function of relative penetration depth for three values of dynamic coefficient, n

The computational experiment consisted in determining the kinematic coefficient, λ, using numerical solution of equation (25). The moment M_O^* was expressed in terms of parameters λ and ξ with the help of equalities (20) and (17). The moment m_O^* was expressed in terms of λ and ξ parameters using equalities (20), (6), (8) and (3). This experiment showed that the relative error of approximation of the kinematic coefficient by expression (30) in the range of $0.1 \le \xi \le 0.7$ and $0 \le n \le 10$ did not exceed 1.5 %.

The power that must be applied to the FRDK to overcome the forces of the soil reaction is:

$$W(\lambda^*, \xi) = -R_x(\lambda^*, \xi)v_f.$$

Dependence of the dimensionless horizontal projection of the resultant soil reaction forces acting on the DK depending on the relative penetration depth and the coefficient n

characterizing the soil properties was obtained by substituting (30) in (29):

$$R_x^*(\lambda^*,\xi) = -W^*(\lambda^*,\xi). \quad (31)$$

Graphs of this projection depending on the relative penetration depth ξ for three values of n are shown in Fig. 4.

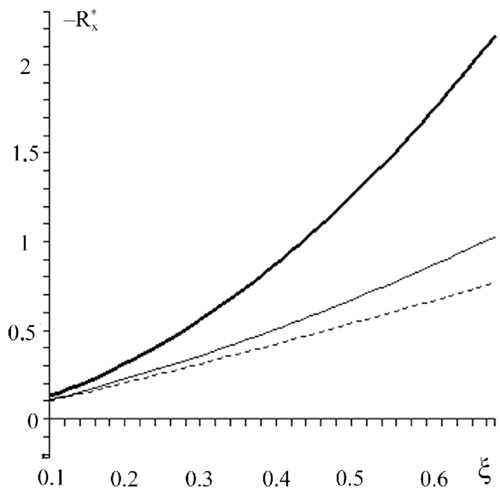

Fig. 4. Graphs of the dimensionless traction force of a freely rotating DK depending on the relative penetration depth ξ for three values of the coefficient n

As follows from equalities (22), (19) and (15), the dimensionless vertical projection of the resultant soil reaction forces acting on the disk is:

$$R_z^* = \left|1-\lambda^*\right| - \sqrt{(1-\lambda^*)^2 + 2\lambda^*\xi}, \quad (32)$$

where λ^* is determined from equality (30).

The graphs of the dimensionless vertical projection of the resultant soil reaction forces acting on the DK are shown in Fig. 5.

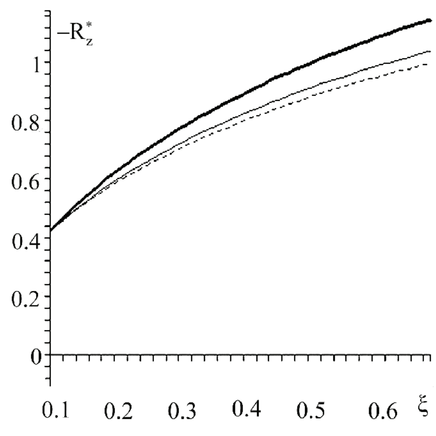

Fig. 5. Graphs of the dimensionless deepening force of a freely rotating DK depending on the relative penetration depth ξ for various values of the coefficient n

Theoretical dependences of the mathematical model using formulas (31) and (30) are compared in Figs. 6, 7, with the experimental results obtained by the dynamometry of a disk knife with diameter D=380 mm in field conditions when processing stubble [8].

Fig. 8 shows a graph of the angle ψ=arctg(R_z/R_x) vs. relative penetration depth ξ and the experimental points taken from the results of experiments with dynamometry of the disk knife [8].

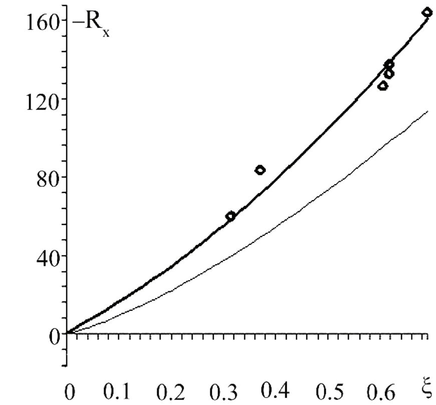

Fig. 6. Graph of FRDK tractive force (thick curve) as a function of relative penetration depth, ξ, and experimental points (circles) according to the data of dynamometry of disk knives

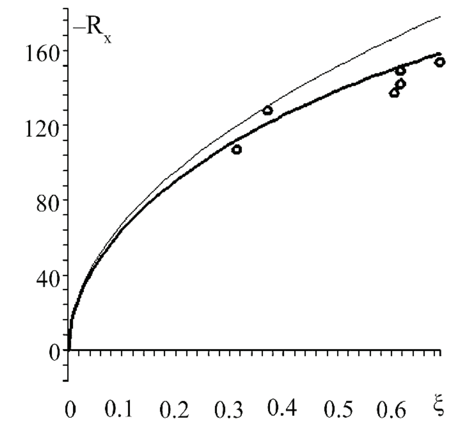

Fig. 7. Graph of the deepening force of the FRDK (thick curve) as a function of relative penetration depth, ξ, and experimental points (circles) according to the data of dynamometry of the disk knives

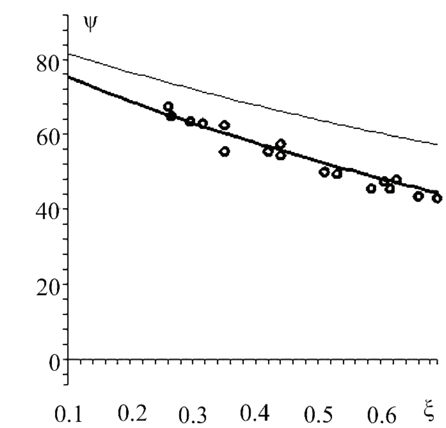

Fig. 8. Graph of the angle ψ variation (thick curve) depending on the relative penetration depth (and experimental points (circles) according to the data of dynamometry of the disk knives)

As follows from Fig. 6–8, the experimental points are located close to the theoretical curves (thick lines) plotted using explicit analytical formulas (29) to (32); discrepancy does not exceed 10–14 %.

6. Discussion of results of analytical approximation of a generalized mathematical model of interaction between a freely rotating disk and soil

Fig. 3 shows the graphs of variation of the kinematic parameter λ^* of a freely rotating DK with variation of penetration depth, ξ, for three values of the dimensionless dynamic coefficient n=0.1 (dashed curve), n=1 (thin line) and n=5 (thick line). It follows from the graphs that for small values of the coefficient n, dependence of λ^* on ξ is almost linear and quadratic for large n. Note that the parameter λ^* for small n decreases relatively quickly with an increase in relative penetration depth, ξ, therefore, this variation should be taken into account when finding power characteristics of the FRDK. The kinematic parameter, λ^*, of the disk monotonically increases with an increase in the dimensionless dynamic coefficient n.

Fig. 4 shows the graphs of the dimensionless tractive force of FRDK depending on penetration depth, ξ, for the same values of the dimensionless dynamic coefficient n=0.1 (dashed curve), n=1 (thin line) and n=5 (thick line). It follows from the graphs that this dependence is also close to linear dependence for small values of n and close to a quadratic dependence for substantial values of this coefficient. The traction force essentially depends on the coefficient n: it increases by 100 % at n=5 when ξ increases from 0.35 to 0.7.

The graphs of the dimensionless vertical projection depending on penetration depth, ξ, are presented in Fig. 5 for the same values of the dynamic coefficient n=0.1 (dashed curve), n=1 (thin line) and n=5 (thick line). With an increase in n from 0.1 to 5, the vertical total projection of the soil reaction forces increases only by about 10 %.

Thick curves in Fig. 6, 7 were plotted based on the experimental data with the use of the least square method at Q=8 kg/cm, n=1.1 and the relative approximation error was 10 to 14 % which is permissible for field tests. Thin lines in these figures were constructed for a constant value of λ=1. Calculations show that the tractive force determined under this condition was 30 % less than the real one and, therefore, it is necessary to take in account the change in the kinematic parameter for the FRDK. Fig. 8 also confirms adequacy of the obtained approximation model of interaction of the FRDK with soil since the errors in approximation of the angle ψ were less than 10 %.

The computational experiment with a generalized mathematical model consisted in determining the kinematic coefficient λ by numerical solution of equation (25) and determination of the dimensionless projections of the resultant soil reaction forces acting on the coordinate axes. These projections were found by formulas (19) after substituting the found kinematic coefficient into equalities (3), (13) and (14). The computational experiment showed that for practically used relative penetration depths of $0.1 \le \xi \le 0.7$ and the range of values of the dynamic coefficient $0 \le n \le 10$, the error in analytical expressions of the power characteristics of the FRDK did not exceed 2 %. Such an error is quite acceptable for the engineering practice.

An important advantage of the constructed approximation model of interaction of the FRDK with soil lies in simplicity of the obtained explicit expressions for power characteristics of the FRDK in soil. This makes it easier to formulate and solve problems of optimizing geometric and operating parameters of a disk at one and many evaluation criteria. Another important advantage of this model is that the constructed model has practically no limitations for all the above-mentioned parameters.

Limitations of the proposed model follow from the assumptions that the soil is sufficiently uniform, and the DK moves in soil with a constant penetration. These assumptions are realized in practice only approximately.

In the future, it is supposed to transfer the proposed approach to construction of a mathematical model of interaction with soil of a disk installed at an angle of attack to the direction of translational motion.

7. Conclusions

1. Two-dimensional analytical approximations have been constructed in the form of fourth-degree polynomials depending on parameters λ and ξ of power consumption required to overcome soil friction forces acting on the side face of the DK and the power needed to overcome forces of soil resistance to their crushing by the DK. The relative approximation error in the range of $0.85 \le \lambda \le 1.15$ and $0.2 \le \xi \le 0.6$ turned out to be less than 1 %.

2. An analytical approximation of the FRDK kinematic parameter was constructed in the form of a rational function of parameters ξ and n. Using a computational experiment, it was shown that the relative approximation error in the range of $0.1 \le \xi \le 0.7$ and $0 \le n \le 10$ did not exceed 1.5 %.

3. An analytical approximation of the horizontal and vertical projections of the resultant soil reaction forces acting on the FRDK was constructed using parameters ξ and n. It was shown by means of the computational experiment that the relative error of approximation of the obtained analytical expressions did not exceed 2 % in the range of $0.1 \le \xi \le 0.7$ and $0 \le n \le 10$.

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